Solidification Analysis - 1

Outline

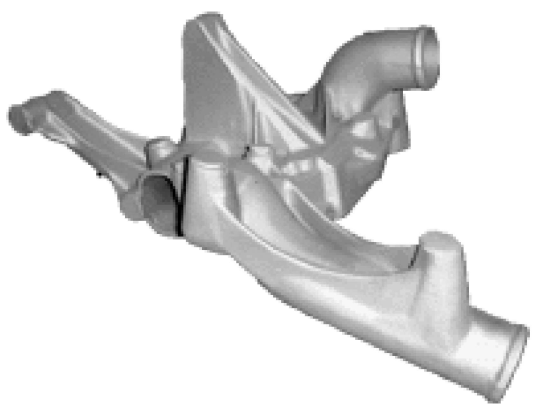
- Casting basics
- Patterns and molds
- Melting and pouring analysis
- Solidification analysis
- Casting defects and remedies

Casting Basics

- A casting is a metal object obtained by *pouring molten* metal into a *mold* and allowing it to *solidify*.

Aluminum manifold

Gearbox casting



Magnesium casting

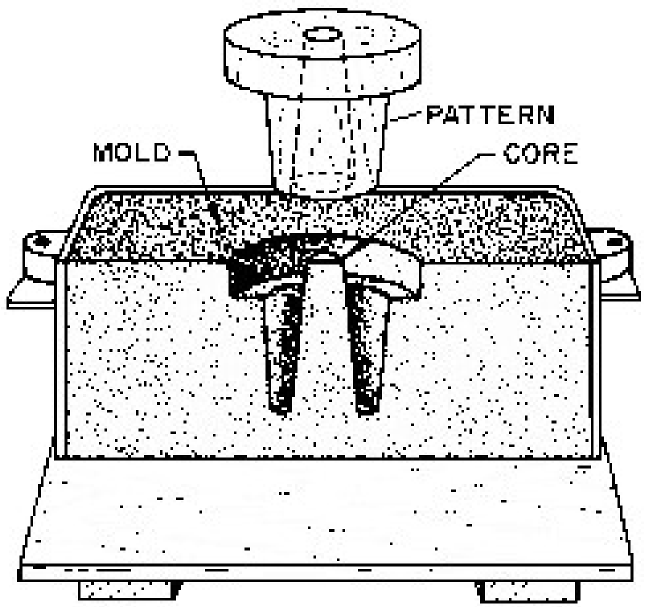
Cast wheel

Casting: Basic Steps

- Basic steps in casting are:
  - Preparation of pattern(s), core(s) and mold(s)
  - Melting and pouring of liquefied metal
  - Solidification and cooling to room temperature
  - Removal of casting - shakeout
  - Inspection (for possible defects)

Pattern Making

- Pattern is a replica of the exterior surface of part to be cast – used to create the mold cavity
- Pattern materials – wood, metal, plaster



Pattern Making

- Pattern usually larger than cast part
  Allowances made for:
  - ***Shrinkage***: to compensate for metal shrinkage during cooling from freezing to room temp

    Shrinkage allowance = $\alpha L(T_f - T_0)$

    expressed as *per unit length* for a given material

α = coeff. of thermal expansion, T_f = freezing temp

T_0= room temp

e.g. Cast iron allowance = 1/96 in./ft

aluminum allowance = 3/192 in./ft

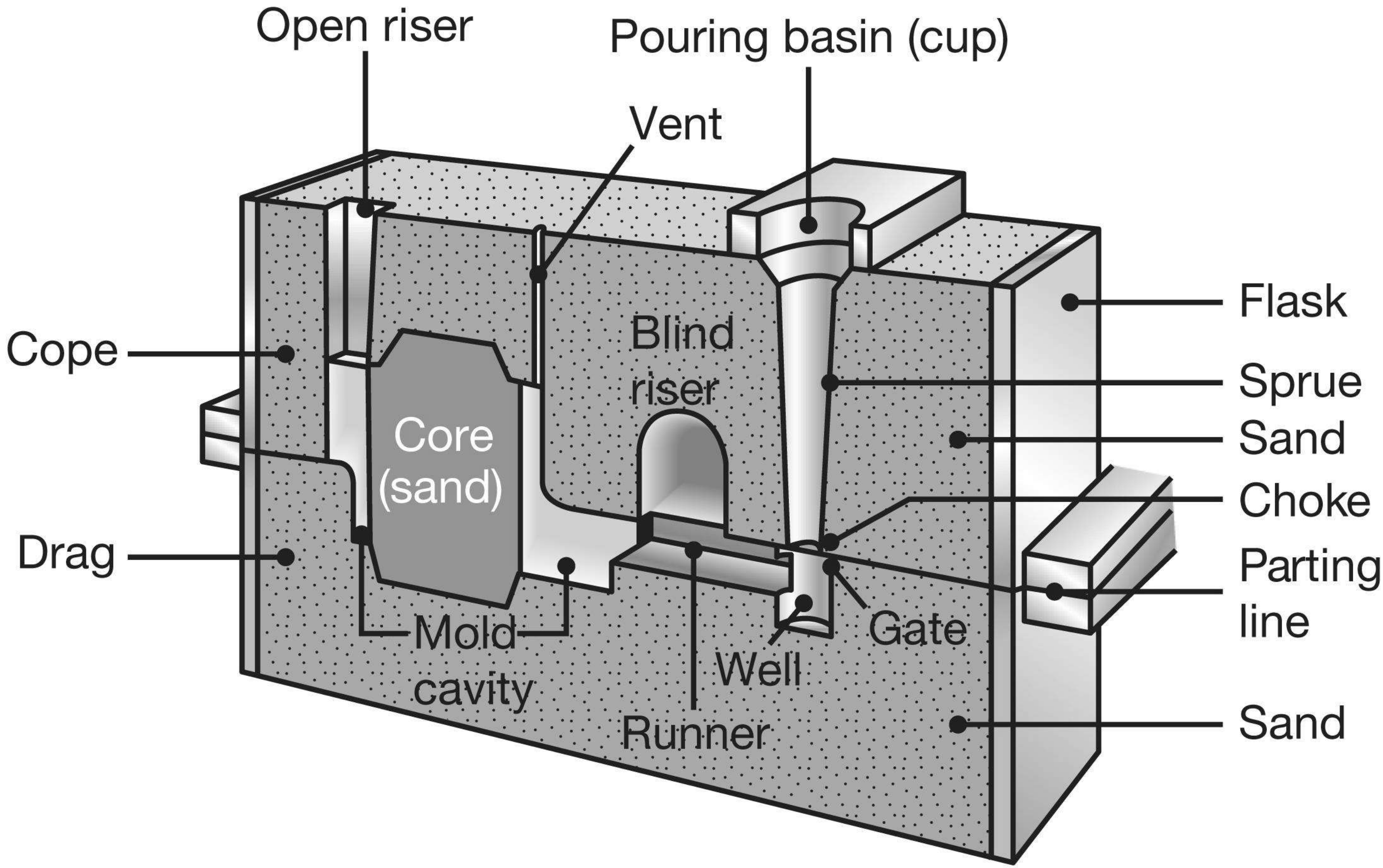
Pattern Making

- Pattern allowances made for:
  - ***Machining***: excess dimension that is removed by machining; depends on part dimension and material to be cast

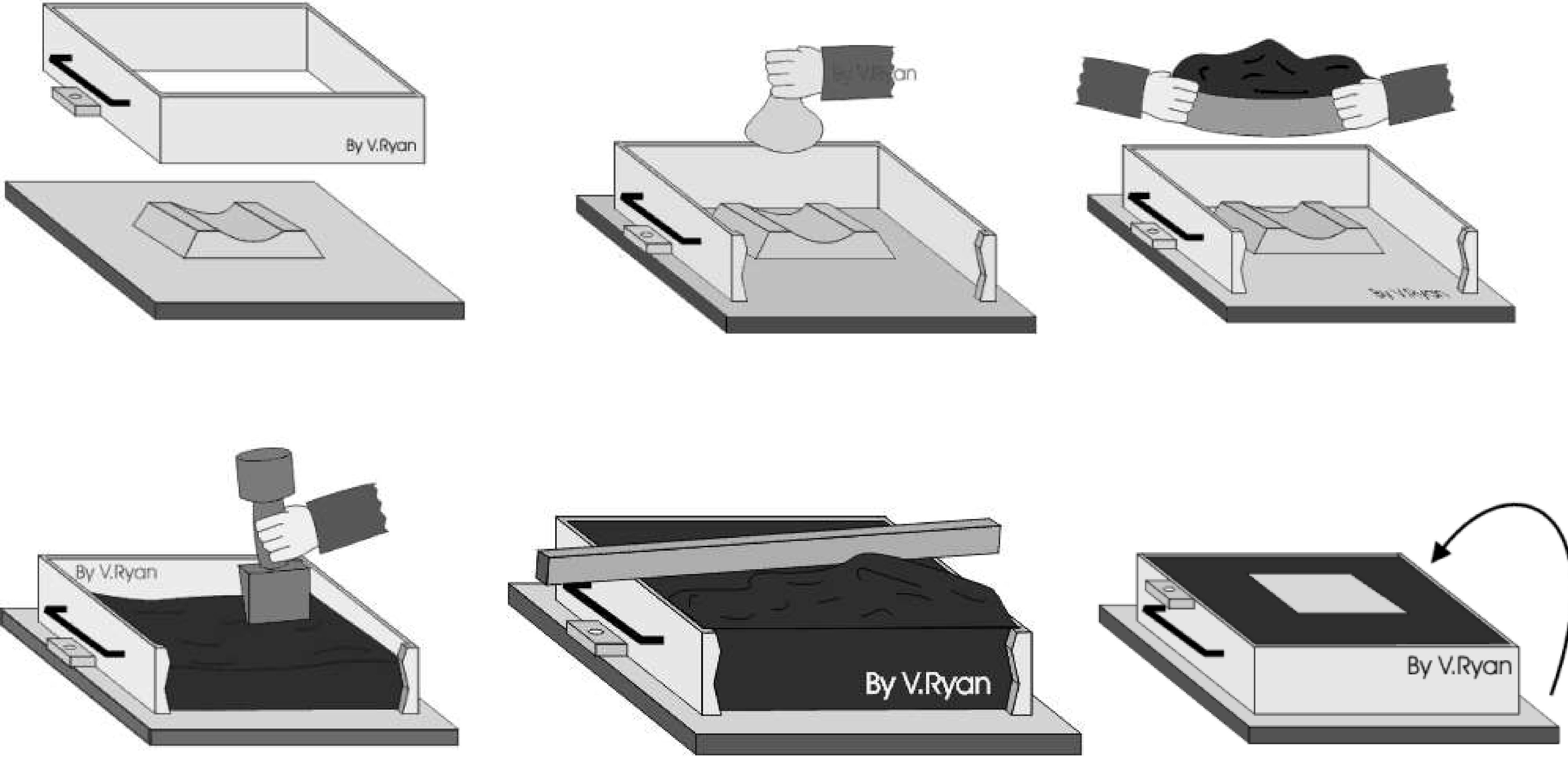
    e.g. cast iron, dimension 0-30 cm, allowance = 2.5 mm; aluminum, allowance = 1.5 mm

