Calculus6 Taylor expansion テイラー展開

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1 式 Formula

関数が多項式になっている。

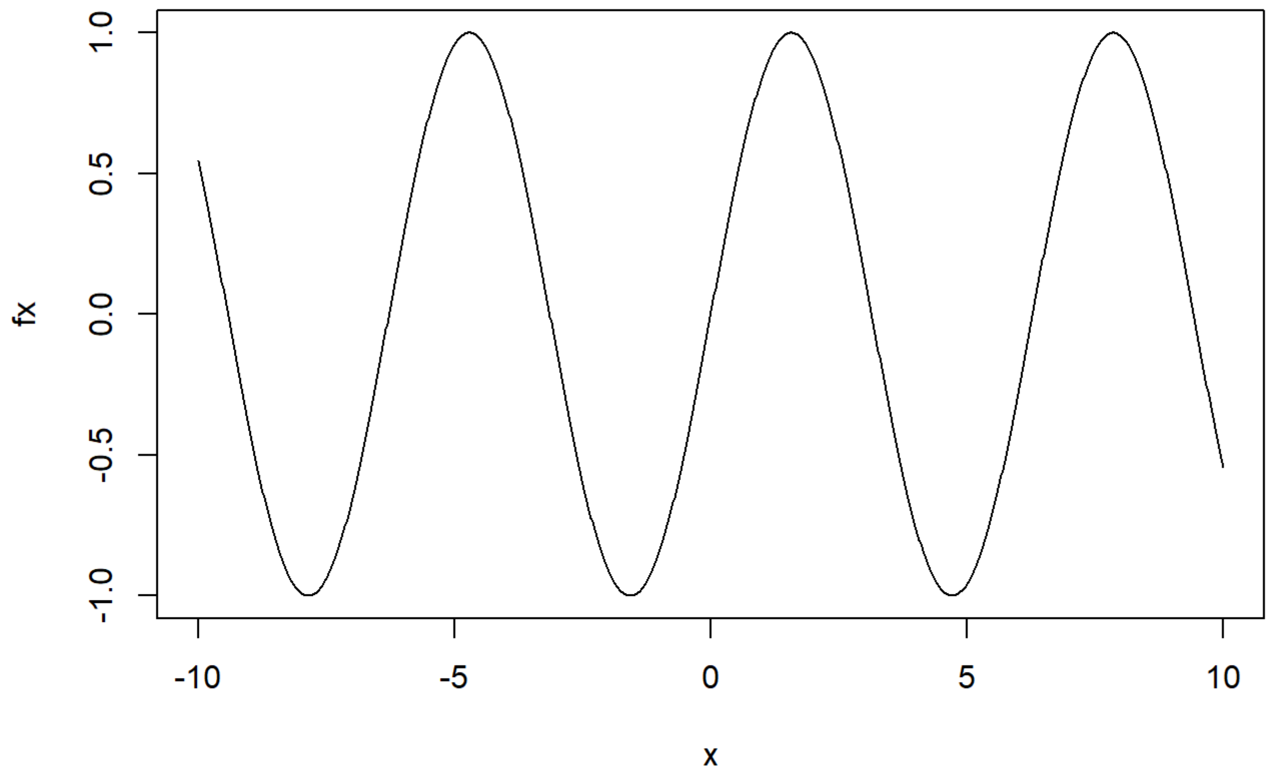
A function is expressed as a polynomial function.

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

2 Examples

$$f(x) = \sin x$$

```
x <- seq(from=-10,to=10,length=1000)
fx <- sin(x)
plot(x,fx,type="l")
```



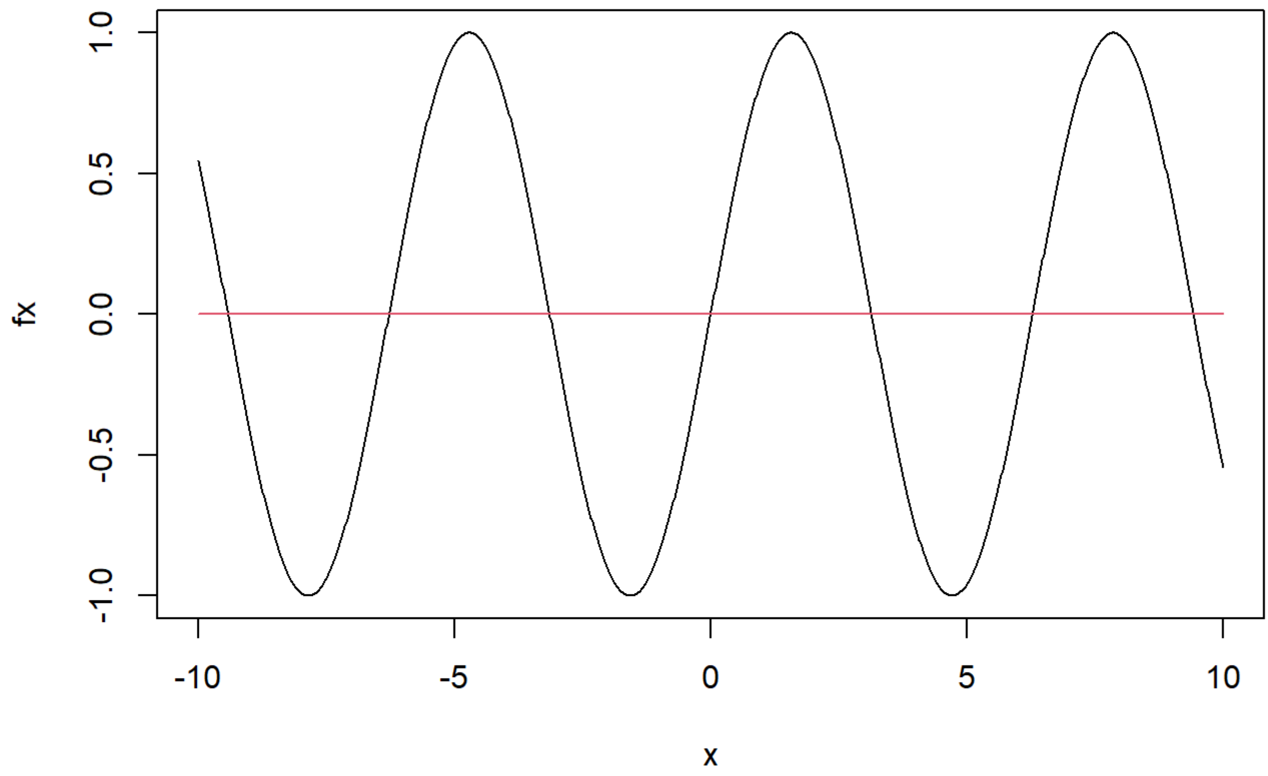
$$
\begin{aligned}
f^{(0)}(x) &= \sin x \\
f^{(1)}(x) &= \cos x \\
f^{(2)}(x) &= -\sin x \\
f^{(3)}(x) &= -\cos x \\
f^{(4)}(x) &= \sin x = f^{(0)}(x)
\end{aligned}
$$

$$
\begin{aligned}
f^{(4k)}(x) &= \sin x \\
f^{(4k+1)}(x) &= \cos x \\
f^{(4k+2)}(x) &= -\sin x \\
f^{(4k+3)}(x) &= -\cos x
\end{aligned}
$$

