



## Recursion

Part 3

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## Program Structure

How they work

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### Program Structure

- When writing a program, you must be aware how it works "behinds the scenes"
- In particular, you must understand memory and how it is used.



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### Program Structure

- There are possible issues that can arise that can negatively impact your programs
- ... and possibly make them unresponsive



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### Some Terminology

- When you call a function, you can specify pieces of data called *arguments*
- These match the format of the function – which is specified in its *parameters*
- Basically
  - arguments are *passed* to the parameters
  - they match, in order, on a one-to-one basis
  - arguments → parameters

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### Scope



- *Scope* refers how a variable/function is bound (i.e. visible to the rest of your program)
- Data is often stored differently, based on its scope

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## Global Variables



- You can declare variables outside functions
- These are visible to all functions in the class (or module)
- These are known as *global variables*

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## Global Variables



- They can be useful for sharing data between functions
- However, it can be problematic
  - variables can be modified in ways that cause side effects in your program
  - it is better to use local variables and pass them to other functions

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## Global Variables

```
int total;  
  
void printTotal()  
{  
    System.out.println(total);  
}  
  
int main()  
{  
    total = 1000;  
    ...  
}
```

Visible to all functions!

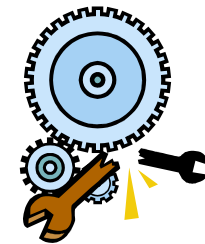
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## Global Variable Problems



- They can be useful for sharing data between functions
- However, it can be problematic
  - variables can be modified in ways that cause side effects in your program
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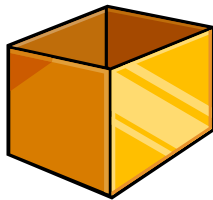
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## Local Variables

- When you create functions, each can have *local* variables
- These are only "visible" to the function in which they are declared
- So, other functions cannot access them



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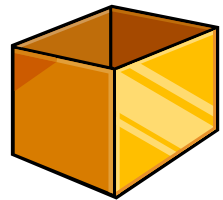
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## Variable Scope

- Different functions can have local variables with the same name
- Why?
  - they can't "see" each other
  - they are different variables, anyway
  - ... so, there is no problem



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## Variable Scopes

```
int hello()
{
    int x;
}

int main()
{
    int x;
}
```

Not the same variable

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## Variable Scopes

```
int hello()
{
    double x;
}

int main()
{
    int x;
}
```

Don't have to be the same type  
(they are different variables)

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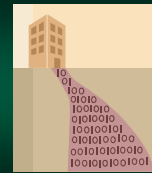
## Example: Average Function

```
double average(double a, double b)
{
    double avg;

    avg = (a + b) / 2;
    return avg;
}
```

Parameters are also local variables

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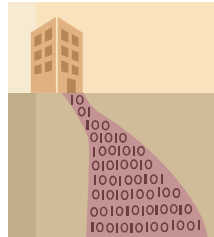
## The System Stack & Heap

Making the Functions Function & Data Delightful

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## The System Stack & Heap

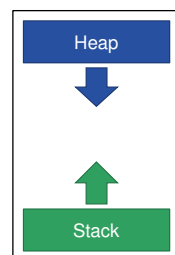
- Computers maintain two types of memory for running programs: *The Stack* and *The Heap*
- Each has a specific purpose, and, in tandem, they make modern programs possible



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## The System Stack & Heap

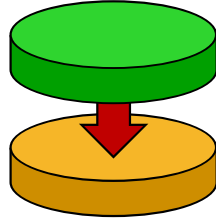
- Each is stored in your computer's main memory
- They grow "towards" each other (and, hopefully, will never meet)



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## The System Stack

- The System Stack is used to store local variables and allow your program to support functions
- So, anytime you call a function or declare a local variable, a stack is used



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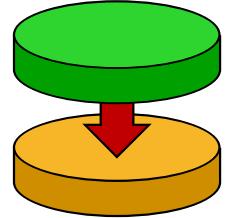
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## The System Stack

- Each time a function calls another function an *Activation Record* is placed on the *stack*
- It contains all the information that the instance of a function requires



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## Contents of the Activation Record

- The Activation Record contains:
  - parameters
  - local variables
  - return address (used by the processor)
- Data in an activation record is temporary to that "instance" of a function
- In other words, data does not persist after the function ends

