Binary Search & Sorting

Part 4

1

Binary Search

Cutting the problem in half… many times

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Binary Searching

- A *binary search* is a fast and efficient way to search an array
- Algorithm works like the classic "secret number game"
- Requires that the array is sorted before the search

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How it Works

- Starts knowing the max & min values
  - in the case of arrays, this is the min and max index
  - in the number game, it is the min and max value
- Algorithm continues
  - it looks at the midpoint between the first and last
  - if the value > target, the max is set to the midpoint
  - if the value < target, the min is set to the midpoint
  - *this eliminates half of the numbers each iteration*

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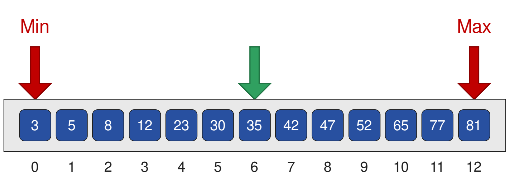
Binary Example: Find 30



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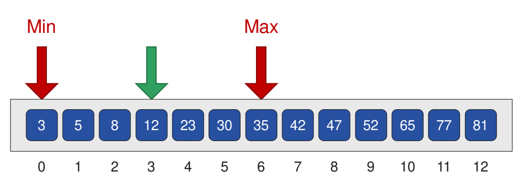
Binary Example: Find 30



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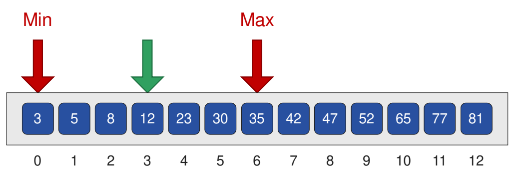
Binary Example: Find 30



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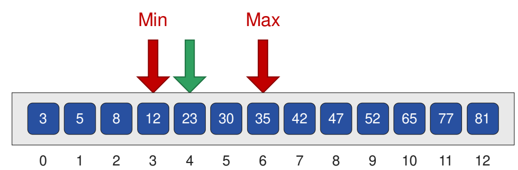
Binary Example: Find 30



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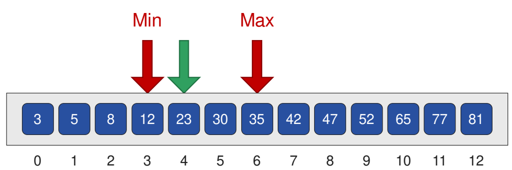
Binary Example: Find 30



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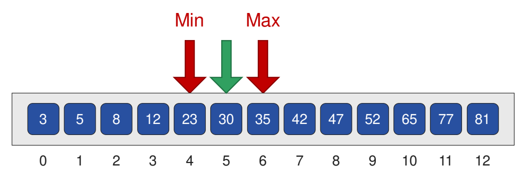
Binary Example: Find 30



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Benefits

- The binary search is incredibly efficient and **absolutely necessary** for large arrays
- Any item can be found only $\log_2(n)$ searches! It is O(log n)
- However, since array must be sorted, sorting algorithms are equally vital

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Maximum # of Searches

Array Size	Sequential	Binary
10	10	4
100	100	7
1,000	1,000	10
10,000	10,000	14
100,000	100,000	17
1,000,000	1,000,000	20
10,000,000	10,000,000	24
100,000,000	100,000,000	27
1,000,000,000	1,000,000,000	30

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Sorting

Bringing Order of "Chaos"

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Sorting

- It is useful (and efficient) to *sort* a list of data – to put it in specific order
- There are multiple sorting algorithms which get complex as they become more efficient

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Sorting

- Examples:
  - sorting scores by highest to lowest
  - sorting filenames in alphabetical order
  - sorting students by their student-id

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Sorting Algorithm Attributes

1. Time Complexity
   - Big-O classification
   - naturally, the smallest classification is better
2. Auxiliary space
   - how extra much memory is needed to run the algorithm
   - some algorithms require extra memory – *perhaps as large as the array itself*

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Sorting Algorithm Attributes

3. Stable
   - what happens when two array elements, a and b, have the same sort value?
   - if a is initially stored before b, a "stable" sort will not change their relative positions
4. Online
   - elements can be added at the same time that the data is being sorted
   - data can be *streamed* into the array at runtime

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Bubble Sort

Carbonated Sorting

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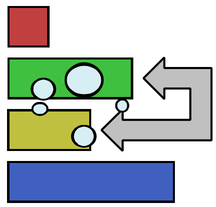
Bubble Sort

- The *bubble sort* is one of the least efficient algorithms …but it is easy to understand
- Basic approach
  - "lighter" elements "bubble up" to the top of the array
  - "heavier" items sink to the bottom



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How It Works

- Consists of two For Loops
- Outer loop runs from the first to the last
- Inner loop …
  - runs from the bottom of the array *up* to the top (well, the position of the first loop)
  - it checks every two neighbor elements, if the they are out of order, it swaps them
  - so, the smallest element moves up the array

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The Bubble Sort (Java-ish)

```
for(i = 0; i < count-1; i++)
{
   for(j = count-1; j > i; j--)
   {
      if (array[j-1] < array[j])
      {
         //swap array[j-1] and array[j]
      }
   }
}
```


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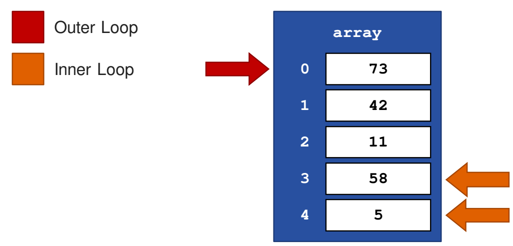
Bubble Sort Example



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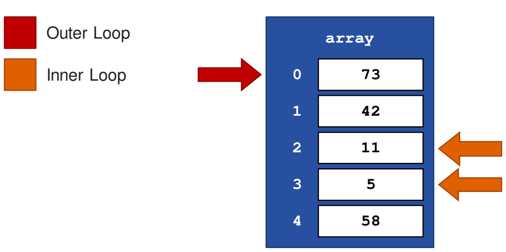
Bubble Sort Example



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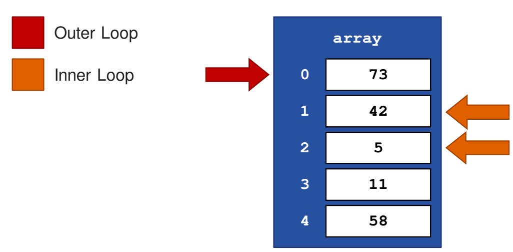
Bubble Sort Example



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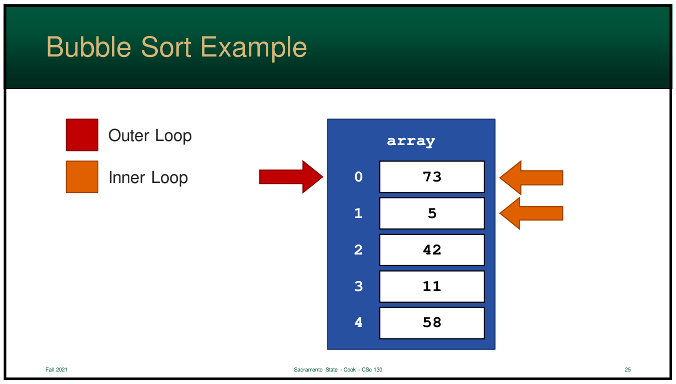

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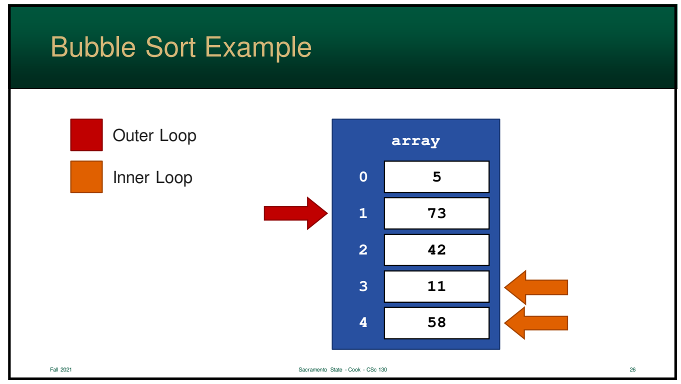

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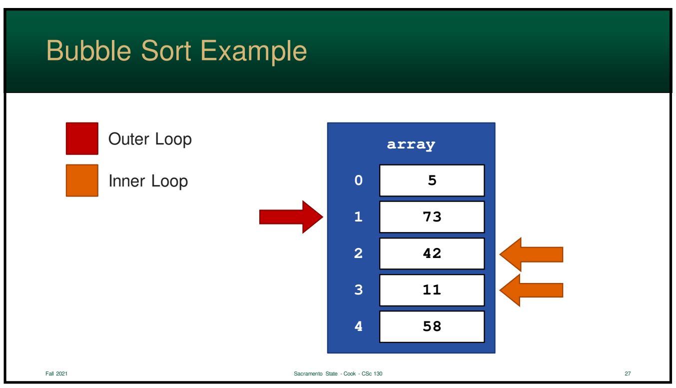

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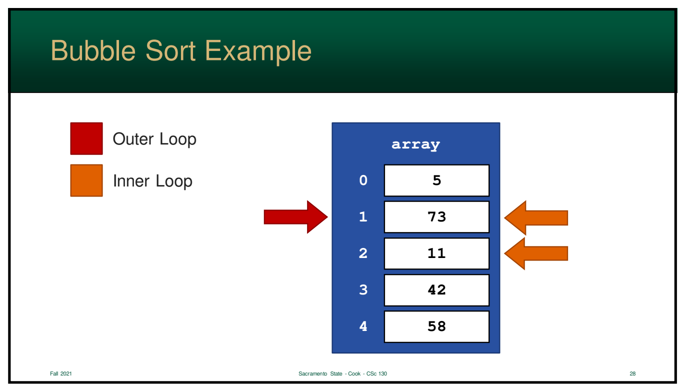