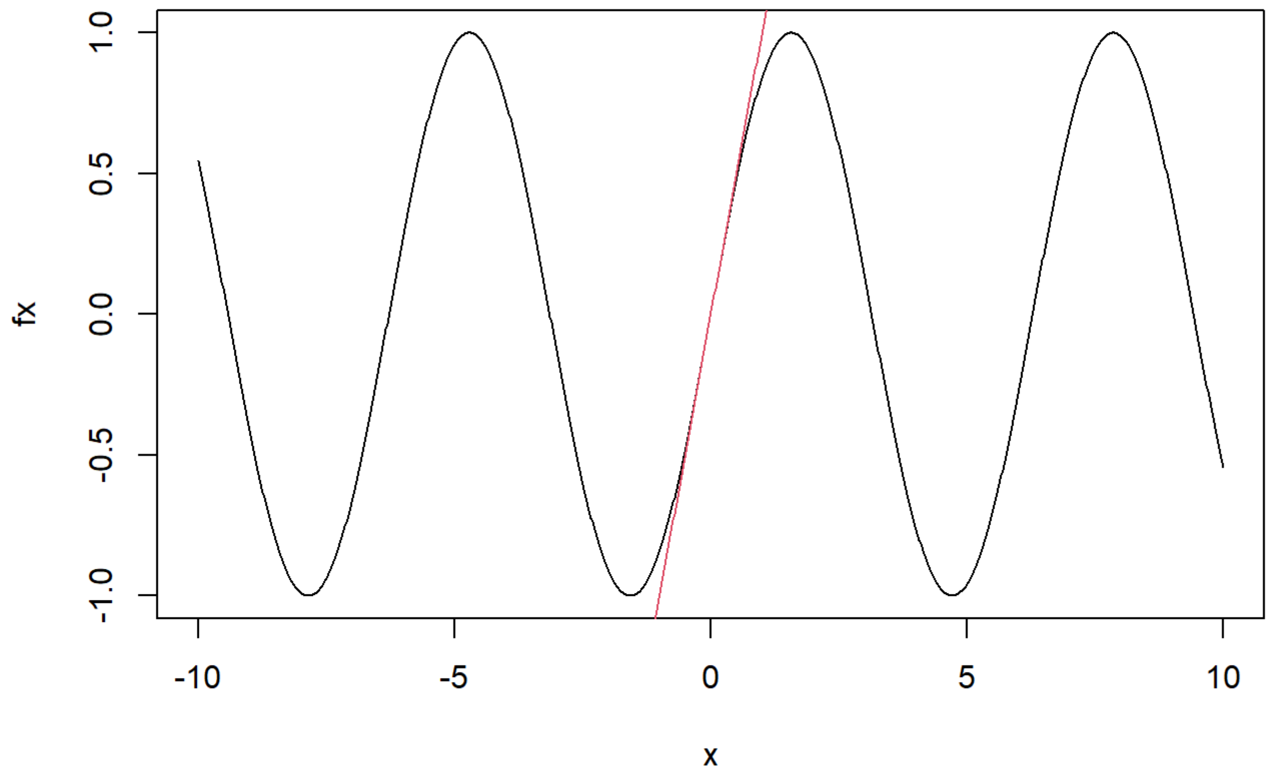
$a = 0$のとき、When $a = 0$

$$
\begin{aligned}
f^{(4k)}(0) &= \sin 0 = 0 \\
f^{(4k+1)}(0) &= \cos 0 = 1 \\
f^{(4k+2)}(0) &= -\sin 0 = 0 \\
f^{(4k+3)}(0) &= -\cos 0 = -1
\end{aligned}
$$

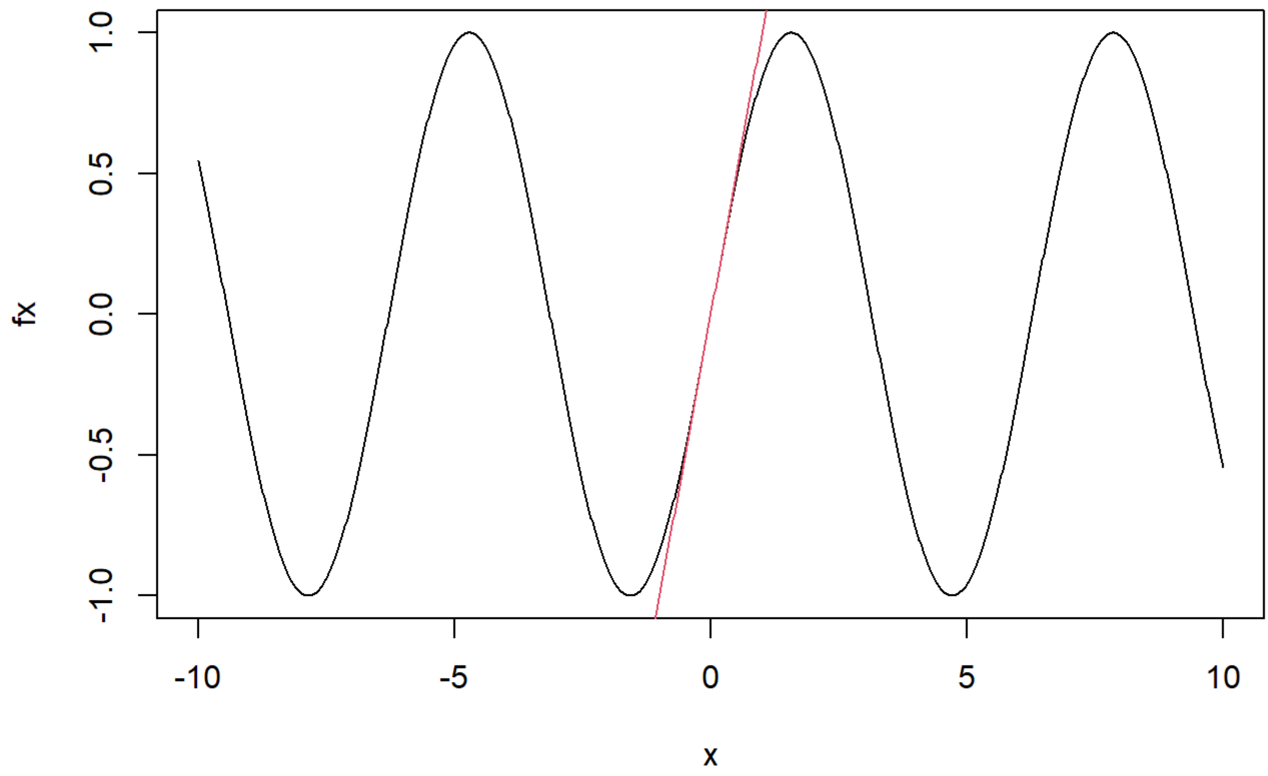
```
fx.0 <- 0/factorial(0) * x^0
plot(x,fx,type="l")
points(x,fx.0,type="l",col=2)
```



