Hashing

Part 12

1

Hashing

We have a need… a need for speed!

2

Hashing

- Array elements can be accessed in O(1)
- Why? The memory address of any element can be calculated mathematically
- … however, this doesn't work for dictionary keys

3

Hashing

- We can use a nice balanced tree to store the data
- … but that is O(log n) – which is still excellent
- Is it possible to get the time complexity down to O(1)?

4

Hashing

- What if we came up with a *"magic function"* that converted keys into array indexes?
- A *hash function* takes a key object as an argument and returns a numeric index
- hash(key) → index

5

Hashing

- Given a specific key the hash function would compute the *exact* index of the element
- This will given dictionaries O(1) access

6

Hash Mathematics

- A hash function maps keys into indexes in a *hash table*
- For element e with key k and h is hash function…
  - e is stored in position $h(k)$ in the hash table *(an array)*
  - to find/store for e, compute $h(k)$ to locate position

7

Using a Hash

Hash function

Key	Hash
cat	0
turkey	4
dog	1
possum	5
human	9

Hash Table

Index	Data
0	cat object
1	dog object
2	
3	
4	turkey object

8

Example Hash Function

- `hash("Rick") = 5`
- `hash("Morty") = 1`
- `hash("Jerry") = 0`
- `hash("Beth") = 4`

Array	
0	Jerry
1	Morty
2	
3	
4	Beth
5	Rick

9

But, there are Problems

- Simple hash functions
  - work for implementing dictionaries
  - but most apps have key ranges that are too large for 1-1 mapping between hashes and keys
- Example:
  - key range from 0 to 65,535
  - collection will have no more than 100 items at any given time
  - impractical to use a hash table with 65,536 slots!

10

Finding the Hash Function

- There is **no** magic function
  - **only** in rare cases, with a limited key range, a perfect function can exist
  - however, for real World cases, there is no function possible
- So, we can take a different approach
  - don't use the hash value as a finishing point
  - use it as a location to start looking

11

Collisions

- When two keys hash to the same array location, this is called a *collision*
- What do we do?
  - normally collisions are "first come, first serve"
  - the first key that hashes to the location gets it
  - so, we need to decide what do with the second item that hash to the same location
  - there are two solutions…

12

Example Hash Function

- `hash("Ash") = 5`
- `hash("Jesse") = 1`
- `hash("Brock") = 0`
- `hash("Misty") = 4`
- `hash("James") = 1`

	Array
0	Brock
1	COLLISION
2	
3	
4	Misty
5	Ash

13

Closed Hashing

Chaos... good news

14

Collision Solution: Closed Hashing

- With *closed hashing*, we use the existing array and search for an empty position
- Use the hash value as start position – we start searching here

15

Collision Solution: Closed Hashing

- If the array element is a occupied...search down and look for an empty element
- The search must also…
  - wrap-around to the top
  - and be aware if the search cycles through the entire array – *we ran out of space*

16

Closed Hash Example

- Let assume we are adding "Pacman" which has a hash value of **0**
- This collides with "Dig Dug"
- We search down for the next empty element

0	Dig Dug
1	Q-Bert
2	
3	
4	Fix-It Felix, Jr
5	Frogger

17

Closed Hash Example

- Now, at index **1**, we collide with Q-Bert
- We search down for the next empty element

0	Dig Dug
1	Q-Bert
2	
3	
4	Fix-It Felix, Jr
5	Frogger

18

Closed Hash Example

- Finally, we find an empty array element
- Pacman is stored here
- Note, this is **2** positions off from the original hash

0	Dig Dug	✖
1	Q-Bert	✖
2	**Pacman**	
3		
4	Fix-It Felix, Jr	
5	Frogger	

19

Closed Hashing Clustering

- One problem with the closed hashing is the tendency to form *clusters*
- A cluster is a group of continuous used array elements – with no open slots
- What happens?
  - the bigger a cluster gets, the more likely it is that new keys will hash into it *(and collide)*
  - it then grows larger and larger
  - the hash will eventually degrade to **O(n)**

20

Efficiency of Closed Hashing

- Hash tables are surprisingly efficient
- Although collisions cause searching, tables, items can be found near O(1)
- Even if the table is nearly full (leading to long searches), efficiency is still quite high

21

Closed Hashing Pitfalls

- Closed hashing is not the best solution
- It requires a static array
  - the array cannot be increased at runtime (or the hash fails)
  - hence, the array is finite
- Clustering causes O(n) degradation

22

Closed Hashing Pitfalls

- You cannot delete items
  - it creates empty slots in clusters!
  - this can prevent an item, *added in a cluster*, from being found below the gap
  - there are work-rounds, but it gets **convoluted**