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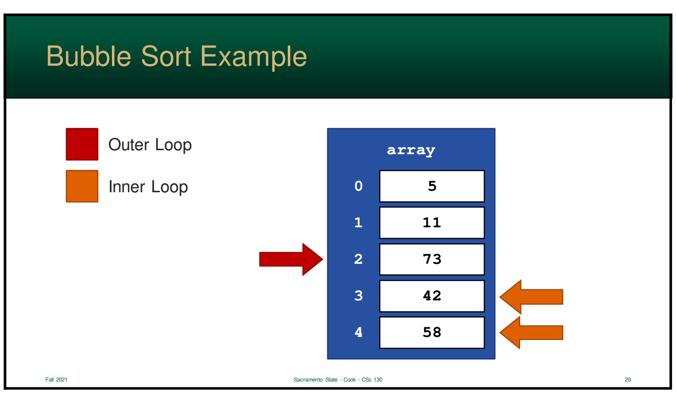

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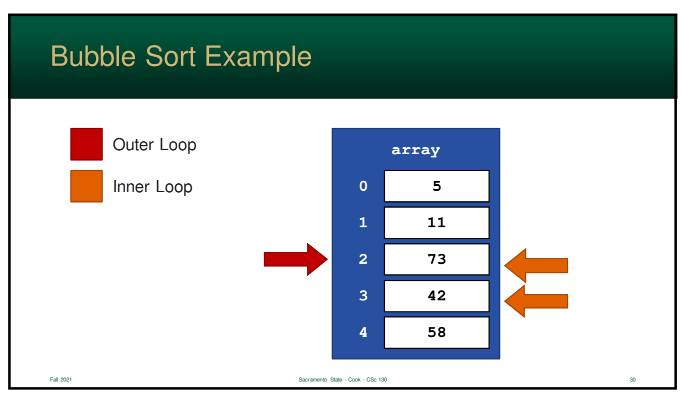

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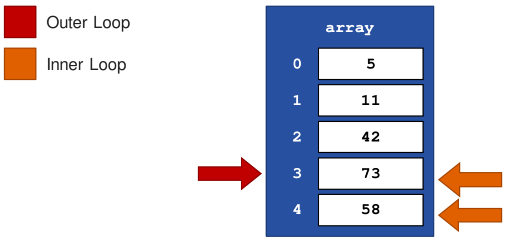
Bubble Sort Example



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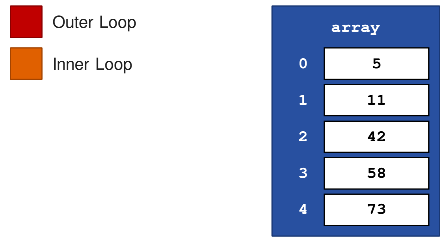
Bubble Sort Example



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Efficiency of the Bubble Sort

- The Bubble Sort is **extremely** inefficient and only good for tiny arrays
- Since Bubble Sort uses two embedded loops
  - the outer loop looks at all *n* items
  - the inner loop looks at basically *n* items
  - the resulting algorithm gets exponentially less efficient as *n* increases

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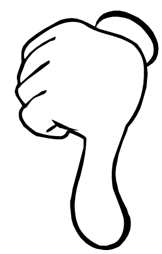
Efficiency of the Bubble Sort

- The Bubble Sort $O(n^2)$
- … two embedded loops that are based on *n*
- … and all that swapping doesn't help either!



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Bubble Sort Summary

Bubble Sort	
Time Average	$O(n^2)$
Time Best	$O(n^2)$
Time Worst	$O(n^2)$
Auxiliary space	O(1)
Stable	Yes – Equal element order preserved
Online?	No – Entire array in use

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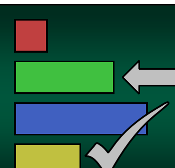

Selection Sort

The Human Way

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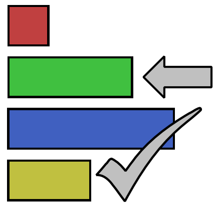
Selection Sort

- The *Selection Sort* is a similar to the Bubble Sort
- However…
  - rather than "bubble up" smaller items, it scans the entire array
  - it finds the smallest element
  - only *then* does it swap the values



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Selection Sort

- Like the Bubble Sort, it consists of two For Loops – one outer and one inner
- Outer loop runs from the first to the last
- Inner loop …
  - starts at the position of the outer loop
  - scans down and finds the *smallest* value
- Then, after the scan, do a single swap

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The Selection Sort

```
for(i = 0; i < count-1; i++)
{
    best = i;
    for(j = i; j < count; j++)
    {
        if (array[j] < array[best])
        {
            best = j;
        }
    }
    //swap array[i] and array[best]
}
```


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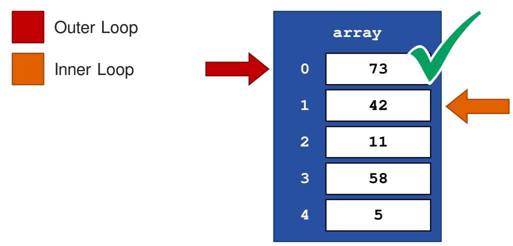
Selection Sort Example



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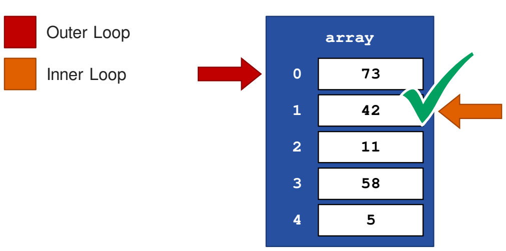
Selection Sort Example: New Best



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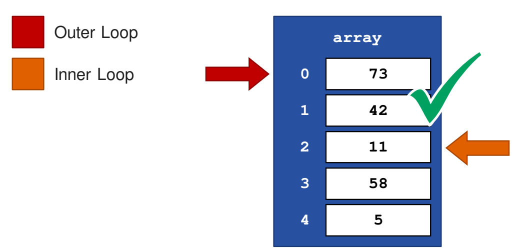
Selection Sort Example



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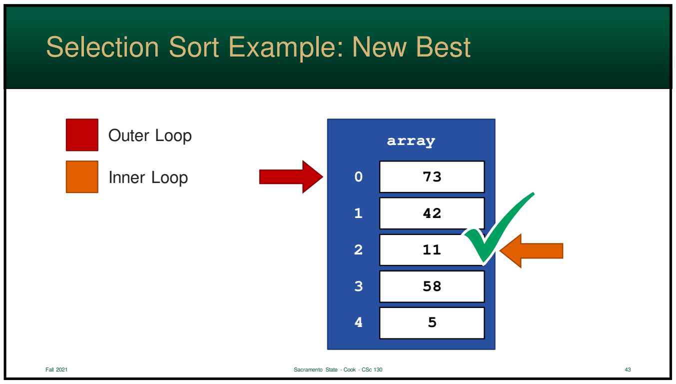

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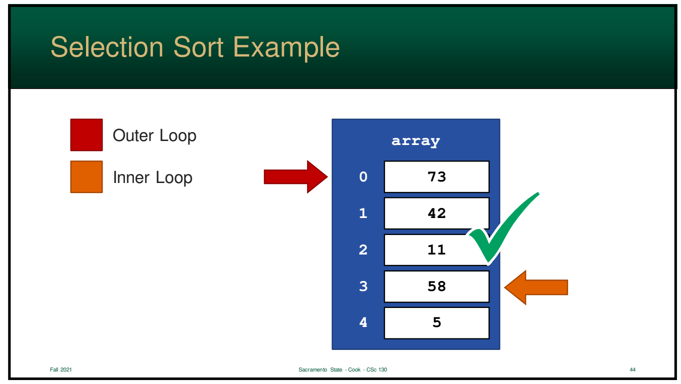

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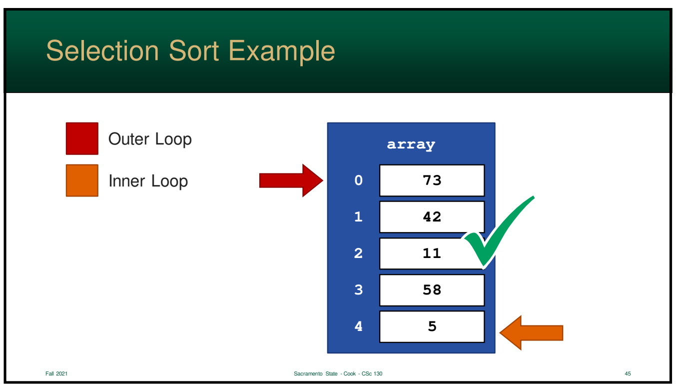

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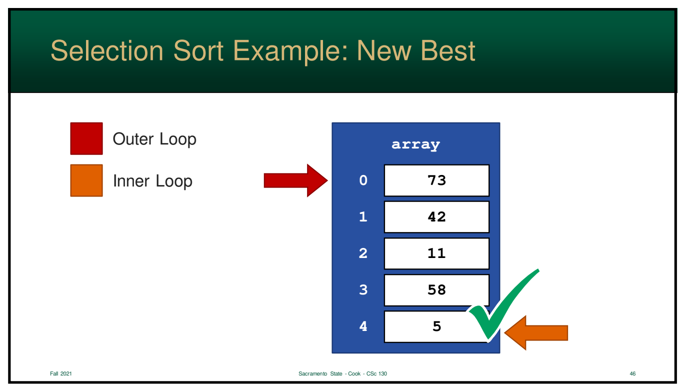

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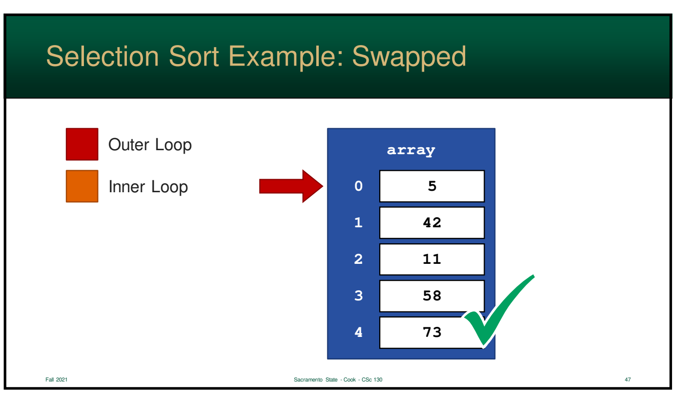