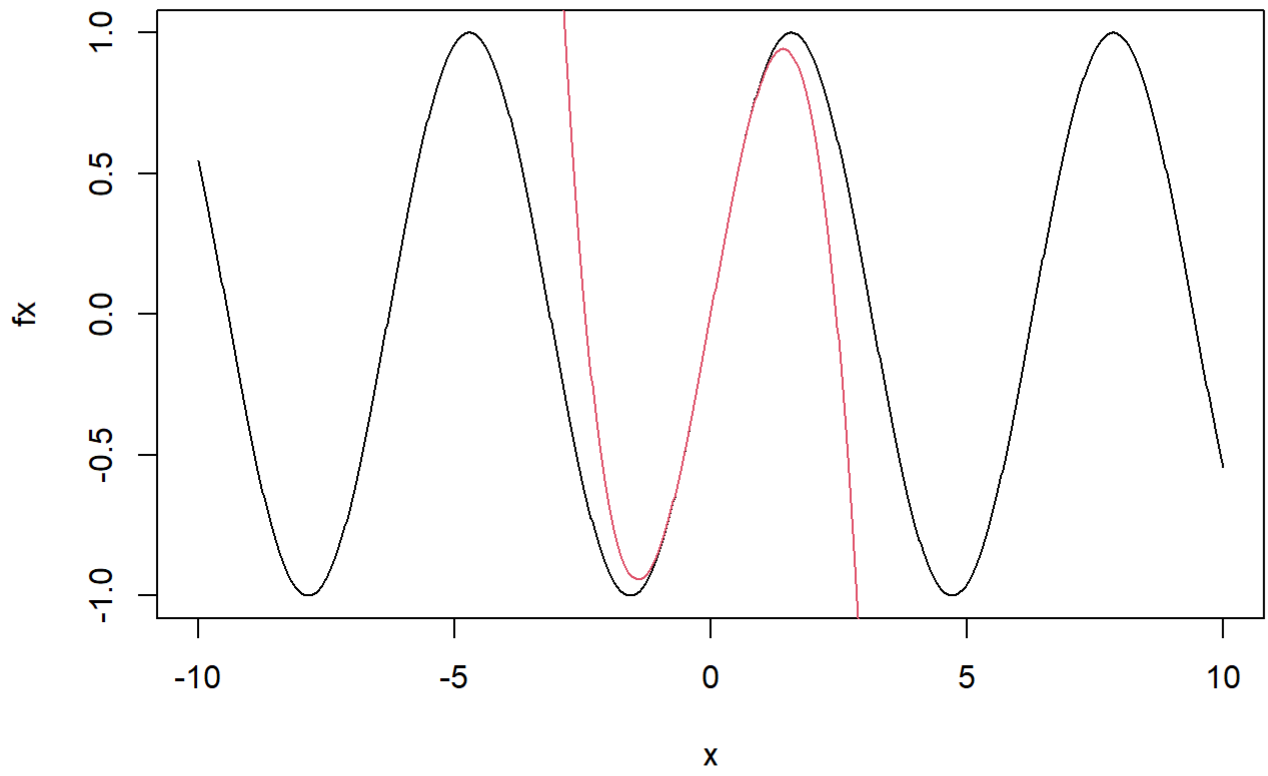

```
fx.1 <- fx.0 + 1/factorial(1) * x^1
plot(x,fx,type="l")
points(x,fx.1,type="l",col=2)
```



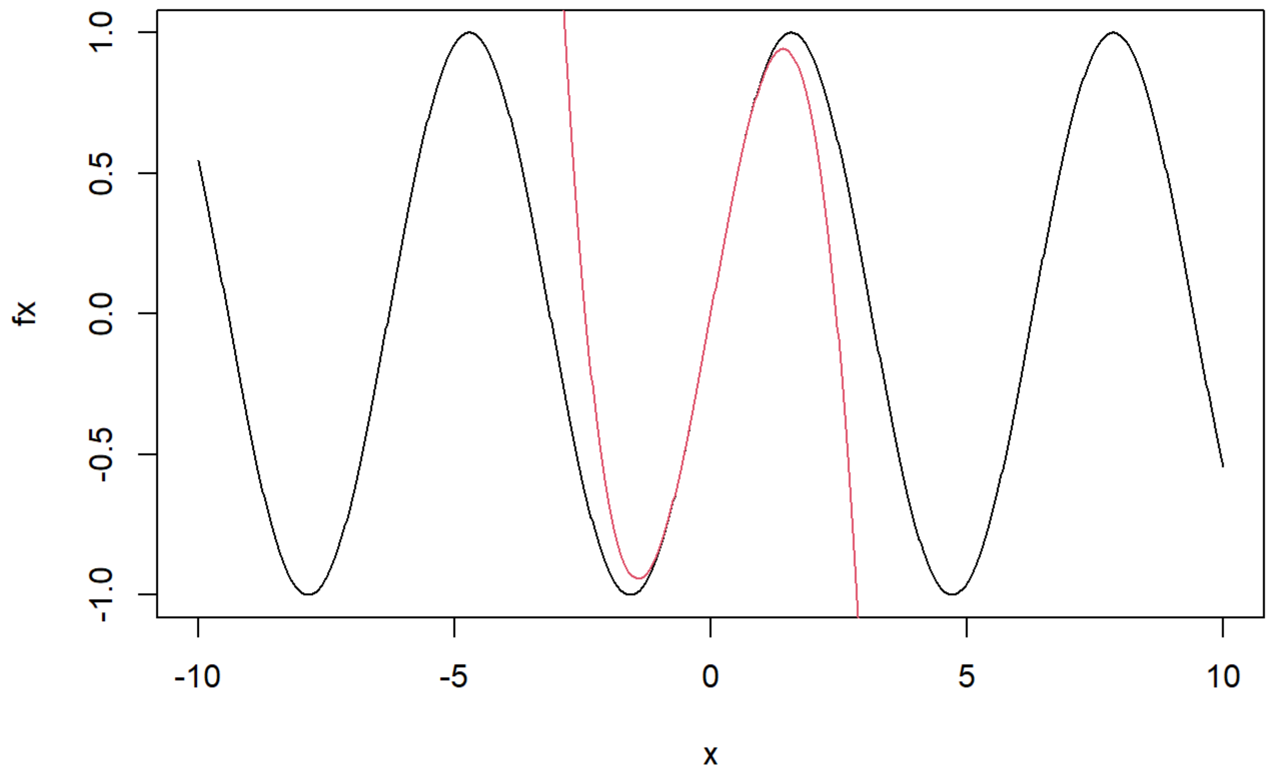

```
fx.2 <- fx.1 + 0/factorial(2) * x^2
plot(x,fx,type="l")
points(x,fx.2,type="l",col=2)
```



