Range Accrual Notes

- In general, a Range Accrual Note (RAN) linked to a stock has the following two features.

- Feature 1: At maturity time T, the issuer redeems the note by paying its face value D if the reference stock price S_T is above or equals the strike price K. If S_T is below K, the issuer delivers D/K shares of the stock. In both cases, the total coupon C is also paid out.

General Description

- Feature 1: Payoff at time T is

$$Payoff_T = \begin{cases} C + \frac{D}{K}\, K, if\ S_T \geq K \\ C + \frac{D}{K}\, S_T, otherwise \end{cases}$$

i.e. $$Payoff_T = C + \frac{D}{K} \min\{K, S_T\}$$

General Description

- Feature 2: The coupon accumulates only on each day i that the closing stock price S_i is greater than or equal the barrier price B. This holds for all coupon periods, with the exception of the first period when the full coupon (coupon rate * D) is paid at the first coupon payment date.

General Description

- A European RAN has Feature 1, but not Feature 2. The full coupon amount, (coupon rate * D), is paid at each coupon payment date.

- An American RAN has the same features as an European RAN. However, its holder can redeem it before maturity and get the full debt amount D plus the accrued coupon up to that point in time.

- A callable RAN has both Feature 1 and Feature 2. In addition, if the closing stock price on any of the coupon payment dates is greater than or equal to the trigger price TG, the contract is automatically ended and the full amount of debt D plus the accrued coupon up to that point in time are paid to the holder.

For this assignment

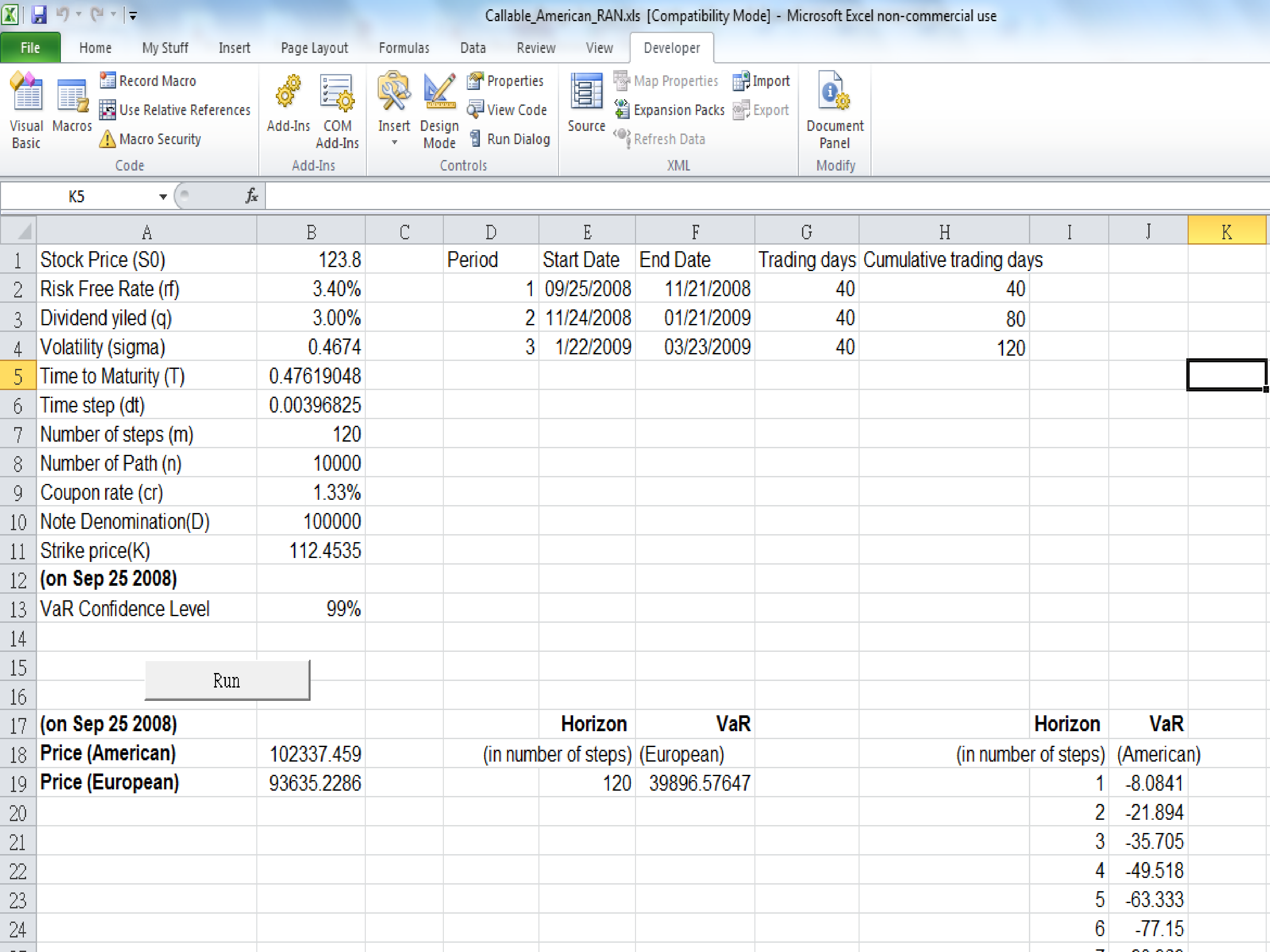
- Write a macro that estimates VaR for American and European RANs by
  - Simulating multiple paths for a geometric brownian motion stock price process
  - Estimating the value of the American RAN at every step along each path
  - Estimating the VaR of the American RAN for each horizon ending at every step