  - ***Draft***: taper on side of pattern parallel to direction of extraction from mold; for ease of pattern extraction; typically 0.5~2 degrees

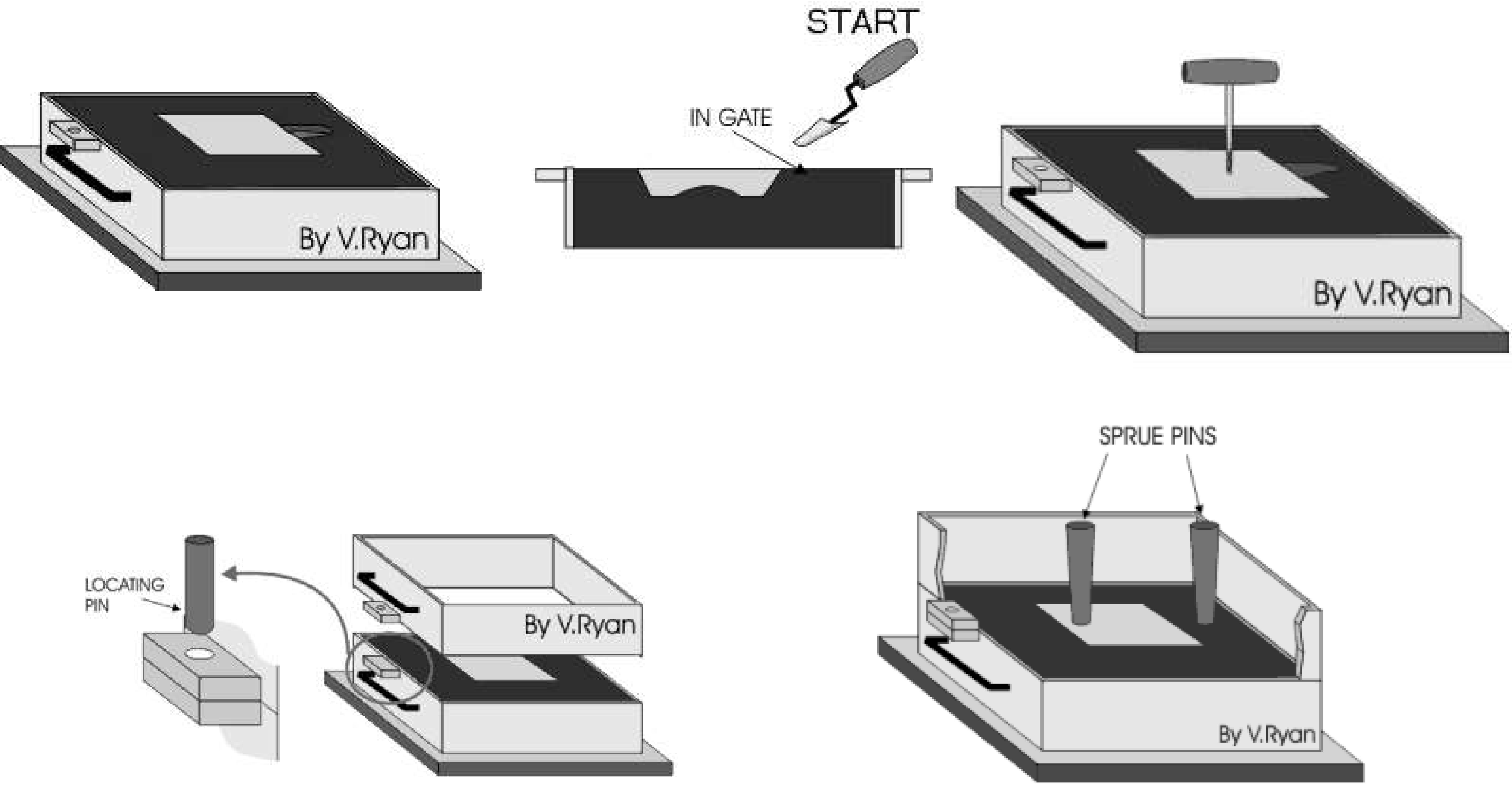
(Green) Sand Mold



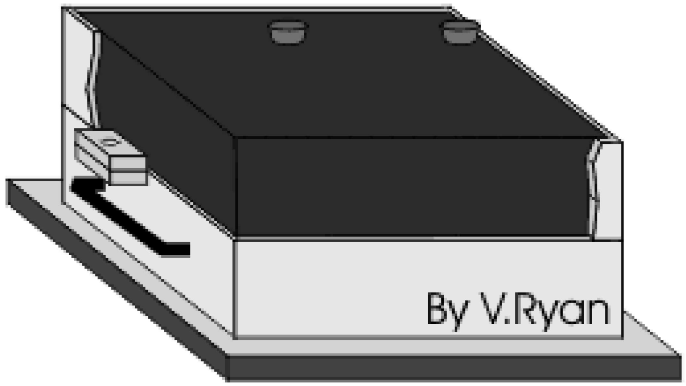
Mold Making: Sand Casting

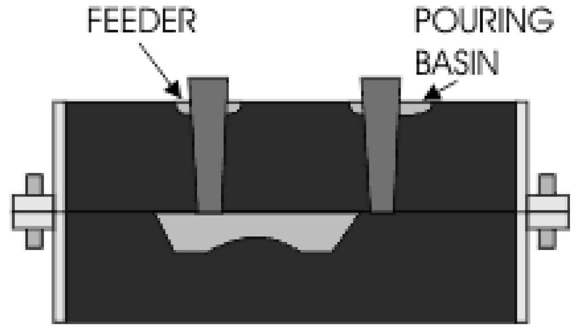


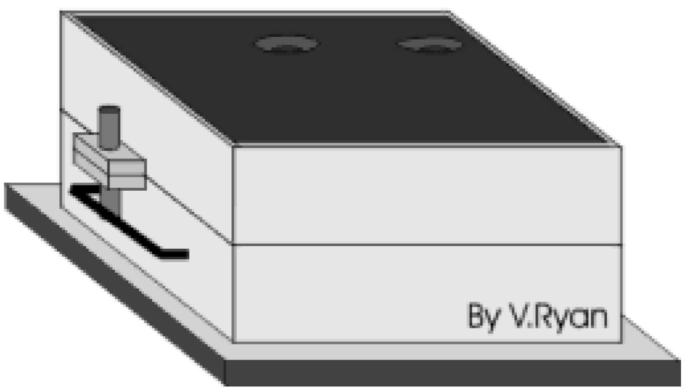
Mold Making: Sand Casting

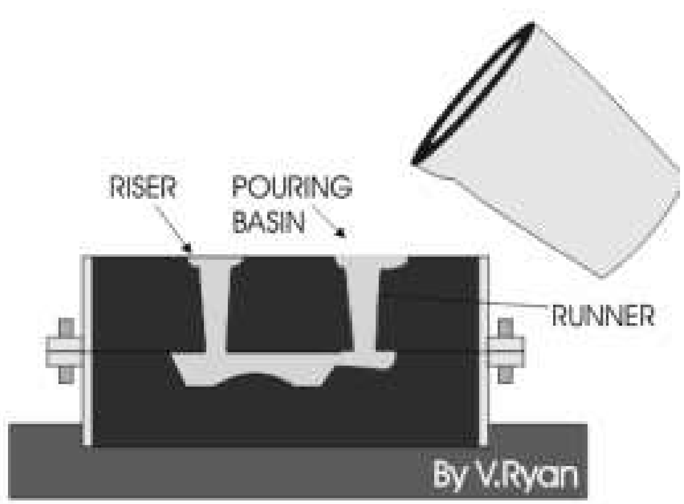


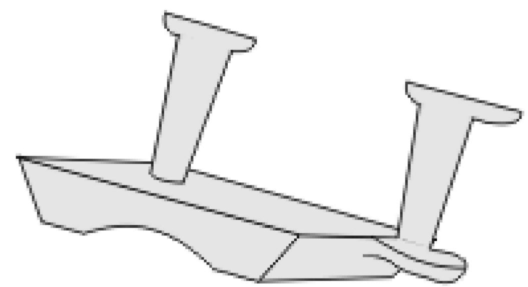
Mold Making: Sand Casting











Sand Casting

- Green sand mold:

  sand + clay + water + additives
- Typical composition (by wt.):
  - 70-85% sand, 10-20% clay, 3-6% water, 1-6% additives
- Important properties of molding sand:
  - Strength
  - Permeability
  - Deformation
  - Flowability
  - Refractoriness

Melting

- For a pure metal:

  total heat energy required, H =

  energy to raise temp of metal to melting point, T_m + heat of fusion, H_f + energy to raise temp of liquid metal to pouring temp, T_p

$$H = \rho V\,[c_s(T_m - T_0) + H_f + c_l(T_p - T_m)]$$

- Heat required for alloys more complex
- Gas fired, electric arc and induction furnaces used to melt metal

Melting

- Solubility of gases (hydrogen and nitrogen) in molten metal an issue
- Solubility of H_2, *S*:

$$S = C \, exp \, [-E_s/(k\theta)]$$

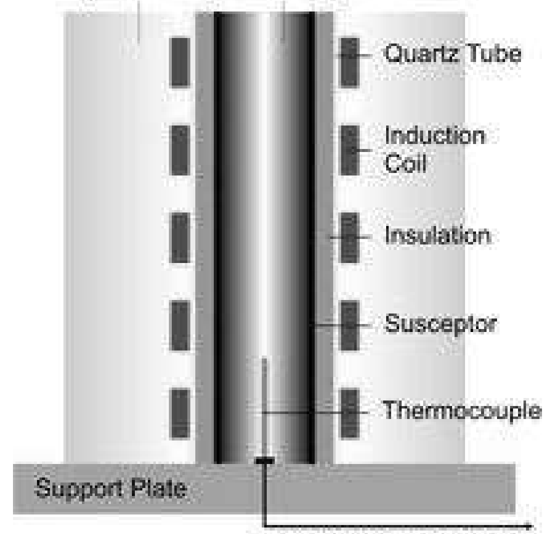
E_s = heat of solution of 1 mol of H_2

θ = absolute temp, *C* and *k* are constants

e.g. 1 atm pressure, liquid solubility of H_2 in iron = 270 cc/kg; in aluminum = 7 cc/kg

