

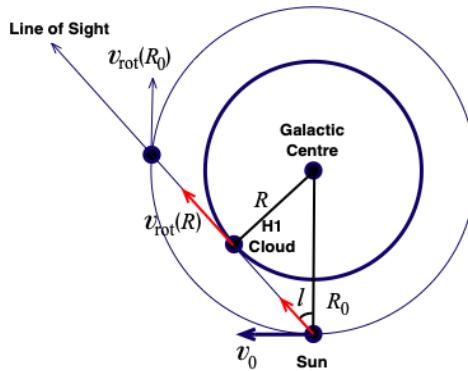
Uncovering Galactic dark matter using a radio telescope

The Universe consists of a large amount of dark matter. You will discover the dark matter in our Galaxy by measuring the tangential rotation velocity of gas in the disk of our Galaxy. The gas in the disk of the Milky Way galaxy contains neutral atomic hydrogen (HI) that can be detected using its spin flip transition, corresponding to a rest frequency of $f_0 = 1420.40575$ MHz. The velocity of rotation of the gas in the Galaxy can be measured using the Doppler shift of this transition.

Your task is to measure the spectra of radio emission between 1419.0 MHz and 1421.0 MHz emitted by gas in the Galactic disc at different longitudes in the Galactic plane using a radio telescope provided to you. You will analyze these data to measure the rotation curve for the Milky Way galaxy. Based on the measured rotation velocities, you will estimate the enclosed mass within different Galacto-centric distances, compare them to the known baryonic mass in the Galaxy within corresponding radii and attribute any difference to dark matter.

Theoretical background

Assume that all the gas in the Galaxy moves clockwise (as seen from the North Galactic Pole) in circular orbits at different distances from the Galactic Center as shown in the figure below. Consider gas moving with a rotation velocity ($v_0 = 220$ km s $^{-1}$) at the position of the Sun. The reference frame rotating at this speed is called the local standard of rest (LSR). Note that the Sun moves with respect to the LSR.



When observing along the line-of-sight towards longitude ℓ , the observer sees emission from gas at different distances from the Galactic Center. Assuming that the rotation velocity $v_{\text{rot}}(R)$ of the gas does not increase significantly with increasing radius, the gas whose total velocity vector is along the observing direction will have a maximum net magnitude of the line-of-sight velocity, v_{LSR}^{\max} . By geometry,

$$v_{\text{rot}}(R) = v_{\text{LSR}}^{\max}(\ell) + v_0 \sin(\ell),$$

where $R = R_0 \sin \ell$, R_0 is the distance of the Sun from the Galactic Center (8.5 kpc) and $v_{\text{rot}}(R)$ is the tangential rotation velocity of gas at distance R from the Galactic Center. We will infer $v_{\text{LSR}}^{\max}(\ell)$ using the observed HI emission line data. For Galactic longitudes $20 < \ell < 90$ degrees, this corresponds to the maximum redshifted emission.

Since both the Sun and Earth are not at rest with respect to the LSR, observed HI line velocities must be corrected for (a) Earth's rotation, (b) its revolution around the Sun, and (c) the Sun's motion relative to the LSR. These motions combine into a line-of-sight correction velocity (v_{corr}), which depends on the observer's location, viewing direction, and date and time of observation. You will be provided tools to compute (v_{corr}), so that measured velocities ($v_{\text{Earth}}^{\text{obs}}$) can be converted to velocities with respect to LSR, v_{LSR} , such that

$$v_{\text{LSR}} = v_{\text{Earth}}^{\text{obs}} + v_{\text{corr}}.$$

The quantity $v_{\text{Earth}}^{\text{obs}}$ can be determined using the observed frequency f_{obs} which is maximally redshifted from f_0 , such that

$$v_{\text{Earth}}^{\text{obs}} = c \left[\frac{f_0 - f_{\text{obs}}}{f_0} \right].$$

Thus observations of HI emission from the Galactic disk allow us to determine the Milky Way's rotation curve, $v_{\text{rot}}(R)$, which can be used to infer the enclosed mass at various distances (R) from the Galactic Center.

Calibration of telescope power output:

The received radio emission from a source is commonly expressed in terms of an equivalent temperature T^{src} (called the brightness temperature) of a (hypothetical) blackbody which would emit the same intensity at a given frequency over the solid angle of the source. In the Rayleigh-Jeans regime, $P = k_B T^{\text{src}} \Delta f$, where P is the average power received from a source with temperature T^{src} in a frequency bin with width Δf . Power and temperature are used equivalently in radio-astronomy.