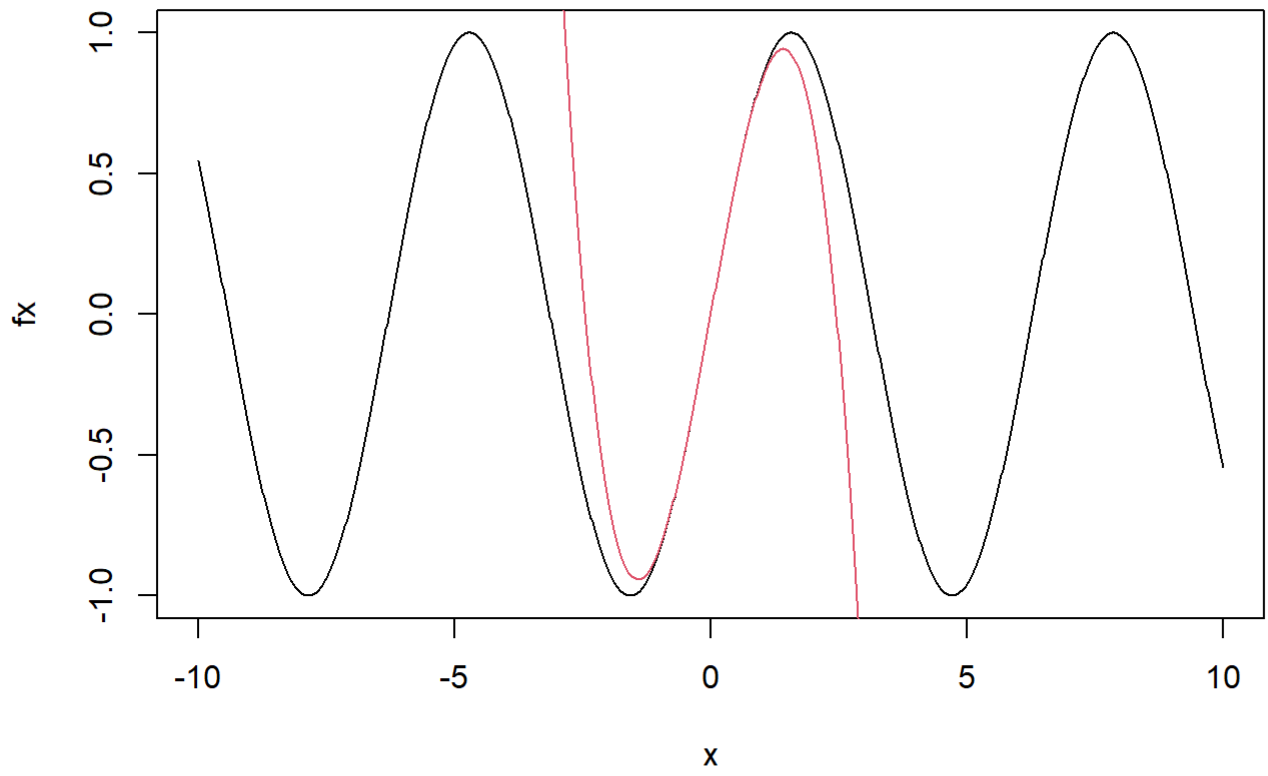

```
fx.3 <- fx.2 + (-1)/factorial(3) * x^3
plot(x,fx,type="l")
points(x,fx.3,type="l",col=2)
```



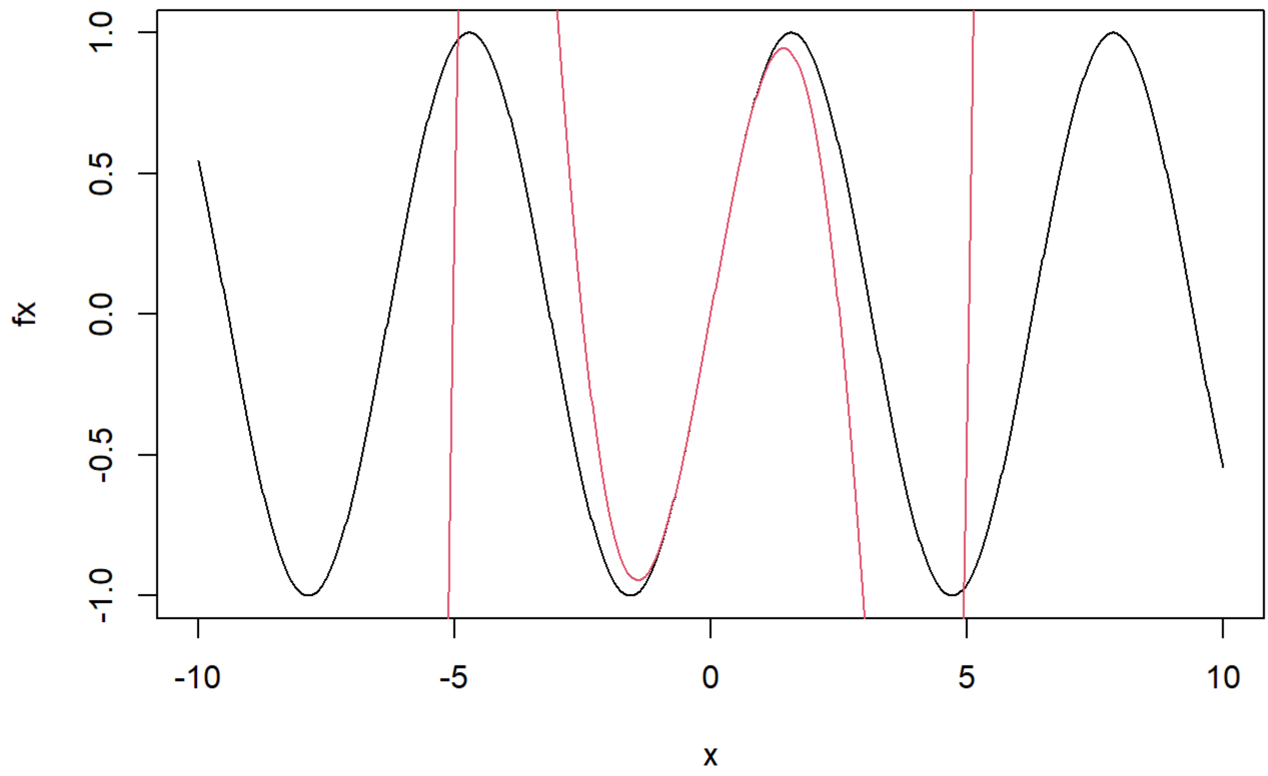

```
fx.4 <- fx.3 + 0/factorial(4) * x^4
plot(x,fx,type="l")
points(x,fx.4,type="l",col=2)
```



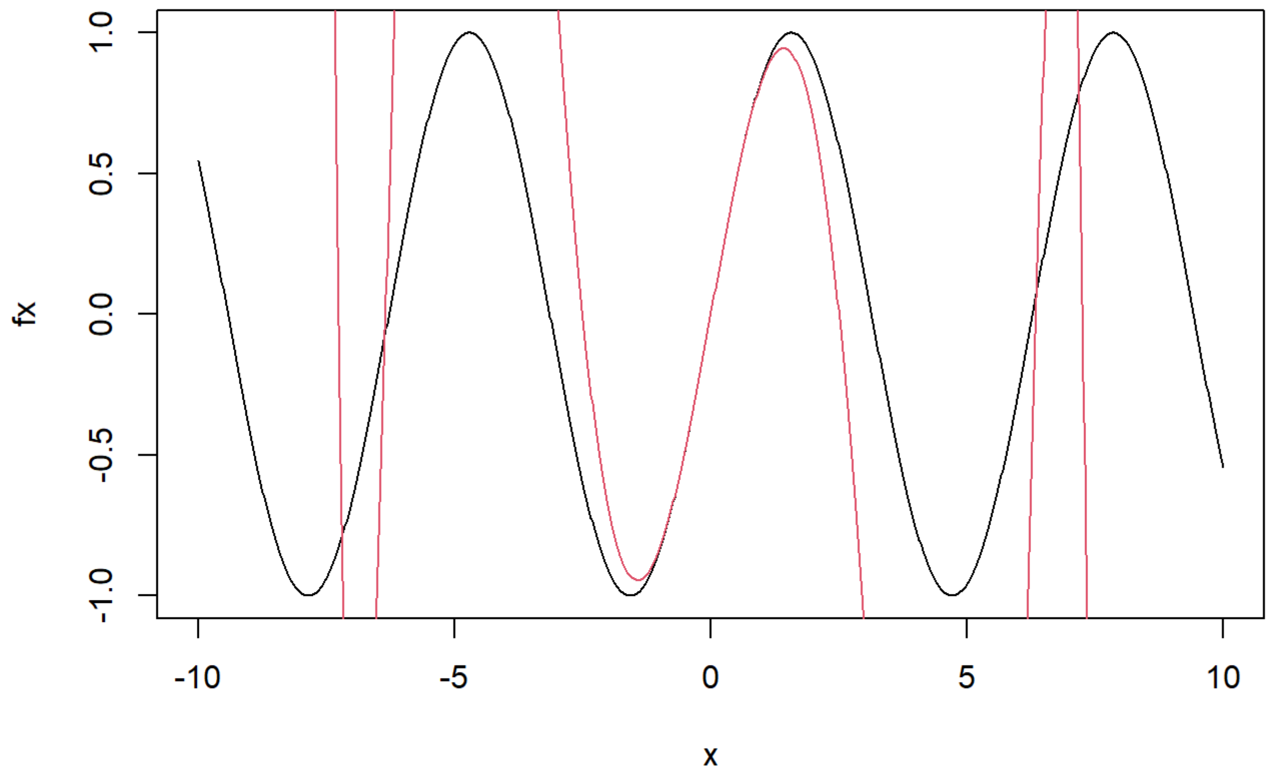

```
fx.5 <- fx.4 + 0/factorial(5) * x^5
plot(x,fx,type="l")
points(x,fx.5,type="l",col=2)
```




```
fx.6 <- fx.5
fx.7 <- fx.6 + 1/factorial(7) * x^7
plot(x,fx,type="l")
points(x,fx.7,type="l",col=2)
```




```
fx.8 <- fx.7
fx.9 <- fx.8 + (-1)/factorial(9) * x^9
plot(x,fx,type="l")
points(x,fx.9,type="l",col=2)
```