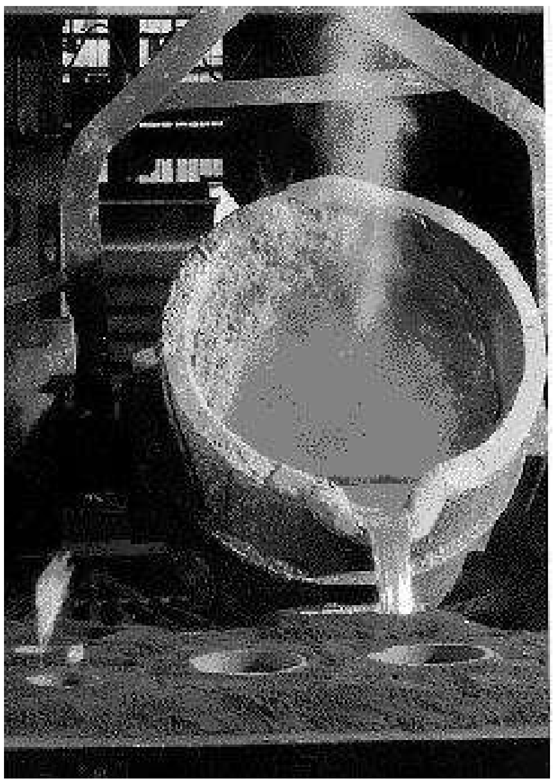
Melting Furnaces

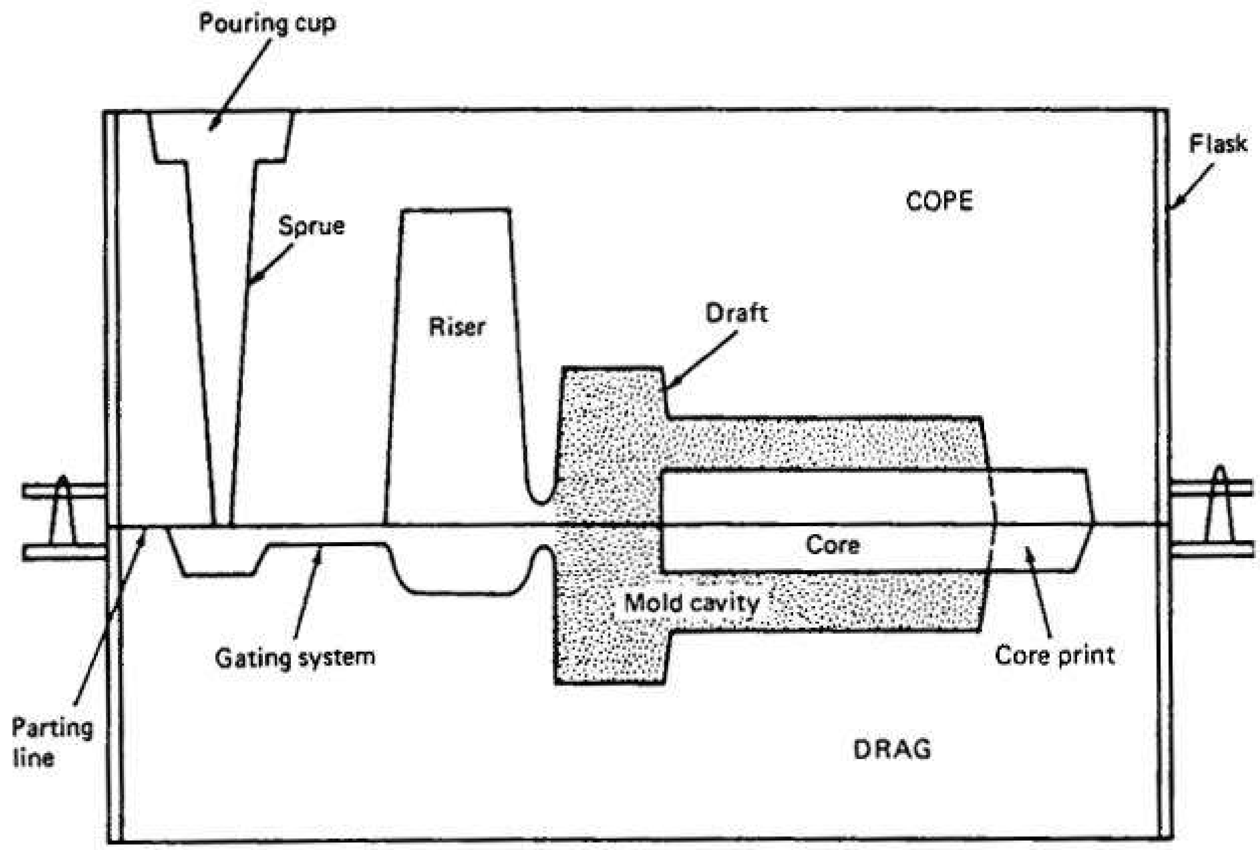
Induction Heating



Pouring

- An important step in casting since it impacts mold filling ability and casting defects





Pouring

- Key aspects of pouring
  - Pouring rate
    - Too slow → metal freezes before complete mold filling
    - Too fast → inclusion of slag, aspiration of gas, etc.
    - Reynolds number: Laminar versus turbulent flow

$$\mathrm{Re} = \frac{\rho V D}{\eta}$$

ρ = density of liquid, V = mean flow velocity, D = tube diameter, η = dynamic viscosity of liquid

    - Most steels reach mildly turbulent flow conditions easily (Re > 3500)
  - Superheat ~ $(T_p - T_m)$; T_p = pouring temp
    - Too high → increased gas solubility → porosity problems

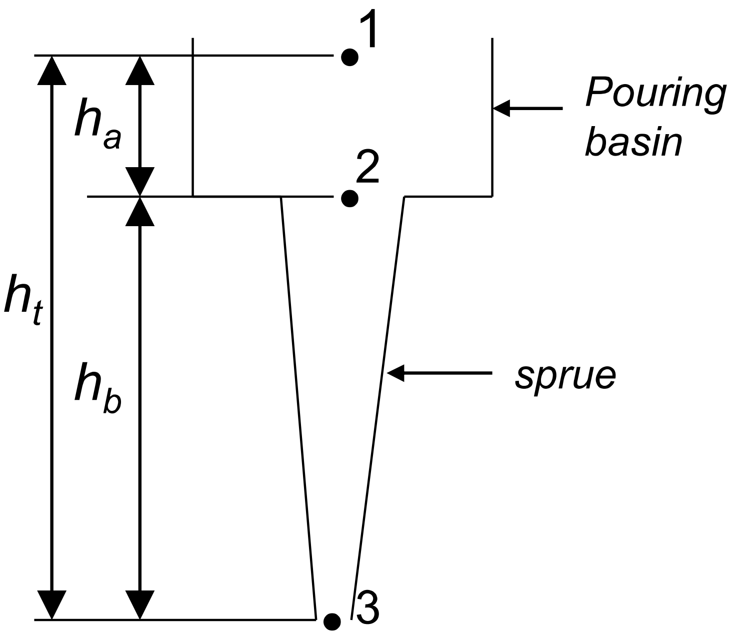
Pouring Analysis (Sprue/Gating Design)

- Fluid flow in sprue/gating/mold can be analyzed using energy balance i.e. Bernoulli's theorem

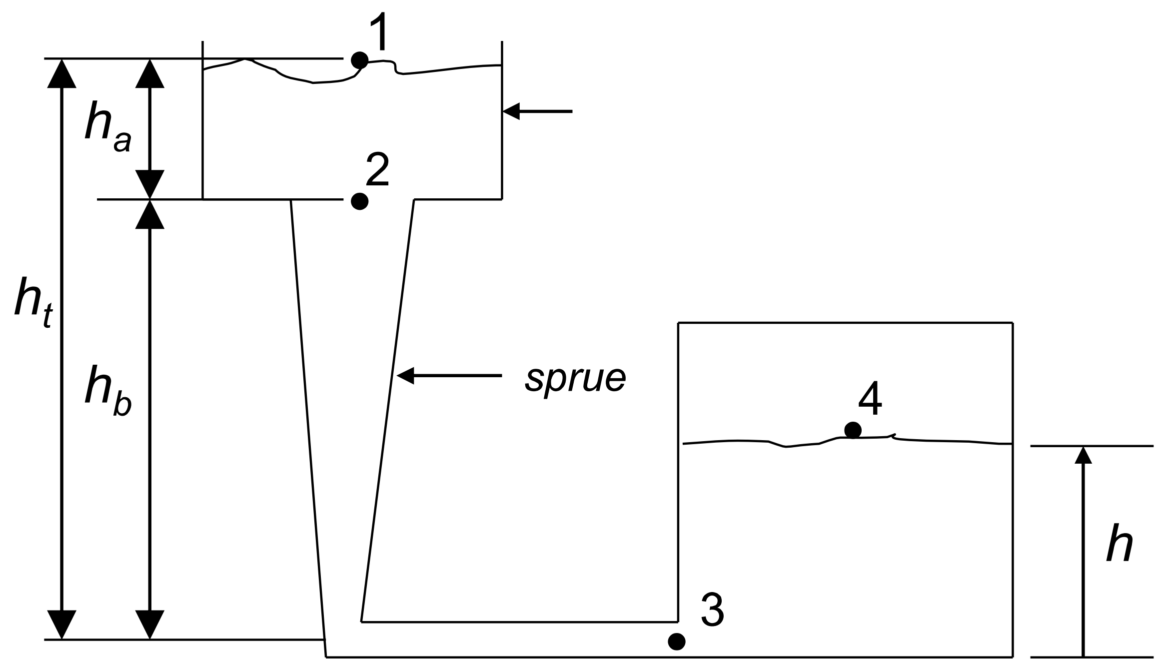
$$h_1 + \frac{V_1^2}{2g} + \frac{P_1}{\rho g} + F = const.$$

- Assumptions of analysis
  - Incompressible fluid
  - Negligible frictional losses
  - Entire mold is at atmospheric pressure

Pouring Analysis (Sprue/Gating Design)



Top Gated Mold

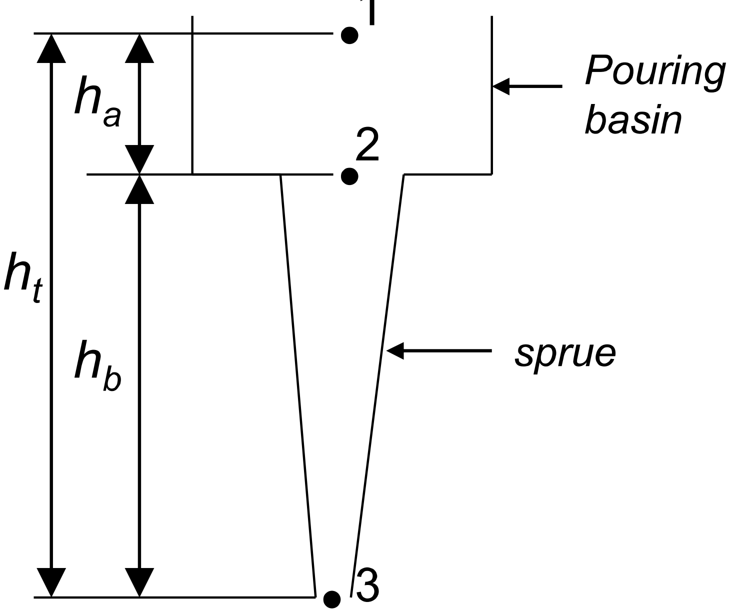


Bottom Gated Mold

- Design of sprue and gating system (runners + gates) based on Bernoulli's theorem