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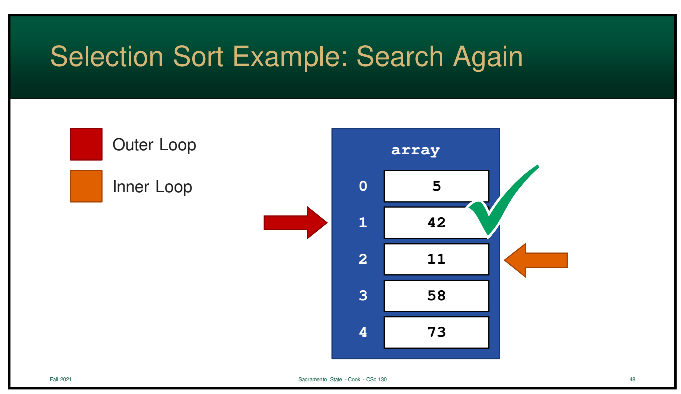

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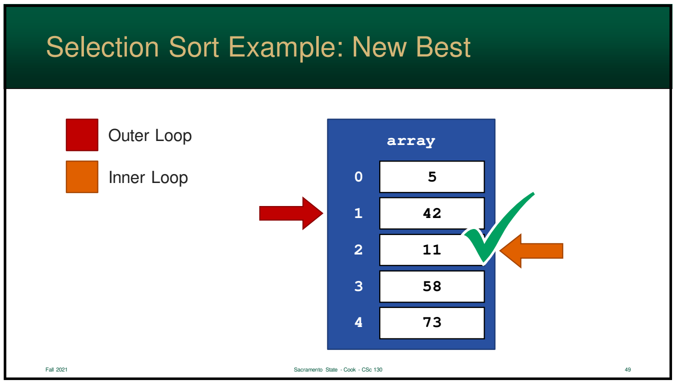

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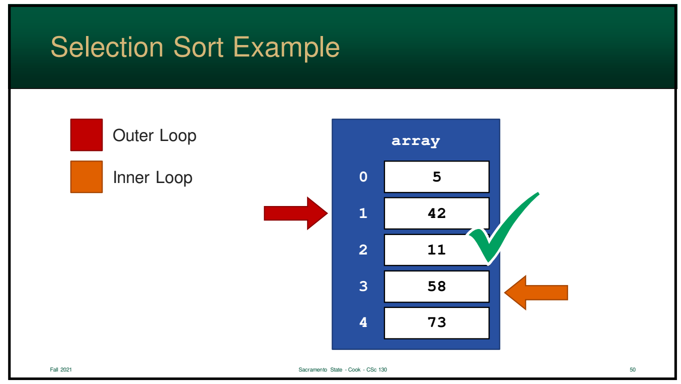

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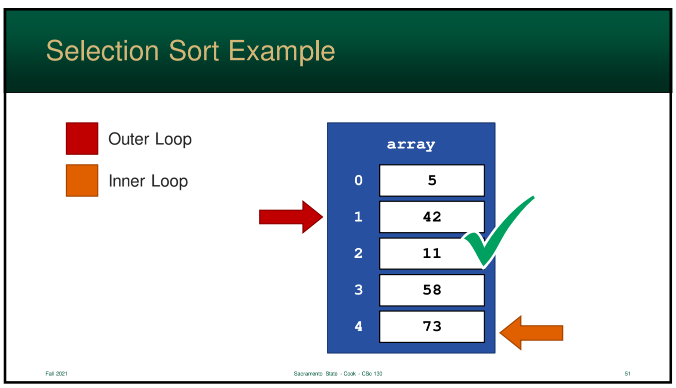

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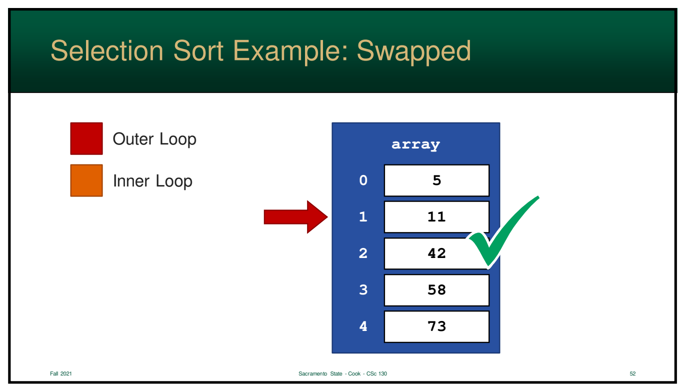

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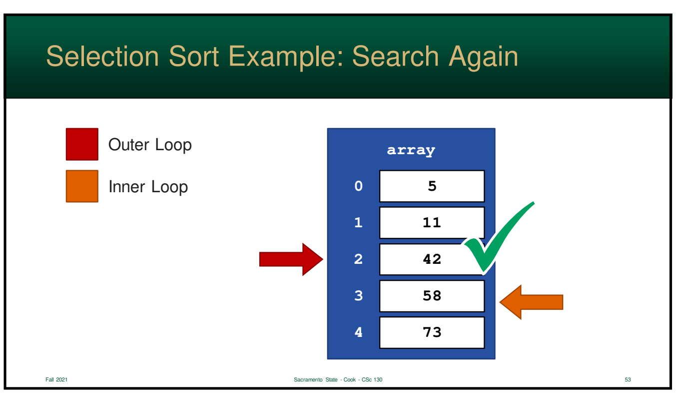

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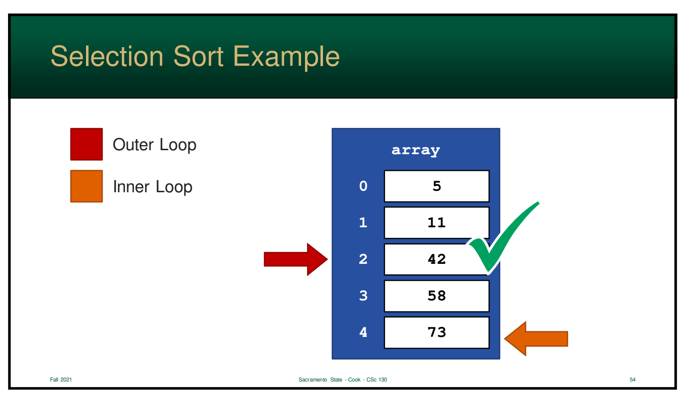

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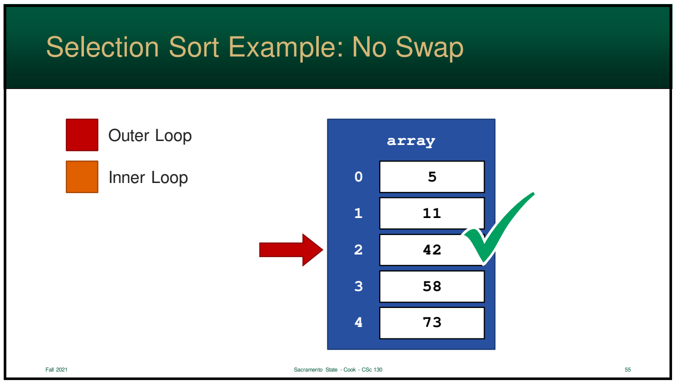

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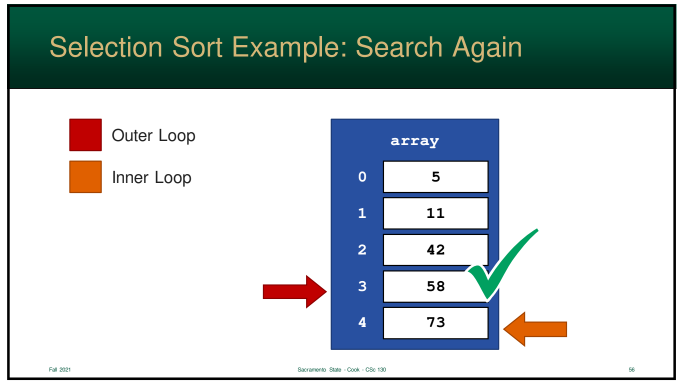

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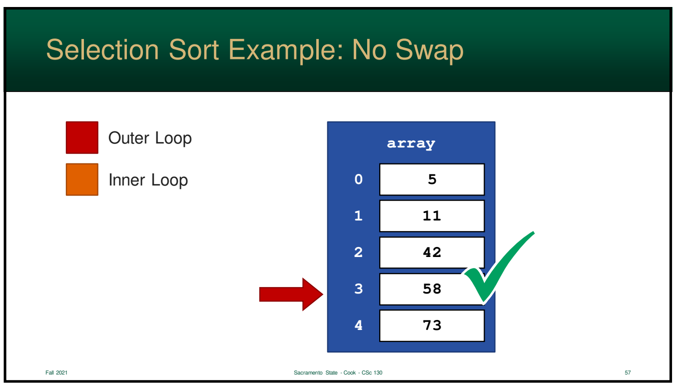

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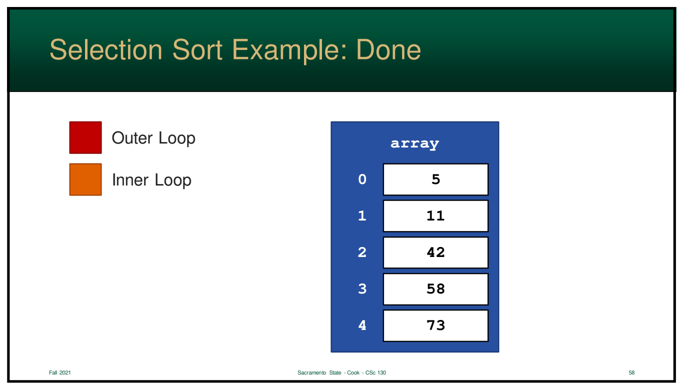

