Set Data Structures

Part 13

1

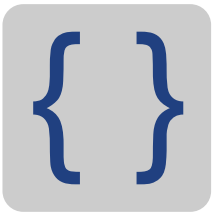
Importance of Sets

Organizing Information

2

Importance of Sets

- A *set* is an unordered collection of "objects"
- Sets are used in computer science is a wide variety of ways



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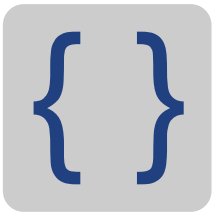
Importance of Sets

- Depending on the attributes of the set, and how we use it, there are different approaches
- Programmers must choose the best model given how it will be used



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Set Review: Membership

- Set notation uses a special symbols to denote if an object is a member of a set
- Below, the set V contains vegetables

```
potato ∈ V
bacon ∉ V
```


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Set Review: Union

- A union of two sets combines all members of each set into a new one
- So, the result is two merged sets

$$A \cup B = \{ x \mid x \in A \text{ or } x \in B \}$$

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Set Review: Intersection

- The intersection of two sets contains only those elements that are found in both sets
- So, the result is where the two sets overlap

$$A \cap B = \{ x \mid x \in A \text{ and } x \in B \}$$

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Set Review: Difference

- *Difference* (aka exclusion) removes all items found in set from another
- Typically, it is written either A – B or A \ B

$$A - B = \{ x \mid x \in A \text{ and } x \notin B \}$$

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Set Review: Complement

- The *complement* of a set A, is all elements in the Universe, not in A
- Typically written as A' or A^c
- Alternatively written using an overbar

$$A' = \{ x \mid x \notin A \}$$

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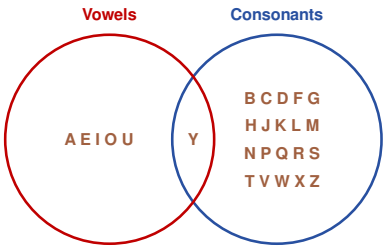
Example: Letters in English



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Multiset

- A *multiset* is closely related to a set, but permits duplicate elements (as does a tuple)
- The Bag ADT, from the beginning of the semester, supports a basic multiset

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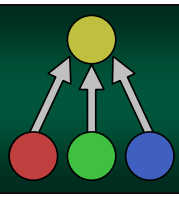

Union-Find

When it absolutely, positively has to be unioned overnight … er… O(1)

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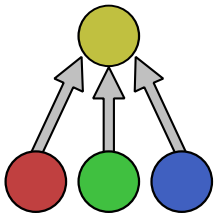
Union-Find

- Often there are situations where we have a number of known objects
- … i.e. a finite countable set
- … and we want to quickly union them together



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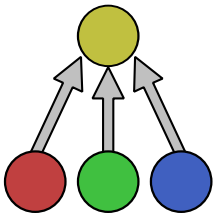
Union-Find

- *Union-Find* data structure maintains a list of nodes *partitioned* into *disjoint* subsets
- e.g. { {a} {b} {c} {d} {e} } is a full partition of {a,b,c,d,e}



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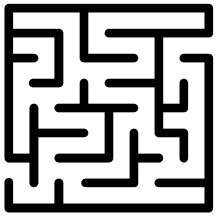
Example Applications

- Kruskal's Tree-spanning Algorithm – uses a set of known edges
- Maze creation algorithms use it to track sets of connected paths – so the maze is always solvable



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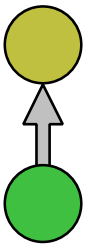
The Approach

- Sets are stored as a variation of the classic tree
- However, in this approach children link to their parents
- So, the branches point *"backwards"* from standard trees – i.e. upwards



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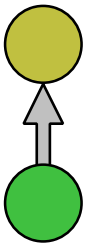
Arrangement of Nodes



- A parent node can have multiple children - this is **not** a binary tree!
- Every node - in the tree - is part of the same set

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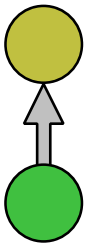
Arrangement of Nodes