3 Exercise 1

3.1 Exercise 1-1

Do the same for $a = 1$ of $\sin x$.

3.2 Exercise 1-2

Do the same for $f(x) = e^x$.

3.3 Exercise 1-3

Do the same of $f(x) = (x-1)(x-2)(x-3)(x-4)$

4 Multiple testing correction

N個の独立な検定を行いN個のp-値を得るとする。 帰無仮説が成り立つとき、すべてのp-値が a以上である確率は

Assume N independent tests. When null hypothesis is true for all, probability that all p-values are equal or more than a is;

$$f(a) = (1-a)^N$$

この式の$a = 0$周辺の$f(a)^{(1)}$までのテイラー展開は

Taylor expanstion around $a = 0$ upto $f(a)^{(1)}$ is;

$$f(a) \sim \frac{1}{0!}a^0 + (-1)\frac{N}{1!}a^1 = 1 - Na$$

この近似を用いたのがボンフェロニ補正。

Bonferroni's correction is based on this approximation.

5 Exercise 2

5.1 Exercise 2-1

以下の関数を$a=0$周辺で$f(a)^k; k=0,1,2,...$までテイラー展開したときのkと近似値との関係をプロットし、ボンフェロニ補正が「保守的」であることを説明せよ。

Expand $f(a)$ around $a=0$ upto $f(a)^k; k=0,1,2,...$ and plot the relation between k and the approximated values, then describe that Bonferroni's correction is conservative using the plot.

$$f(a) = (1-a)^N$$

6 積率母関数 Moment generating function

6.1 Exercise 3

6.1.1 Exercise 3-1

このサイト https://www.probabilitycourse.com/chapter6/6_1_3_moment_functions.php (https://www.probabilitycourse.com/chapter6/6_1_3_moment_functions.php) を読み、積率母関数とテイラー展開との関係を説明せよ。

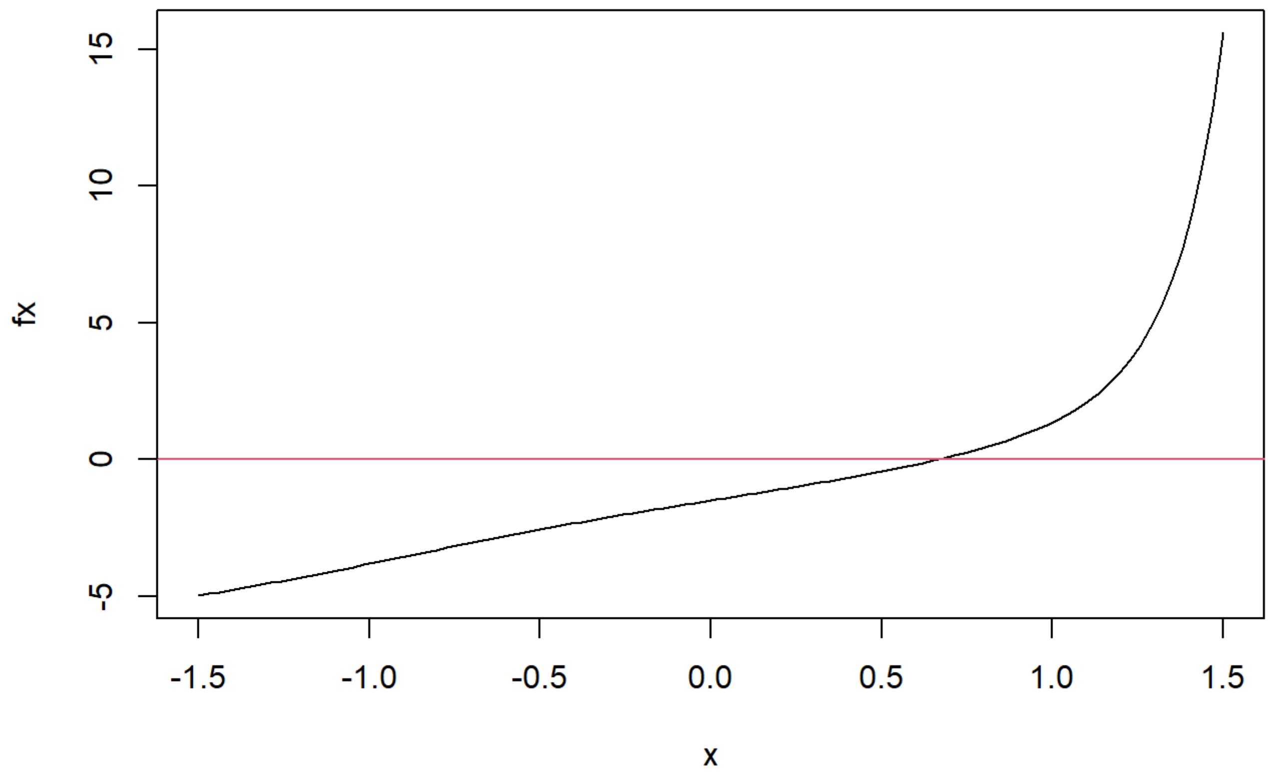
Read the site https://www.probabilitycourse.com/chapter6/6_1_3_moment_functions.php (https://www.probabilitycourse.com/chapter6/6_1_3_moment_functions.php) .

Describe relation between moment generating function and Taylor expansion.

7 ニュートン法　Newton's method

$$f(x) = \frac{1}{2}e^{x^3} + 2x - 2$$

```
x <- seq(from=-1.5,to=1.5,length=100)
my.fx <- function(x){
  exp(x^3)/2 + 2*x - 2
}
fx <- my.fx(x)
plot(x,fx,type="l")
abline(h=0,col=2)
```



$f(x) = 0$の解を近似的に求める。

Solve $f(x) = 0$ numerically.