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Selection Sort Summary

Selection Sort	
Time Average	$O(n^2)$
Time Best	$O(n^2)$
Time Worst	$O(n^2)$
Auxiliary space	$O(1)$
Stable	Yes – Equal element order preserved
Online?	No – Entire array in use

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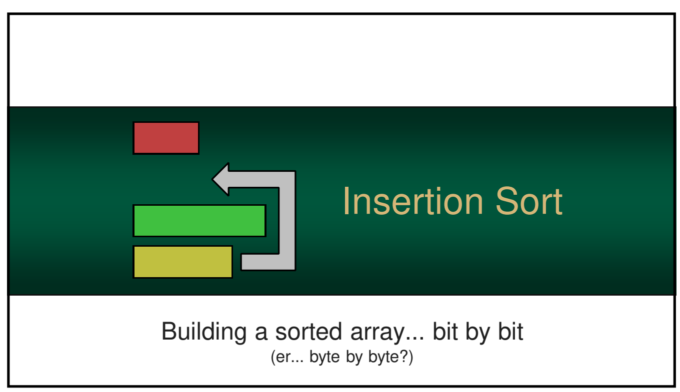

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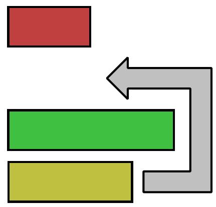
Insertion Sort

- The *Insertion Sort* is a $O(n^2)$ sorting algorithm with several advantages over bubble-sort and selection-sort
- While it is still $O(n^2)$ is far more efficient than the other two



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Deck of Cards

- Often, it is compared to sorting a deck of cards
- This is how you would manually sort a row of cards
  - if you start sorting on the left side, you will find a card, move it, and shift the rest of the cards right
  - you build a sorted list a bit at a time – on the left side of your row

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How it Works

- The algorithm consists of two loops – one embedded within the other
- The outer loop starts and the top of the array and moves down
- The algorithm builds a sorted array above the outer loop.

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How it Works

- Current array value is saved into a temporary variable
- Inner loop then searches all the values that come before it in the array
- If the value, being looked at, is larger than the saved value, it's moved down

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The Insertion Sort

```
for (i = 1; i < count; i++)
{
   value = array[i];

   j = i - 1;
   while (j >= 0 && array[j] > value)
   {
      array[j + 1] = array[j];
       j --;
   }
   array[j + 1] = value;
}
```


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Insertion Sort Example

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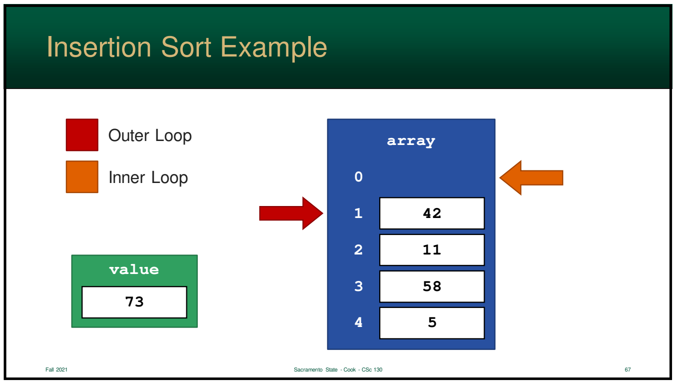

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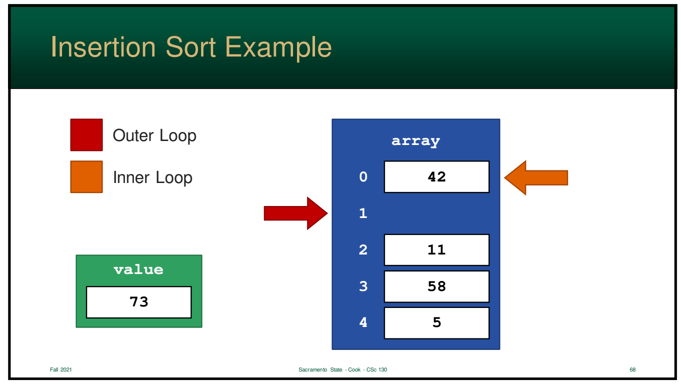

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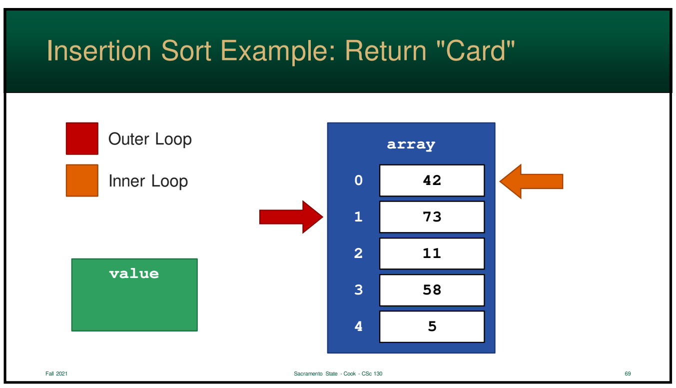

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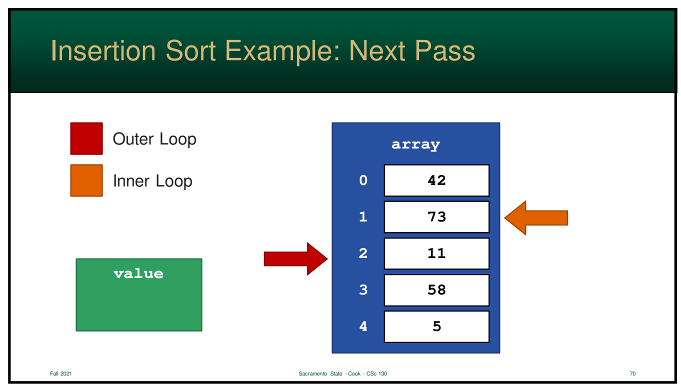

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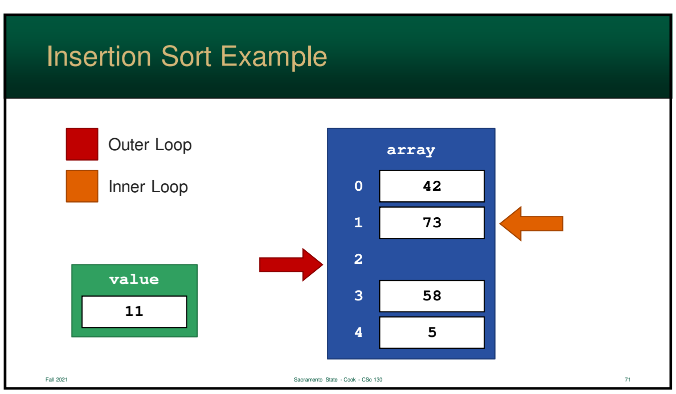

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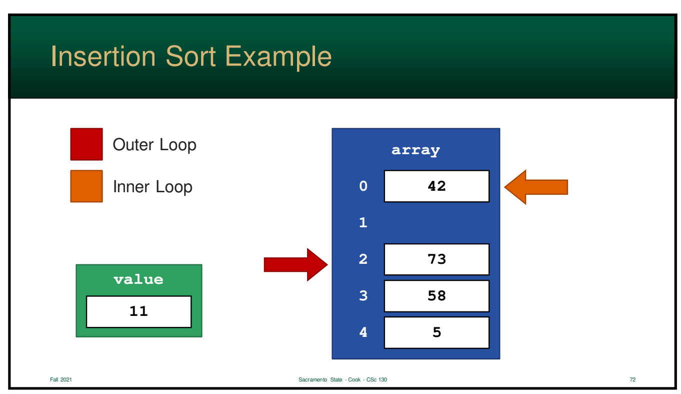

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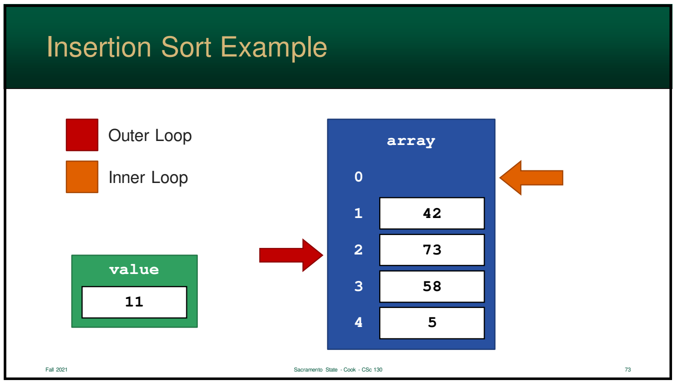

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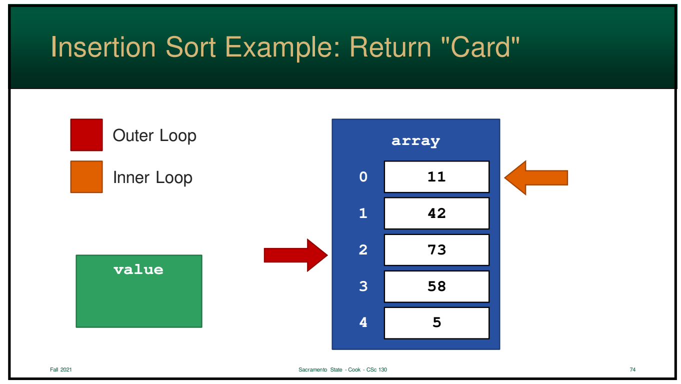

