



Graph Shortest Path

Part 16

1



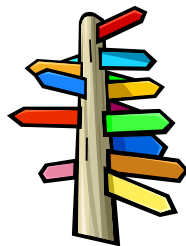
Dijkstra's Algorithm

Finding the best route!

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Getting the Shortest Path

- The Kruskal and Prim Algorithms can efficiently create minimum spanning trees
- And these are the "best" trees that *span all nodes*



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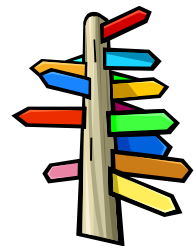
Seccombe, Stein, Gask, CS51, 100

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Getting the Shortest Path

- However, it doesn't necessarily produce the minimum path for *each* vertex to reach other vertices
- In other words, the shortest path for each, *between two vertices*, might not be reflected in the MST



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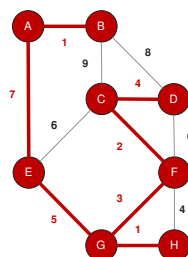
Seccombe, Stein, Gask, CS51, 100

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Prim-Jarník's Algorithm Example

- This is the spanning tree created by both the Prim and Kruskal Algorithms
- Let's say we want to get the minimal path from **B** to **D**



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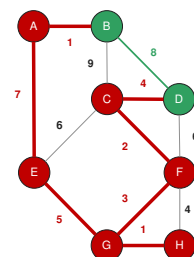
Seccombe, Stein, Gask, CS51, 100

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Prim-Jarník Algorithm Example

- The incident edge from vertex **B** to **D** has a weight of **8**
- However, following the spanning tree, the total weight would be **22**



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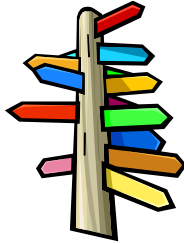
Seccombe, Stein, Gask, CS51, 100

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Getting the Shortest Path

- We need to compute the best path for **each** vertex
- This is called the *distance*
- Given 2 vertices *a* and *b* ...
 - find a path with the "best" total weight between *a* and *b*
 - ...with the minimum sum



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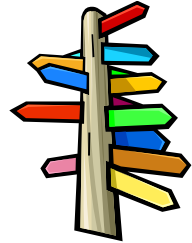
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Some Real World Examples

- Internet packet routing
 - fastest route – faster downloads
 - load distribution
- Driving directions
 - shortest route in miles
 - fastest route – given traffic and speed limit data



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Shortest Path Attributes

- A sub-path of a shortest path
 - ...is itself a shortest path
 - so, *any subset of a optimal solution is optimal*
- There is a tree of shortest paths
 - there are multiple solutions from the start vertex to all the other vertices
 - might be multiple equally-optimal solutions

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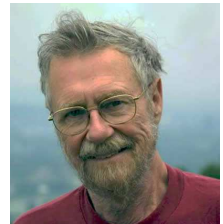
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Edsger Dijkstra

- The algorithm used today to find the optimal path was written by Edsger Dijkstra
- In fact, his algorithm runs the Internet – a self healing, load distributing, network
- **No Dijkstra → No Internet**



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Dijkstra's Algorithm

- *Dijkstra's Shortest Path First Algorithm* computes the distances of all the vertices from a given start vertex *s*
- Works on directed and undirected graphs
- However...
 - all edges must have nonnegative weight, and...
 - the graph is connected

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Dijkstra's Algorithm

- Dijkstra's Algorithm is *greedy*
 - *always* takes the *best* immediate or local solution while finding an answer
 - they find optimal solutions for some optimization problems very efficiently
 - ...but may find less-than-optimal solutions
- Extra analysis is needed to determine if they work - *Dijkstra's does*

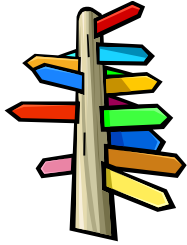
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Dijkstra's Algorithm Logic



- Each vertex has its own "distance" table
- The table contains the best weight to each other vertex - as well as the best path

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Dijkstra's Algorithm Logic

- Initialize a distance table
 - table represents the best distance (weight) to get to the specific vertices
 - set all to infinity (worst possible)
 - it will be updated as we see more vertices
- Analyze vertices until all are known (visited)
 - always select the one with the smallest distance
 - treat the vertices as being in a priority queue*

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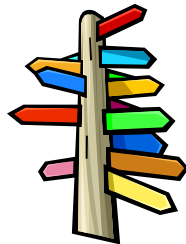
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Dijkstra's Algorithm Logic

- Given table D, vertex v , edge weight w with target t :
 $D(t) = D(v) + w$
- For example:
 - if it takes 12 to get to v
 - ... and an edge of 6 goes to t
 - it takes $12 + 6$ total to get to t



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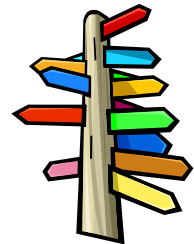
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Dijkstra's Algorithm Logic

- For each vertex v ...
 - compute the total distance to get each vertex t that is adjacent to v
 - if the value is better than the current table value then update the table



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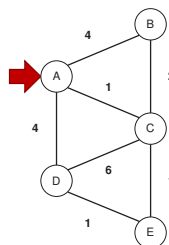
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Example 1

- Unknown
- Known
- Examining
- Visible edge



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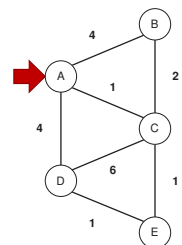
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Example 1: Vertex A Table

Vertex	Best Path	Distance	Look
A	A	0	
B		∞	
C		∞	
D		∞	
E		∞	



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Example 1: Check Edges

Vertex	Best Path	Distance	Look
A	A	0	
B		∞	$0 + 4$
C		∞	$0 + 1$
D		∞	$0 + 4$
E		∞	

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Example 1: Update Table

Vertex	Best Path	Distance	Look
A	A	0	
B	A B	4	$0 + 4$
C	A C	1	$0 + 1$
D	A D	4	$0 + 4$
E		∞	

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Example 1: Select Next

Vertex	Best Path	Distance	Look
A	A	0	
B	A B	4	
C	A C	1	
D	A D	4	
E		∞	

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Example 1: Check Edges

Vertex	Best Path	Distance	Look
A	A	0	
B	A B	4	$1 + 2$
C	A C	1	
D	A D	4	$1 + 6$
E		∞	$1 + 1$

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Example 1: Update Table

Vertex	Best Path	Distance	Look
A	A	0	
B	A C B	3	$1 + 2$
C	A C	1	
D	A D	4	$1 + 6$
E	A C E	2	$1 + 1$

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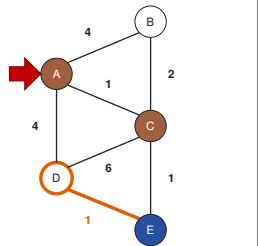
Example 1: Select Next

Vertex	Best Path	Distance	Look
A	A	0	
B	A C B	3	
C	A C	1	
D	A D	4	
E	A C E	2	

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Example 1: Check Edges

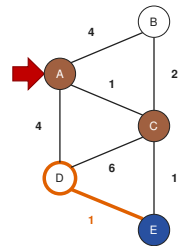
Vertex	Best Path	Distance	Look
A	A	0	
B	ACB	3	
C	AC	1	
D	AD	4	2 + 1
E	ACE	2	



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Example 1: Update Table

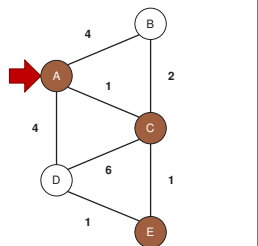
Vertex	Best Path	Distance	Look
A	A	0	
B	ACB	3	
C	AC	1	
D	ACED	3	2 + 1
E	ACE	2	



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Example 1: Select Next

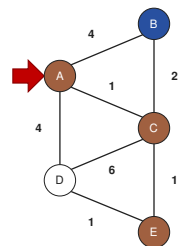
Vertex	Best Path	Distance	Look
A	A	0	
B	ACB	3	
C	AC	1	
D	ACED	3	
E	ACE	2	