- The root is called the *representative* of the set
- This node is **not** special nor is its value special
- … it just happens to be the node the represents the set

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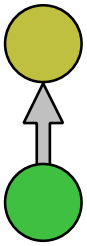
Arrangement of Nodes



- Any node can "find" its representative – by following the upwards branches
- If any two nodes have the same representative, then they are in the same set

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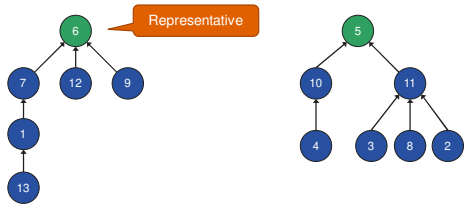
Examples



{ 1, 6 ,7, 9, 12, 13 }

{ 2, 3, 4, 5, 8, 10, 11 }

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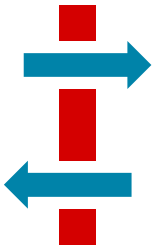
Union-Find Operations

- The Union-Find data structure contains 3 operations that handle sets
- All 3 can modify the structure of the tree – which automatically optimizes itself into **O(1)**



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Union-Find API

```
public class UnionFind
    void    makeSet(object value)
    void    union(object a, object b)
  object    find(object a)
```


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Makeset

- Creates a set (within the Union-Find instance) that contains only a single element
- Also known as a *singleton*
- Essentially, this is the same as an "add" for the ADT

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Find

- The *find operation* starts with a node and locates its *representative*
- Travels up the tree until it finds a node without a link (which is the representative)

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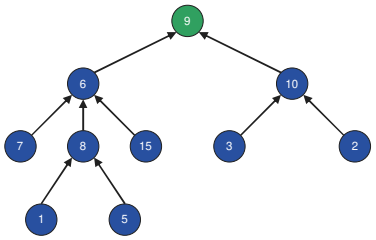
Find(5) → 9



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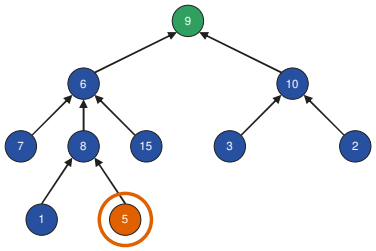
Find(5) → 9



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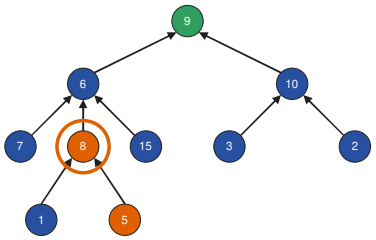
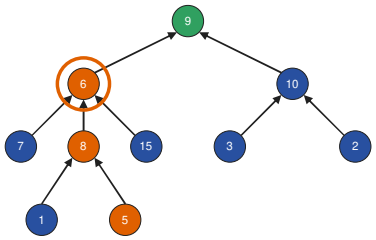
Find(5) → 9



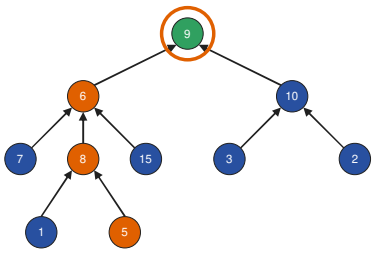
27

Find(5) → 9



28

Find(5) → 9



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Find Automatically Optimizes the Tree

- After the answer is found, all nodes, along the path, are pointed to the representative
- This means the tree optimizes by becoming "flattened" (and faster) with each find

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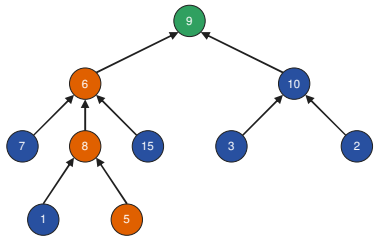
Connect Each Node to Representative