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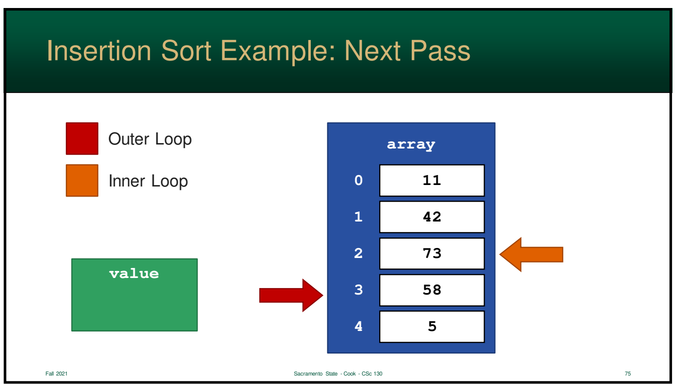

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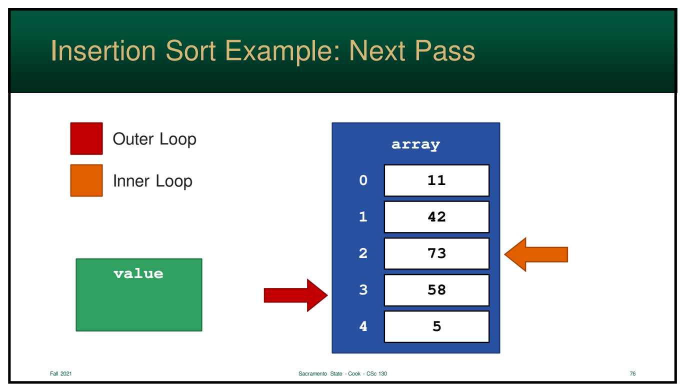

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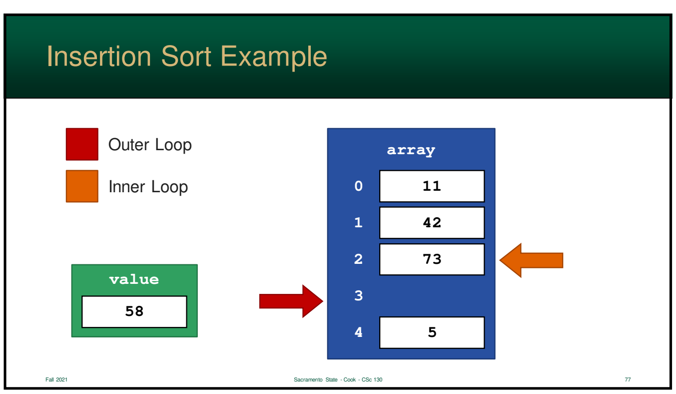

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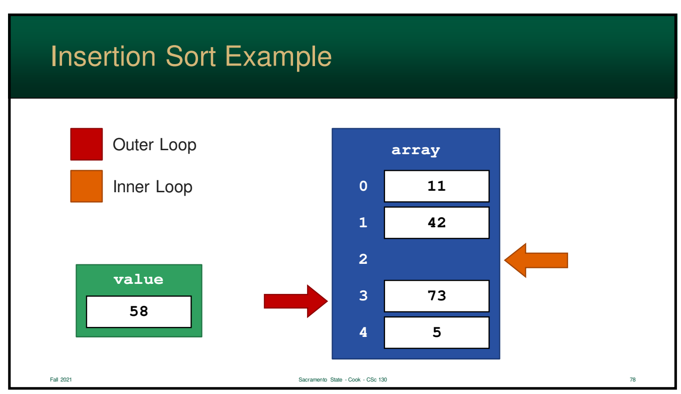

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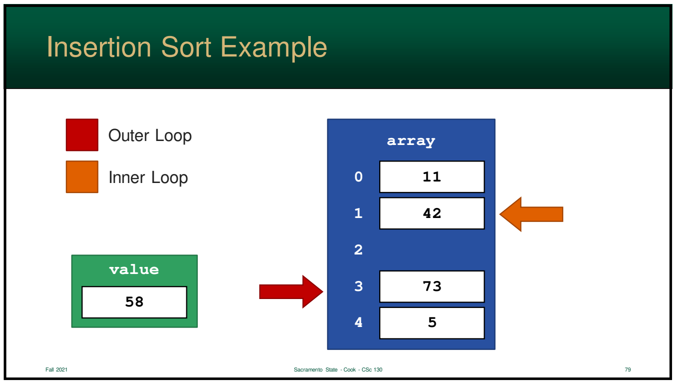

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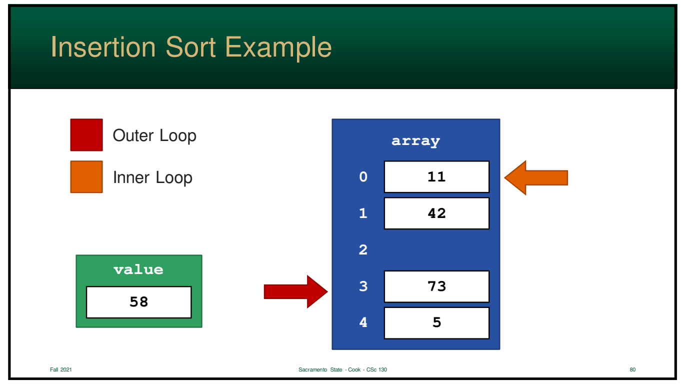

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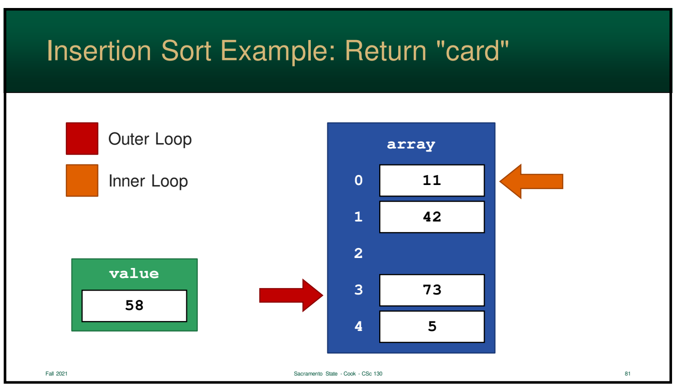

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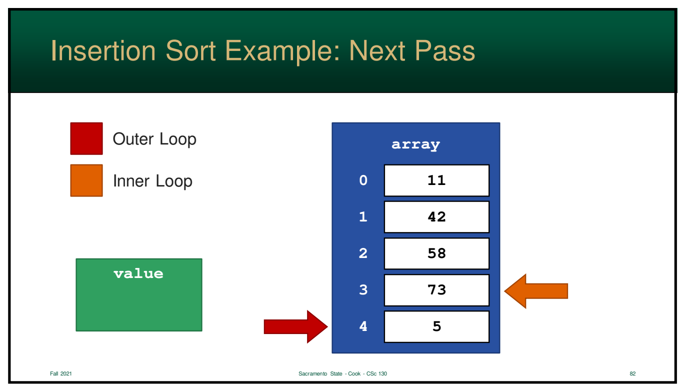

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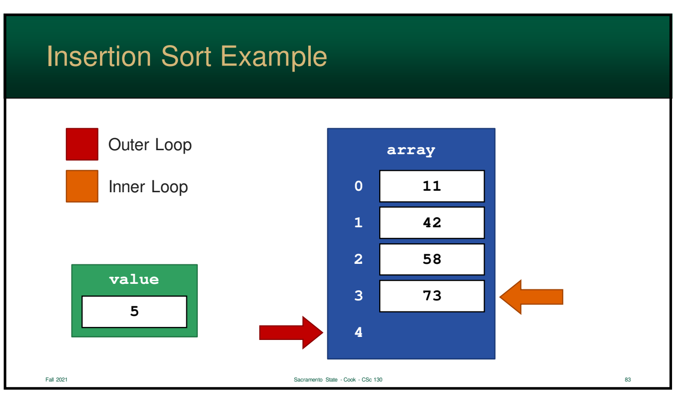

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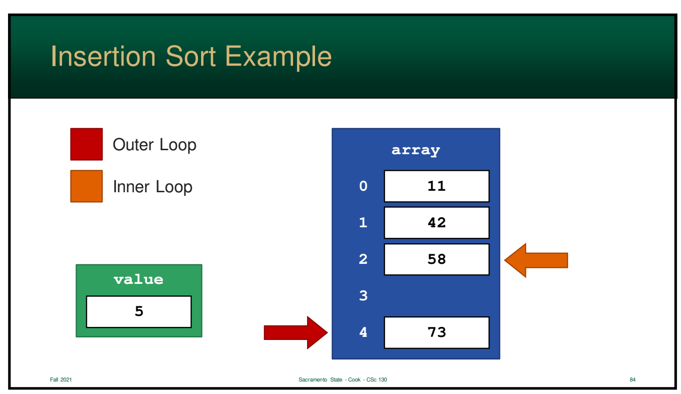

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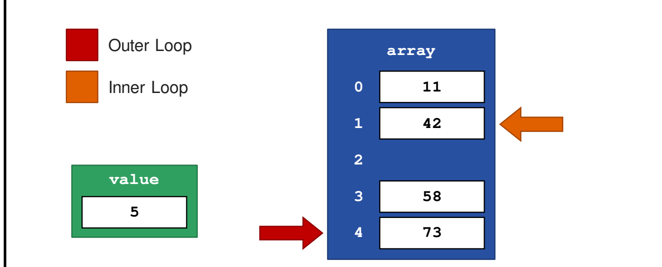
Insertion Sort Example



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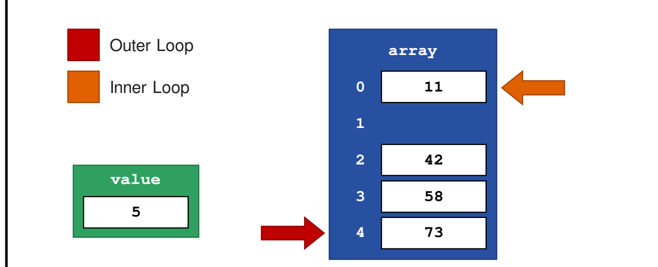
Insertion Sort Example



