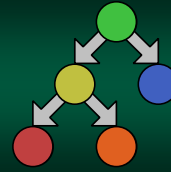




## Binary Search Trees

Part 10

1



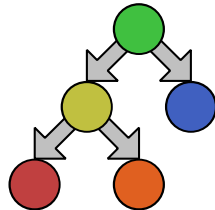
## Binary Search Trees

Climbing a tree has never been so fun

2

### Binary Search Trees

- *Binary Search Tree (BST)* is a special type of binary tree that sorts nodes by value
- For each node...
  - all the nodes on the left branch are less than it
  - all the nodes on the right branch are greater than it



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### Searching the Tree

- Since the tree divides the problem progressively by two, the time complexity is only  $O(\log n)$
- Which gives that all the benefits of a sorted array
- Worst case is  $O(n)$  – if the tree is a list-like chain



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### Search logic (looking for S)

- Looking for If **S** is equal to the current node, you found it
- If **S** is smaller than the current node, take the left branch
- If **S** is bigger than the current node, take the right branch
- If there are no branches, **S** was not found



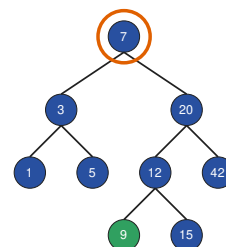
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### Searching for 9



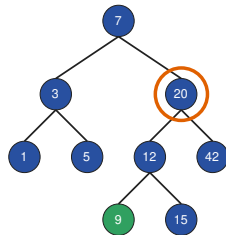
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## Searching for 9



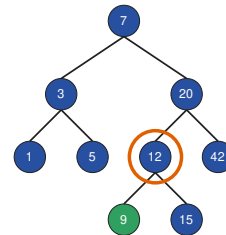
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## Searching for 9



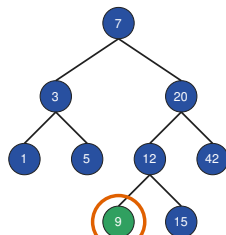
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## Searching for 9



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## Inserting into the Tree

- Inserting is handled exactly like a search
- The only difference is that if the item is not found, the node is added
- If we reach a leaf...
  - we are already at the max-depth of the tree
  - we are at the node that needs to be updated
  - so, add a left or right node (based on value)
  - ... wow, this is easy!

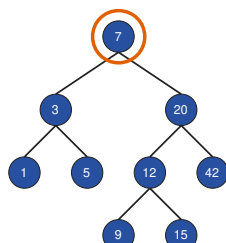
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## Inserting 4



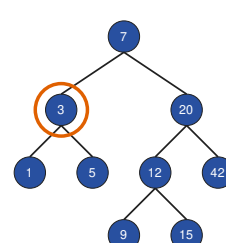
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## Inserting 4



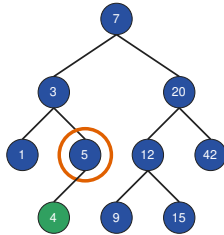
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## Inserting 4



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## Binary Search Trees vs. Arrays

- Insert into a Binary Search Tree
  - traverses down the tree
  - then add itself as a leaf
  - it requires only  $O(\log n)$
- Insert into an Array
  - must expand when new elements are added
  - ...and compacted when elements are removed
  - these requires  $O(n)$

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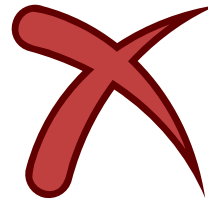
## Deleting From a BST

Not as easy as Heaps

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## Deleting From a BST



- Deleting from a BST is a tad more tricky
- When the node is deleted, the tree needs to be re-linked to *still preserve the ordering*

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## Deleting From a BST



- Fortunately, while the logic might seem a tad hard, it is fairly straight forward
- There are 3 different scenarios that must be taken into account based on the node being deleted

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## Case 1: The Node is a Leaf

- If the node (to be deleted) is a leaf, it can simply be removed from the parent
- This is due to, the fact that, nothing is linked from the node



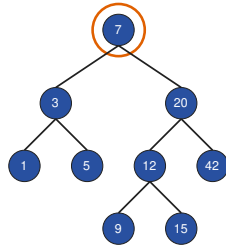
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### Case 1: Deleting 5



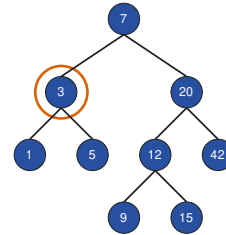
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### Case 1: Deleting 5



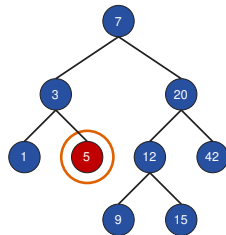
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### Case 1: Deleting 5



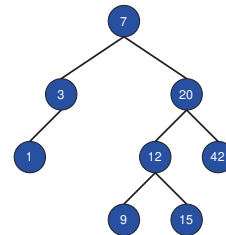
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### Case 1: Deleting 5



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### Case 2: One Child

- If the deleted node has a single child, then that child can be "*promoted*"
- The deleted node's child is moved into its position
- ... so, the subtree moves into the space created by the deleted node



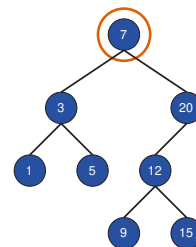
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### Case 2: Deleting 20



