SACRAMENTO
STATE

CSc 28
Discrete Structures

Chapter 10
Matrix Multiply

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Syllabus

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Definition

- Matrix is a two-dimensional data object, organized in rows and columns
- Row and columns are used to identify individual matrix elements
- Common operations on matrices are the "Matrix Multiply", and "Cramer's Rule" for solving multiple unknowns in linearly independent equations
- Many other uses!
- Focus here getting acquainted with simple matrices operations
- Implement two projects: P1 Store Max Value, and project: P2 Matrix Multiply

Goal of P1: Store Max Value

- Project P1 Store Max Value initializes all elements of a 2-Dim square integer matrix
- For each row, P1 extracts the row's largest integer, and stores it in an extra element at the end of that row, at index a[SZ]
  - SZ being the symbolic integer constant of the matrix size
- Actual data structure for P1 this is not a square matrix, but a rectangular matrix, with one extra element per row
- Finally, P1 prints the "almost square" matrix with all max values in the last position of each row

Goals of P2: Matrix Multiply

- Project P2 Matrix Multiply is a square matrix multiply problem
- Both source matrices are square, simplifying the upper bound computation
- All elements are integers
- Pre-assign the source matrices via initialization, not by reading values from files or from the console
- Compute result into suitably-sized matrix c[ ][ ]
- AKA c[ ][ ] = a[ ][ ] × b[ ][ ]
- With × being the matrix multiply operator, not the common, dyadic multiply operation!

Project P1: Store Max Value

Specify P1

- Matrix a[SZ][SZ + 1] is an integer matrix, sized via symbolic constant SZ, AKA macro in C++
- The a[][] is printed twice, once before finding the maximum value of each row, and once after placing the max value of a[row] into position a[row][SZ]

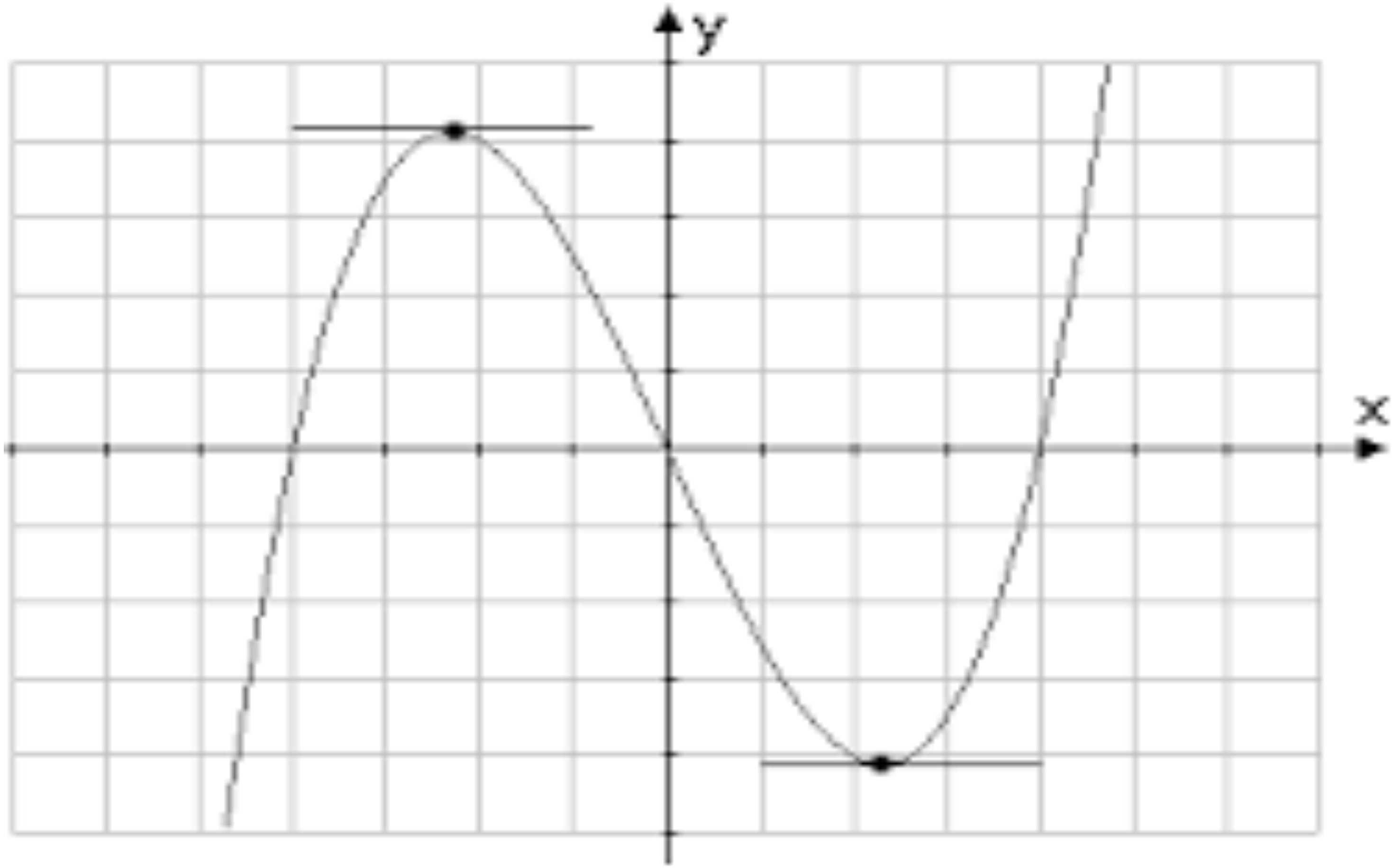
Implement P1, Initialize C++

```
#include <iostream>
#define SZ 5                                // small matrices
typedef int m_tp[ SZ ][ SZ + 1 ];// use typedef!

// actual data in rectangular matrix a[ ][ ]:
m_tp a = { {  1,-2, 3,-4, 2, 0 },
           {  8, 7,-6, 5, 9, 0 },
           {  6,-5, 4,-3, 0, 0 },
           { -4, 5,-6, 7, 8, 0 },
           {  6,-5, 4, 3, 1, 0 } };

void print( char * msg, m_tp m )
{ // print
   cout << "Printing " << msg << endl;
   for( int row = 0; row < SZ; row++ ) {
       for( int col = 0; col < SZ + 1; col++ ) {
           cout << m[ row ][ col ] << " ";
           // no newline: array "known to be small"☺
       } //end for
       cout << endl;
    } //end for
    cout << endl;
} //end print
```