- Estimating the current value of the European RAN for each path
- Estimating the VaR of the European RAN for the horizon ending at maturity

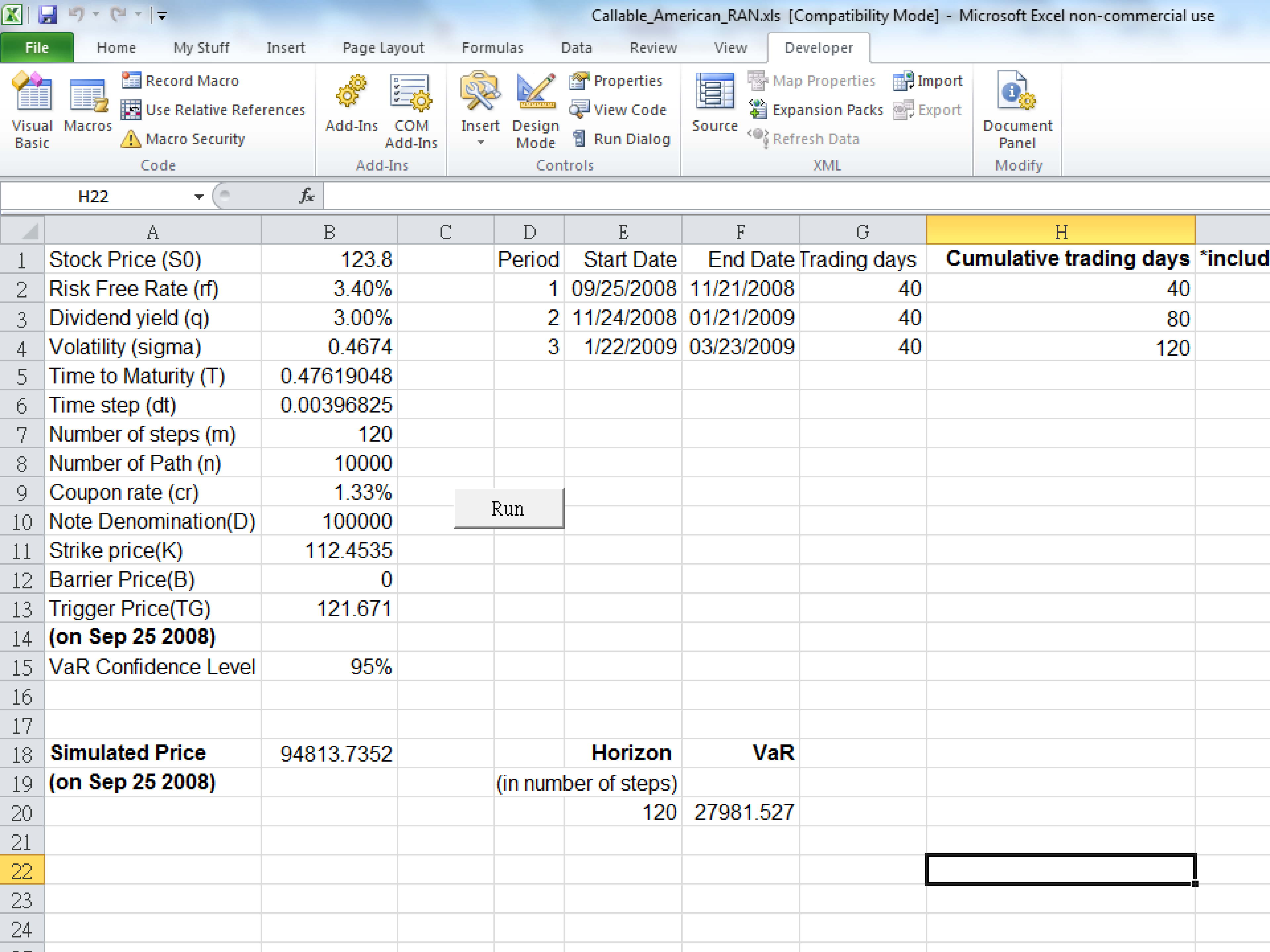
- Write a macro that estimates VaR for Callable RANs by
  - Simulating multiple paths for a geometric brownian motion stock price process
  - Estimating the current value of the Callable RAN for each path
  - Estimating the VaR of the Callable RAN for the horizon ending at maturity