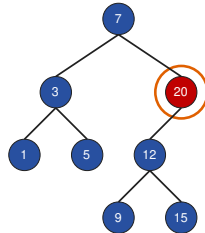
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### Case 2: Deleting 20



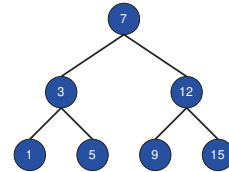
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### Case 2: Deleting 20



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### Case 3: Two Children

- However, if the deleted node has two children, then things get a tad more complex
- We need to find a node to "*promote*" to the deleted node's position



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### Case 3: Two Children

- We need to find a value that is mathematically next to the deleted value
- There are two equally valid options
- Choose one:
  - find the **maximum** node of the **left** (smaller) branch
  - ... or find the **minimum** node of the **right** (larger) branch

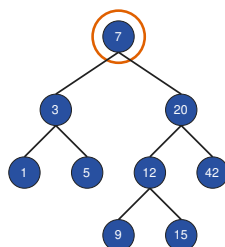
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### Case 3: Deleting 20



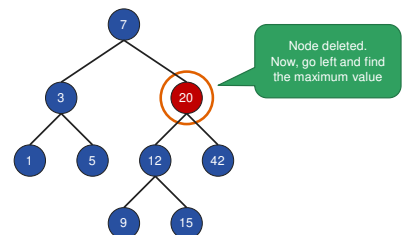
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### Case 3: Deleting 20



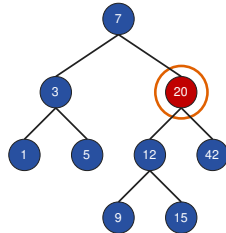
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### Case 3: Deleting 20 (Going Left)



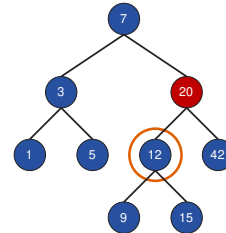
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### Case 3: Deleting 20 (Going Left)



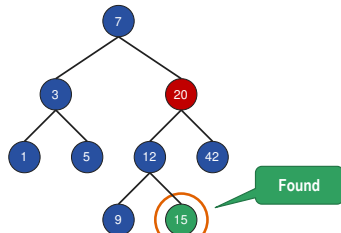
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### Case 3: Deleting 20 (Going Left)



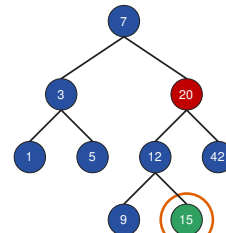
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### Case 3: Deleting 20 (Going Left)



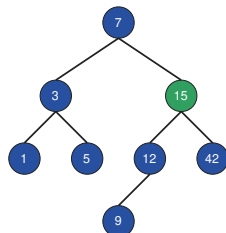
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### Case 3: Deleting 20 (Going Left)



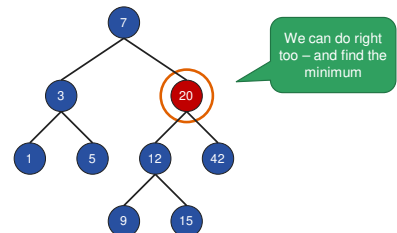
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### Deleting 20 : Going Right



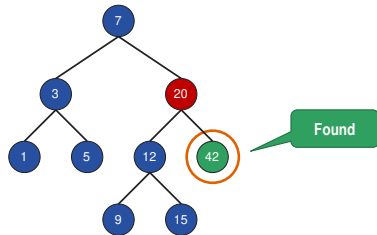
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## Deleting 20 : Going Right



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## The Problem with BSTs

Climbing a tree has never been so fun

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## The Problem with BSTs

- In a BST, internal nodes **never** change – and have a profound affect on the rest of the tree
- There are cases where the tree is unbalanced – one particular path contains most of the data



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## The Problem with BSTs

- This can easily occur if the data is not truly random (which is generally the case)
- When this happens, **the time complexity slowly deteriorates to  $O(n)$**



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## Let's Try Adding...

- Let's add the following number to a binary search tree
- How did this tree look?

<input type="radio"/>	
<input type="radio"/>	7, 8, 21, 43, 76
<input type="radio"/>	

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## Let's add: 7, 8, 21, 43, 76



42

Let's add: 7, 8, 21, 43, 76



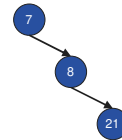
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Let's add: 7, 8, 21, 43, 76



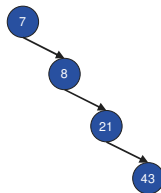
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Let's add: 7, 8, 21, 43, 76



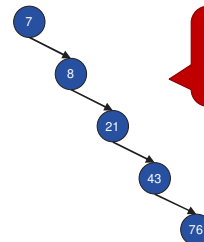
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Let's add: 7, 8, 21, 43, 76



Our tree is unbalanced & no better than a linked list!

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## Sort in a Binary Search Tree?

- Unfortunately, while this might seem like a good idea, this is not a great solution
- Binary Search Trees can deteriorate into linked lists
- So...
  - $O(\log n)$  search can quickly deteriorate to  $O(n)$
  - ...and  $O(n \log n)$  sort can deteriorate to  $O(n^2)$

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