Pouring Analysis (Sprue/Gating Design)

- Applying energy balance between points 1 and 3



Top Gated Mold

$$h_1 + \frac{V_1^2}{2g} + \frac{P_1}{\rho g} = h_3 + \frac{V_3^2}{2g} + \frac{P_3}{\rho g}$$

Assuming entire mold is at atmospheric pressure and velocity of melt at point 1 ~ 0

$$V_3 \approx \sqrt{2gh_t}$$

Pouring Analysis (Sprue Design)

- Consider the geometry of freely falling liquid from the pouring basin; also assume permeable walls (e.g. sand mold)

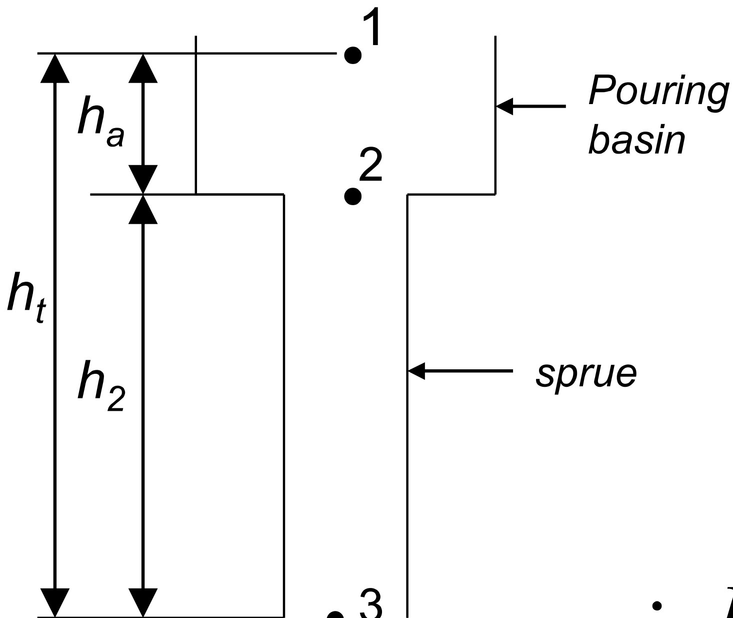
$$V_2 \approx \sqrt{2gh_a} \qquad V_3 \approx \sqrt{2gh_t}$$

- Assuming continuity of fluid flow, flow rate at point 2 = flow rate at point 3:

$$A_2 V_2 = A_3 V_3 \Rightarrow \boxed{\frac{A_3}{A_2} = \frac{V_2}{V_3} = \sqrt{\frac{h_a}{h_t}}}$$

Aspiration Effect – Sprue Design

- For permeable molds (e.g. sand), the pressure at any point in the liquid metal flow $\geq$ atmospheric pressure (Why?)



Straight Sprue

$$P_1 = P_3 = Atmospheric$$

$$P_2 + \frac{\rho v_2^2}{2} + \rho g h_2 = P_3 + \frac{\rho v_3^2}{2}$$

$$P_2 = P_3 - \rho g h_2 \quad (\text{since } v_2 = v_3)$$

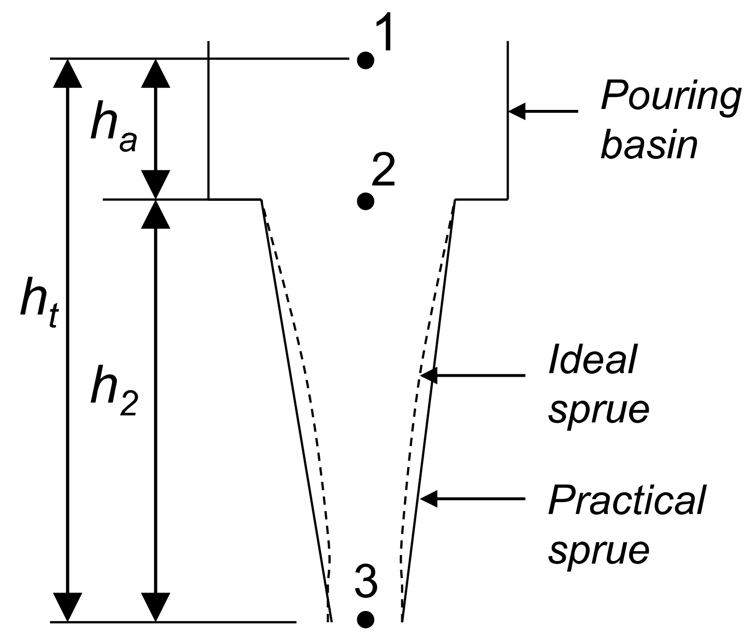
$$\therefore P_2 < Atmospheric \Rightarrow Aspiration!$$

Sprue Design

- To prevent aspiration, ideally,

$$P_2 = P_1 = P_3 = Atmospheric$$

$$\frac{A_3}{A_2} = \frac{v_2}{v_3} = \sqrt{\frac{h_a}{h_t}}$$



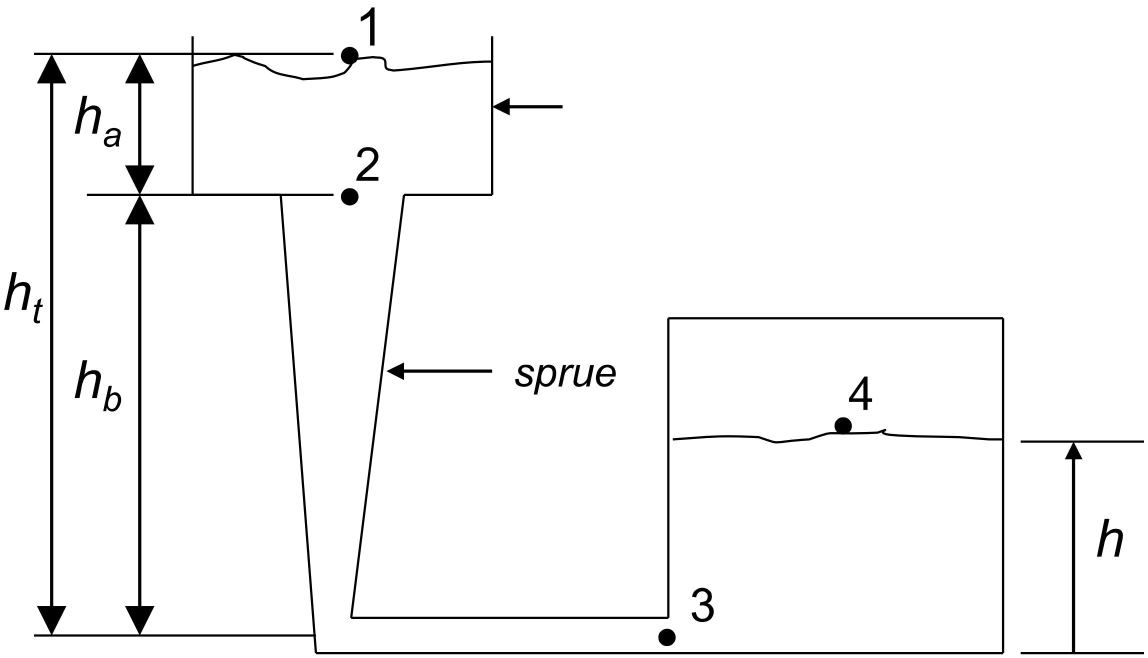
- In practice, sprue is tapered to prevent aspiration of gases into the (permeable) mold → minimizes porosity

Pouring Analysis (Sprue/Gating Design)

- As metal is poured into mold, the effective "head" decreases

- Velocity of metal at point 3:

$$V_3 \approx \sqrt{2g(h_t - h)}$$



Bottom Gated Mold

Mold Filling Analysis

- <u>Bottom gated mold</u>: In time *dt* increase in volume of metal in mold = $A_m dh$, where A_m = cross-section of the mold cavity
- Volumetric flow rate of metal delivered to mold at point 3 (gate) = $A_3 V_3$
- Volume balance at point 3:

$$A_m dh = A_3 \sqrt{2g(h_t - h)}\,dt$$

- Mold filling time, t_f

$$\frac{1}{\sqrt{2g}} \int_0^{h_m} \frac{dh}{\sqrt{h_t - h}} = \frac{A_3}{A_m} \int_0^{t_f} dt \Rightarrow \boxed{t_f = \frac{2A_m}{A_3\sqrt{2g}}\left(\sqrt{h_t} - \sqrt{h_t - h_m}\right)}$$

Mold Filling Analysis

- Mold filling time, t_f

$$\frac{1}{\sqrt{2g}}\int_0^{h_m}\frac{dh}{\sqrt{h_t-h}}=\frac{A_3}{A_m}\int_0^{t_f}dt \Rightarrow \boxed{t_f=\frac{2A_m}{A_3\sqrt{2g}}\left(\sqrt{h_t}-\sqrt{h_t-h_m}\right)}$$

- Mold filling time for <u>top gated mold</u>

$$t_f=\frac{Mold\ \ Volume}{Flow\ Rate}=\frac{A_m h_m}{A_g V_g}$$

- Above calculations represent the <u>minimum</u> time necessary

Example Problem 1

Given a top gated mold with the following:

Sprue height, h_t = 20 cm

Cross-section of sprue base, A_3 = 2.5 cm^2

Volume of mold cavity, V = 1560 cm^3

Find:

a) Flow velocity at sprue base

b) Flow rate of metal into mold cavity

c) Mold filling time

Example Problem 1 (contd)

Solution:

a) Velocity at sprue base

$$V_3 = \sqrt{2gh_t} = \sqrt{2(981)(20)} = 198.1 \text{ cm/s}$$

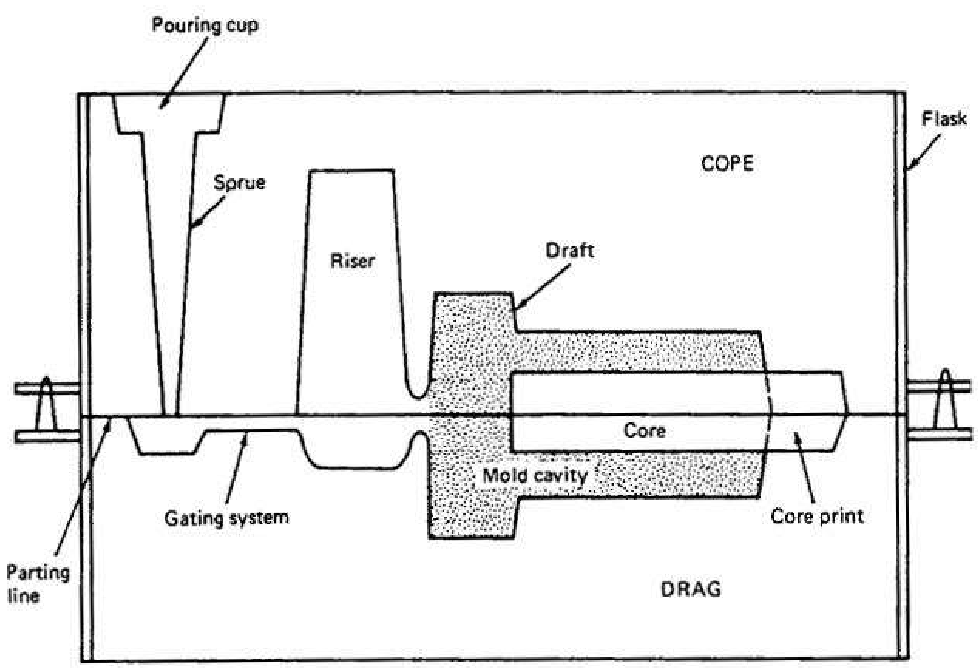
b) Flow rate, $Q = A_3 V_3 =$ $(2.5)(198.1) = 495 \text{ cm}^3/\text{s}$

c) Mold filling time

$$t_f = \frac{1560}{495} = 3.2 \text{ s}$$

Example Problem 2

- Consider the sand mold shown below. You wish to pour molten iron so that the flow into the mold cavity is not very turbulent. Determine the diameter of the gate for the given problem data.



