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Appendix D from S. Peischl et al., "Expansion Load and the Evolutionary Dynamics of a Species Range" (Am. Nat., vol. 185, no. 4, p. 000)

Effect of Parameters on Equilibrium Mean Fitness

We study here the dependence of the equilibrium mean fitness on the wave front on the other parameters. The mean fitness at equilibrium is monotonically increasing or decreasing with respect to a parameter if the partial derivative with respect to that parameter is positive or negative, respectively. The partial derivatives of $\tilde{w}_{\mathrm{f}}$ with respect to the parameters of the model are given by

$$
\begin{aligned}
\frac{\partial \tilde{\bar{w}}_{\mathrm{f}}}{\partial R_0} &= -\frac{8F_0 s \log(K_0/F_0)}{R_0^3 \log[\varphi_{\mathrm{d}}/(1-\varphi_{\mathrm{d}})]} - \frac{1}{R_0^2} < 0,\\
\frac{\partial \tilde{\bar{w}}_{\mathrm{f}}}{\partial \varphi_{\mathrm{d}}} &= -\frac{4F_0 s \log(K_0/F_0)}{R_0^2(1-\varphi_{\mathrm{d}})\varphi_{\mathrm{d}} \log[\varphi_{\mathrm{d}}/(1-\varphi_{\mathrm{d}})]^2} < 0,\\
\frac{\partial \tilde{\bar{w}}_{\mathrm{f}}}{\partial s} &= \frac{4F_0 \log(K_0/F_0)}{R_0^2 \log[\varphi_{\mathrm{d}}/(1-\varphi_{\mathrm{d}})]} > 0,\\
\frac{\partial \tilde{\bar{w}}_{\mathrm{f}}}{\partial F_0} &= \frac{4F_0 s[\log(K_0/F_0)-1]}{R_0^2 \log[\varphi_{\mathrm{d}}/(1-\varphi_{\mathrm{d}})]} > 0 \ \text{ if } \ F_0 \ll K_0,\\
\frac{\partial \tilde{\bar{w}}_{\mathrm{f}}}{\partial K_0} &= \frac{4F_0 s}{R_0^2 K_0 \log[\varphi_{\mathrm{d}}/(1-\varphi_{\mathrm{d}})]} > 0.
\end{aligned}
$$

Note that we can write $\sigma = u[\int_{-1}^{\infty}(2s - s^2)\varphi(s)N(\tau)p(s,\tau)\mathrm{d}s]$, where $\varphi(s)$ denotes the probability that a randomly chosen mutation has effect s and u is the total nonneutral mutation rate. The total mutation rate u thus affects mainly the rate of convergence to equilibrium. We cannot exclude the possibility that large u may lead to limit cycles or more complex behavior, but note that this was never observed in our numerical investigations. Furthermore, if selection is weak, we can approximate the difference equation for the evolution of mean fitness by an ordinary differential equation (see weak selection approximation in, e.g., Nagylaki 2012), which excludes the possibility of cyclic—or more complex—behavior.