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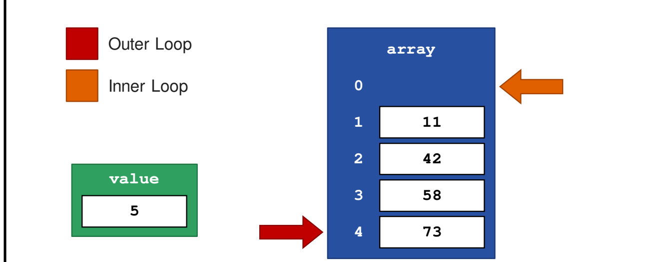
Insertion Sort Example



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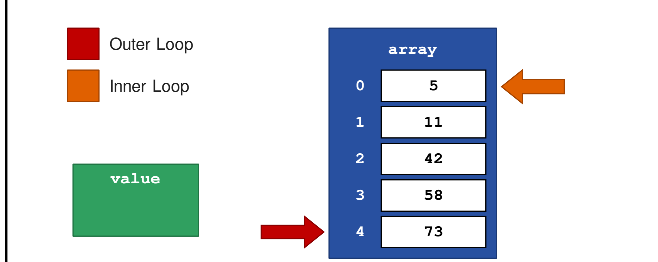
Insertion Sort Example: Return "Card"



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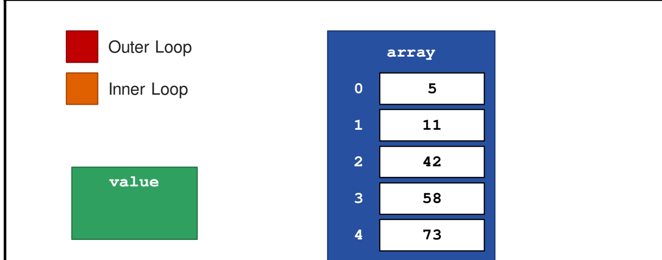
Insertion Sort Example: Done



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Advantages

- Because Insertion Sort creates a sorted array above the outer loop
  - inner loop, on average, only needs to move 1/2 positions up – far faster!
  - data can be sent during the sorting process
  - this means the algorithm is considered *"online"* – i.e. it can sort *streaming* data

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Advantages

- Insertion sort does not "swap" values
  - most of the overhead of bubble and selection-sort is swapping
  - insertion sort moves data as it sorts, so, there is little unnecessary overhead
- Little to no auxiliary storage overhead
  - like Bubble-Sort and Selection-Sort, Insertion-Sort requires little storage overhead
  - so, in regards to *n*, storage complexity is O(1)

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Advantages

- Insertion sort is O(n) on sorted lists
  - inner loop stops when the current array value cannot be moved up
  - the more sorted the list, the more the inner loop approaches O(1)

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Insertion Sort Summary

Insertion Sort	
Time Average	$O(n^2)$
Time Best	**O(n)**
Time Worst	$O(n^2)$
Auxiliary space	O(1)
Stable	Yes – Equal element order preserved
Online?	Yes – Can sort streamed data

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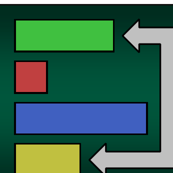

Shell Sort

Insertion Sort with an identity crisis

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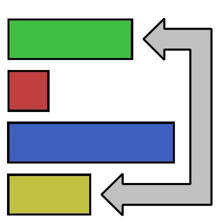
Shell Sort

- *Shell-Sort* is a version of the Insertion-Sort created by *Donald Shell* in 1959
- Yes, it is named after the guy, *not a shell metaphor*
- But, ironically, that metaphor works



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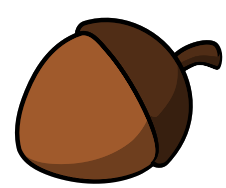
Shell Sort

- It was the first algorithm to break the $O(n^2)$ barrier
- For a few years, this was the fastest sort algorithm available – until O(n log n) was invented



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What is Going On?

- With insertion sort, each time we insert an element, the rest are moved <u>one</u> step closer to where they belong
- Can we move elements a <u>larger</u> distance than just one?
- **Yes…** Shell Sort works like Insertion Sort, but works on elements at large distances
- This distance is called the *gap*

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What's Going On?

- Gap changes with each outer loop iteration
  - the distance between comparisons <u>decreases</u> as the sorting algorithm runs
  - in the last iteration, the <u>gap is 1</u>
  - so, at that point, adjacent elements are compared – so it is a regular Insertion Sort
- Shell Sort is also known as a "diminishing increment sort"

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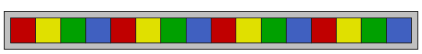
Sorting "Shells"

- Shell Sort orders elements that are spaced a relative distance from each other
- So, the red cells above are sorted *relative* to each other, as are the yellow, green, and blue cells



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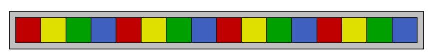
Sorting "Shells"

- The decreasing gaps are a sequence
- The notation $h_1, h_2, h_3, \ldots, h_t$ represents a sequence of <u>increasing</u> integer values which will be used (from right to left)
- Any sequence works if it $h_n > h_{n-1}$ and $h_1 = 1$



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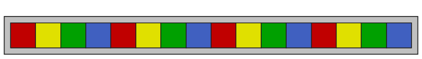
Gap Mathematics

- *h_k-sorted array* - all elements with gap h_k are sorted relative to each other
- So, after each phase of h_k…
  - for each i, we have array[i] ≤ array[i + h_k]
  - all elements spaced h_k apart are sorted.



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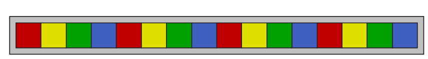
Gap Mathematics

- Shell-Sort <u>only</u> works because an array that is h_k-sorted…
- …remains h_k-sorted when h_{k-1}-sorted.



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So, What are Gap Values?

- For $h_1, h_2, h_3, \ldots, h_t$ we need to determine what the actual values will be
- Some sequences as better than others
- Shell's original design...
  - starts at N / 2 (where N is the size of the array)
  - cuts the gap in half for each iteration
- There are several competing sequences

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So, What are the Gap Values

- The algorithm is most efficient when...
  - the gap sequence are *relatively prime*
  - i.e. the sequence does not share any divisors
- However....
  - using a prime sequence is often not practical in a program – too much to store!
  - so, real, practical solutions attempt to approximate a relatively prime sequence

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So, What are Gap Values?

Creator	Sequence
Shell	1, ..., (n / 8), (n / 4), (n / 2)
Hibbard	1, 3, 7, 15, 31, ..., 2^k - 1
Knuth	1, 4, 13, 40, 121, ..., (3^k - 1) / 2
Sedgewick	1, 5, 19, 41, 109, ..., (4^k - 3 * 2^k + 1)

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The Shell Sort: Original

```
for (gap  = count / 2;  gap > 0; gap /= 2)
{
  for(i =  gap; i < count; i++)
  {
     value = array[i];
     j = i;
     while (j >= gap && array[j - gap] >  value)
     {
        array[j] = array[j -  gap];
      j -=  gap;
     }

     a[j] = value;
  }
}
```


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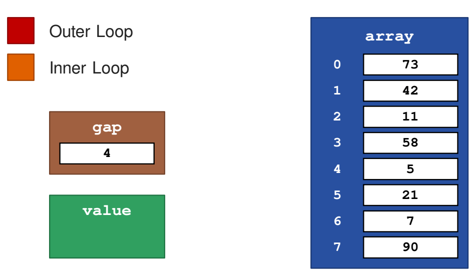
Gap = 4, First Outer Loop Pass



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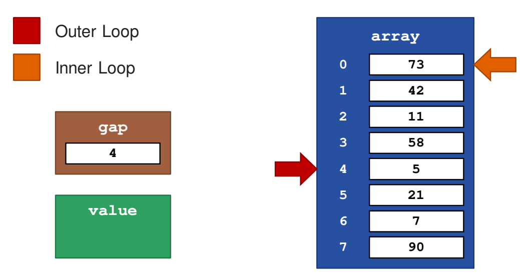
Gap = 4, Outer Loop starts at 4 (the gap)



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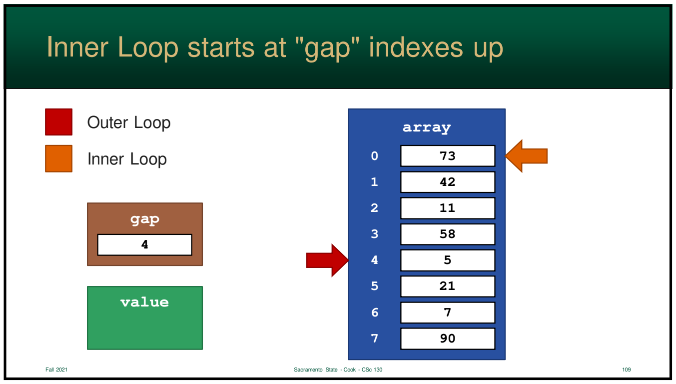

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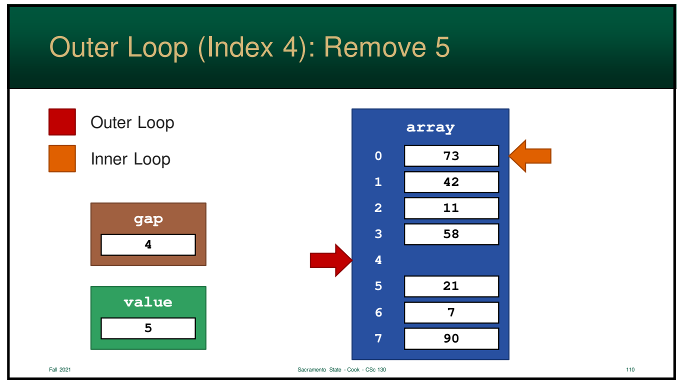