23

Closed Delete Problem

- "Dig Dug" and "Pacman" hash to **0**
- When "Pacman" was added it had to be stored at **2**
- This is a cluster

0	Dig Dug
1	Q-Bert
2	Pacman
3	
4	Fix-It Felix, Jr
5	Frogger

24

Closed Delete Problem

- If "Dig Dug" is deleted, the array element is empty
- If there is a search for "Pacman", it will hash to **0** and it <u>won't</u> be found

0	
1	Q-Bert
2	Pacman
3	
4	Fix-It Felix, Jr
5	Frogger

25

Open Hashing

The Merging of Concepts

26

Open Hashing

- With *open hashing*, we don't store individual objects in each array element
- Instead, each array element is a linked list <u>or</u> a tree (preferably balanced)

27

Open Hashing

- So, our hash table is an array of either linked lists or trees
- This approach is also known as *bucket hashing* since each list/tree acts a "hash bucket"

28

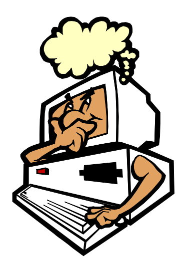
Collision Solution: Open Hashing

- When a collision occurs the item is added to the list/tree
- So, this list/tree will contain <u>**all**</u> the objects with the <u>same</u> hash value
- We don't need to look for conflicts

29

When an Object is Looked Up…

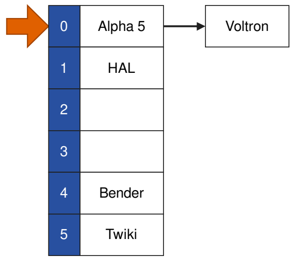
- Compute the hash value
- And then search the targeted list/tree
- For example, for a balanced tree, searching would be O(1) at best and O(log n) at worst



30

Open Hash Example

- Adding "Voltron" with a hash of 0
- This collides with "Alpha 5"
- Open hashing just adds the item to the linked list

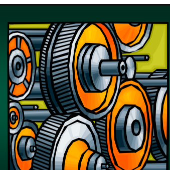


31

Open Hashing Benefits

1. Open hashing will not fill up the array and can grow *indefinitely*
2. Far faster access time than closed hashing
3. No clustering
4. Objects can be deleted

32



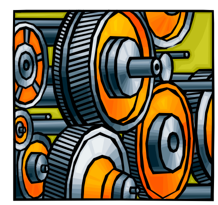
Hash Functions

Techniques for Spreading the Data

33

Hash Functions

- A hash function can be *anything*
- However, it is best to...
  - find one that spreads items evenly over the array
  - ... and one that limits collisions



34

Random Hashing

- Most hashing algorithms use a pseudo-random number generator
- This essentially scatters the items “randomly” throughout the hash table
- ... but there is no real "random" numbers in computers – only chaotic series

35

Popular Algorithm: Modulus

- Uses the formula: $h(k) = k \bmod N$
  - k is a raw key value produced by some internal function
  - we don't care "how" this was produced
  - N is the size of the array
- Selecting N
  - table size N is usually a prime number
  - it prevents patterns – which can cause collisions

36

Popular Algorithm: MAD

- Based on multiply, add, and divide (MAD)
- Uses the formula: $h(k) = (a * k + b) \bmod N$
  - a and b are both constants
  - eliminates patterns provided $a \bmod N \neq 0$
  - this is the same formula used to create (pseudo) random number generators

37

Let's Hash The Students!

Closed hashing

38

Let's Hash The Students!

- Let's add your names to a closed hash table
- First, let's figure out *how* we want to hash your names
- Then, I'll ask for 5 volunteers

39

39

Hash by Name Digits mod 6 (Section 1)

	Name	Hash
1	Ganesh	0
2	Quincy	0
3	Zachary	1
4	Sebastian	3
5	Justin	0

➡

Hash Table	
0	Ganesh
1	Quincy
2	Zachary
3	Sebastian
4	Justin
5	

40

Hash by Age mod 6 (Section 1)

	Name	Hash
1	Sorea	2
2	Javier	2
3	Ganesh	3
4	Hormoz	4
5	Justin	

➡

Hash Table	
0	Justin
1	
2	Sorea
3	Javier
4	Ganesh
5	Hormoz

41

Hash by Name Digits mod 6 (Section 2)

	Name	Hash
1	Timothy	1
2	Nav	3
3	Andleep	1
4	Delia	5
5	Dante	

➡

Hash Table	
0	Dante
1	Timothy
2	Andleep
3	Nav
4	
5	Delia

42

Hash by Age mod 6 (Section 2)

	Name	Hash
1	Nav	3
2	Elliott	4
3	Delia	1
4	Likhith	4
5	Cook	5

Hash Table	
0	Cook
1	Delia
2	
3	Nav
4	Elliot
5	Likhith

43