Infinite Sequence

Definition

An infinite sequence of real numbers is a function whose domain is N.

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A sequence a_n converges to l if for every $\varepsilon > 0$ there is a natural number N such that, for all natural number n,
if $n > N$, then

$$|a_n - l| < \varepsilon$$

Theorem 1

Let f be a function defined in an open interval containing c, except perhaps at c itself, with

$$\lim_{x \to c} f(x) = l.$$

Suppose that a_n is a sequence such that

(1) each a_n is in the domain of f.
(2) each $a_n \neq c$,
(3) $\lim_{n \to c} a_n = c$.

Then the sequence $f(a_n)$ satisfies

$$\lim_{n \to \infty} f(a_n) = l.$$

Conversely, if this is true fir every sequence a_n satisfying the above conditions, then $\lim_{x \to c} f(x) = l$.

proof
Suppose first that