Example 2 (contd.)

Data for problem:

- Iron data:
  - density = 7860 kg/m^3
  - viscosity at pouring temp. = 2.25×10^{-3} $N.s/m^2$
- Sprue height (including pouring cup) = 6 in. or 0.15 m
- Diameter of sprue base = 0.5 in. or 12.7 mm
- Assume that the runners and gates are uniform in cross-section; ignore riser

Example 2 (contd.)

- Iron data:
  - density (ρ) = 7860 kg/m^3
  - viscosity at pouring temp (μ) = 2.25 cp = 2.25 x 10^{-3} N*s/m^2
- h_1 = 6 in. = 0.15 m
- h_3 = 0 m
- g = 9.8 m/s^2
- $A_1 = \pi r^2 = 3.14 * 0.00635^2 =$

  1.27×10^{-4} m^2

Example 2 Solution

- Solved in class

Flow Velocity Distribution

- Actual metal flow in mold has non-uniform velocity distribution
  - zero at walls, max. at center

- Depends on conduit shape and nature of flow (laminar or turbulent)

Flow Velocity Distribution

- Account for velocity distribution by replacing v^2 in Bernoulli's equation by

$$\frac{\bar{V}^2}{\alpha}$$

where $\bar{V}$ = avg. velocity

- For circular conduit,
  α = 0.5 (laminar) or 1 (turbulent)

Energy Losses

- Real metal flow has energy losses due to
  - Wall surface roughness
  - Nature of flow (laminar vs. turbulent)
  - Sudden expansion or contraction of flow, entry/exit (minor losses)

Energy Losses

- Frictional energy (pressure) loss (per unit mass) in a circular channel (Darcy-Weisbach equation)

$$E_1 = f_D \frac{L}{D}\frac{\bar{V}^2}{2}$$

D, L = diameter and length of channel

f_D= Darcy friction factor (depends mostly on the nature of flow)

Energy Losses

- For a smooth conduit,

$$f_D = 64/\text{Re} \qquad \text{Re} < 2000$$

$$f_D = 0.316(\text{Re})^{-0.25} \qquad 2000 < \text{Re} < 10^5$$

Energy Losses

- For a rough conduit (Haaland's equation),

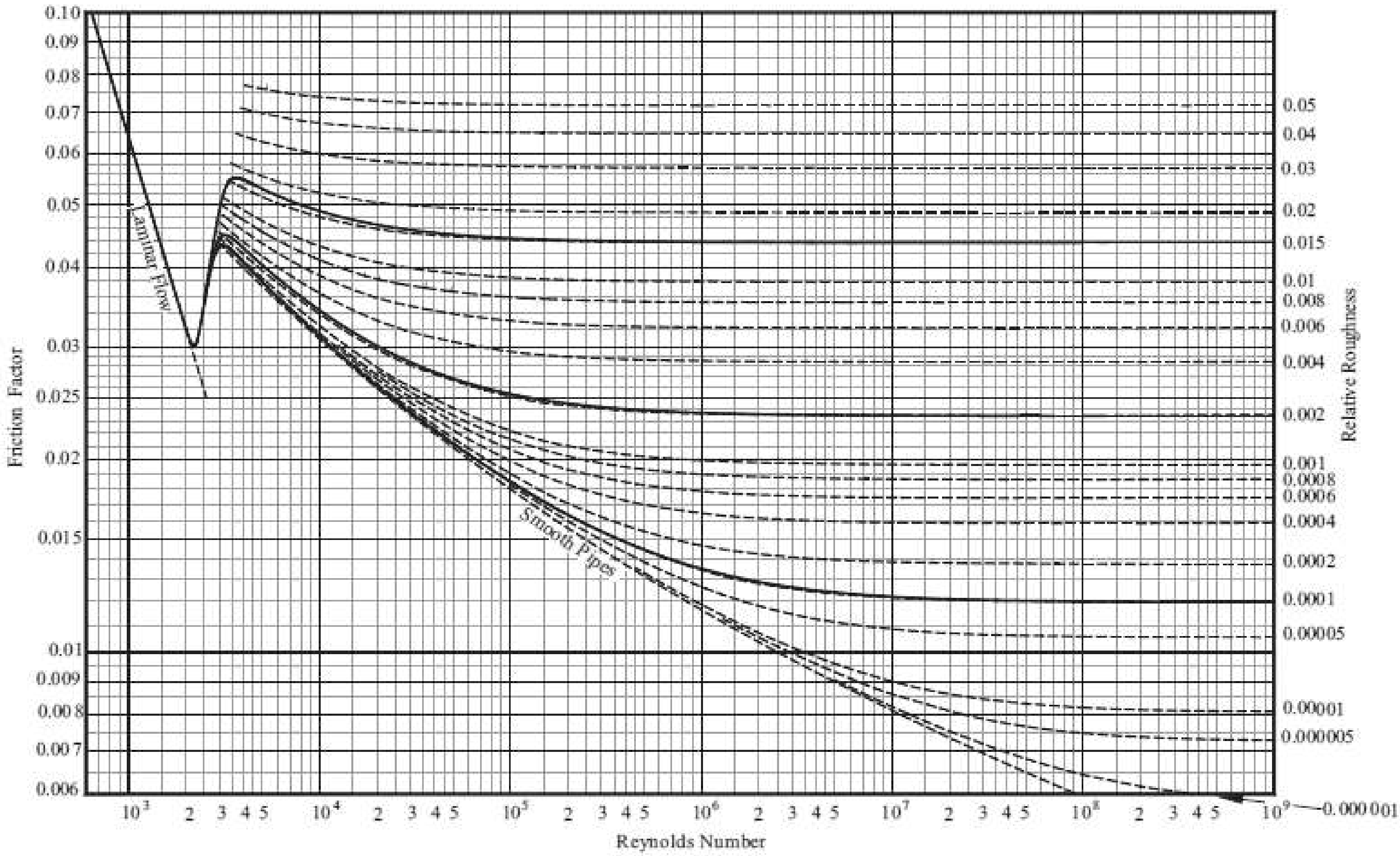
$$f_D = \left\{-1.8\log_{10}\left[\left(\frac{\epsilon/D}{\mathrm{Re}}\right)^{1.11} + \frac{6.9}{\mathrm{Re}}\right]\right\}^{-2}$$, valid for

Re > 2000

Where $\epsilon/D =$ relative roughness, ϵ is the average roughness of the conduit

Note: there are several alternate formulae available in pipe flow handbooks

Friction Factor Chart



- Charts like the above were used historically

(Source: Rennels, D.C. and Hudson, H. M., 2012)

Energy Losses

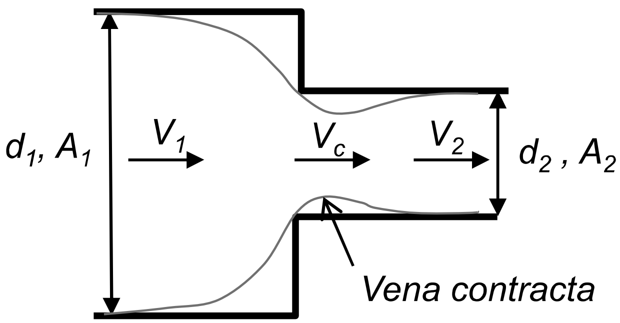
- Energy loss (per unit mass) for expansion or contraction of flow

$$E_2 = e_f \frac{\bar{V}^2}{2}$$

e_f = loss factor, $\bar{V}$ = avg. flow velocity in the smaller cross-section

- Loss factor depends on the ratio of flow areas (A_1/A_2 for expansion; A_2/A_1 for contraction) and the Reynolds number

Loss Factor for Sudden Contraction



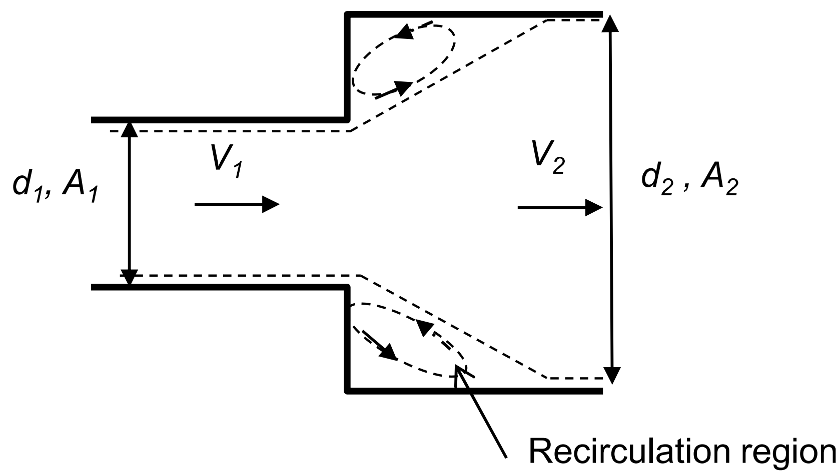
$$e_f = 0.0696(1-\beta^5)\lambda^2 + (\lambda - 1)^2$$

Where $\beta = d_2/d_1 = \sqrt{A_2/A_1}$, and

λ = jet velocity ratio $= 1 + 0.622(1 - 0.215\beta^2 - 0.785\beta^5)$

Note: jet velocity ratio is the inverse of the jet contraction ratio (= A_c/A_2)

