Basket Default Swaps

- In an *n*-th-to-default swap, the seller compensates the buyer for any loss of principal and the accrued interest on the n-th asset in the reference basket to default.
- In turn, the buyer pays a fixed amount of money periodically.

Definition

Pricing

- Current time = t_0
- N underlying loans
- Payments for the *n*-th-to-default swap = spread $s^{(n)}$ at dates $t_1, t_2, \ldots, t_M$
- Present value of these payments, while the asset has not defaulted, is the premium leg $PL^{(n)}$ Additional payment for part of the default period is the accrued payment $AP^{(n)}$
- Payment to the buyer in the case of default is the default leg $DL^{(n)}$
- Δ = period between payments, as a fraction of year

- $P(t_0, t_j)$ = discount factor for maturity t_j, $j = 1,\ldots.,M$
- $P(t_0, t_j) = \exp(-rt_j)$
- R = recovery rate
- The default time for loan i has a CDF $F_i(t) = 1 – \exp(-ht)$ where h is the default intensity rate
- Correlation between default times = ρ
- Joint distribution modeled as a Gaussian or t copula
- *n*-th default time = τ_n
- N(t)= number of defaults in the basket before t

$$E[\mathrm{PL}^{(n)}] = \mathrm{s}^{(n)}\Delta \sum_{i=1}^{M} \mathrm{P}(\mathrm{t}_0, \mathrm{t}_\mathrm{i})\mathrm{E}[1_{[N(t_i)<n]}]$$

Premium Leg

$$E[\mathrm{AP}^{(n)}] = s^{(n)} \sum_{i=1}^{M} \mathrm{E}[(\tau_n - t_{i-1})\, \mathrm{P}(t_0, \tau_n)\, 1_{[t_{i-1} < \tau_n \le t_i]}]$$

Accrued Payment

$$E[\mathrm{DL}^{(n)}] = (1 - R)E[P(t_0, \tau_n)\, 1_{[\tau_n \leq t_M]}]$$

Default Leg

- Set $E[\mathrm{PL}^{(n)}] + E[\mathrm{AP}^{(n)}] = E[\mathrm{DL}^{(n)}]$

- Solve for $s^{(n)}$

Valuation

$$s^{(n)} = \frac{(1 - R)E[P(t_0, \tau_n)\, 1_{[\tau_n \le t_M]}]}{\Delta\ \sum_{i=1}^{M} P(t_0, ti)E[1_{[N(t_i)<n]}] + \sum_{i=1}^{M} E[(\tau_n - t_{i-1})\, P(t_0, \tau_n)\, 1_{[t_{i-1}<\tau_n \le t_i]}]}$$

Fair Spread Equation

- Write an application that estimates the fair spreads for the 1-, 2-,…,N-th- to-default swaps for a portfolio of N loans

- Inputs: number of loans, maturity of swaps, payment period, recovery rate, risk-free rate, default intensity rate, correlation coefficient, degrees of freedom for t-copula, number of simulations

- Outputs: Fair spread for each swap

Assignment

Number of loans	7
Maturity of swap (years)	7
Payment period (years)	1.00
Recovery rate	0.50
Risk-free interest rate	0.03
Default intensity	0.01
Correlation coefficient for all loans	0
Degrees of freedom (for t copula)	1
Number of simulations	10000

Start simulating basket default swaps

Number to default	Spread (in basis points)
1	357.8938
2	54.9036
3	5.5999
4	0.2080
5	0.0000
6	0.0000
7	0.0000

Algorithm

- Run S simulations of a N-dimensional vector of correlated uniform random variables U = (U1,…,UN) from a selected copula (Gaussian or t)

- For each simulation, compute the default time for each loan i as $\tau_i = -\frac{1}{h} Log(1 - U_i)$

- For each simulation, sort the default times to determine the n-th default time τ_n

- Use the Fair Spread Equation to estimate $s^{(n)}$ for each n. Compute each average E[X] as the sum of X over the S simulations, divided by S

- C = loan default time correlation matrix

- Compute the Cholesky decomposition L of C from

$C = LL^T$

- Generate a random standard normal vector $Z = (Z_1,\ldots Z_N)T$ where the Z_i's are independent normal variables

- Compute $X = LZ$

- $U = (CDF(X_1),\ldots,CDF(X_N))$ represents the joint distribution of X, where CDF is normal

Gaussian Copula

- Compute the Cholesky decomposition L of C from

$C = LL^T$

- Generate a random standard normal vector $Z = (Z_1, \ldots Z_N)T$ where the Z_i's are independent normal variables

- Generate a random variable Y from the Chi-square distribution with *d* degrees of freedom

- Compute $X = LZ\sqrt{\frac{d}{Y}}$

- $U = (CDF(X_1), \ldots, CDF(X_N))$ represents the joint distribution of X, where CDF is a t distribution with *d* degrees of freedom

T- Copula

- $C = [a_{ij}]$ and $L = [l_{ij}]$

- Set $l_{11} = \sqrt{a_{11}}$
- For j=2 to N, set $l_{j1} = a_{j1}/l_{11}$
- For i=2 to N-1, perform the next 2 steps
- Set $l_{ii} = [a_{ii} - \sum_{k=1}^{i-1} l_{ik}{}^2]^{1/2}$
- For j=i+1 to N, set $l_{ji} = \frac{1}{l_{ii}} [a_{ji} - \sum_{k=1}^{i-1} l_{jk} l_{ik}]$
- Set $l_{nn} = [a_{nn} - \sum_{k=1}^{n-1} l_{nk}{}^2]^{1/2}$

Cholesky Decomposition