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Example 1: Vertex A Table

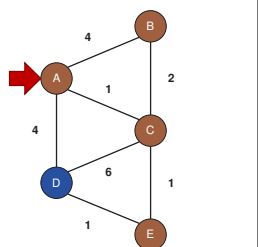
Vertex	Best Path	Distance	Look
A	A	0	
B	ACB	3	
C	AC	1	
D	ACED	3	
E	ACE	2	



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Example 1: Vertex A Table

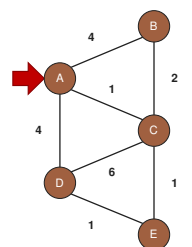
Vertex	Best Path	Distance	Look
A	A	0	
B	ACB	3	
C	AC	1	
D	ACED	3	
E	ACE	2	



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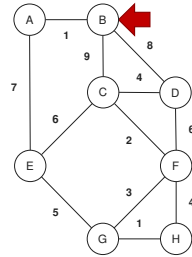
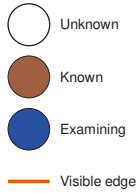
Example 1: Vertex A Table

Vertex	Best Path	Distance
A	A	0
B	ACB	3
C	AC	1
D	ACED	3
E	ACE	2



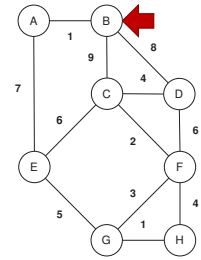
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Example 2



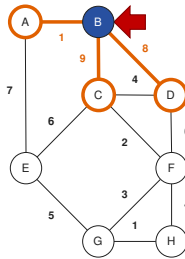
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Vertex	Best Path	Distance	Look
A		∞	
B	B	0	
C		∞	
D		∞	
E		∞	
F		∞	
G		∞	
H		∞	



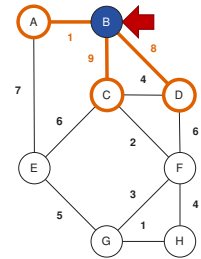
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Vertex	Best Path	Distance	Look
A		∞	0 + 1
B	B	0	
C		∞	0 + 9
D		∞	0 + 8
E		∞	
F		∞	
G		∞	
H		∞	



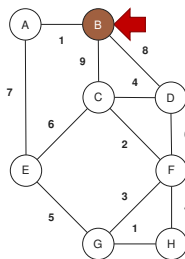
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Vertex	Best Path	Distance	Look
A	B A	1	0 + 1
B	B	0	
C	B C	9	0 + 9
D	B D	8	0 + 8
E		∞	
F		∞	
G		∞	
H		∞	



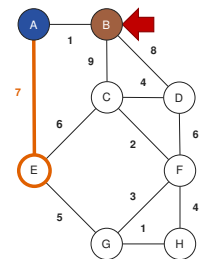
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Vertex	Best Path	Distance	Look
A	B A	1	
B	B	0	
C	B C	9	
D	B D	8	
E		∞	
F		∞	
G		∞	
H		∞	

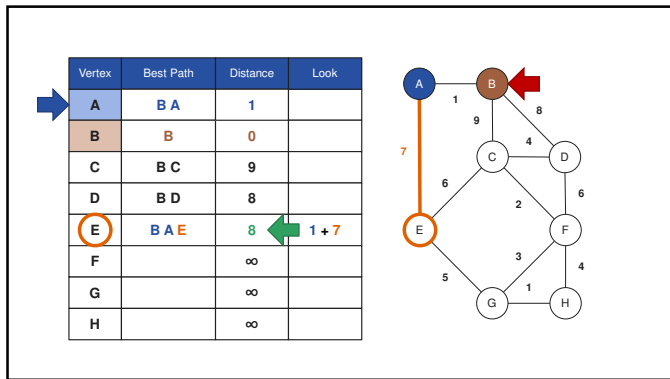


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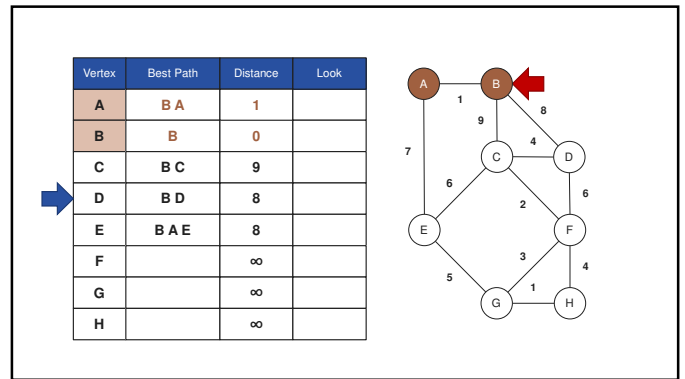
Vertex	Best Path	Distance	Look
A	B A	1	
B	B	0	
C	B C	9	
D	B D	8	
E		∞	1 + 7
F		∞	
G		∞	
H		∞	