- Once you have simulated multiple paths for the geometric brownian motion stock price process, transform these paths through the “Emartingale” public function that is provided, before using them.
- The function is an implementation of “empirical martingale simulation”, a corrective method that aims to ensure the martingale property of the simulated results holds, i.e. the current price at time 0 equals the expected PV of any future time t price.

Note



	A	B	C	D	E	F	G	H	I	J	K
1	Stock Price (S0)	123.8		Period	Start Date	End Date	Trading days	Cumulative trading days			
2	Risk Free Rate (rf)	3.40%		1	09/25/2008	11/21/2008	40	40			
3	Dividend yiled (q)	3.00%		2	11/24/2008	01/21/2009	40	80			
4	Volatility (sigma)	0.4674		3	1/22/2009	03/23/2009	40	120			
5	Time to Maturity (T)	0.47619048									
6	Time step (dt)	0.00396825									
7	Number of steps (m)	120									
8	Number of Path (n)	10000									
9	Coupon rate (cr)	1.33%									
10	Note Denomination(D)	100000									
11	Strike price(K)	112.4535									
12	**(on Sep 25 2008)**										
13	VaR Confidence Level	99%									
14											
15											
16											
17	**(on Sep 25 2008)**				**Horizon**	**VaR**			**Horizon**	**VaR**	
18	**Price (American)**	102337.459			(in number of steps)	(European)			(in number of steps)	(American)	
19	**Price (European)**	93635.2286			120	39896.57647			1	-8.0841	
20									2	-21.894	
21									3	-35.705	
22									4	-49.518	
23									5	-63.333	
24									6	-77.15	


Callable_American_RAN.xls [Compatibility Mode] - Microsoft Excel non-commercial use

	A	B	C	D	E	F	G	H	I
1	Stock Price (S0)	123.8		Period	Start Date	End Date	Trading days	**Cumulative trading days**	***includ**
2	Risk Free Rate (rf)	3.40%		1	09/25/2008	11/21/2008	40	40	
3	Dividend yield (q)	3.00%		2	11/24/2008	01/21/2009	40	80	
4	Volatility (sigma)	0.4674		3	1/22/2009	03/23/2009	40	120	
5	Time to Maturity (T)	0.47619048							
6	Time step (dt)	0.00396825							
7	Number of steps (m)	120							
8	Number of Path (n)	10000							
9	Coupon rate (cr)	1.33%	Run						
10	Note Denomination(D)	100000							
11	Strike price(K)	112.4535							
12	Barrier Price(B)	0							
13	Trigger Price(TG)	121.671							
14	**(on Sep 25 2008)**								
15	VaR Confidence Level	95%							
16									
17									
18	**Simulated Price**	94813.7352			**Horizon**	**VaR**			
19	**(on Sep 25 2008)**			(in number of steps)					
20					120	27981.527			