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## The Power of Stacks

- Because the stack is a First-In-Last-Out structure, it allows function nesting
- And even a more powerful concept – *recursion*
- Examples
  - web browser "back button"
  - undo sequence in a text editor

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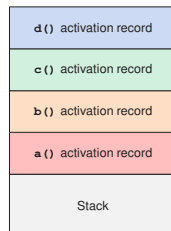
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## Nesting Activation Records

- For example:
  - `main()` calls `a()`
  - `a()` calls `b()`
  - `b()` calls `c()`
  - `c()` calls `d()`
- Each activation record is pushed onto the stack



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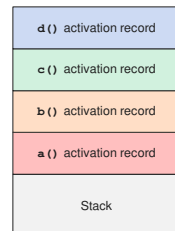
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## Nesting Activation Records

- When a function "returns", its activation record is pop'd and discarded
- The local variables cease to exist
- Only the return value is passed to the caller



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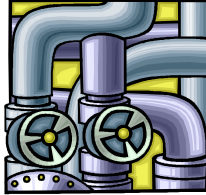
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## The Heap

- Nothing on the system stack persists forever – it is quite temporary
- So, how do we make data last indefinitely? ...or, as long as our program is active



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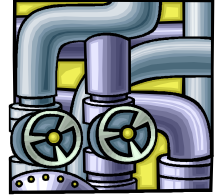
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## The Heap

- *The Heap* is used to store dynamic allocation
- It is allocated *as needed*
- ... not to be confused with the Heap Data Structure (which we will cover later)



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## The Heap

- Anytime you create objects using *"new"*...
  - the heap is used to allocate storage
  - system performs garbage collection after the memory is no longer needed
- Unlike the stack, data persists regardless of function calls

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## Reference Types



The objective of Object Oriented Programs

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## Reference Types



- Most languages are based on largely based on building abstract data types called *reference types*
- They are links to nebulous *objects* – whose contents & implementation are unknown

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## Reference Types



- This is known as *object-oriented programming*
- ... and is the basis of all modern programming languages

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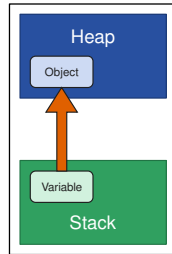
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## Reference Types

- So, local variables exist on the stack
- But... they reference objects stored on the system heap



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## Garbage Collection

- Programming languages use *garbage collection* to reclaim unused data from the heap
- Policy is to reclaim the memory used by objects that *can no longer be accessed* (i.e. no references)



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## Garbage Collection

- So, languages maintain a counter on each object
  - if you add a reference, it increments
  - if a reference is removed, it decrements
- When it reaches zero, the object can be removed



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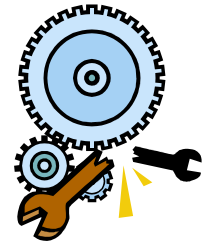
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## Loitering

- It is possible to "remove" an item from the ADT, but accidentally keep a reference (link) to it
- The item is effectively an *orphan* - it will be never be accessed again by the ADT



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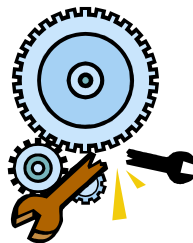
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## Loitering

- The garbage collector has no way to know unless it's overwritten
- So, under this condition, the object is said to *loiter* - stay in memory with no purpose
- This can negatively affect performance



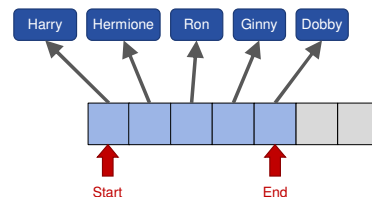
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## Array Storing a List (partially filled)



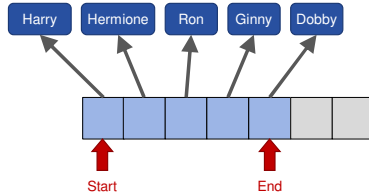
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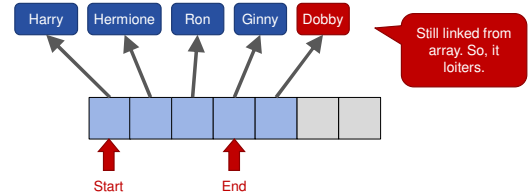
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## Delete Last – Move End



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## Dobby is still linked...