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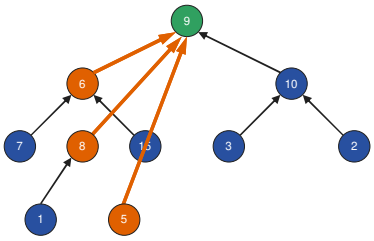
Connect Each Node to Representative



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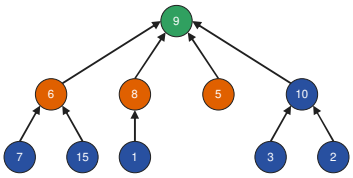
Updated, Flattened, Tree



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Union

- Combines two sets into a single set
- This is accomplished by updating a single parent link
- So, one representative will be pointed at another representative

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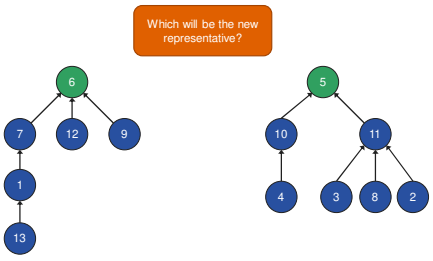
So, which one do we choose?



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Approach #1: Union by Weight

- Using *Union by Weight*, the tree with the fewer nodes (not edge weight) is made a subtree of the larger tree
- This helps create a more balanced union in terms of weight
- Again, the Find operation will optimize the tree

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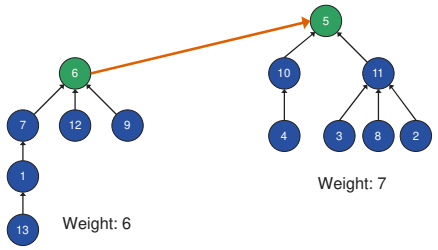
So, which one do we choose?



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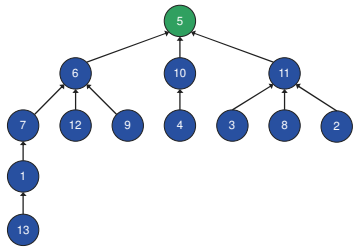
Approach #1: Result



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Approach #2: Union By Height

- Using *Union by Height*, the tree with the smaller height is made a subtree of the taller tree
- This helps create a more balanced union in terms of height
- But, Find, will fix this automatically

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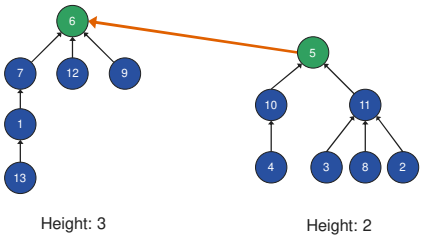
Approach #2: Union By Height



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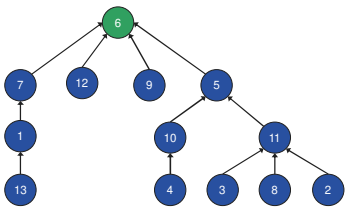
Approach #2: Result



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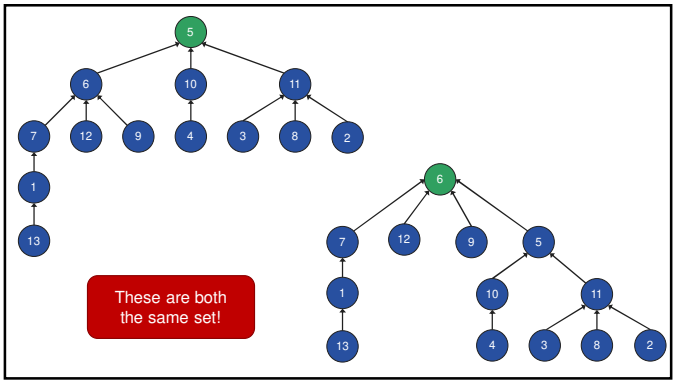

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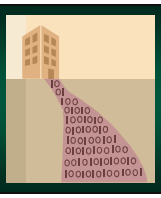

Bit Vectors

Sets and Bits

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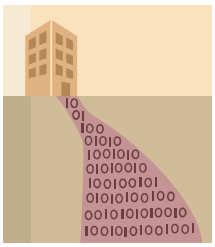
Bit Vectors