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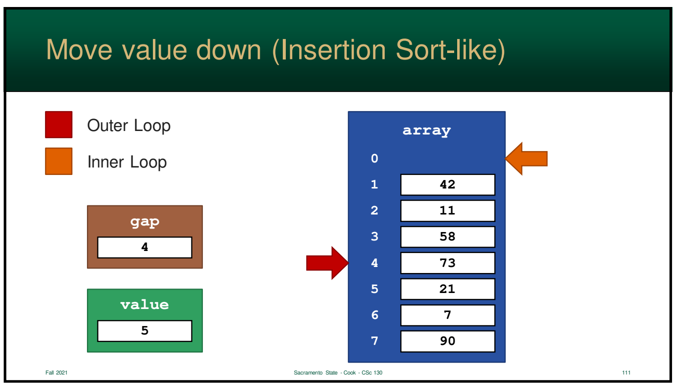

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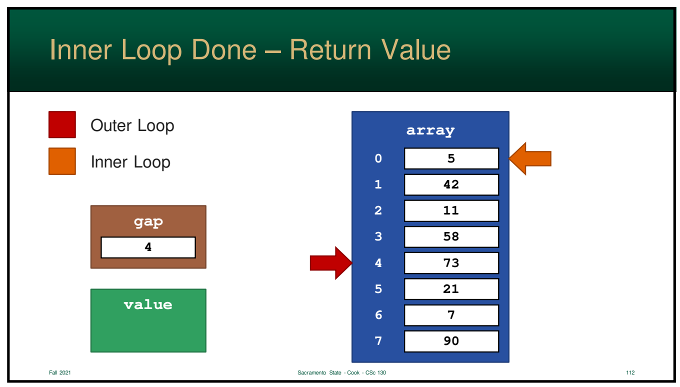

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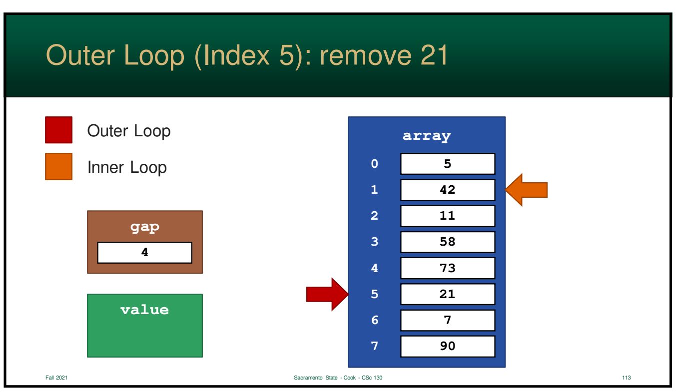

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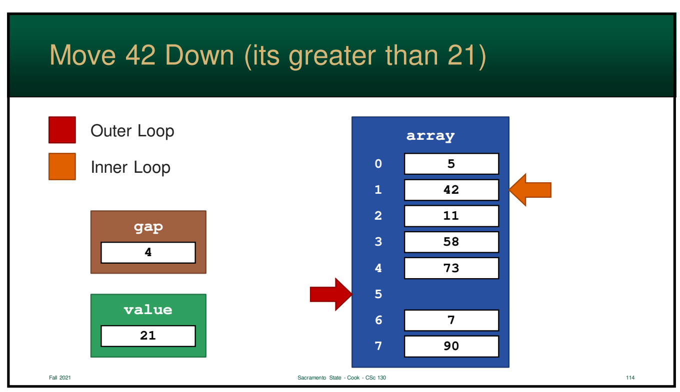

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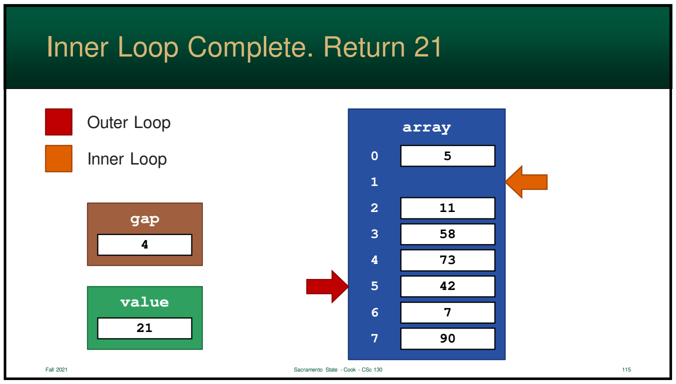

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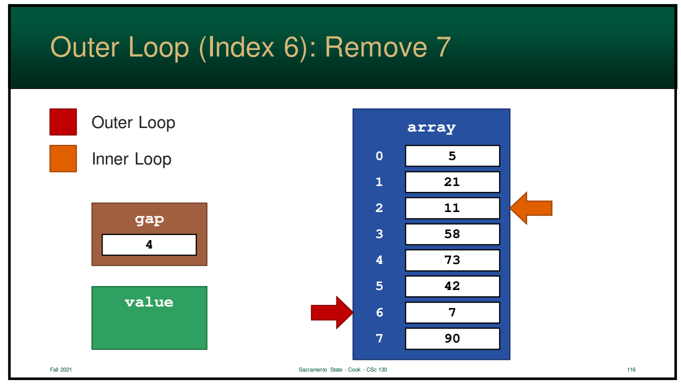

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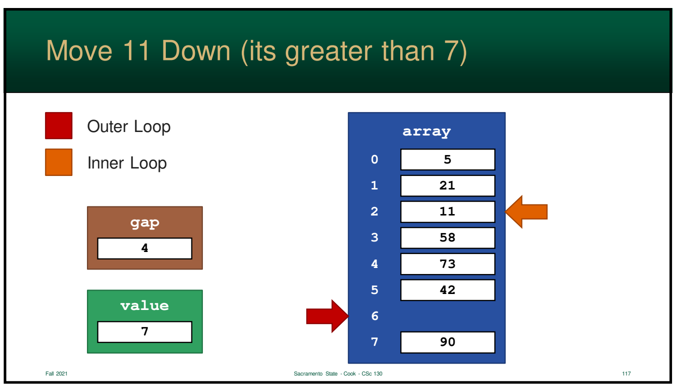

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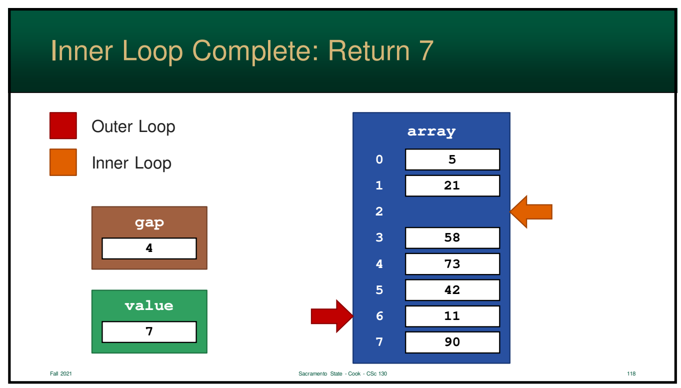

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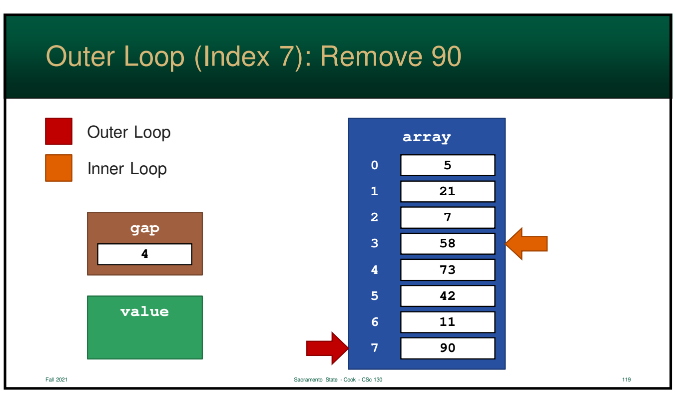

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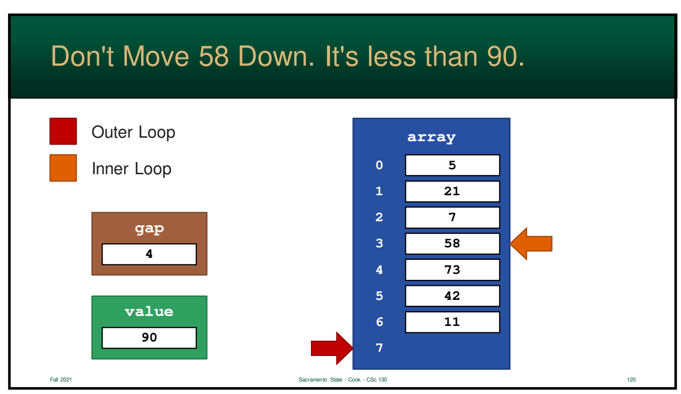

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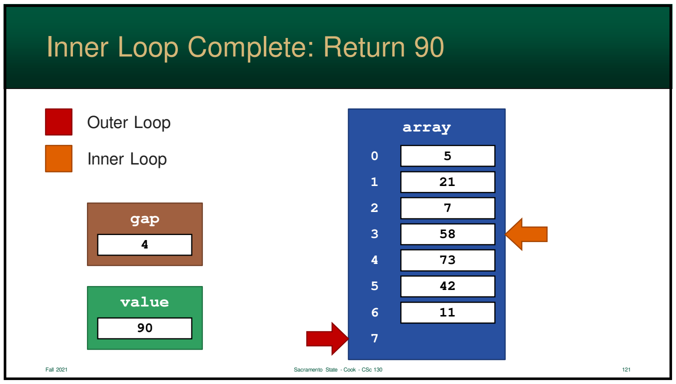

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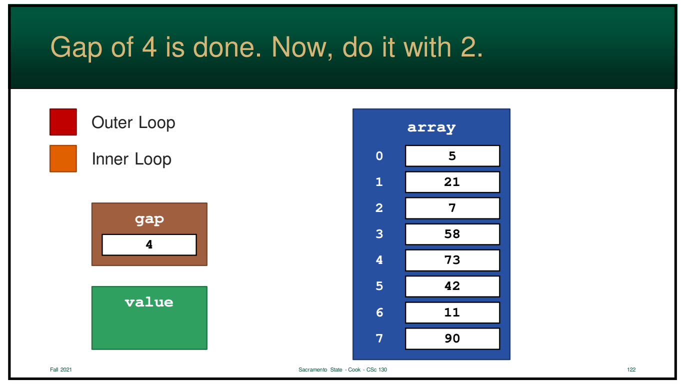

