# Objective Function

$$\text{Objective: } \sum_{\text{lengths}} \sum_{\text{widths}} (t_2 \times \text{num\_scans})$$

Where:

$$t_2 = \frac{\text{scan\_length}}{\text{max\_feed\_rate}}$$

$$\text{max\_feed\_rate} = \frac{0.866 \times d}{t_1}$$

$$t_1 = \frac{E_c}{P_a}$$

$$P_a = \frac{P}{A}$$

$$A = \pi \left(\frac{d}{2}\right)^2$$

$$\text{num\_scans} = \left\lceil \frac{\text{scan\_width}}{\left(\frac{d}{2}\right)} \right\rceil$$

# Normalized Enthalpy Constraint

$$\left(\frac{\alpha \cdot P}{\rho \cdot H \cdot \sqrt{\pi \cdot \kappa \cdot v \cdot d}}\right) = 30$$

# Minimum Laser Power Constraint

$$P \geq \rho \cdot V \cdot \left[c_s \cdot (T_m - T_0) + H_f \cdot 10^3 + c_l \cdot (T_p - T_m)\right]$$

# Solubility Constraint

$$1 - \left(\frac{P_a}{v \cdot t_l}\right) \geq 0$$

# Shrinkage Constraint

$$0.02 - (\beta \cdot (T_m - T_0)) \geq 0$$

## Melt Pool Depth Calculation

$$E(z) = E_0 \exp\left(-\frac{z}{D_p}\right)$$

$$E(z) = \frac{P}{A} \exp\left(-\frac{z}{D_p}\right) - E_c$$

Where:

$$E_0 = \frac{P}{A}$$

$$D_p = \text{penetration depth}$$

$$z = \text{melt pool depth}$$

$$E_c = \text{threshold energy density}$$

## Melt Pool Diameter Calculation

$$d_m = 2\sqrt{\frac{P}{\rho \cdot c_p \cdot v}}$$

Where:

$$\rho = \text{density}$$

$$c_p = \text{specific heat capacity of the solid}$$

$$v = \text{scan speed}$$

## Paraboloid Volume Calculation

$$V = \frac{1}{2}\pi r^2 h$$

Where:

$$r = \frac{d_m}{2}$$

$$h = \text{melt pool depth}$$

## Variable Definitions

- $\alpha$: absorptivity
- $P$: laser power
- $\rho$: density
- $H$: enthalpy
- $\kappa$: thermal diffusivity

- $v$: scan speed
- $d$: beam diameter
- $V$: volume of the melt pool (paraboloid)
- $c_s$: specific heat capacity of the solid
- $T_m$: melting temperature
- $T_0$: initial temperature
- $H_f$: heat of fusion
- $c_l$: specific heat capacity of the liquid
- $T_p$: pouring temperature
- $P_a$: power density
- $t_l$: layer thickness
- $\beta$: thermal expansion coefficient