- A *bit vector* can store finite, countable sets using bits
- Also known as a *bit array*, *bit set*, and *bit map*
- Compact format that can perform a set operations with a single operation *(fast!)*



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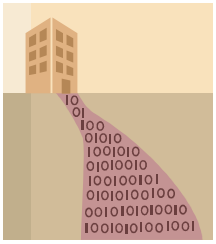
Bit Vectors



- Each object in the universe is represented as a single bit in the string of bits
- If x ∈ A, then the bit is **1**, otherwise **0**

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Example 1

- U = { fry, leela, zoidberg, bender, hermes }
- A = { fry, leela, bender }

```
U = 11111
A = 11010
```


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Example 2

- U = { 2, 3, 5, 7, 11, 13, 17, 19}
- A = { 3, 5, 11, 19 }

```
U = 11111111
A = 01101001
```


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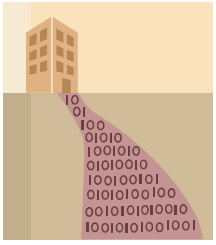
Why this is useful



- Computers can easily perform and & or operations on bytes (or multiple bytes)
- This means set operations can be performed amazingly fast

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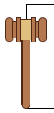
Let's look at the definitions again…

- The definitions of union and intersection are nearly identical
- The relationship between the elements is defined using an *AND* or *OR*



$A \cup B = \{ x \mid x \in A \text{ or } x \in B \}$
$A \cap B = \{ x \mid x \in A \text{ and } x \in B \}$

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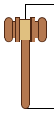
Let's look at the definitions again…

- We can apply *bit-wise-and* & *bit-wise-or*
- It will apply the operation to each of the bits in matching columns



$A \cup B = \{ x \mid x \in A \text{ or } x \in B \}$
$A \cap B = \{ x \mid x \in A \text{ and } x \in B \}$

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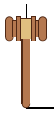
Let's look at the definitions again…

- So, each bit in A will be compared to its matching bit in B
- Bit match can do sets!



$A \cup B = \{ x \mid x \in A \text{ or } x \in B \}$
$A \cap B = \{ x \mid x \in A \text{ and } x \in B \}$

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Example: Union (using or)

```
U = {a, b, c, d, e, f, g}
A = {b, c, d} = 0111000
B = {d, e, f} = 0001110

    0111000
 or 0001110
    0111110 = {b, c, d, e, f}
```


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Example: Intersection (using and)

```
U = {a, b, c, d, e, f, g}
A = {b, c, d} = 0111000
B = {d, e, f} = 0001110

    0111000
and 0001110
    0001000 = {d}
```


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Complement

- Then, how do we do a complement of a set A?
- We must flip all the bits from 1 to 0, and 0 to 1
- We can use a *binary-not* or the *XOR* operation

$A' = \{ x \mid x \notin A \}$

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Example: Complement (using not)

```
U = {a, b, c, d, e, f, g}
A = {b, c, d} = 0111000

not 0111000
    1000111 = {a, e, f, g}
```


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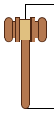
Exclusion

- Finally, how do we do set difference?
- The "subtract" operator will not work
- Let's look at the definition a bit more closely



$$A - B = \{ x \mid x \in A \text{ and } x \notin B \}$$

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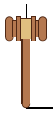
Exclusion

- It's essentially the definition of *intersection*
- Except, the second operand is the definition of *complement*.



$$A - B = \{ x \mid x \in A \text{ and } x \notin B \}$$

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Example: Intersection (using and)

```
U = {a, b, c, d, e, f, g}
A  = {b, c, d, f} = 0111010
B  = {d, e, f}    = 0001110
B' = not 0001110  = 1110001

    0111010
and 1110001
    0110000 = {b, c}
```


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Java/C Code

```
Intersection :  a & b
Union        :  a | b
Complement   :  ~a
Exclusion    :  a & ~b
```

`&&` and `||` are Boolean operators. These are bit-wise.

The tilde ~ is a bitwise not.

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