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Appendix B from S. Peischl et al., "Expansion Load and the Evolutionary Dynamics of a Species Range" (Am. Nat., vol. 185, no. 4, p. 000)

Dynamics of Mean Fitness on the Wave Front

Here we derive equation (3) in the main text. We use here a continuous distribution of fitness effects such that mutations of effect s occur at rate $u(s)$, and the total mutation rate is $u = \int_{-1}^{\infty} u(s)\mathrm{d}s$. Results for discrete distribution of fitness effects can be obtained by replacing the integrals by sums.

We assume that the per locus mutation rate $u/n \ll 1/N$, such that we can ignore the occurrence of multiple mutations at the same locus within a single generation. Then, at any given locus, the probability that a new mutation with effect s enters the population in deme $d_\mathrm{f}(t)$ at generation τ is $2N_\mathrm{f}(\tau)u(s)/n$. We further assume that mutations are either fixed or lost within a single generation after they appear with probability $p(s, \tau)$ and $1 - p(s, \tau)$, respectively. The probability that a mutation with effect s enters the population in generation τ and becomes fixed is then

$$P(s, \tau) = \frac{2N_\mathrm{f}(\tau)p(s, \tau)u(s)}{n}.$$

Mean fitness $\bar{w}$ then evolves according to the following stochastic recurrence relation:

$$\bar{w}_\mathrm{f}(t+1) = \bar{w}_\mathrm{f}(t)\prod_{i=0}^{n}[1 + S_i(\tau)],$$

where the S_i's are a family of independent and identically distributed random variables that take the value $2s + s^2$ with probability $P(s, \tau)$ and 0 with probability $1 - \int_{-1}^{\infty} P(s, \tau)\mathrm{d}s$. We next calculate the expected value of $\bar{w}_\mathrm{f}(t+1)$ conditioned on $\bar{w}_\mathrm{f}(t)$:

$$E[\bar{w}_\mathrm{f}(t+1)|\bar{w}_\mathrm{f}(t)] = \bar{w}_\mathrm{f}(t)[1 + \sigma(\bar{w}_\mathrm{f}(t), \tau)]^n,$$

where

$$\sigma[\bar{w}_\mathrm{f}(t), \tau] = E[S(\tau)|\bar{w}_\mathrm{f}(t)] = \int_{-1}^{\infty}(2s + s^2)P(s, \tau)\mathrm{d}s$$

is the expected per locus relative change in fitness per generation on the wave front.

We can then approximate the evolution of the mean fitness on the wave front with the recurrence relation

$$\bar{w}_\mathrm{f}(t+1) \approx [1 + \sigma(\bar{w}_\mathrm{f}(t))]^n \bar{w}_\mathrm{f}(t),$$

where we omit the dependence on τ (see also eq. [C2]). Using $\bar{w}_\mathrm{f}(0) = 1$, this is equivalent to equation (3) in the main text.