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Appendix C from S. Peischl et al., “Expansion Load and the Evolutionary Dynamics of a Species Range” (Am. Nat., vol. 185, no. 4, p. 000)

Mean Fitness at Equilibrium

The probability that a mutation appears in a deme of age τ and then goes to fixation is

$$P(s,\tau) = u(s)2N(\tau)\frac{\exp\left[-4FTsx_0(\tau)\right]-1}{\exp\left[-4FTs\right]-1}, \tag{C1}$$

where

$$x_0(\tau) = \frac{\{1/[2N(\tau)]\}(1+s)^{T-\tau}}{\{1/[2N(\tau)]\}(1+s)^{T-\tau}+1-\{1/[2N(\tau)]\}}.$$

Developing the arguments of the exponentials in equation (C1) into Taylor series around $s = 0$ and ignoring second- and higher-order terms in s yield

$$P(s,\tau) \approx u(s)2N(\tau)\frac{\exp\{-4FTs/[2N(\tau)]\}-1}{\exp(-4FTs)-1}.$$

Using

$$\exp\left[-\frac{4FTs}{2N(\tau)}\right] \approx 1-\frac{4FTs}{2N(\tau)},$$

it follows that

$$P(s,\tau) \approx u(s)\frac{4FTs}{1-\exp(-4FTs)} = P(s) \tag{C2}$$

and that $P(s, \tau)$ depends only weakly on τ. We therefore ignore the dependence of P on τ and use equation (C2). Assuming that mutations have symmetric effects $\pm s$, we obtain

$$\sigma \approx 2s(1-\varphi_d)P(s) - 2s\varphi_d P(-s).$$

Solving $\sigma = 0$ for φ_d reveals that the critical fraction φ_c of deleterious mutations is given by

$$\varphi_c = \frac{1}{1+\exp(-4FTs)}, \tag{C3}$$

which corresponds to equation (4) in the main text.

The mean fitness at equilibrium can now be calculated by using

$$T \approx \frac{\log(K_0/F_0)}{\log(R_0\bar{w}_f)},$$

$$F = F_0\bar{w}_f,$$

where F_0 is the number of founders if $\bar{w}_f = 1$. Solving equation (C3) for $\bar{w}_f$ then yields

$$\tilde{\bar{w}}_f \approx -\frac{\log[1/(1-\varphi_d)]W(-[4F_0 s\log(K_0/F_0)]/\{R_0\log[1/(1-\varphi_d)]\})}{4F_0 s\log(K_0/F_0)}, \tag{C4}$$

where $W(x)$ is the product logarithm of x. Developing equation (C4) into a Taylor series in s around $s = 0$ leads to equation (5) in the main text.