7.1 単調性の確認 Monotonicity

8 Exercise 4

8.1 Exercise 4-1

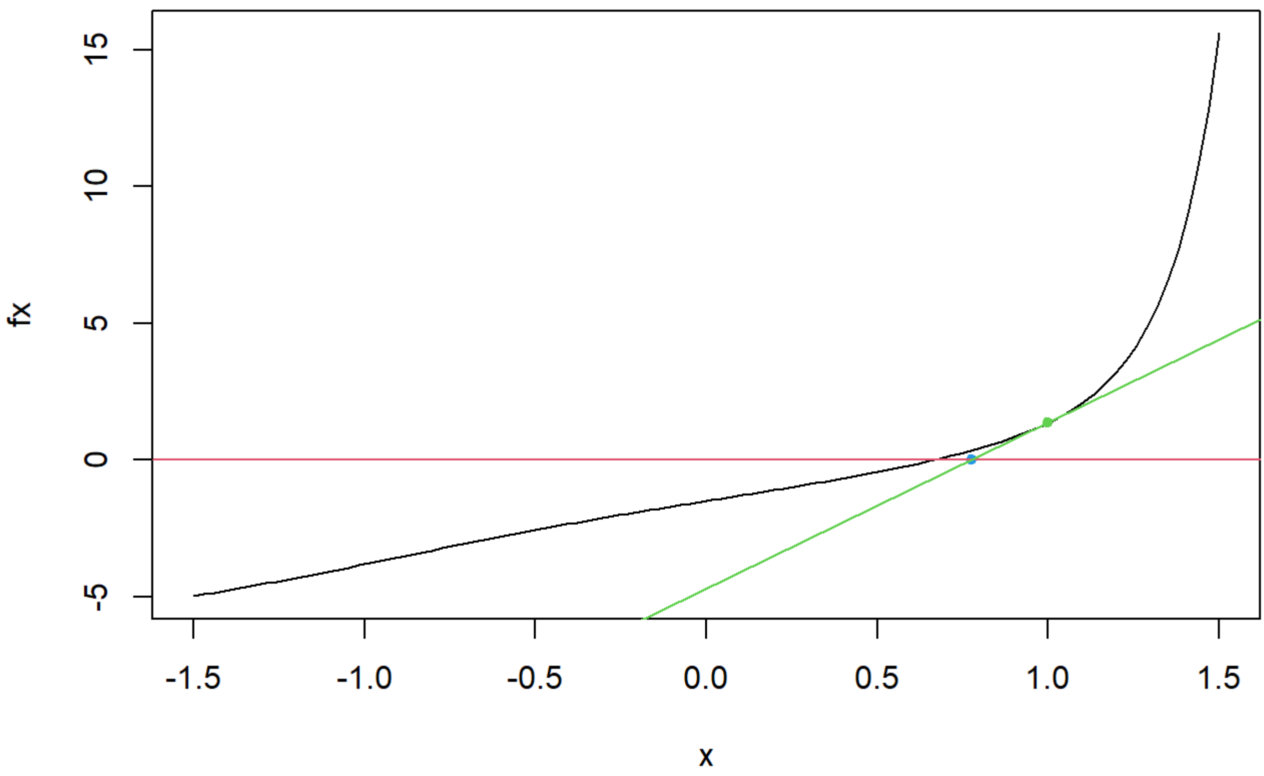
$f(x)$が単調増加関数であることを示せ。 Show that $f(x)$ is a monotonically increasing function.

8.2 ニュートン法 Newton's method

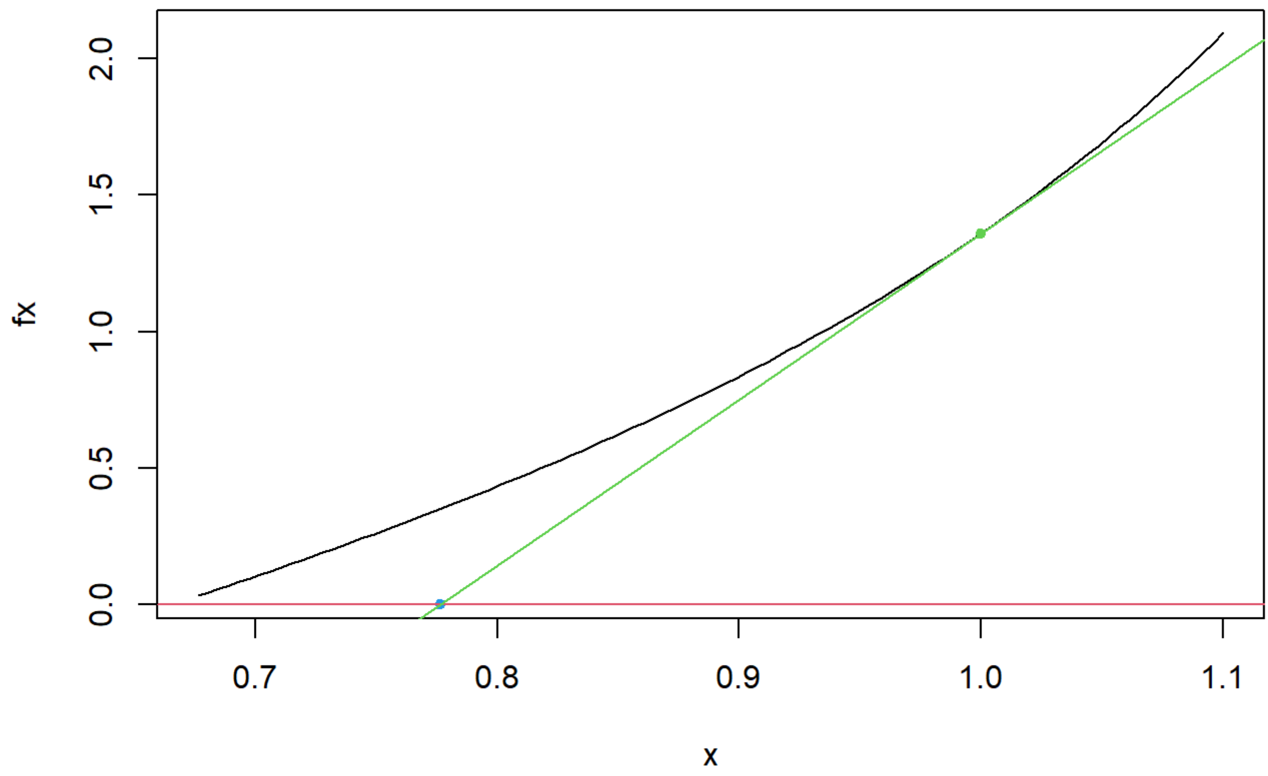
適当な値x_0からスタートし、$\frac{d}{dx}f(x)$を使って、$f(x) = 0$の解に次第に近づく。

Take an arbitrary valuex_0 and get closer to the solution of $f(x) = 0$ step-by-step.

```
x0 <- 1

my.dfxdx <- function(x){
  3/2 * x^2 * exp(x^3) + 2
}

this.x <- x0
this.y <- my.fx(this.x)

this.dfxdx <- my.dfxdx(this.x)
this.intercept <- this.y -this.x*this.dfxdx

new.x <- this.x - this.y/this.dfxdx

plot(x,fx,type="l")
points(this.x,this.y,pch=20,col=3)
points(new.x,0,pch=20,col=4)
abline(h=0,col=2)
abline(this.intercept,this.dfxdx,col=3)
```




```
x <- seq(from = new.x -0.1,to=this.x+0.1,length=100)
fx <- my.fx(x)
plot(x,fx,type="l")
points(this.x,this.y,pch=20,col=3)
points(new.x,0,pch=20,col=4)
abline(h=0,col=2)
abline(this.intercept,this.dfxdx,col=3)
```

