



Basket Default Swaps

- In an n -th-to-default swap, the seller compensates the buyer for any loss of principal and the accrued interest on the n -th asset in the reference basket to default.
- In turn, the buyer pays a fixed amount of money periodically.

Definition

Pricing

- Current time = t_0
 - N underlying loans
 - Payments for the n -th-to-default swap = spread $s^{(n)}$ at dates t_1, t_2, \dots, t_M
 - Present value of these payments, while the asset has not defaulted, is the premium leg $PL^{(n)}$ Additional payment for part of the default period is the accrued payment $AP^{(n)}$
 - Payment to the buyer in the case of default is the default leg $DL^{(n)}$
 - Δ = period between payments, as a fraction of year
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- $P(t_0, t_j)$ = discount factor for maturity t_j , $j = 1, \dots, M$
 - $P(t_0, t_j) = \exp(-rt_j)$
 - R = recovery rate
 - The default time for loan i has a CDF $F_i(t) = 1 - \exp(-ht)$ where h is the default intensity rate
 - Correlation between default times = ρ
 - Joint distribution modeled as a Gaussian or t copula
 - n -th default time = τ_n
 - $N(t)$ = number of defaults in the basket before t
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$$E[PL^{(n)}] = s^{(n)}\Delta \sum_{i=1}^M P(t_0, t_i)E[1_{[N(t_i)<n]}]$$

Premium Leg

$$E[AP^{(n)}] = s^{(n)} \sum_{i=1}^M E[(\tau_n - t_{i-1}) P(t_0, \tau_n) 1_{[t_{i-1} < \tau_n \leq t_i]}]$$

Accrued Payment

$$E[DL^{(n)}] = (1 - R)E[P(t_0, \tau_n) 1_{[\tau_n \leq t_M]}]$$

Default Leg

- Set $E[PL^{(n)}] + E[AP^{(n)}] = E[DL^{(n)}]$
- Solve for $s^{(n)}$

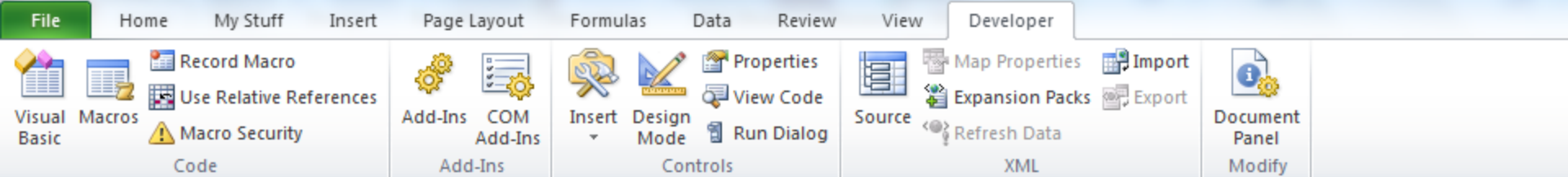
Valuation

$$S^{(n)} = \frac{(1 - R)E[P(t_0, \tau_n) 1_{[\tau_n \leq t_M]}]}{\Delta \sum_{i=1}^M P(t_0, t_i)E[1_{[N(t_i) < n]}] + \sum_{i=1}^M E[(\tau_n - t_{i-1}) P(t_0, \tau_n) 1_{[t_{i-1} < \tau_n \leq t_i]}]}$$

Fair Spread Equation

- Write an application that estimates the fair spreads for the 1-, 2-, ..., N-th- to-default swaps for a portfolio of N loans
- Inputs: number of loans, maturity of swaps, payment period, recovery rate, risk-free rate, default intensity rate, correlation coefficient, degrees of freedom for t-copula, number of simulations
- Outputs: Fair spread for each swap

Assignment

[illegible]

Algorithm

- Run S simulations of a N -dimensional vector of correlated uniform random variables $U = (U_1, \dots, U_N)$ from a selected copula (Gaussian or t)
 - For each simulation, compute the default time for each loan i as $\tau_i = -\frac{1}{h} \text{Log}(1 - U_i)$
 - For each simulation, sort the default times to determine the n -th default time τ_n
 - Use the Fair Spread Equation to estimate $s^{(n)}$ for each n . Compute each average $E[X]$ as the sum of X over the S simulations, divided by S
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- C = loan default time correlation matrix
- Compute the Cholesky decomposition L of C from $C = LL^T$
- Generate a random standard normal vector $Z = (Z_1, \dots, Z_N)^T$ where the Z_i 's are independent normal variables
- Compute $X = LZ$
- $U = (\text{CDF}(X_1), \dots, \text{CDF}(X_N))$ represents the joint distribution of X , where CDF is normal

Gaussian Copula

- Compute the Cholesky decomposition L of C from $C = LL^T$
- Generate a random standard normal vector $Z = (Z_1, \dots, Z_N)^T$ where the Z_i 's are independent normal variables
- Generate a random variable Y from the Chi-square distribution with d degrees of freedom
- Compute $X = LZ \sqrt{\frac{d}{Y}}$
- $U = (\text{CDF}(X_1), \dots, \text{CDF}(X_N))$ represents the joint distribution of X , where CDF is a t distribution with d degrees of freedom

T- Copula

- $C = [a_{ij}]$ and $L = [l_{ij}]$
- Set $l_{11} = \sqrt{a_{11}}$
- For $j=2$ to N , set $l_{j1} = a_{j1}/l_{11}$
- For $i=2$ to $N-1$, perform the next 2 steps
- Set $l_{ii} = [a_{ii} - \sum_{k=1}^{i-1} l_{ik}^2]^{1/2}$
- For $j=i+1$ to N , set $l_{ji} = \frac{1}{l_{ii}} [a_{ji} - \sum_{k=1}^{i-1} l_{jk}l_{ik}]$
- Set $l_{nn} = [a_{nn} - \sum_{k=1}^{n-1} l_{nk}^2]^{1/2}$

Cholesky Decomposition