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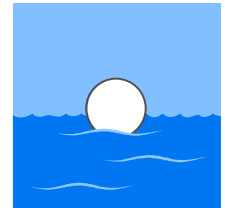
## Pools

Okay, now it's getting weird

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## Pools

- Creating and destroying objects is expensive on the heap
- So, we want to minimize the constant creation and deletion of new nodes



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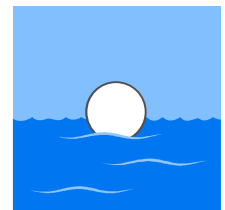
## Why?

- Arrays can be wasteful ...
  - in space – when there are partially
  - in time – created and destroyed frequently
- Linked lists can be wasteful...
  - require memory to be allocated each time a node is created
  - puts a lot of work on the heap

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## Jump in the Pool

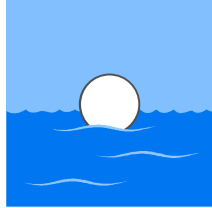
- One solution is to maintain a *pool*
- This is a collection of nodes that are allocated early and are used as, kind of, a recycling bin



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## Jump in the Pool

- If a node is needed, one is removed from the pool
- If a node is removed, and the array has room, it is placed back in the array (after the data field is set to null, of course)



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## Even more approaches

- You can also use a "pool" for linked lists
- So, your Linked List class
  - would have a linked list of valid nodes
  - and another list of unused nodes
  - the danger here is that you don't limit the size of the pool – and it grows **forever**
  - so, if you use two linked lists, keep a pool member count

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## Recursion

The best way to learn recursion...  
is to, first, learn recursion!



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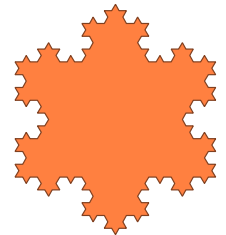
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## Recursion

- *Recursion* occurs when a function directly or indirectly calls **itself**
- This results in a loop
- However, it doesn't use iterative structures such as For or While loops



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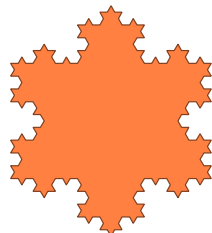
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## Recursion

- This can greatly simplify programming tasks
- Commonly used to traverse a graph, tree, or run complex calculations
- While powerful, it is costly on computer resources
- ...and can also create pitfalls



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## Breaking a Problem Down

- Recursion allows a problem to be broken down **into smaller instances of themselves**
- Each call will represent a smaller, simpler, version of the **same** problem
- Eventually, it will reach a **"base case"** which will **not** require any more recursive calls

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## Where Recursion Shines

- When the program can be broken into smaller pieces, recursion is a great solution
- Examples:
  - graph traversal – searching, etc....
  - state machines
  - sorting
  - many math problems

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## Danger: Never Ending

- If you break down a task into smaller parts... at some point, it should become a single value
- If not, the function will never end and will recurse forever – *at least until the computer runs out of resources*



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## Danger: Accidental Recursion

- Accidental recursion is a common mistake by beginner programmers
- Recursion can be done directly or indirectly
  - for example: A calls B, B calls C, C calls A
  - organize your code carefully!



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## Results of These Dangers...

- Runaway recursion
  - function will recurse *forever*
  - eventually all memory is exhausted
- You will see either...
  - "stack overflow" error
  - "heap exhaustion" error



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## To infinity... but not beyond

```
void toInfinity()  
{  
    System.out.println("To infinity!");  
    toInfinity();  
    System.out.println("and beyond!");  
}
```

We never get here!

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## Designing a Recursive Function

- Does the problem lend itself to recursion?
  - can the problem be broken down into smaller instances of itself?
  - is there a iterative version that is better
- Is there a base case?
  - is there a case where recursion will stop?
  - remember: ALWAYS have a stopping point!