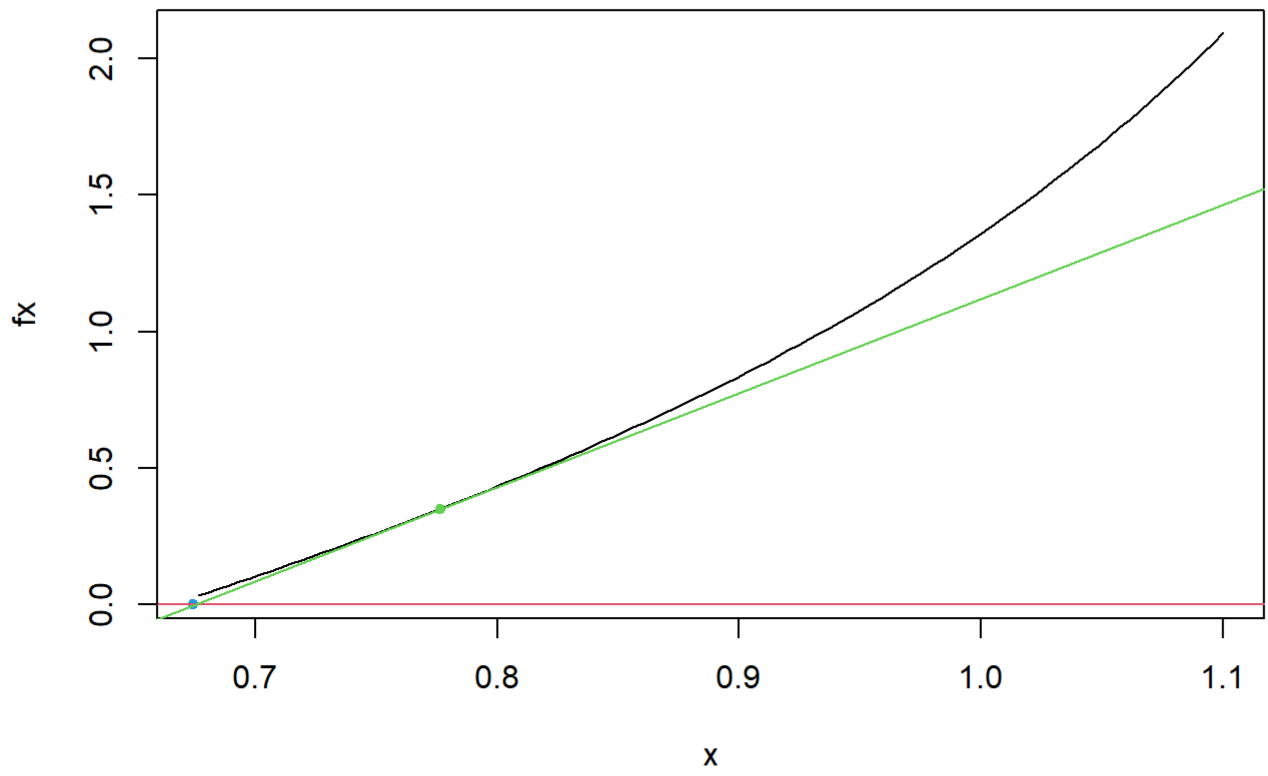


接線とx軸の交点は$f(x) = 0$の解に近づいている。

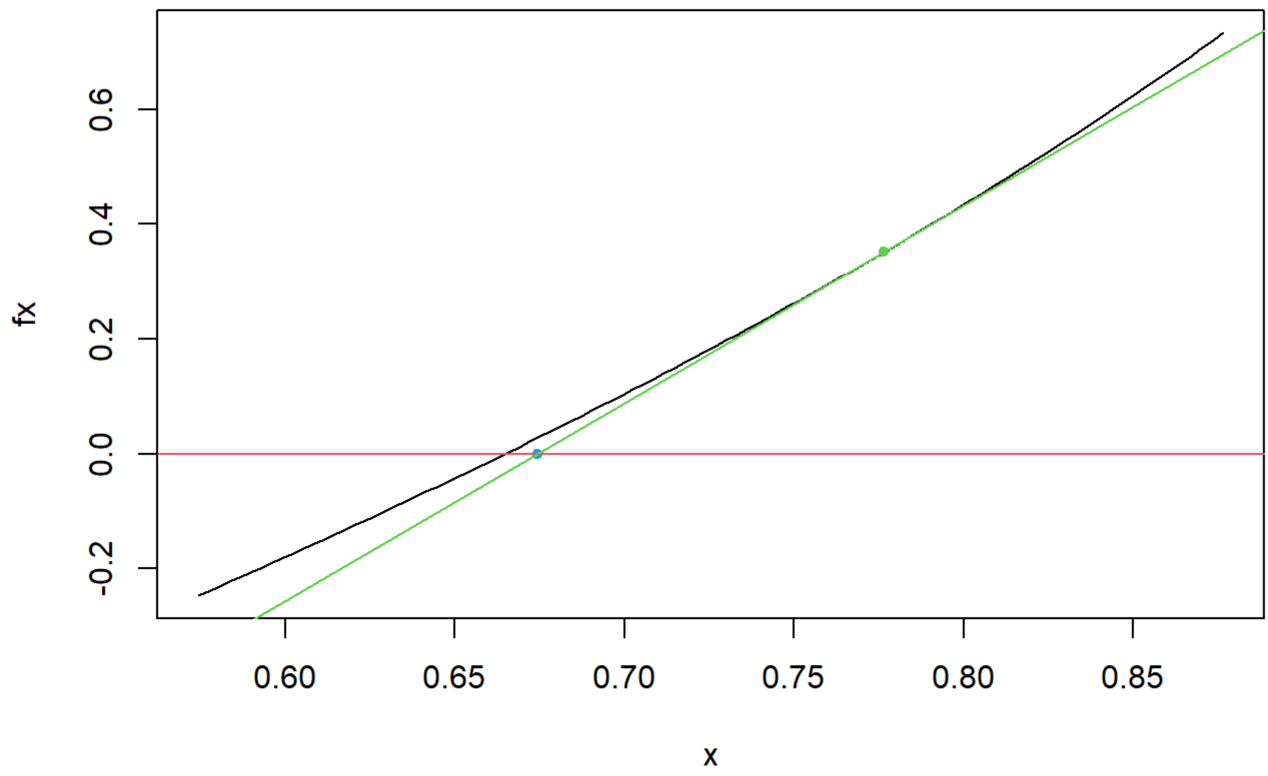
The crossing point with the $y = 0$ line of the tangent line at $(x_0, f(x_0))$ gets closer to the crossing point of $y = f(x)$ with the $y = 0$ line is closer that x_0.

これを繰り返す。Repeat this procedure.