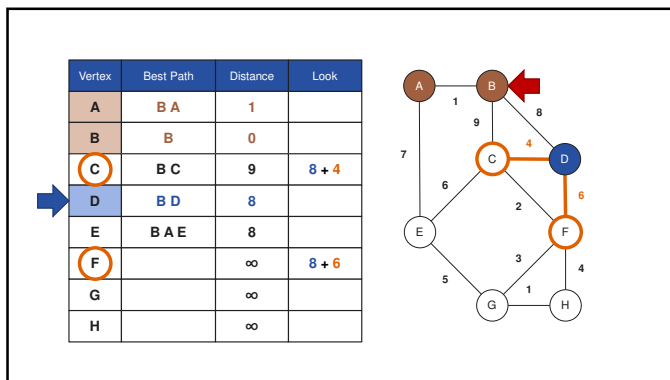
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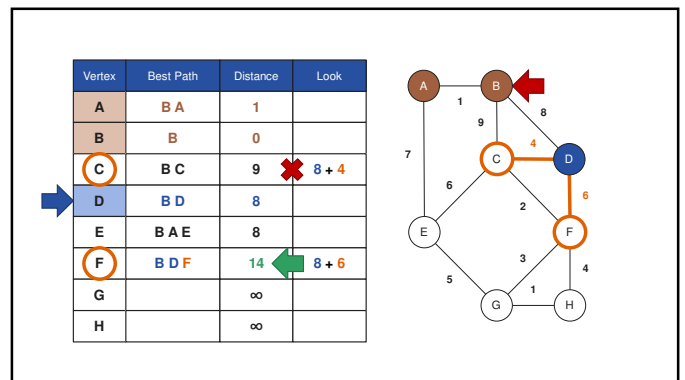
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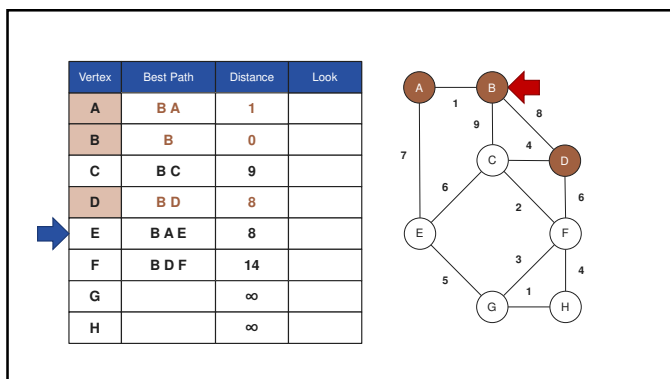
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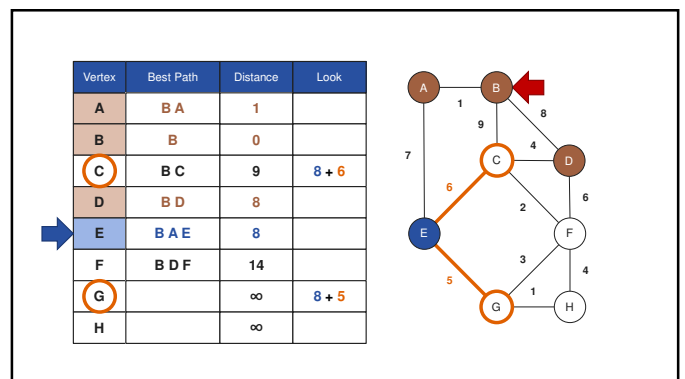
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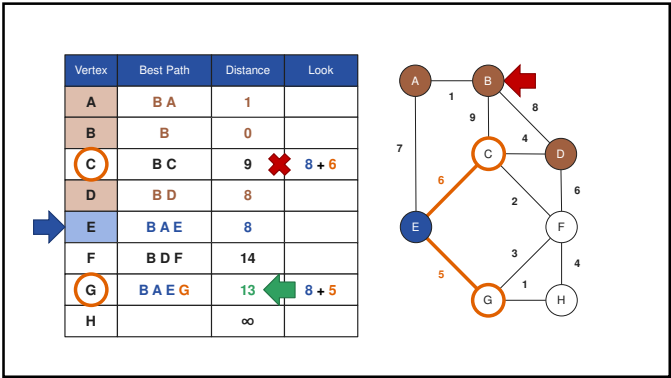
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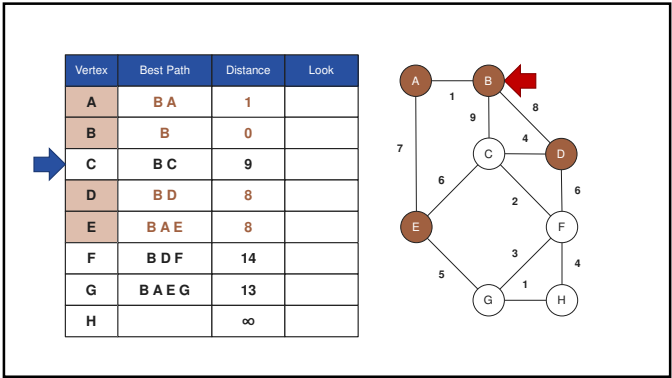
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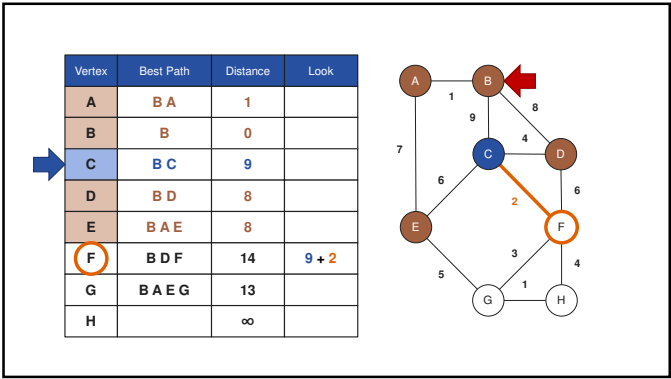
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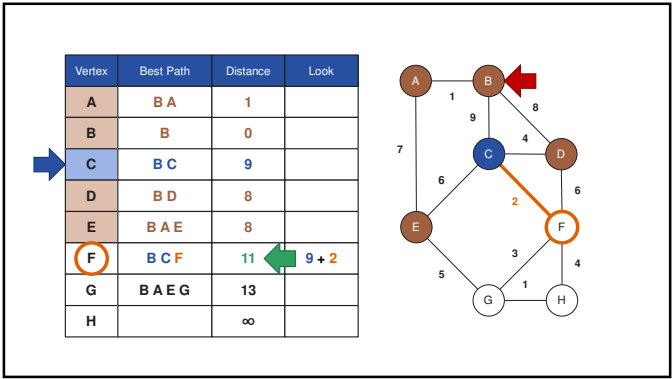
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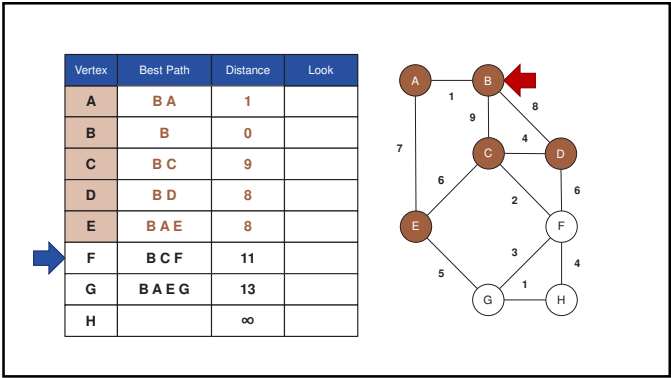
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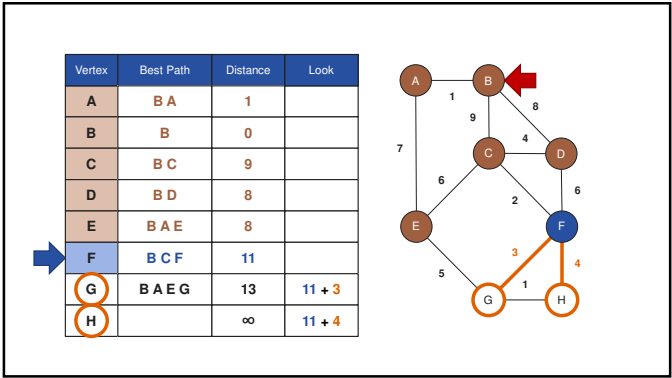
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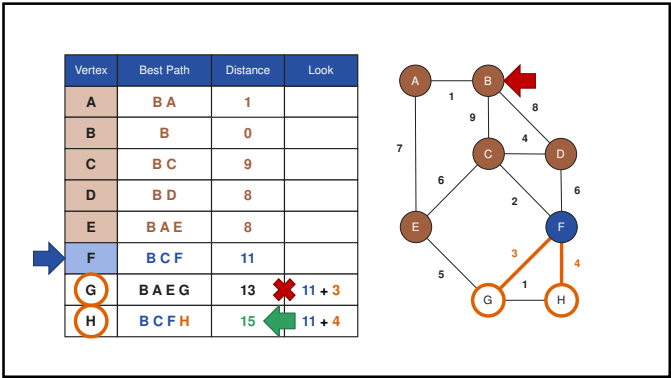
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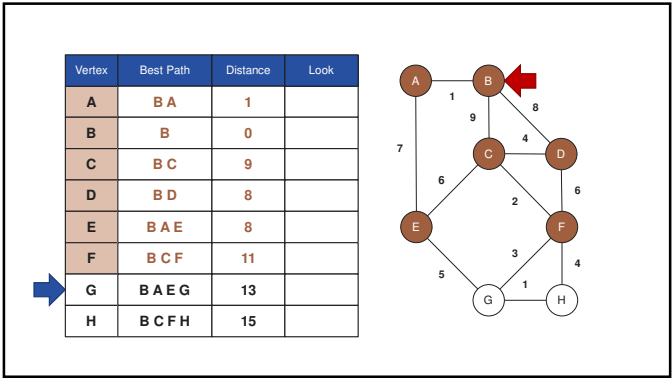
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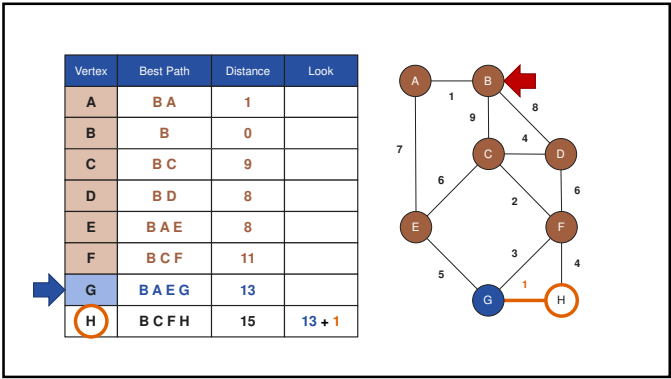
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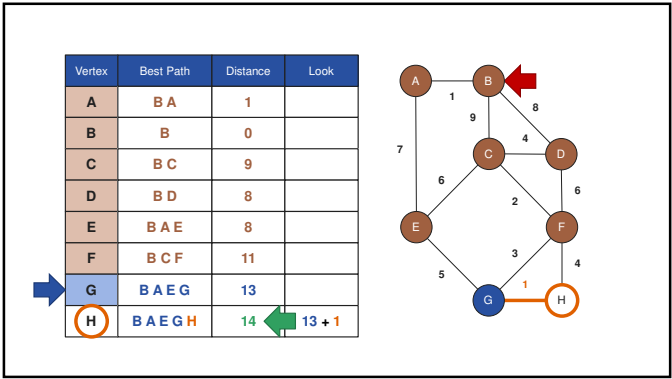
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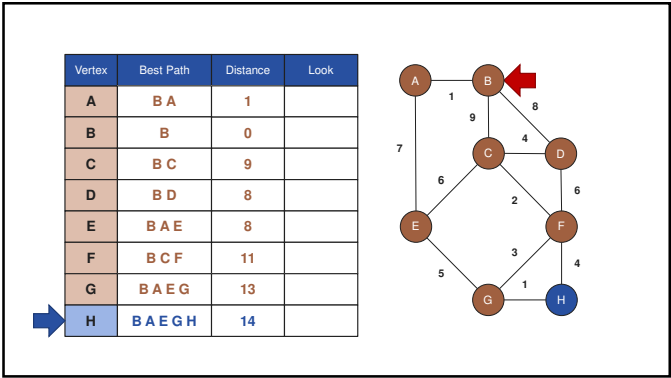
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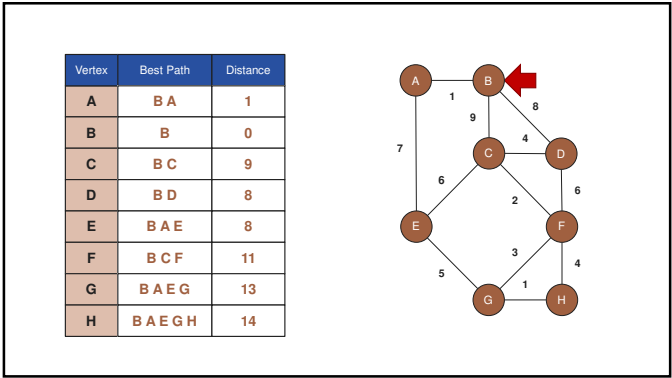
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Let's Look at California