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## Example: Factorials

- Factorials are classic mathematical problem that lends itself easily to recursion
- If you don't remember, a factorial of  $n$  is defined as the value of  $n$  multiplied by all lesser integers  $\geq 1$
- For example:  $5! \rightarrow 5 \times 4 \times 3 \times 2 \times 1 \rightarrow 120$

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## Example: Factorials

- It should be easy to observe that  $n!$  can be defined as  $n \times (n-1)!$
- So,  $n!$  can be computed by multiplying  $n$  by the factorial of one less than it
- $4! \rightarrow 4 \times 3! \rightarrow 4 \times 3 \times 2! \rightarrow 4 \times 3 \times 2 \times 1$

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## Example: Factorials

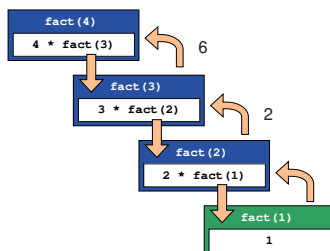
```
int factorial(int n)
{
    if (n == 1) {
        return 1;
    }
    else {
        return n * factorial(n - 1);
    }
}
```

Base case

Recursion

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## Example Factorial



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## Iteration vs. Recursion

- Any** program that can be expressed using recursion, can be done through iteration
- The recursive solution will often be far simpler – **more "eloquent" to read**
- ... but is never more efficient due to the overhead of calling functions

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## Historical Perspective

Some really cool solutions!

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## Some Well-known Problems


- Sorting
- Searching
- Shortest paths in a graph
- Minimum spanning tree
- Primality testing



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## Some Well-known Problems

- Traveling salesman problem
- Knapsack problem
- Chess
- Towers of Hanoi
- Program termination



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## Fibonacci Numbers



- Rabbits tend to reproduce like... well... rabbits
- Mathematician *Fibonacci* analyzed this situation and created a mathematical system to predict this phenomena
- It is used today in finance, simulation, and several computer science algorithms

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## Fibonacci Numbers



- The problem:
  - start with a pair of rabbits
  - at month #2, the rabbits begin to reproduce
  - the female gives birth to a new pair of rabbits: one male and one female
  - babies mature at the same rate and will have more babies
- Fibonacci number sequences predict the total pairs after  $n$  months

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## Fibonacci Numbers

- After two months, the female gives birth creating a new pair... then they get pregnant again!
- This continues forever.....
- Sequence: 1, 1, 2, 3, 5, 8, 13, 21, 34, ...

```

if n == 1 then Fib(n) = 1
if n == 2 then Fib(n) = 1
if n > 2 then Fib(n) = Fib(n-2) + Fib(n-1)
  
```

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## Example 2: Fibonacci Numbers

```
f(3) = f(2) + f(1) = 1 + 1 = 2
f(4) = f(3) + f(2) = 2 + 1 = 3
f(5) = f(4) + f(3) = 3 + 2 = 5
f(6) = f(5) + f(4) = 5 + 3 = 8
```

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## Example: Fibonacci Numbers

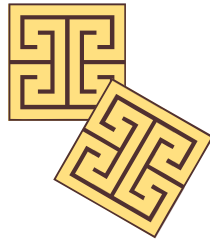
```
int fib(int n)
{
    if (n == 1 || n == 2)
    {
        return 1;
    }
    else
    {
        return fib(n-2) + fib(n-1);
    }
}
```

Recursion

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## Greatest Common Denominator

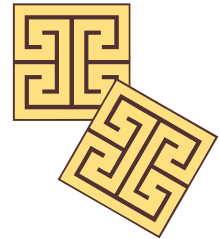
- A common problem in computer science is finding the greatest (or least) common denominator for two integers
- For example: the GCD of 64 and 40 is 8



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## Greatest Common Denominator

- Euclid* created an ingenious algorithm for finding the greatest common divisor
- This is known example of recursion – first solved using geometry using the metaphor of a tile floor



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## Euclid's Algorithm

- Euclid's algorithm is recursive
- You reapply the expression below until the second value of  $\text{gcd}(n, m)$  is zero.
- In this case,  $n$  will be the CGD

```
gcd(n, m) → gcd(m, n mod m)
```

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## Euclid's Algorithm Examples

- 60 and 24
  - $\text{gcd}(60, 24) \rightarrow \text{gcd}(24, 12) \rightarrow \text{gcd}(12, 0)$
  - the result is 12
- 84 and 20
  - $\text{gcd}(84, 20) \rightarrow \text{gcd}(20, 4) \rightarrow \text{gcd}(4, 0)$
  - result is 4
- These might seem trivial, but it can find HUGE numbers quite easily

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