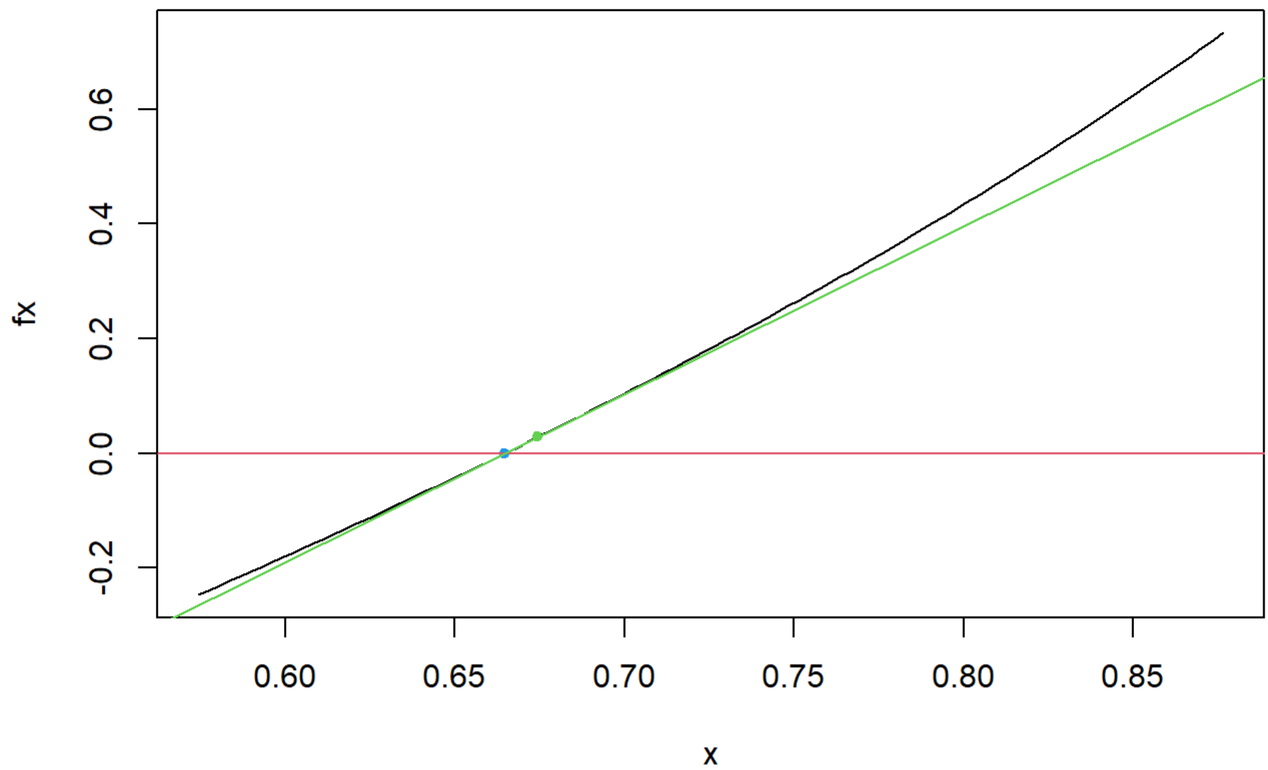
```
this.x <- new.x
this.y <- my.fx(this.x)

this.dfxdx <- my.dfxdx(this.x)
this.intercept <- this.y -this.x*this.dfxdx

new.x <- this.x - this.y/this.dfxdx

plot(x,fx,type="l")
points(this.x,this.y,pch=20,col=3)
points(new.x,0,pch=20,col=4)
abline(h=0,col=2)
abline(this.intercept,this.dfxdx,col=3)
```



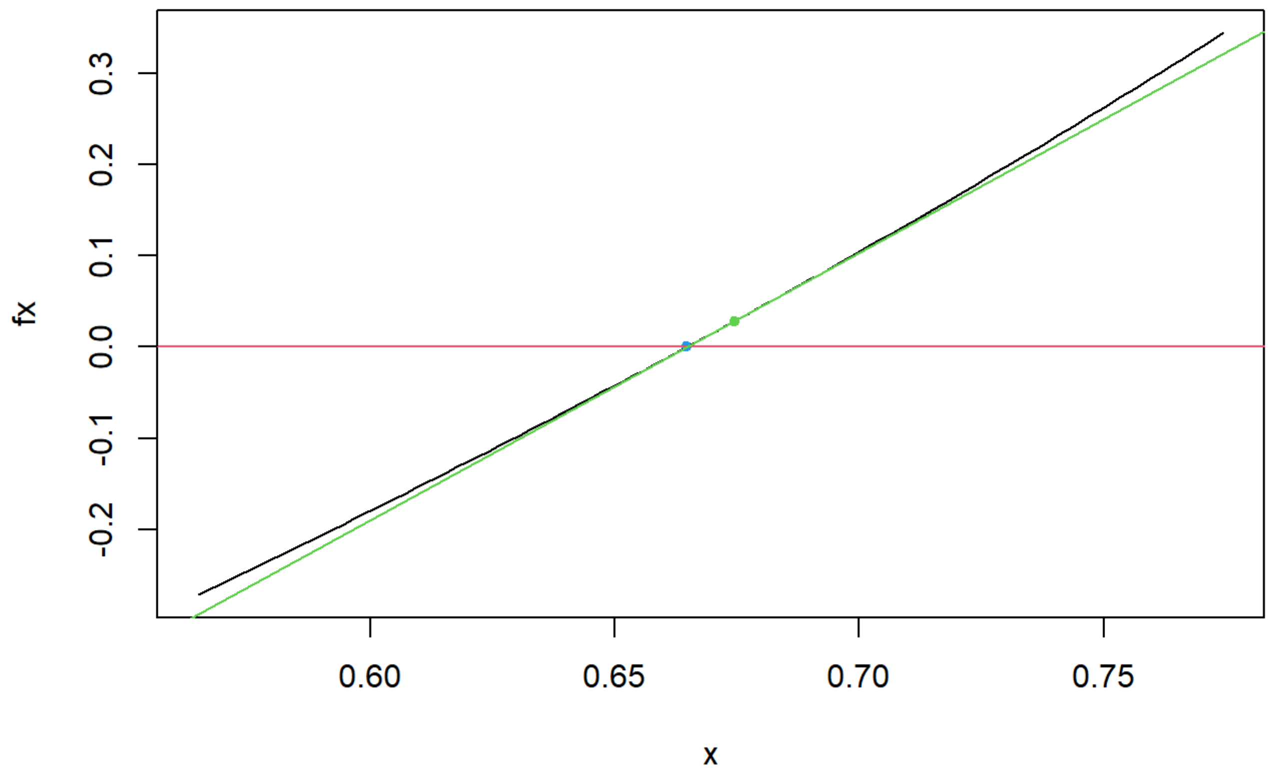

```
x <- seq(from = new.x -0.1,to=this.x+0.1,length=100)
fx <- my.fx(x)
plot(x,fx,type="l")
points(this.x,this.y,pch=20,col=3)
points(new.x,0,pch=20,col=4)
abline(h=0,col=2)
abline(this.intercept,this.dfxdx,col=3)
```




```
this.x <- new.x
this.y <- my.fx(this.x)

this.dfxdx <- my.dfxdx(this.x)
this.intercept <- this.y -this.x*this.dfxdx

new.x <- this.x - this.y/this.dfxdx

plot(x,fx,type="l")
points(this.x,this.y,pch=20,col=3)
points(new.x,0,pch=20,col=4)
abline(h=0,col=2)
abline(this.intercept,this.dfxdx,col=3)
```




```
x <- seq(from = new.x -0.1,to=this.x+0.1,length=100)
fx <- my.fx(x)
plot(x,fx,type="l")
points(this.x,this.y,pch=20,col=3)
points(new.x,0,pch=20,col=4)
abline(h=0,col=2)
abline(this.intercept,this.dfxdx,col=3)
```



fx
0.3
0.2
0.1
0.0
-0.1
-0.2
0.60
0.65
0.70
0.75
x


``` }
```