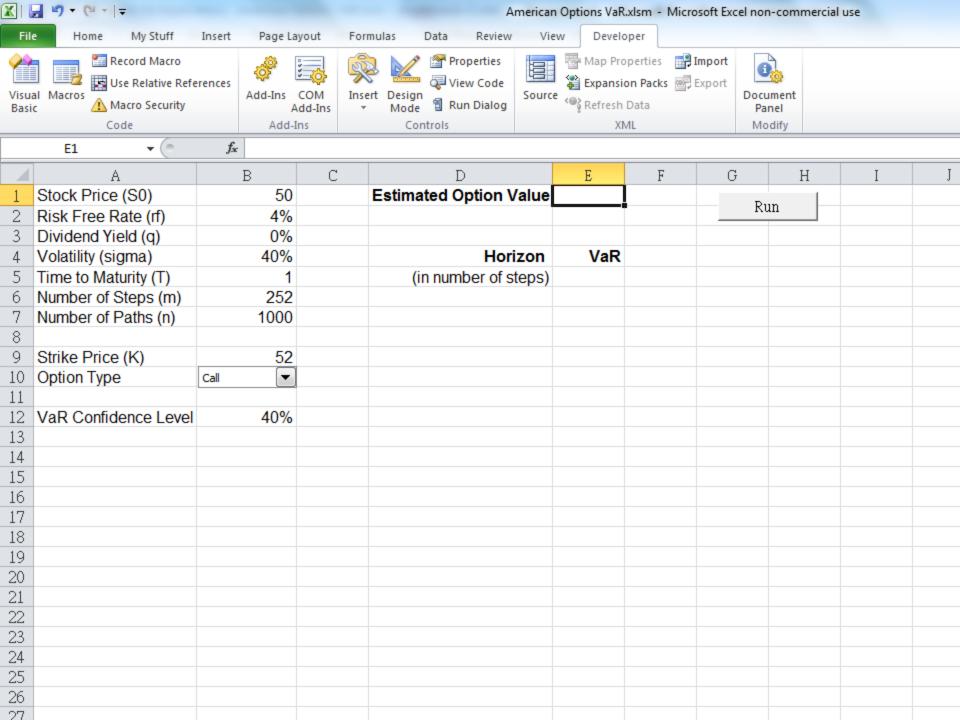
Var Simulation for American Options

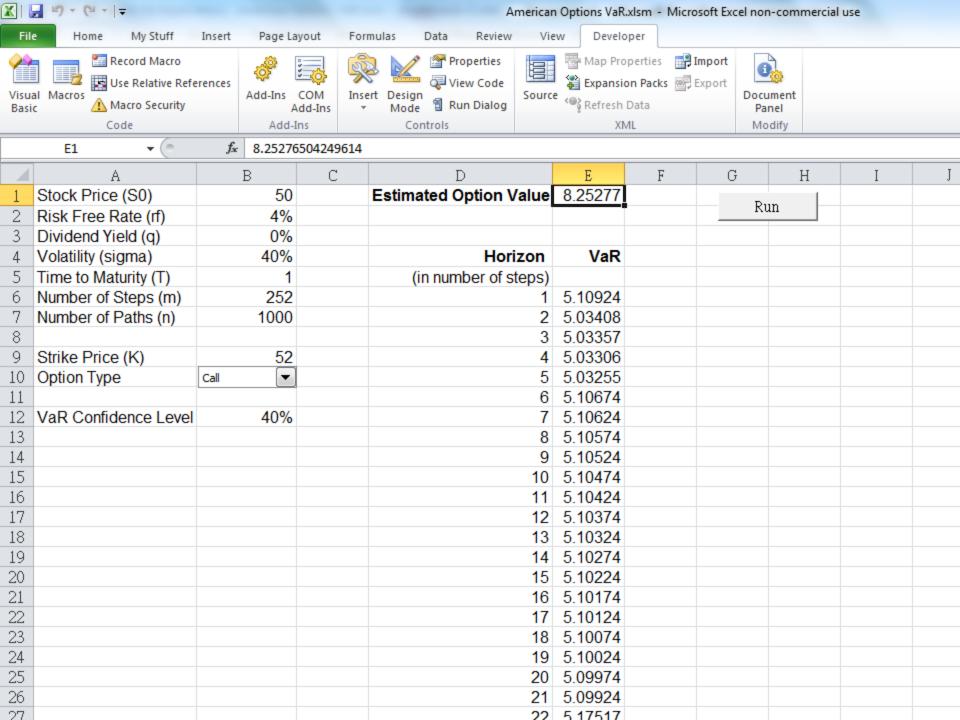
 Write a macro that estimates VaR for American calls, puts, and straddles by

• Simulating multiple paths for a geometric brownian motion asset price process

• Estimating the value of the option at every step along each path

• Estimating the VaR of the option for each horizon ending at every step





• At step j + 1 of a path, price S changes as follows:

$$S_{j+1} = S_j + (r - q)S_j\Delta t + \sigma S_j\sqrt{\Delta t}Z_j$$

- r is the risk-free rate
- q is the asset dividend yield
- Δt is the time between 2 steps
- σ is the annual asset volatility
- Z_j is a random value from a standard normal distribution
- $(Z_j = \text{WorksheetFunction.NormInv}(\text{Rnd}(), 0, 1)))$

Asset Price Dynamics

- For each path, compute the option value V_m at terminal step m: $\max(K S_m, 0)$
- Work back from step m to step m-1 to step m-2 to...to step 1 to get V₁ for each path
- At each step i-1, the option value, V_{i-1} , is the highest of the exercise value, $\max(K S_{i-1}, 0)$ and the continuation value, $E[e^{-r\Delta t}V_i \mid S_{i-1}]$

Put Valuation

- Set V_0 to $e^{-r\Delta t}V_1$ for each path
- Average V_0 over all paths to estimate the option value

Put Valuation

• At intermediate step i-1, estimate the regression coefficients in $e^{-r\Delta t}V_i = \alpha + b S_{i-1} + c S_{i-1}^2$ by using the observations from in-the-money paths $(K - S_{i-1} > 0)$

• For each of the n paths, plug S_{i-1} into the regression formula to come up with the continuation value, $E[e^{-r\Delta t}V_i \mid S_{i-1}]$

Continuation Value

• If the continuation value $E[e^{-r\Delta t}V_i \mid S_{i-1}]$ is greater than the exercise value $\max(K - S_{i-1}, 0)$, set the option value V_{i-1} to $e^{-r\Delta t}V_i$ in order to reduce standard errors

Continuation Value

• For horizon i, the absolute VaR at the $(1-\alpha)$ confidence level is -1 times (the α -th percentile of the distribution of V_i across the n paths, minus the initial average value V_0)

VaR Estimation

Data Structure

Option Explicit

```
Type BS PathType
    SO As Double
                     'initial asset price
                     'dividend vield
    g As Double
    sigma As Double 'volatility
    rf As Double
                     'risk free rate
    t As Double
                     'period length
    dt As Double
                     'time step
                     'no of steps
    m As Long
    n As Long
                     'no of paths
End Type
```

Dim A As BS_PathType

A.S0 = Cells(1, 2)

OLS Regression

```
Public Function OLS(x As Variant, y As Variant) As Variant
    Dim Beta As Variant
    Dim i As Integer
    With Application
        Beta = .MMult(.MInverse(.MMult(.Transpose(x), x)), .MMult(.Transpose(x), y))
    End With
    ReDim res(0 To UBound(Beta) - 1) As Double
    For i = 0 To UBound(res)
        res(i) = Beta(i + 1, 1)
    Next i
    OLS = res
End Function
```