Loss Factor for Sudden Expansion



$$e_f = (1 - A_1/A_2)^2$$

Frictional Loss due to Bends

Friction loss due to bends in flow (e.g. 90 deg. bend) can also be accounted for using a equivalent (L/D) ratio

$$E_3 = f_D \left(\frac{L}{D}\right)_{eq} \frac{\bar{V}^2}{2}$$

Energy Balance with Losses

Energy balance between points 1 and 3 in the mold:

$$P_1 + \frac{\rho \bar{V_1}^2}{2\alpha} + \rho g h_1 = P_3 + \frac{\rho \bar{V_3}^2}{2\alpha} + \rho g h_3 + \rho E_1 + \rho E_2 + \rho E_3$$

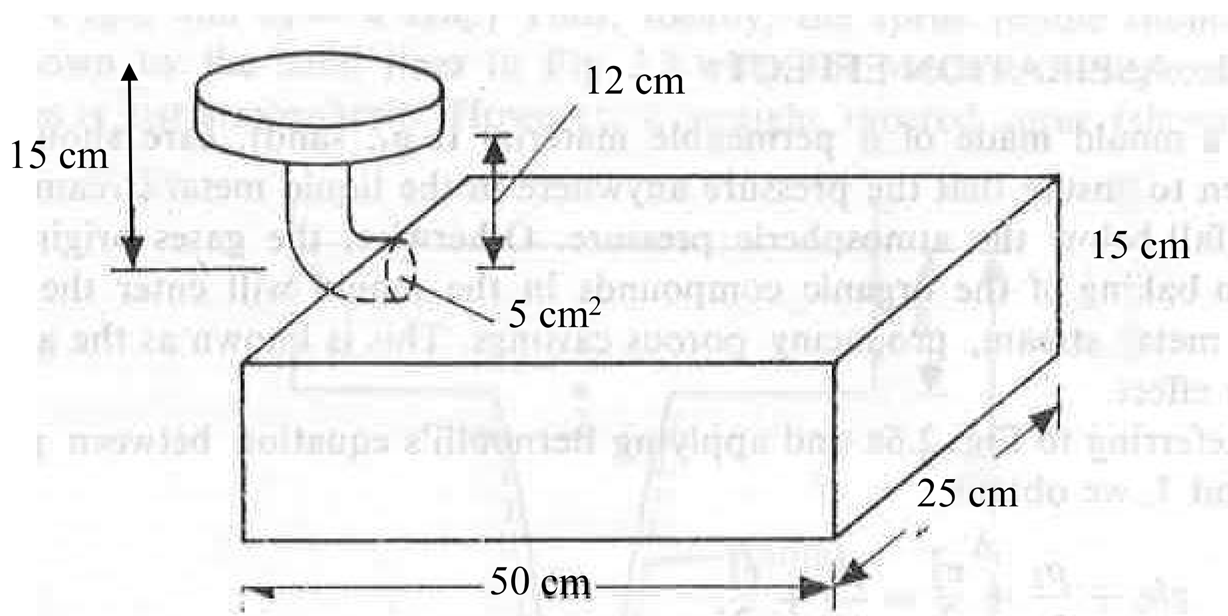
Since $h_1 = h_t$, $h_3 = 0$, and $P_1 = P_3 = P_{atm}$:

$$2gh_t = \bar{V_3}^2 \left\{ \frac{1}{\alpha} + e_f + f_D \frac{L}{D} + f_D \left(\frac{L}{D} \right)_{eq} \right\}$$

$\bar{V}_3 = C_D \sqrt{2gh_t}$ where $C_D = \left\{ \frac{1}{\alpha} + e_f + f_D \frac{L}{D} + f_D \left(\frac{L}{D} \right)_{eq} \right\}^{-0.5}$

Example 3

- Find the filling time for the 50 cm x 25 cm x 15 cm top gated mold shown in the figure: a) without friction, b) with friction. Liquid metal (Fe) properties: density = 7800 kg/m^3, viscosity = 0.00496 kg/m-s. Assume a smooth conduit. For the 90 deg. turn at the end of the sprue, $(L/D)_{eq}$ = 25.

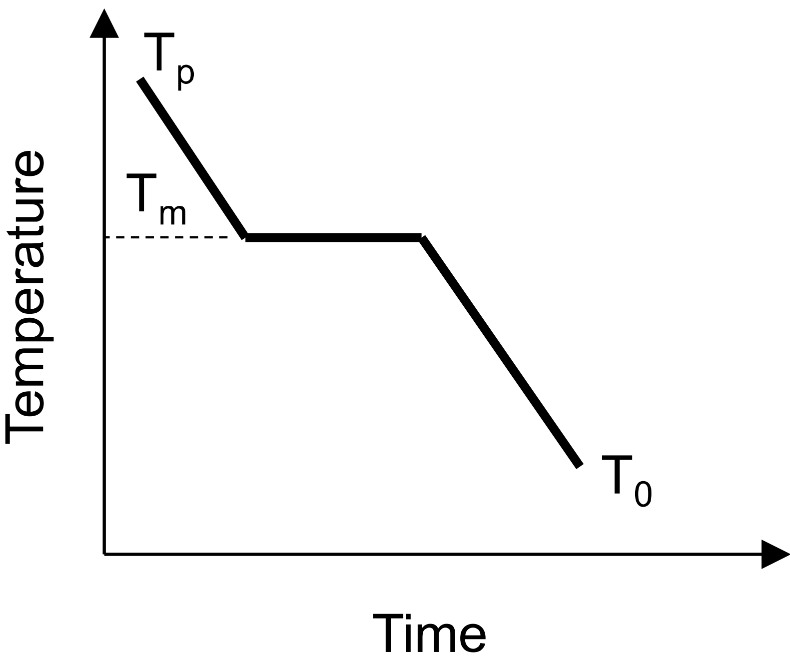


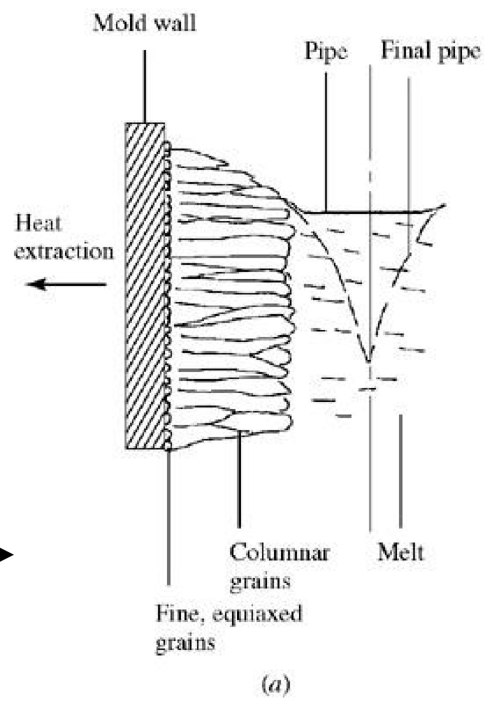
Example 3 Solution

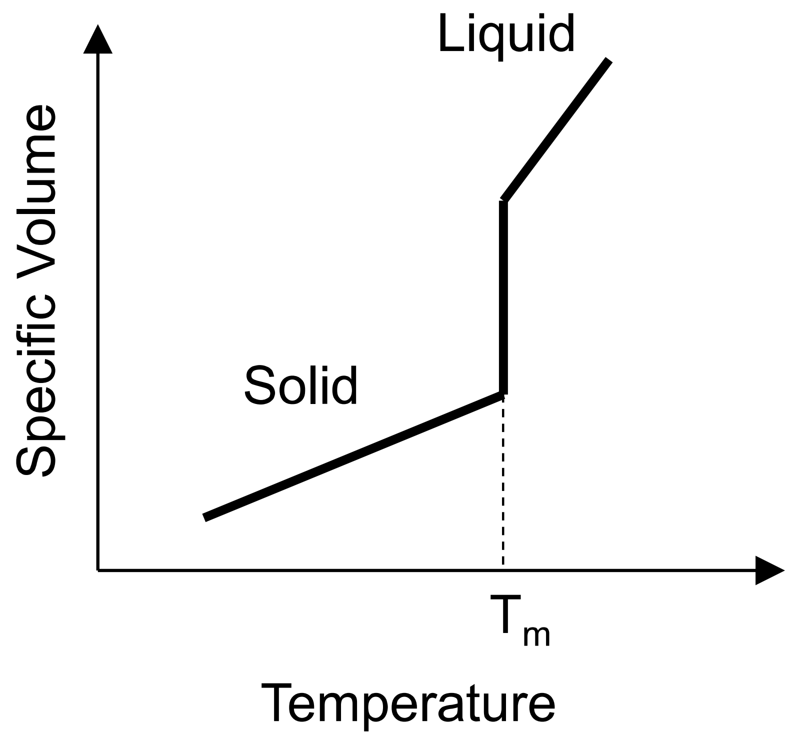
- Solved in class

Solidification

- Pure metals
  - Solidify at approx. constant temperature
  - Initiation of solidification requires undercooling