Driving Around the Golden State

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Let's Look At California

- The cities of California are connected in many different ways
- Whether it is communications, water, power, transportation, etc... each is its own graph

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Let's Look At California

- Let's look at the California freeway system and run the Prim-Jarník and Dijkstra algorithms on it
- So, let's get a MST and a nice shortest-path for Sacramento

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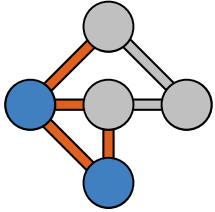
These are the major highways connecting these cities

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Same graph with weight in miles

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Minimal Spanning Tree



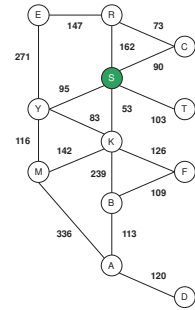
- Now that we have a graph representation of California, let's use Prim-Jarník's Algorithm to compute the minimal spanning tree
- We can start any node, but let's use Sacramento (S)

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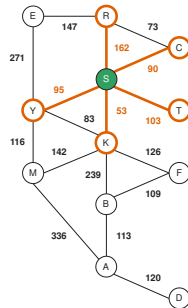
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Prim-Jarník's Minimal Spanning Tree Algorithm



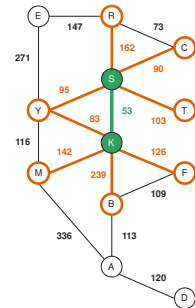
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Prim-Jarník's Minimal Spanning Tree Algorithm



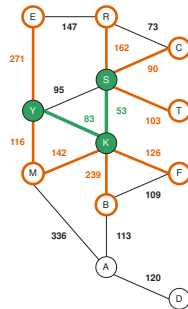
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Prim-Jarník's Minimal Spanning Tree Algorithm



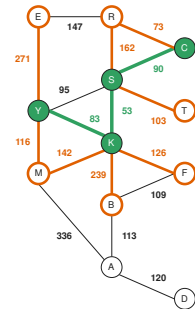
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Prim-Jarník's Minimal Spanning Tree Algorithm



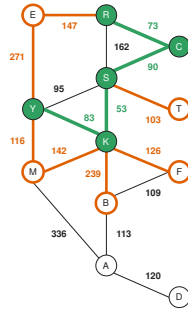
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Prim-Jarník's Minimal Spanning Tree Algorithm



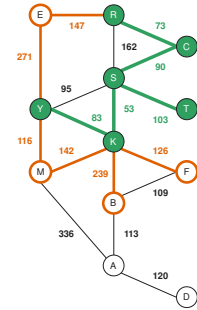
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Prim-Jarník's Minimal
Spanning Tree Algorithm



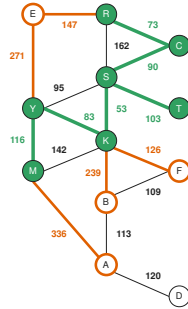
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Prim-Jarník's Minimal
Spanning Tree Algorithm



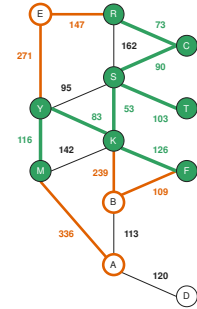
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Prim-Jarník's Minimal
Spanning Tree Algorithm



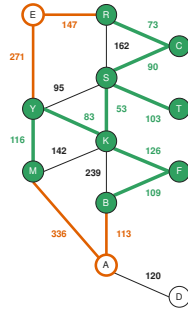
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Prim-Jarník's Minimal
Spanning Tree Algorithm



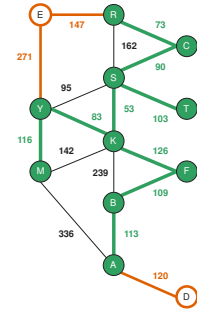
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Prim-Jarník's Minimal
Spanning Tree Algorithm

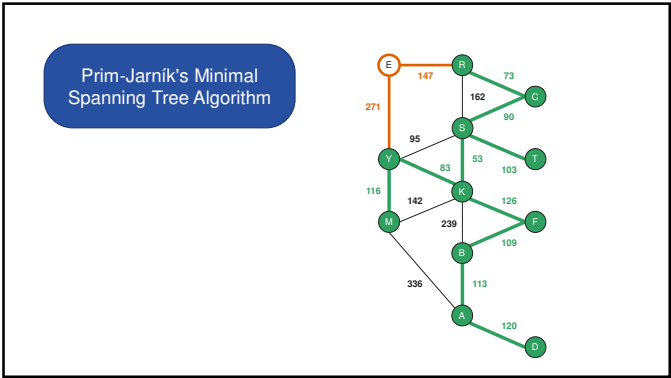


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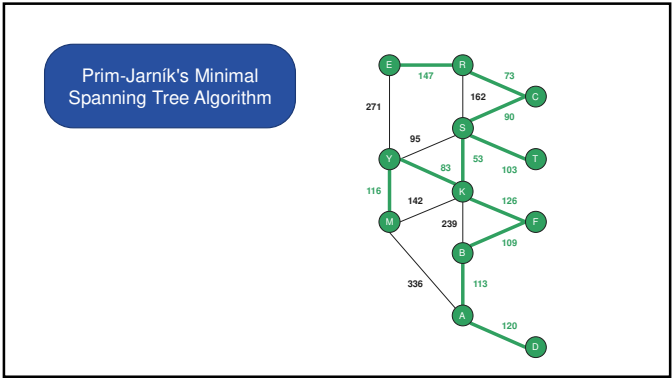
Prim-Jarník's Minimal
Spanning Tree Algorithm



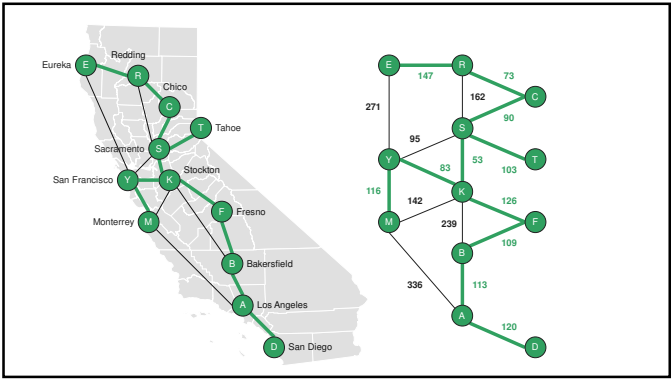
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Let's Compute the Best Path