Find Max & Min



Third Order Function

Implement P1, Find Max

```
// input parameter is a[], a single-dimensional array
// find max value in a[ SZ ] and store on a[ SZ ]
// i.e. object is passed as parameter, no globals

void extract( int a[] ) // a[] passed by reference
{ // extract
   int max = a[ 0 ];    // initial guess: this is max
   for( int col = 1; col < SZ; col++ ) {
      max = ( a[ col ] > max ) ? a[ col ] : max;
   } //end for
  a[ SZ ] = max;  // now we really know max
} // end extract
```

Implement P1, One Row at a Time

```
void extract_mat()
{ // extract_mat
    // handle all rows of global matrix a[][]
    for( int row = 0; row < SZ; row++ ) {
        // pass matrix’s row to function extract()
        extract( a[ row ] ); // handle 1 row at a time
     } // end for
} // end extract_mat

int main()
{ // main
    print( "before", a );
    extract_mat();
    print( "after ", a );
    return 0;
} //end main
```

Project P2: Matrix Multiply

Matrix Multiply

- **Matrix Multiply:** 2 source matrices (here a[][] and b[][]) and a destination matrix (here x[][], or later named c[][])
- Element x[i][j] is sum of all products of all elements in row a[i][*] by corresponding elements in column b[*][i]
- Size of columns of a[][] and rows of b[][] must match
- Rows of a[i][*] and columns of b[*][j] define c[i][j]