Stock Price Dynamics

- At step j + 1 of a path, price S changes as follows:

$$\boldsymbol{S_{j+1} = S_j + \left(r_f - q\right)S_j\Delta t + \sigma S_j\sqrt{\Delta t}Z_j}$$

  - r_f is the risk-free rate
  - q is the asset dividend yield
  - Δt is the time between 2 steps
  - σ is the annual asset volatility
  - Z_j is a random value from a standard normal distribution (NormInv on Excel)

- For each path j, compute the RAN value $\boldsymbol{V_{m,j}}$ at terminal step m:

$$\frac{\boldsymbol{D}}{\boldsymbol{K}} * \mathbf{min}\left(\boldsymbol{S_{m,j}}, \boldsymbol{K}\right) + \boldsymbol{e^{r_f m \Delta t} PV_0 (Ac. Couponm)}$$

- Work back from step m-1 to step 1. For each step i, compute the accrued coupon then the RAN exercise value for path j as:

American RAN Valuation

$$D + e^{r_f i \Delta t} PV_0(Ac.Couponi)$$

- At each step i-1, the RAN value, V_{i-1}, is the highest of the exercise value and the continuation value, $E[e^{-r\Delta t} V_i \mid S_{i-1}]$
- Set V_0 to $e^{-r\Delta t} V_1$ for each path
- Average V_0 over all paths to estimate the RAN value

American RAN Valuation

- At intermediate step i-1, estimate the regression coefficients in $\boldsymbol{e^{-r\Delta t}V_i = a + b\, S_{i-1} + c\, S_{i-1}{}^2}$ by using the observations from all paths

- For each of the n paths, plug $\boldsymbol{S_{i-1}}$ into the regression formula to come up with the continuation value, $\boldsymbol{E}[\boldsymbol{e^{-r\Delta t}V_i} \mid \boldsymbol{S_{i-1}}]$

Continuation Value

Continuation Value

- If the continuation value $\boldsymbol{E}[\boldsymbol{e}^{-\boldsymbol{r}\Delta \boldsymbol{t}}\boldsymbol{V}_{\boldsymbol{i}} \mid \boldsymbol{S}_{\boldsymbol{i}-\mathbf{1}}]$ is greater than the exercise value, set the option value $\boldsymbol{V}_{\boldsymbol{i}-\mathbf{1}}$ to $\boldsymbol{e}^{-\boldsymbol{r}\Delta \boldsymbol{t}}\boldsymbol{V}_{\boldsymbol{i}}$ in order to reduce standard errors

- For each path j, compute the current RAN value $\boldsymbol{V_{0,j}}$ as:

$$\frac{\boldsymbol{D}}{\boldsymbol{K}} * \mathbf{min}\left(\boldsymbol{S_{m,j}}, \boldsymbol{K}\right) * \boldsymbol{e^{-r_f m \Delta t}} + \boldsymbol{PV_0}(\boldsymbol{Ac.Couponm})$$

- Average $\boldsymbol{V_0}$ over all paths to estimate the RAN value

European RAN Valuation

- For each path j, work from step 0 to step m to:
  - Accumulate PV(coupon) every step the stock price is greater than or equal to the barrier price
  - Test if at step i, the stock price is greater than or equal to the target price, in which case the current RAN value is:

$$\boldsymbol{De^{-r_f i\Delta t} + PV_0(Ac.Couponi)}$$

  - If the RAN is not called before maturity then its current value is:

$$\boldsymbol{\frac{D}{K}e^{-r_f m\Delta t} * \min\left(S_{m,j}, K\right) + PV_0(Ac.Couponm)}$$

- Average $\boldsymbol{V_0}$ over all paths to estimate the RAN value

Callable RAN Valuation

- For horizon i, the absolute VaR at the (1-α) confidence level is -1 times (the α-th percentile of the distribution of $\boldsymbol{V_i}$ across the n paths, minus the initial average value $\boldsymbol{V_0}$)

VaR Estimation