- Given we are in Sacramento, let's compute a shortest path table for it using Dijkstra's Algorithm
- This will take awhile... but it will work

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Sacramento, California Example

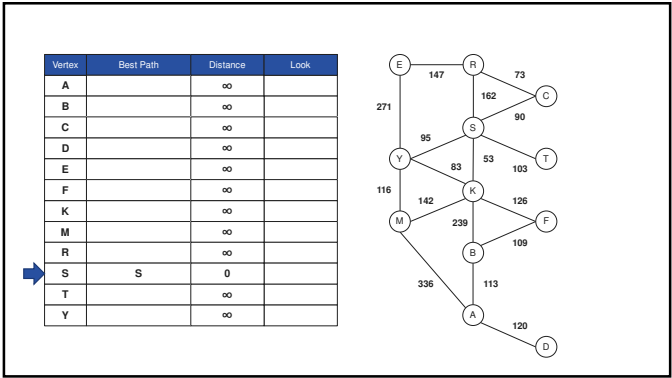
Unknown

Known

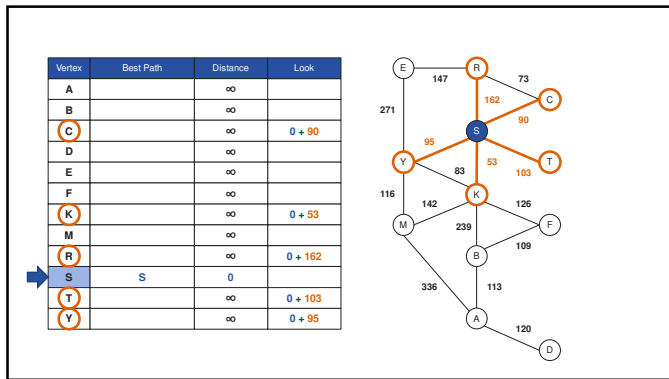
Examining

Visible edge

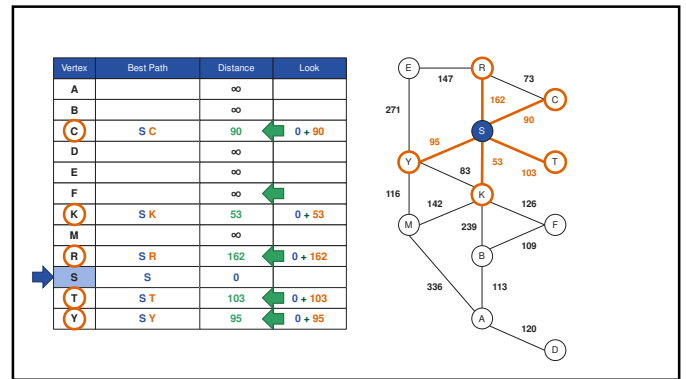
77



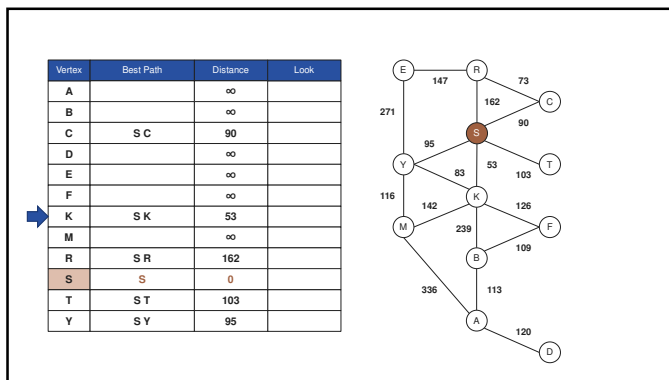
78



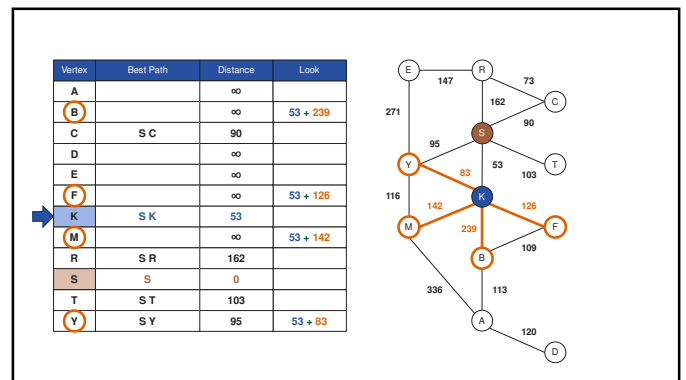
79



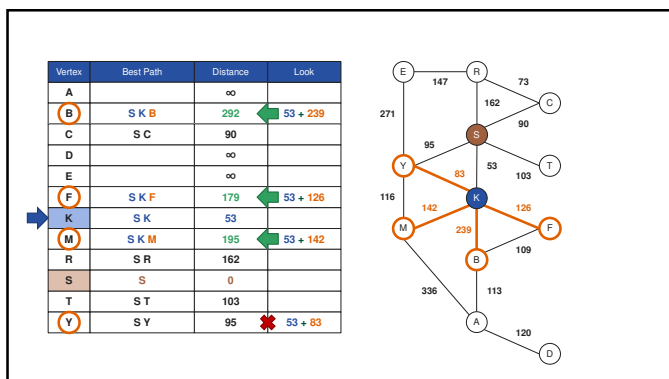
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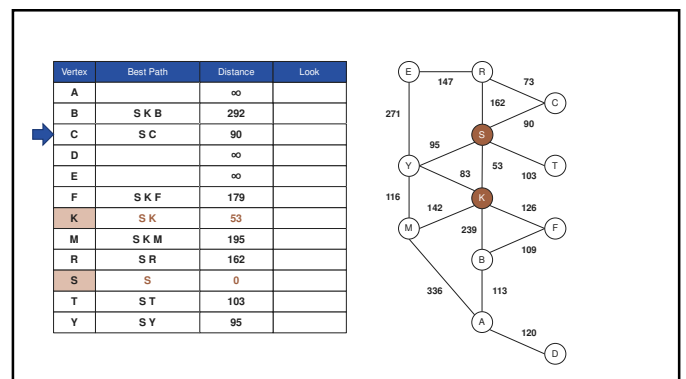
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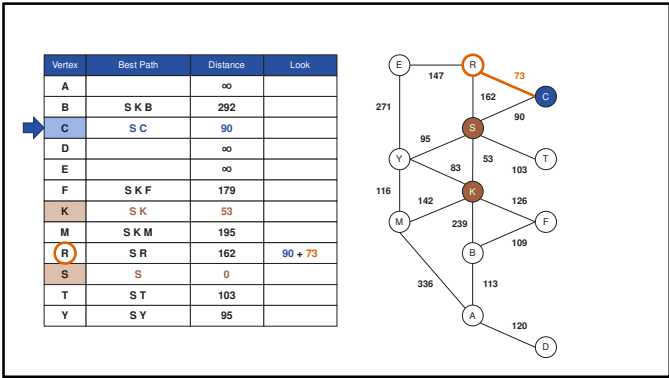
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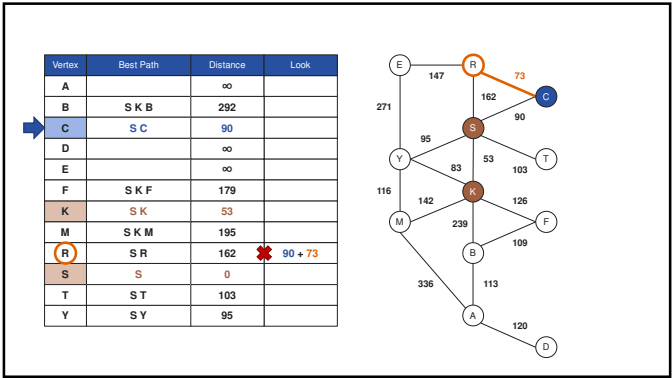
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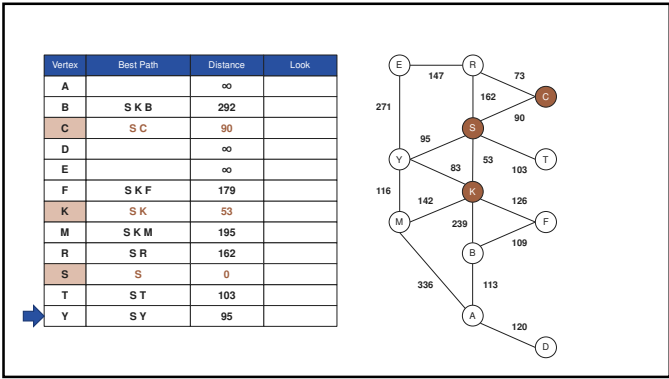
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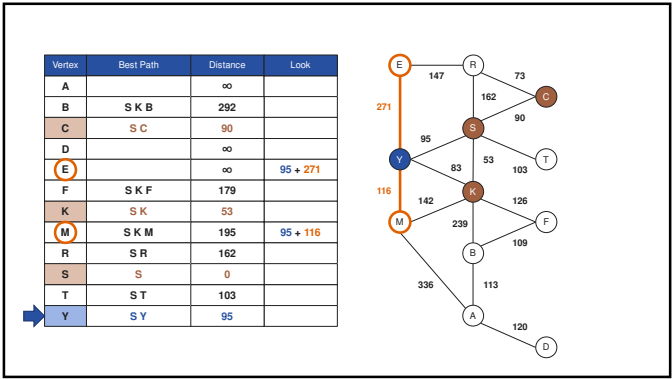
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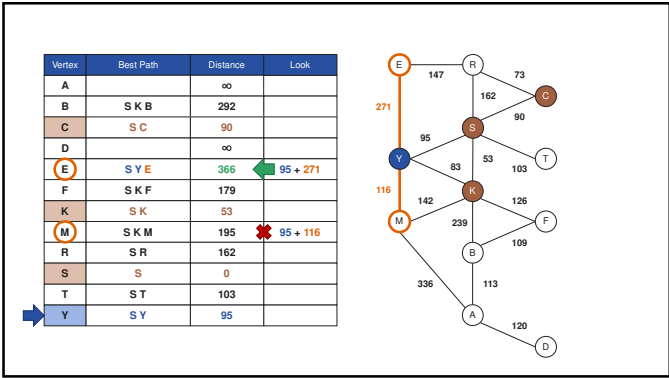