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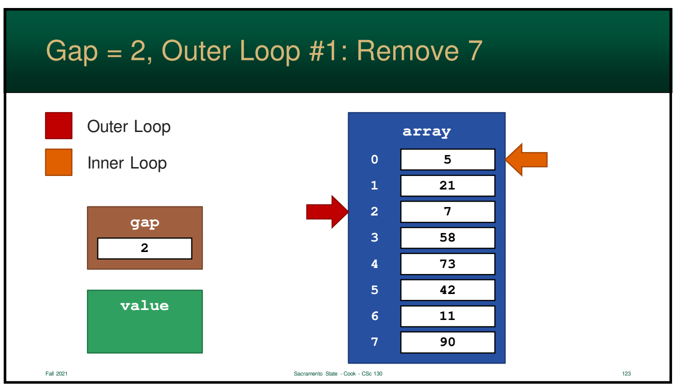

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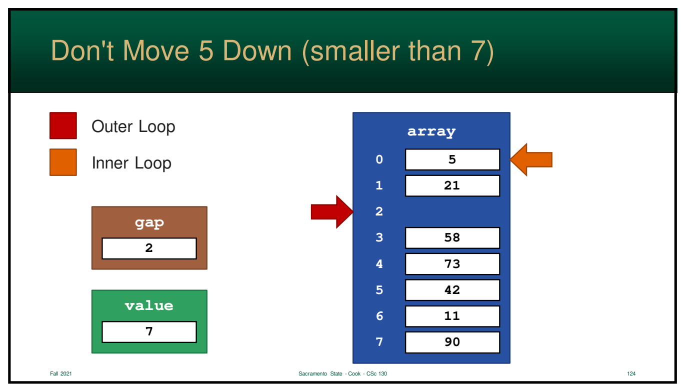

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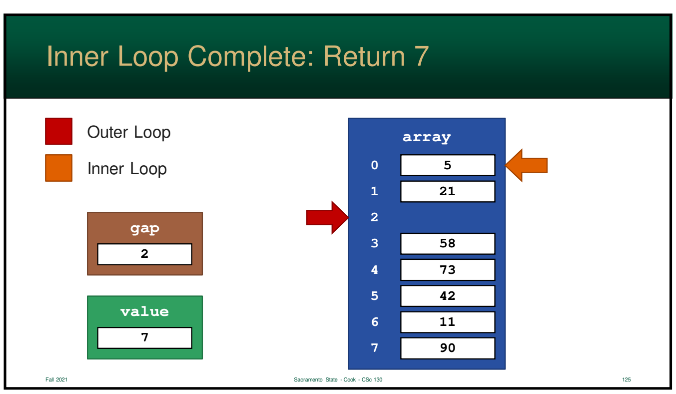

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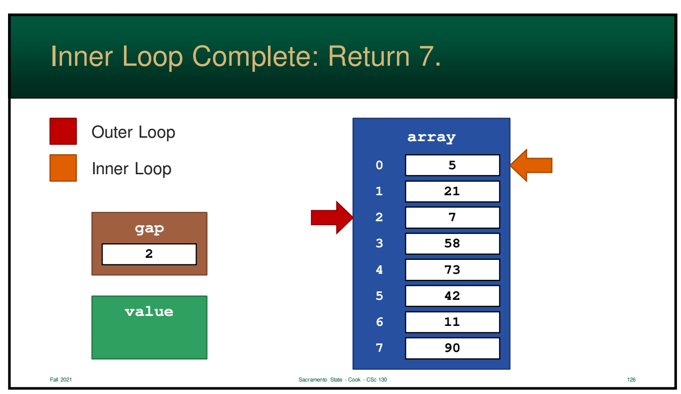

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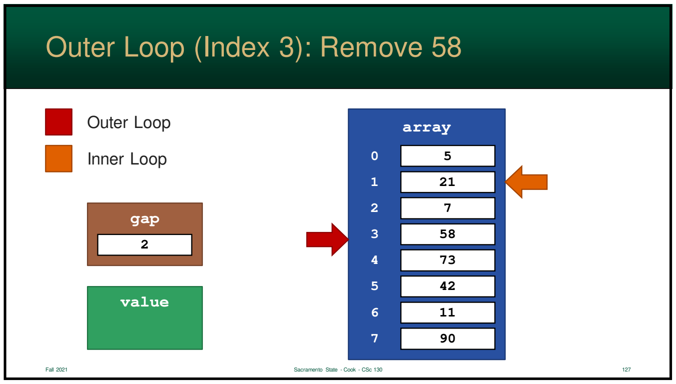

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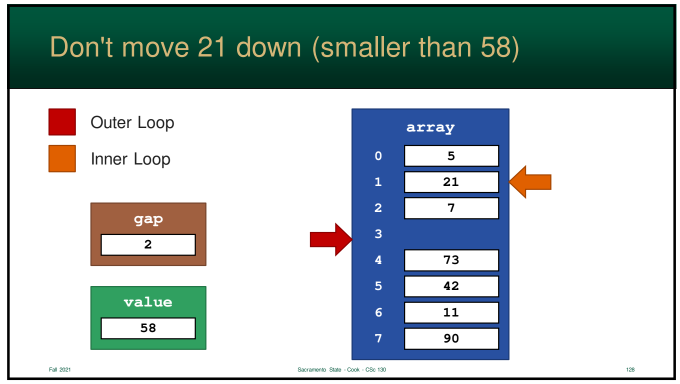

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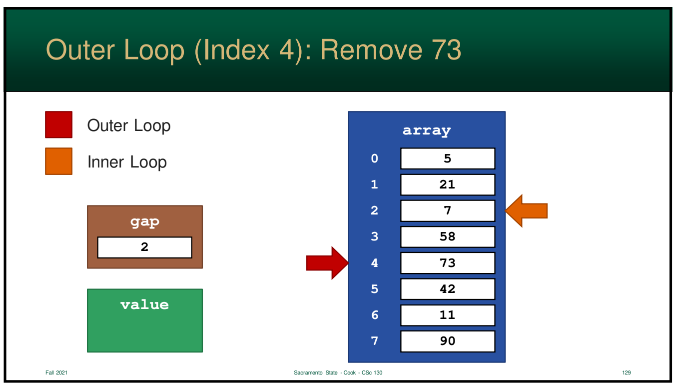

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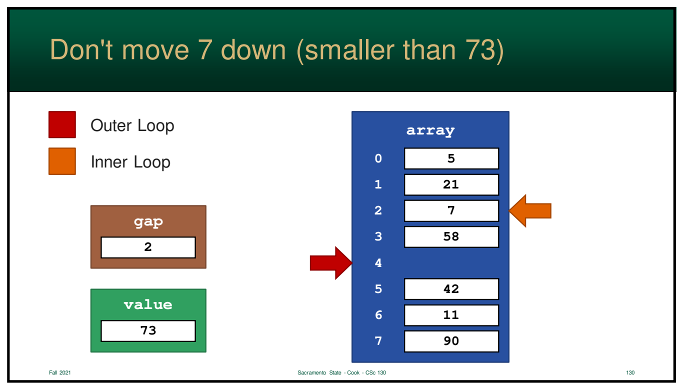

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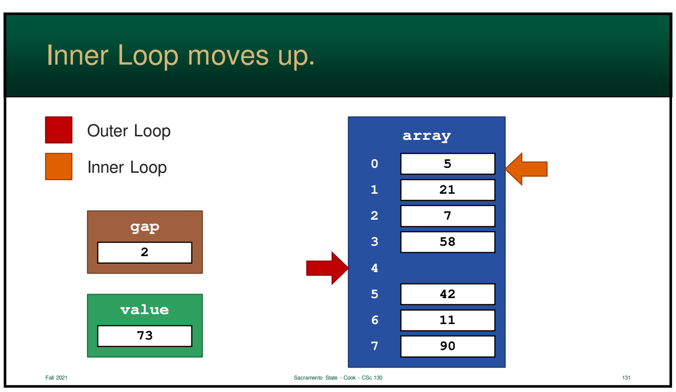

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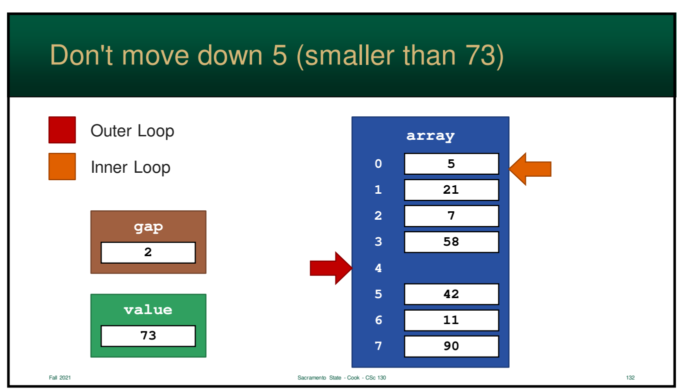

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... and so on....

- The example continues to sort for each h_k
- The outer loop continues to the bottom of the array
- Finally, gap will go to one and the sort acts just like an Insertion-Sort

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Time Complexity

- Time complexity of Shell Sort is up for debate
- Although the algorithm is fairly simple, proving its time complexity is not
- What is known...
  - it is approximately $O(n^r)$ where $1 < r < 2$
  - this is ultimately faster than $O(n^2)$ but worse than $O(n \log n)$

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Time Complexity

- Empirical analysis of the algorithm has given some widely accepted values for average, best, and worst times
- Worst case performance (using Hibbard's sequence) is $O(n^{3/2})$
- Average performance is thought to be about $O(n^{5/4})$

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Shell Sort Summary

Shell Sort	
Time Average	≈ $O(n^{5/4})$
Time Best	≈ O(n log n) – For a near sorted list
Time Worst	≈ $O(n^{3/2})$
Auxiliary space	O(1)
Stable	No – Equal element order not preserved
Online?	No – Entire array in use

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