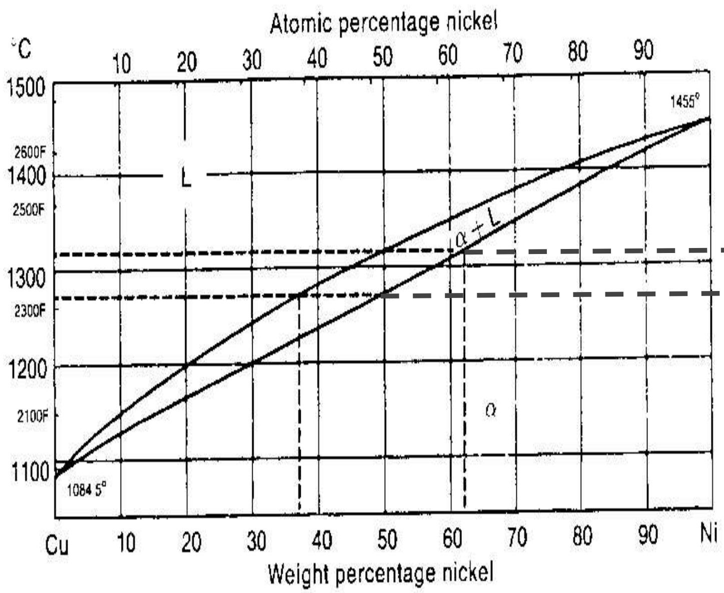




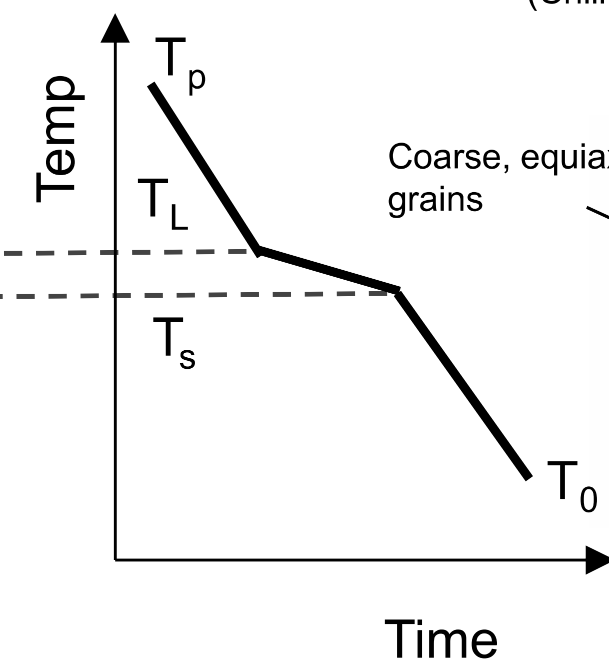


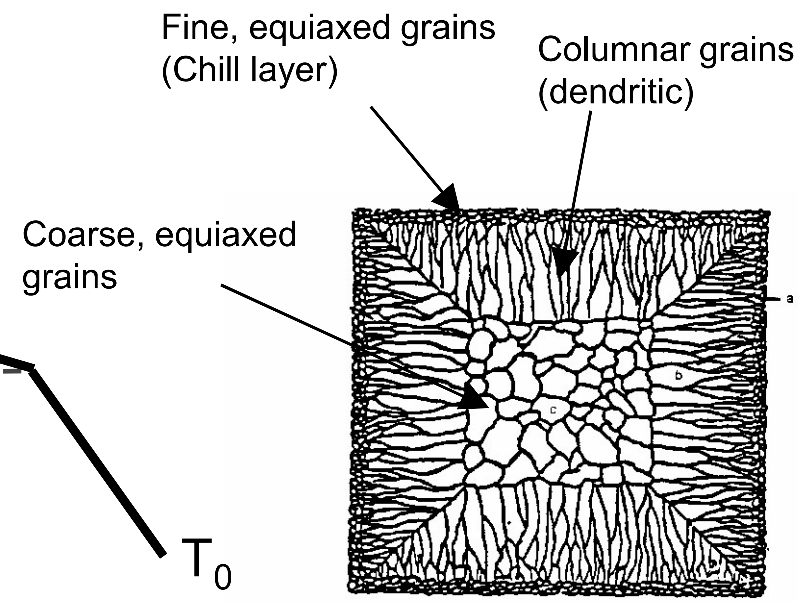
Solidification

- Alloys
  - Solidify over a temperature range
  - Composition and microstructure determined by phase diagram of alloy



Binary Phase Diagram

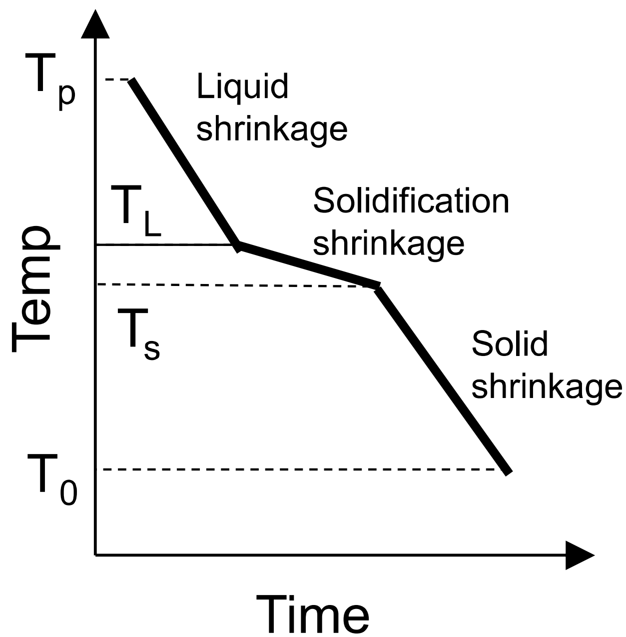




Microstructure

Shrinkage

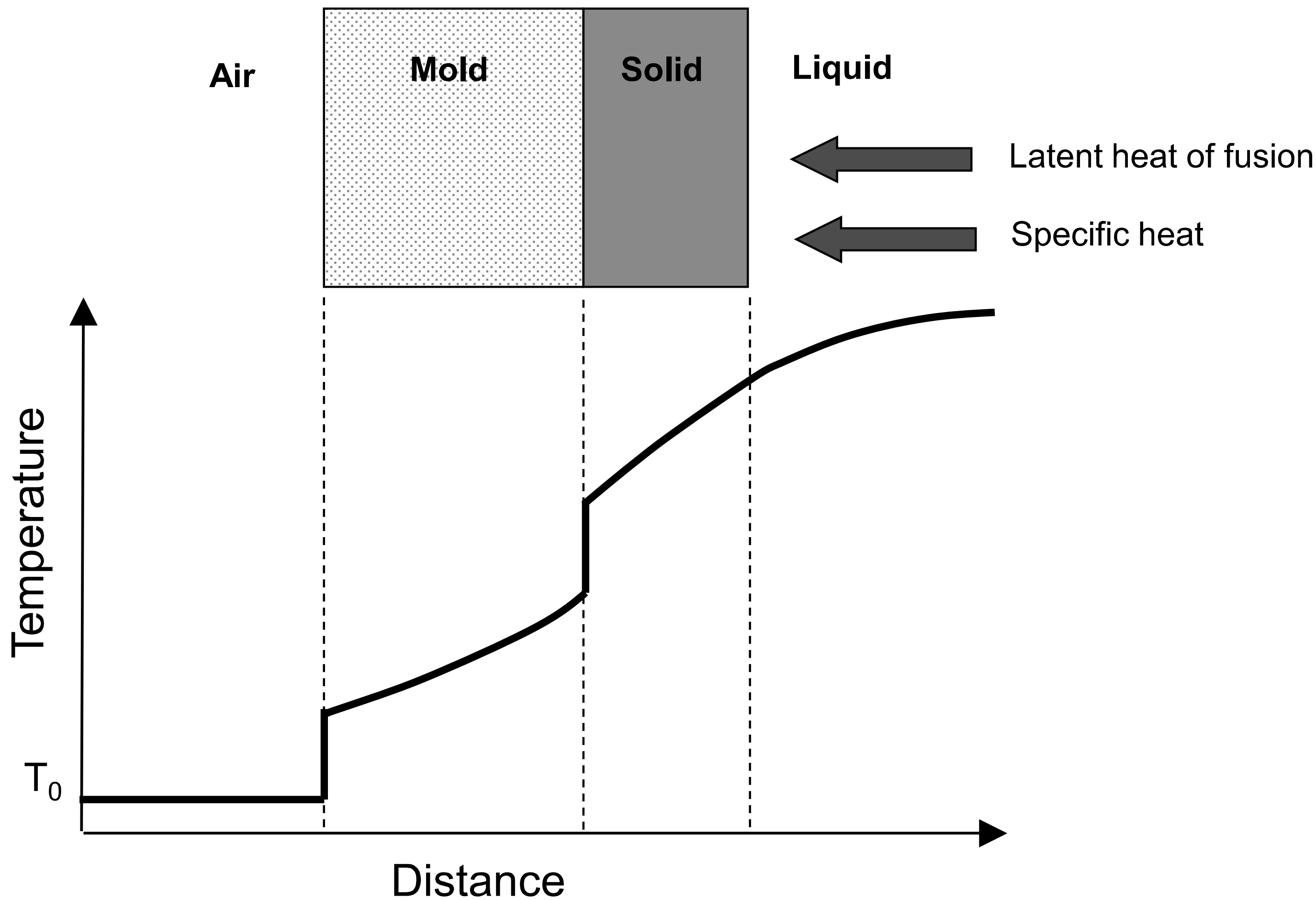
- Shrinkage: most metals shrink when cooled from the liquid state
  - Liquid shrinkage
  - Solidification shrinkage
  - Solid shrinkage



Metal	Solidification Shrinkage (%)	Coeff. Thermal Exp.	Solid Shrinkage (%)
Aluminum alloys	**7**	**25×10^{-6}**	**6.7**
Cast iron	**1.8**	**13×10^{-6}**	**4**
Steel	**3**	**14×10^{-6}**	**7.2**
Copper alloys	**5.5**	**17×10^{-6}**	**6**

Heat Transfer During Solidification

- Casting: non-steady state heat flow
- Consider the 1-d solidification of a pure metal

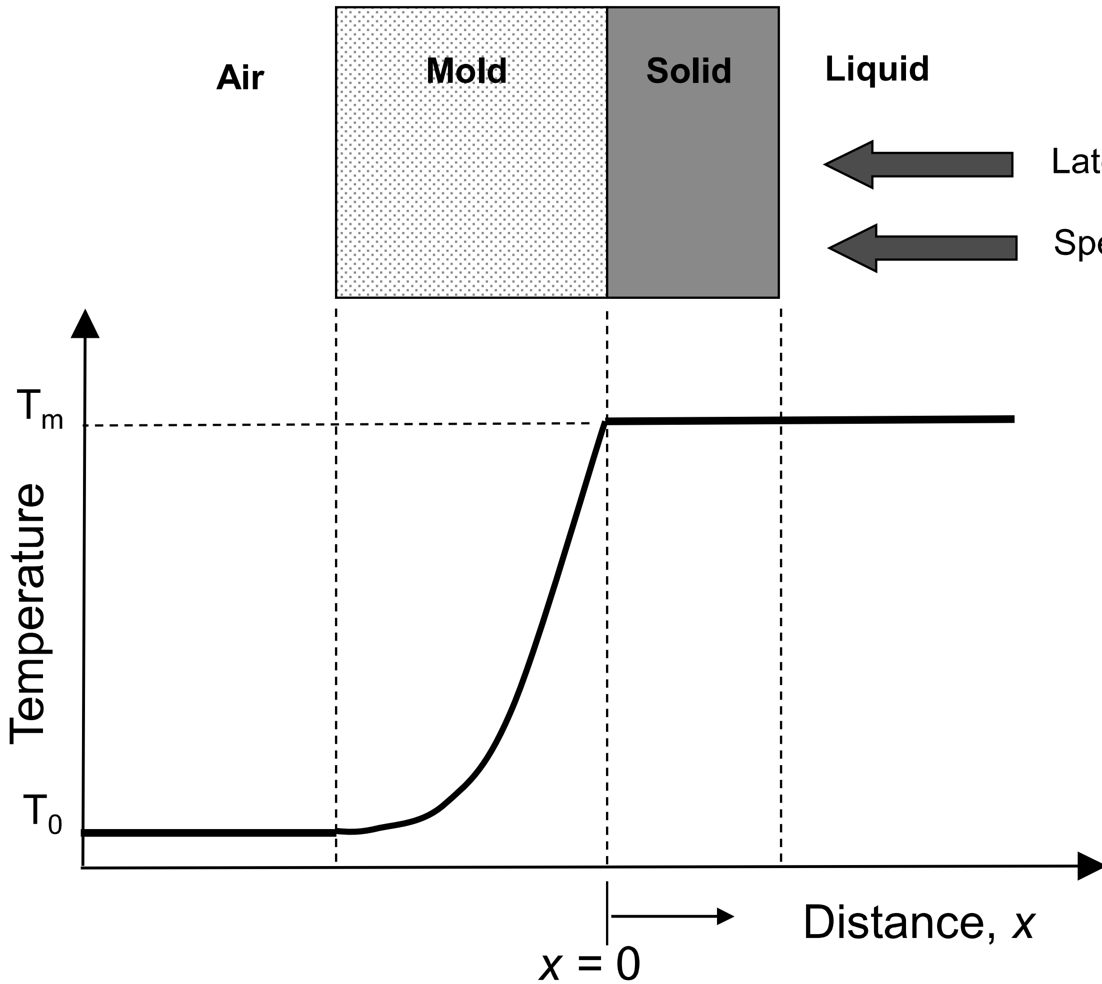


Heat Transfer Analysis: Insulating Molds

- Insulating mold example: sand mold
- Solidification rate for such molds depends primarily on the thermal properties of the mold
- Assumptions of analysis:
  - Pure metal
  - One dimensional heat transfer
  - Mold is semi-infinite in size
  - Mold thermal properties are uniform
  - Zero superheat
  - Uniform thickness of solidified metal
  - No thermal resistance at mold-metal interface
  - No temperature gradient within solid and liquid metal