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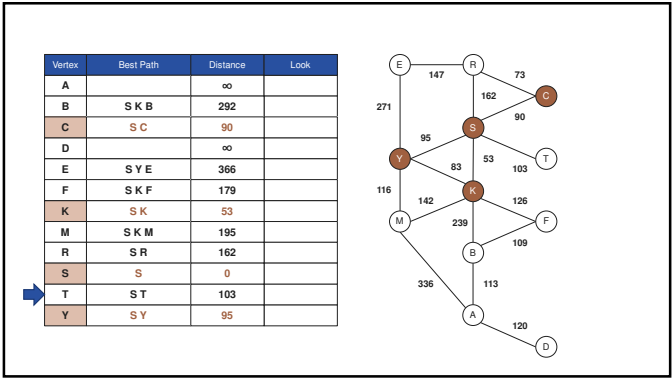
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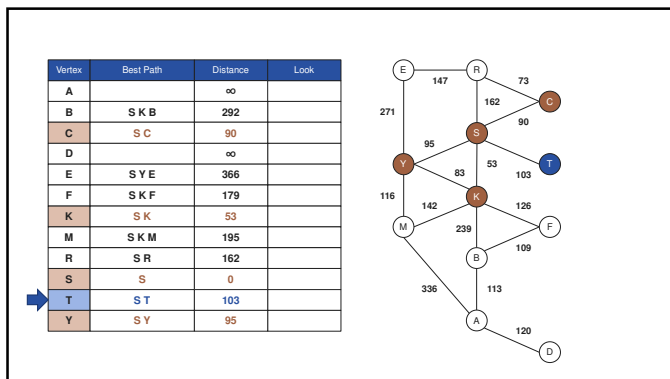
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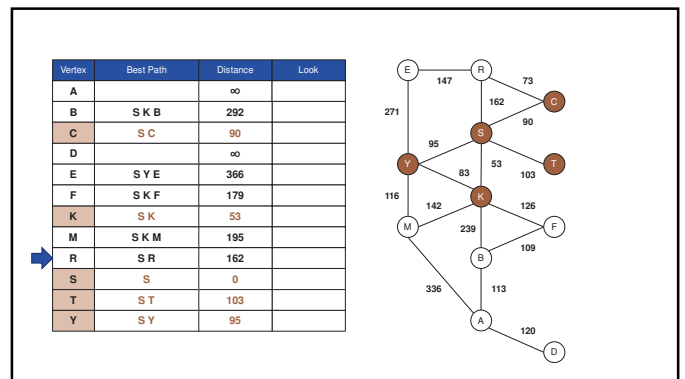
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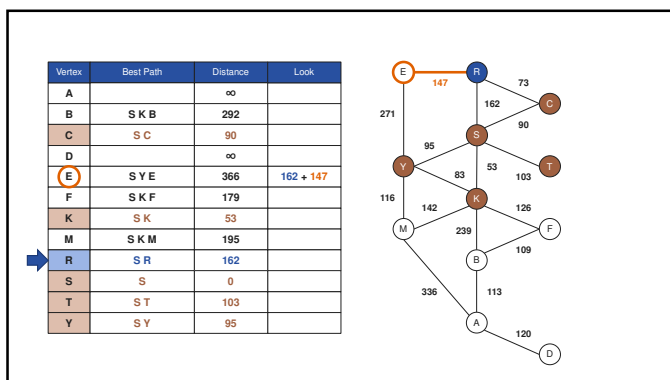
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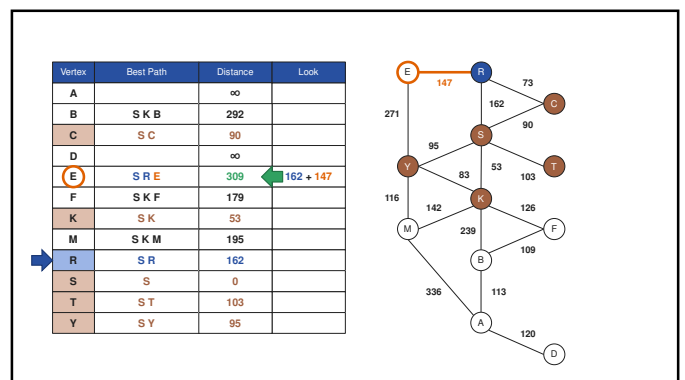
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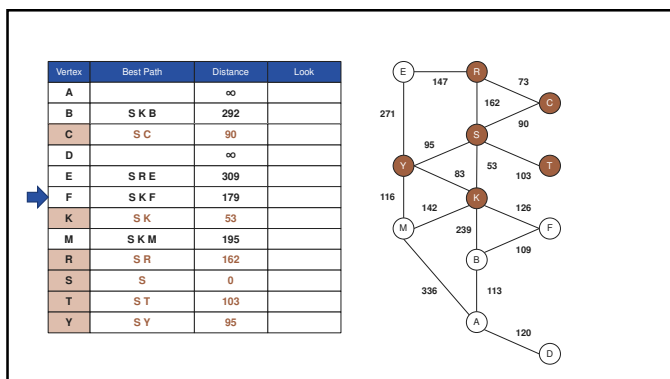
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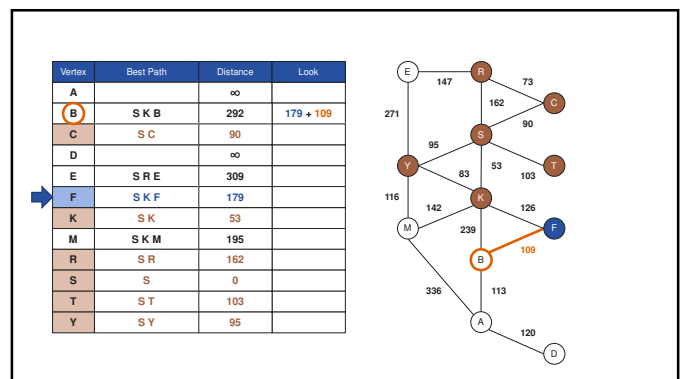
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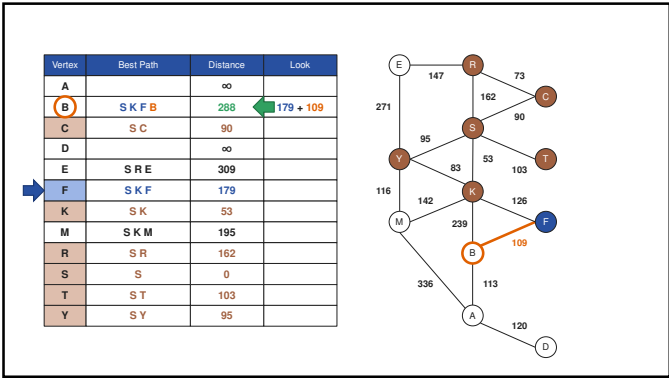
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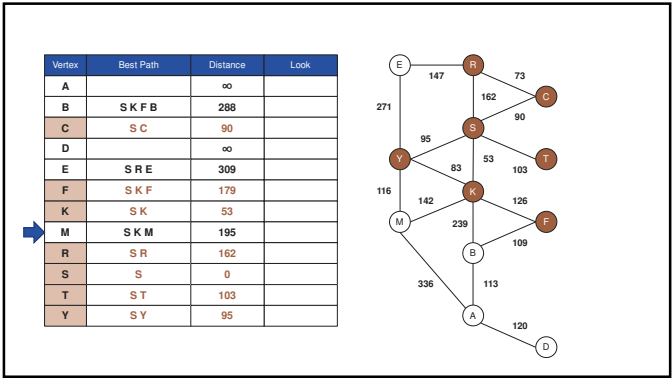
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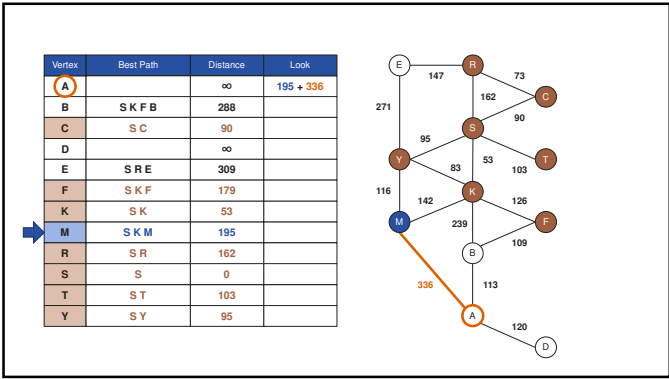
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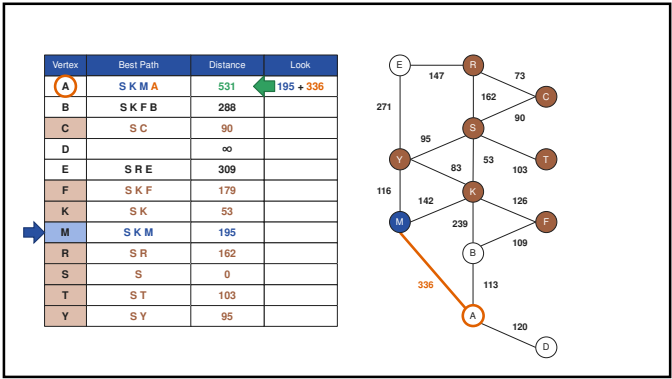
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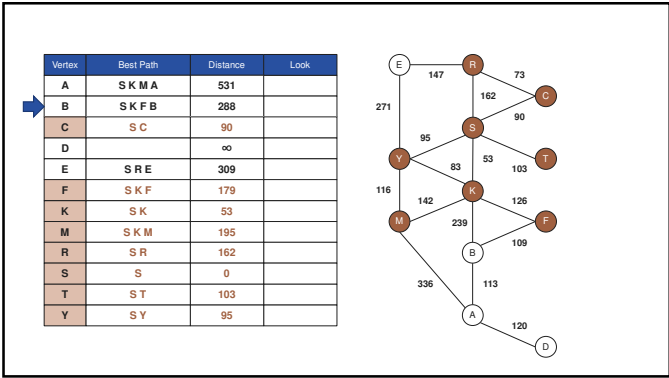
98