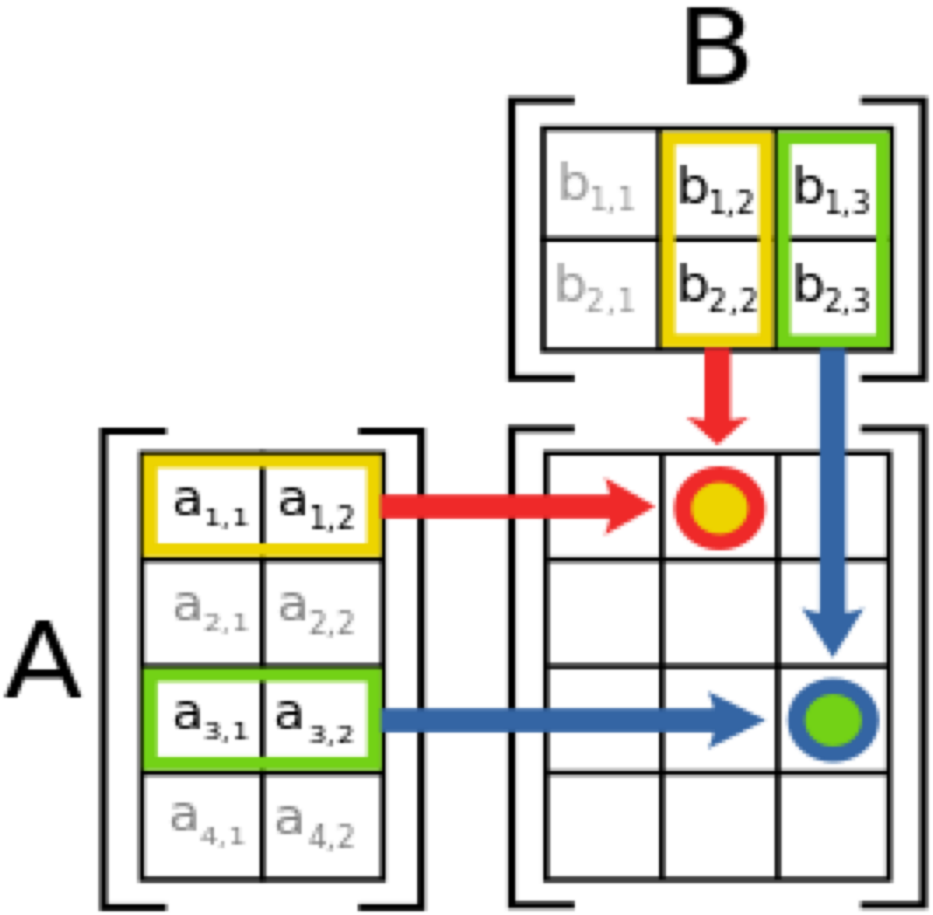
$$\overset{4\times 2\text{ matrix}}{\begin{bmatrix} a_{11} & a_{12} \\ \cdot & \cdot \\ a_{31} & a_{32} \\ \cdot & \cdot \end{bmatrix}} \overset{2\times 3\text{ matrix}}{\begin{bmatrix} \cdot & b_{12} & b_{13} \\ \cdot & b_{22} & b_{23} \end{bmatrix}} = \overset{4\times 3\text{ matrix}}{\begin{bmatrix} \cdot & x_{12} & x_{13} \\ \cdot & \cdot & \cdot \\ \cdot & x_{32} & x_{33} \\ \cdot & \cdot & \cdot \end{bmatrix}}$$

Matrix Multiply

- Matrix multiplication (or matrix product, or MatMult) is an operation that produces a result matrix from two source matrices
- Matrix multiplication: A typical structure and operation of linear algebra
- Respective sizes do not have to be identical, but must match along two pairs of two dimensions:
  - If a[n][m] is an $n \times m$ matrix and b[m][p] is an $m \times p$ matrix, their matrix product a[*][*] × b[*][*] is an $n \times p$ matrix, in which the m entries across a row of a[][] are multiplied with the m entries down a column of b[][] and summed
- . . . To produce one single matrix element c[][] each for the vector product of a[][] × b[][]

Matrix Multiply Sample

- Number of columns of a[][] must match rows of b[][]
- Number of rows of a[][] match number of rows of c[][]
- Number of columns of b[][] match columns of c[][]



Matrix Multiply Sample

Matrix Multiplication

$$\begin{bmatrix} 1 & -2 & 1 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \times 2 + (-2) \times 3 + 1 \times 1 & 1 \times 1 + (-2) \times 2 + 1 \times 1 \\ 2 \times 2 + 1 \times 3 + 3 \times 1 & 2 \times 1 + 1 \times 2 + 3 \times 1 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & -2 \\ 10 & 7 \end{bmatrix}$$

Matrix Multiply

- Size defined via symbolic literal SZ; here 5 –very small
- Square integer matrices a[][], b[][], and c[][] are used to simplify index ranges of *multiply function*:

```
c[ row ][ col ] += a[ row ][k] * b[k][ col ];
```