Heat Transfer Analysis: Insulating Molds

- Implications of assumptions on instantaneous temperature distribution across mold



Heat Transfer Analysis: Insulating Molds

- Governing equation of transient heat transfer:

$$\frac{\partial T}{\partial t} = \alpha_m \frac{\partial^2 T}{\partial x^2} \qquad (1)$$

α_m = thermal diffusivity of mold = $k_m/(\rho_m c_m)$

k_m = thermal conductivity of mold

c_m = specific heat of mold

ρ_m = density of mold

- For the assumed boundary conditions, the general solution to (1) is:

$$\frac{T - T_m}{T_0 - T_m} = erf\left(\frac{-x}{2\sqrt{\alpha_m t}}\right) \qquad (2)$$

where *erf*() = Gaussian error function

- Note that Eq. (2) can be differentiated to obtain temperature gradient within the mold

Heat Transfer Analysis: Insulating Molds

- Quantity of practical interest: solidification time
- Can be obtained from energy balance at mold-solid interface
- Rate of heat flow into mold at mold-solid interface:

$$\dot{Q}_{@x=0} = -k_m A \frac{\partial T}{\partial x}_{@x=0} \qquad (3)$$

A = area of mold-metal interface

$$\dot{Q}\Big|_{x=0} = -A\sqrt{\frac{k_m \rho_m c_m}{\pi t}}\left(T_m - T_0\right) \qquad (4)$$

Heat Transfer Analysis: Insulating Molds

- Heat entering mold is given by solidifying metal as heat of fusion:

$$\dot{Q}\Big|_{x=0} = -\rho_s A H \frac{\partial S}{\partial t} \qquad (5)$$

H = latent heat of fusion

- Equating Eqs. (4) and (5) and integrating from S = 0 at t = 0 to t = t_s

$$S = \frac{2}{\sqrt{\pi}}\left(\frac{T_m - T}{\rho_s H}\right)\sqrt{k_m \rho_m c_m}\sqrt{t_s} \qquad (6)$$

Heat Transfer Analysis: Insulating Molds

- Replacing *S* with (V_s/A), where V_s = volume of metal solidified at time t_s and re-arranging

$$t_s = \left[\frac{\pi}{4} \left(\frac{\rho_s H}{T_m - T_0} \right)^2 \frac{1}{k_m \rho_m c_m} \right] \left(\frac{V_s}{A} \right)^2 \qquad (7)$$

$$\boxed{t_s = C_m \left(\frac{V_s}{A} \right)^2 \qquad (8)}$$

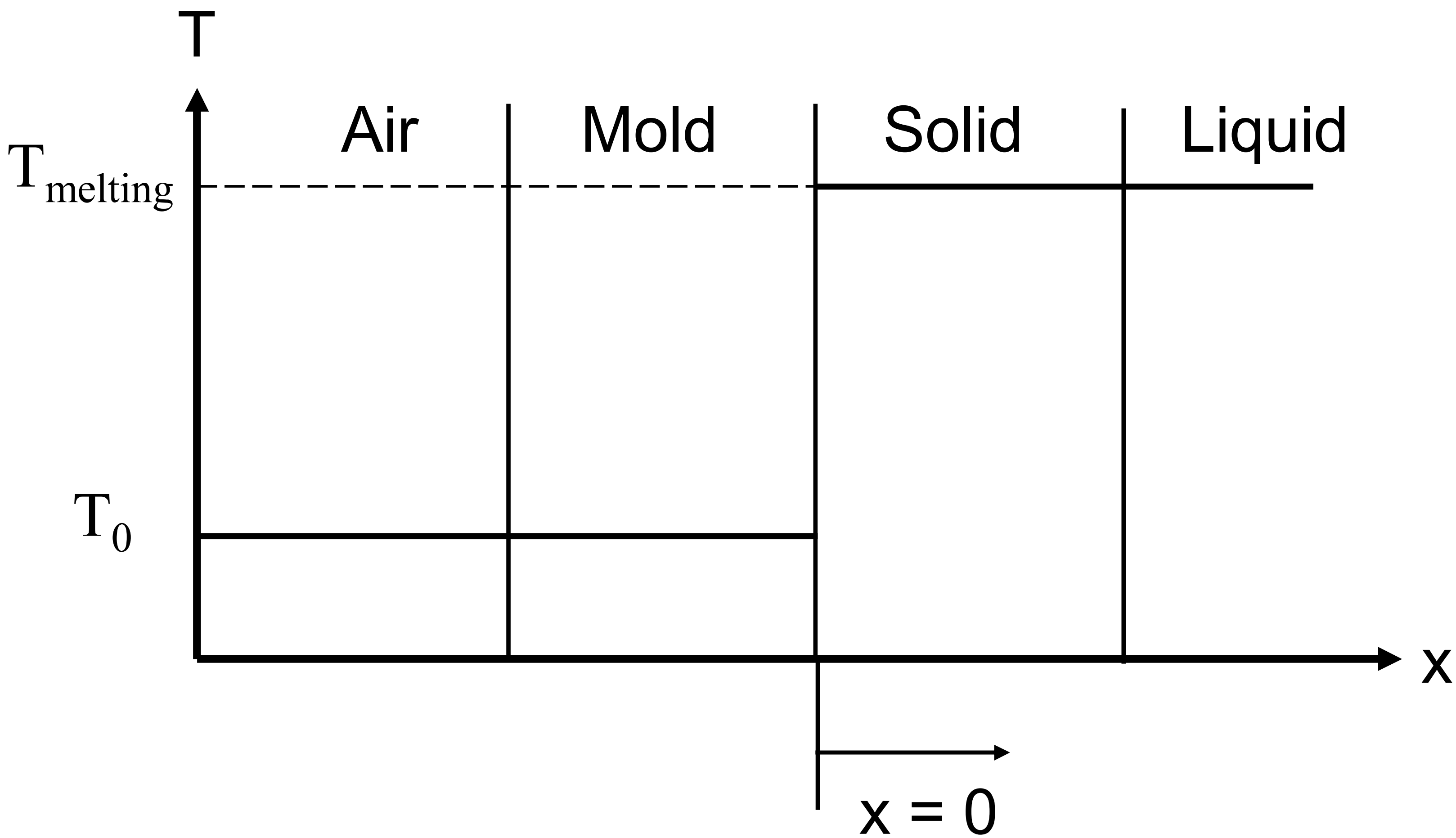
⇨ Chvorinov's Rule

- Chvorinov's rule can be used for more complex castings to provide a first approximation of solidification time

Heat Transfer Analysis: Conducting Mold

- We can perform a similar analysis for when the mold is a good thermal conductor, and the mold-metal interface thermal resistance controls heat transfer.
- Die casting is an example.

Instantaneous Temperature Gradient: Conducting Mold



Conducting Mold Analysis

- To solve this we will equate heat flux into the mold with that required to solidify the casting

$$\left(\frac{q}{A}\right)_{mold_{x=0}} = \left(\frac{q}{A}\right)_{casting}$$

Conducting Mold Analysis

- Assumptions
  - Mold-metal interface resistance dominates

$$Bi\# = \frac{h_{\text{interface}} l_{casting}}{k_{casting}} \ll 1 \quad \text{or} \quad h_{\text{interface}} \ll \frac{k_{casting}}{S}$$

  - If mold is relatively insulating

$$h_{\text{interface}}^2 \ll \frac{k_m \rho_m c_m}{t}$$

Conducting Mold Analysis

- You may also check to see if the mold is conducting
  - $k_{mold} \sim k_{casting}$
  - $\alpha_{mold} \sim \alpha_{casting}$

Conducting Mold Analysis

Heat flux across the mold-metal interface into the mold

$$\left(\frac{q}{A}\right)_{x=0} = -h\left(T_M - T_0\right)$$

Heat flux due to solidification of casting

$$\left(\frac{q}{A}\right)_{casting} = -\rho_{casting} \Delta H_f \frac{dS}{dt}$$

Conducting Mold Analysis

- heat flux away from mold-metal interface = heat flux to mold-metal interface due to solidification

$$\left(\frac{q}{A}\right)_{mold_{x=0}} = \left(\frac{q}{A}\right)_{casting}$$

$$h\left(T_M - T_0\right) = \rho_{casting} \Delta H_f \frac{dS}{dt}$$

- Integrating from S=0 and t=0 to S=S and t=t

Conducting Mold Analysis

$$S = h \frac{T_M - T_0}{\rho_{casting} \Delta H_f} t$$

also S = V/A

Conducting Mold Analysis

$$t = \frac{\rho_{casting} \Delta H_f}{h\left(T_M - T_0\right)} \frac{V}{A}$$

Riser Design

- Riser design is based on Chvorinov's rule
- Function of riser: to feed the mold cavity with molten metal in order to compensate for shrinkage

$$\boxed{t_{riser} \geq t_{mold}} \Rightarrow \left(\frac{V_{riser}}{A_{riser}}\right) \geq \left(\frac{V_{mold}}{A_{mold}}\right)$$

- Design and place risers so that solidification begins in the casting and ends in the riser → shrinkage defects such as voids, porosity are limited to the riser

Riser Design Example

A closed cylindrical riser must be designed for a sand mold. The part to be cast is a 125 mm x 125 mm x 25 mm plate. The foundry person knows from experience that the total solidification time for casting this part is 2 min. It is required that the height-to-diameter ratio of the riser be 1. Find the dimensions of the riser so that its total solidification time is 30% longer than the casting.

Solved in class