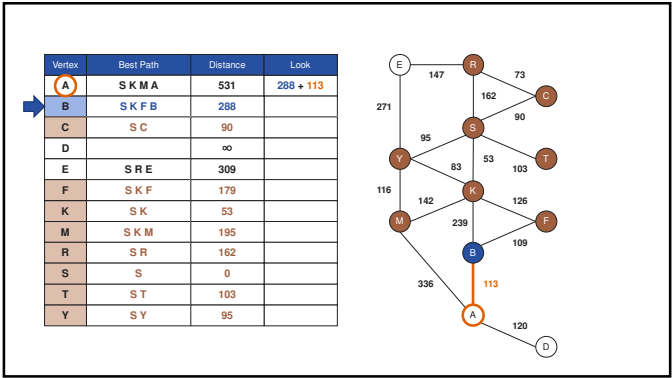
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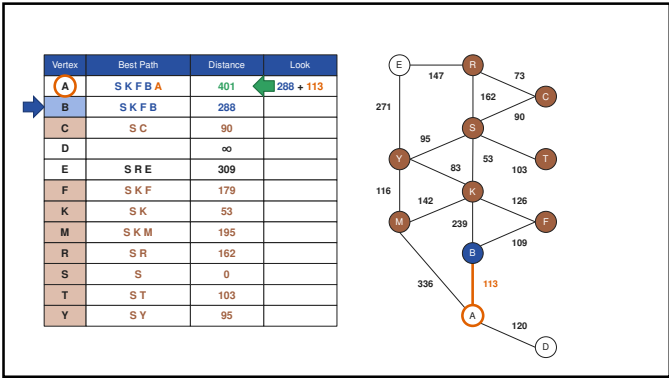
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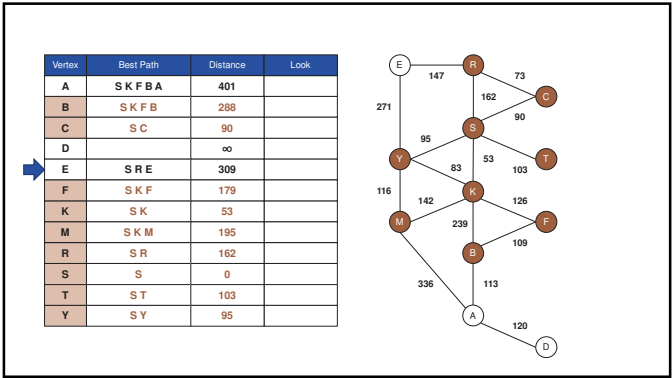
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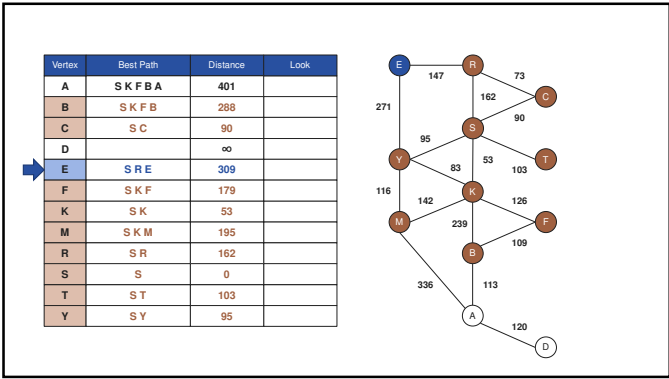
102