- a[SZ][SZ] is initialized at the point of declaration
- b[row][col] = row * SZ + col initialized for all elements; simple code to bypass any input operations
- c[SZ][SZ] initializes all elements to 0 (really needed?)
- Then c[ ][ ] is recomputed via *matrix multiply*
- We'll print matrices before & after matmult ☺ operation

Matrix Multiply in C++

```
// matrix multiply of square integer matrix
// all data pre-computed, not read from console

#include <iostream.h>
#define SZ 5   // tiny: permits printing all on 1 line

typedef int m_tp[ SZ ][ SZ ];    // square matrix type

// Matrix object declarations, a[][] being initialized
m_tp a =
   {
      { 1,2,3,4,5 },     // 5 columns
      { 8,7,6,5,4 },
      { 6,5,4,3,2 },
      { 4,5,6,7,8 },
      { 6,5,4,3,2 }      // 5 rows, 5 columns each row
   };
  // b[][] and c[][] initialized elsewhere
  m_tp b, c;
```

Matrix Multiply in C++

```
// a[][] already set; now initialize b[][] and c[][]
// all 3 matrices are global: caveat!
// Could be passed by ref: would also be efficient!

void init( void )
{ // init
   for( int row = 0; row < SZ; row++ ) {
        for( int col = 0; col < SZ; col++ ) {
            b[ row ][ col ] = row * SZ + col;
            // since c[][] is summed up, needs init!
            // else OK to leave uninitialized!
            c[ row ][ col ] = 0;
        } //end for col
    } //end for row
} //end init
```

Matrix Multiply in C++

```
// output a[][], b[][], and c[][], define C++ width
void print_m( char * msg, m_tp m )
{ // print_m
   cout.width( 6 ); // positions for int output
   cout << "Printing matrix " << msg << endl;
   for( int row = 0; row < SZ; row++ ) {
       for( int col = 0; col < SZ; col++ ) {
          cout << m[ row ][ col ] << " ";
       } //end for col
       cout << endl;       // assume "small" # columns
   } //end for row
   cout << endl;           // end of matrix
} //end print_m

void print( void )
{ // print
    print_m( "a", a );
    print_m( "b", b );
    print_m( "c", c );
} //end print
```

Matrix Multiply in C++

```
void mat_mult( void )
{ // mat_mult
   for( int row = 0; row < SZ; row++ ) {
      for( int col = 0; col < SZ; col++ ) {
         for( int k = 0; k < SZ; k++ ) {
            // note: c[row][col] is loop invariant!
            c[ row ][ col ] = c[ row ][ col ] +
                a[ row ][ k ] * b[ k ][ col ];
         } //end for k
      } //end for col
   } //end for row
}  //end mat_mult

int main( void )
{ // main
   print();    // before multiplication
   mat_mult(); // do work
   print();    // after multiplication
   return 0;
} //end main
```

Matrix Multiply in C++

Inner loop simplification, using Algol68 style operator += inherited in C, C++, and Java

Similar for: -= *= /= etc.

```
void mat_mult( void )
{ // mat_mult
   for( int row = 0; row < SZ; row++ ) {
      for( int col = 0; col < SZ; col++ ) {
         for( int k = 0; k < SZ; k++ ) {
            c[ row ][ col ] += // note C++ style +=
               a[ row ][ k ] * b[ k ][ col ];
         } //end for k
      } //end for col
   } //end for row
}  //end mat_mult
```

“Matrix” in the Movies

Discuss Matrix Multiply

- Key operation inside inner loop:

```
c[ row ][ col ] =
   c[ row ][ col ] +
   a[ row ][ k ] * b[ k ][ col ];
```

- More terse style:

```
c[ row ][ col ] +=
   a[ row ][ k ] * b[ k ][ col ];
```

- Notice C++ specification for output width

```
cout.width( 6 ); . . . cout << m[row][col]
```

- similar to printf() of traditional C, legal in C++ too:

```
printf( "%6d", m[row][col] );
```

Summary

- Multi-dimensional data common in SW Engineering, Math, real world
- Matrix Multiply and related operations are typical
- Matrix Multiply objects are not necessarly square
- Dimensions of a[][] and b[][] in MatMult do define the size of c[][]

References

1. Matrix multiply on Wiki: https://en.wikipedia.org/wiki/Matrix_multiplication
2. How to multiply 2 matrices: https://www.mathwarehouse.com/algebra/matrix/multiply-matrix.php
3. Matrix algebra by Binet: https://en.wikipedia.org/wiki/Jacques_Philippe_Marie_Binet