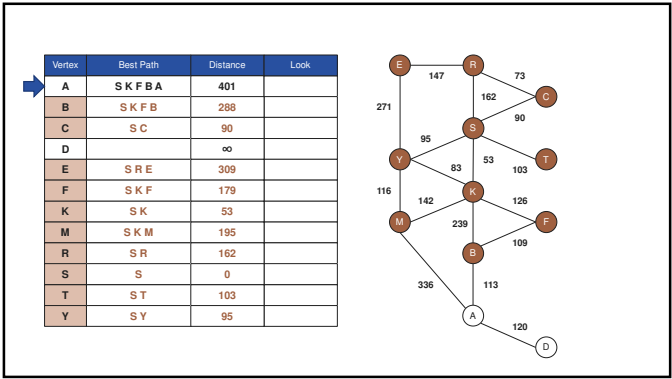
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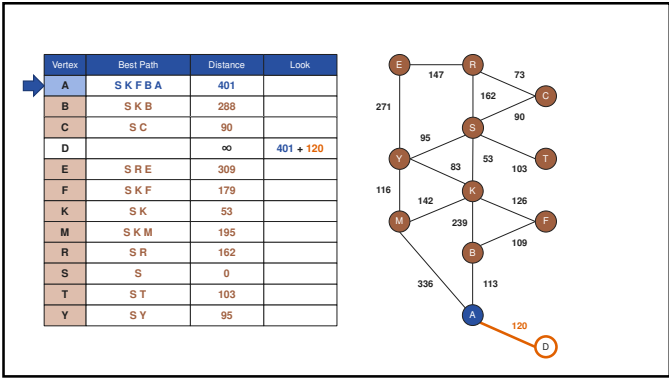
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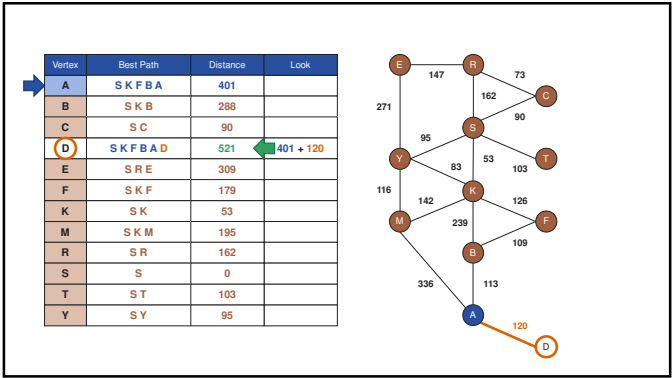
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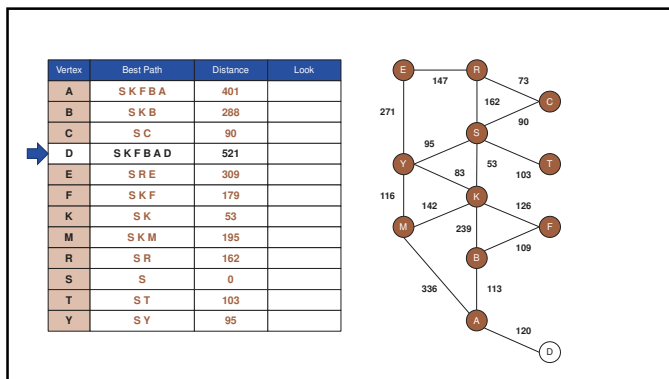
106



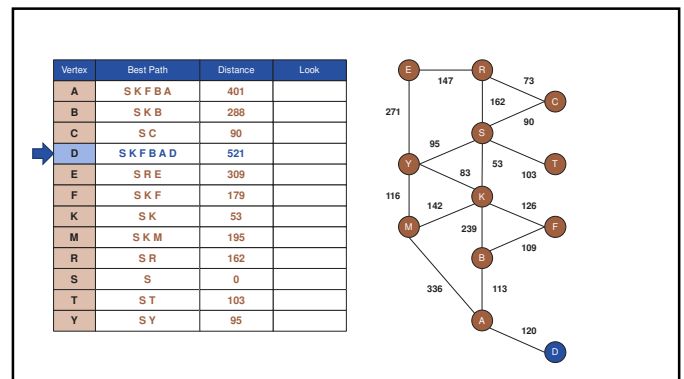
107



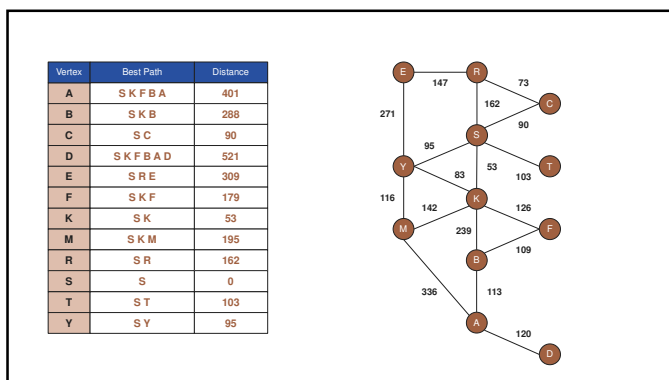
108



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111

Best Path Complete

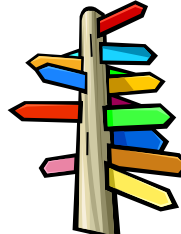


- It appears like the best route from Sacramento to San Diego is on the minimal spanning tree
- That makes sense... no "short cut" roads can be built

FIG 2021 Sacramento State - Gosh - CSX 100 112

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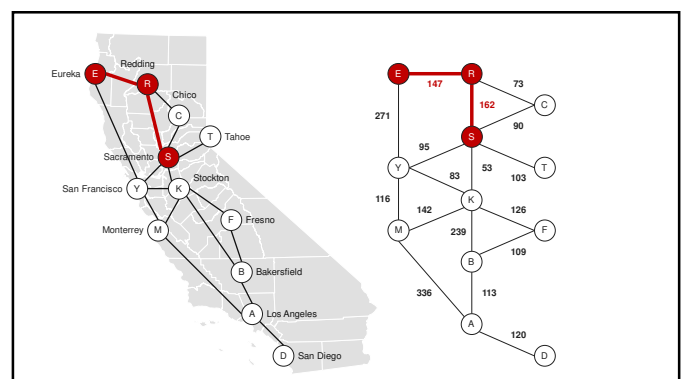
Best Path Complete



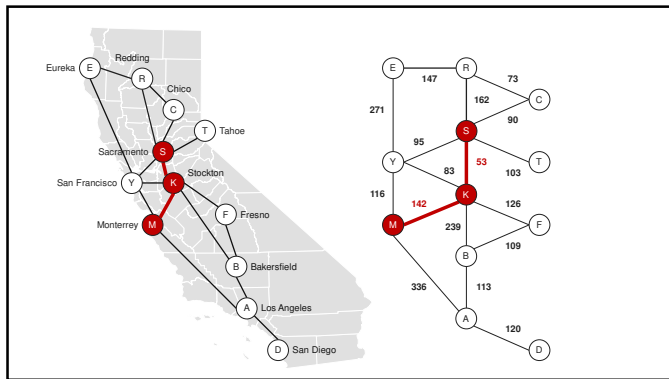
- However, our best path isn't always on the MST
- Path to Eureka skips Chico
- Path to Monterey goes around San Francisco

FIG 2021 Sacramento State - Gosh - CSX 100 113

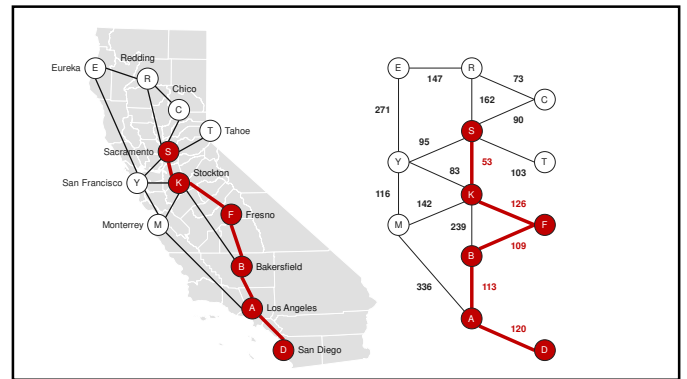
113



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California Result

- But, of course, if you going to go to San Diego, then you might want to take Hwy 5
- It goes around Fresno – which while a great city – does have some traffic (*which will slow you down*)



Slide 117 contains a green header with the text "California Result". Below the header, there is a list of two bullet points. The first bullet point says "But, of course, if you going to go to San Diego, then you might want to take Hwy 5". The second bullet point says "It goes around Fresno – which while a great city – does have some traffic (*which will slow you down*)". To the right of the text is a thumbs up icon. At the bottom of the slide, there is a small footer that reads "Fall 2021 Sacramento State - CSIS - CSIS 100 107".

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