The equivalent temperature of the radio power received by a telescope is an average of T^{src} over a solid angle called the beam area, which is related to the resolution of the telescope. This equivalent temperature is called the antenna temperature, T_{ant} . In addition, the entire telescope system also adds some noise power, described by the so-called receiver temperature, T_{recv} . Together, the antenna temperature and the receiver temperature add up to the system temperature,

$$T_{\text{sys}} = T_{\text{ant}} + T_{\text{recv}},$$

which corresponds to the total power measured by the telescope. The telescope records power after it is amplified by a gain factor G_R such that in a simplified model we can express

$$P_{\text{out}} = k_B G_R [T_{\text{ant}} + T_{\text{recv}}] \Delta f,$$

where P_{out} , G_R , T_{ant} and T_{recv} are all functions of frequency.

Thus at each frequency, there are two unknowns to be determined, G_R and T_{recv} , in order to determine T_{ant} from the measured P_{out} . We will determine these two unknowns by pointing the telescope at two standard sources assuming that they completely fill the telescope's view and have known antenna temperatures, and measuring the powers received.

We will point the telescope to

- the 'ground' which is assumed to have an antenna temperature $T_{\text{ground}} = 300$ K, and
- a cold part of the 'sky' away from the Galactic plane with an assumed antenna temperature $T_{\text{sky}} = 5$ K.

These calibration temperatures can be assumed to be independent of the frequency in the band of interest. Thus one needs to solve the following two equations at each frequency

$$\begin{aligned} P_{\text{out}}^{\text{ground}} &= k_B G_R [T_{\text{ground}} + T_{\text{recv}}] \Delta f \\ P_{\text{out}}^{\text{sky}} &= k_B G_R [T_{\text{sky}} + T_{\text{recv}}] \Delta f. \end{aligned}$$

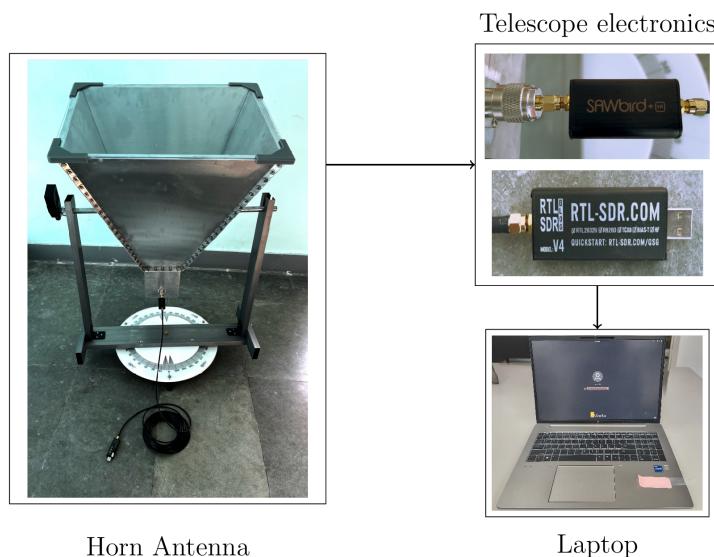
You will be provided tools which will solve these equations and determine the frequency dependent T_{recv} and the G_R which can be used to obtain T_{ant} for further measurements. Note that making both the Ground and the Sky measurements is essential for obtaining a properly calibrated HI emission line spectrum.

Because our telescope has low angular resolution, you may find it hard to point to a region of the sky completely free of HI gas from our Galaxy. Emission from gas outside the Galactic plane and other noise sources may affect the sky measurement. We will mask such spectral range during calibration.

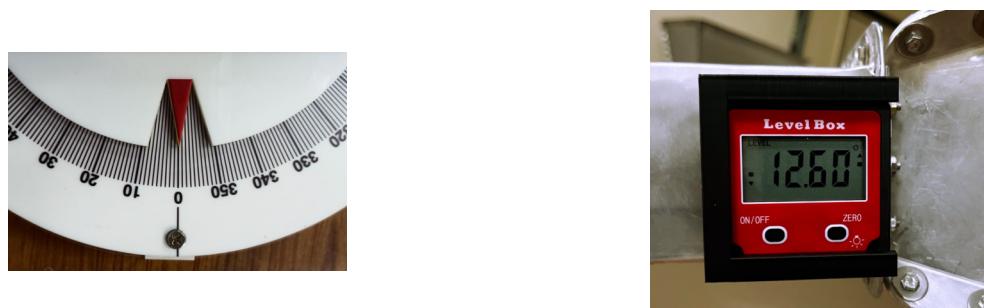
The HI emission line appears as an excess in radio intensity relative to the background at a given frequency and direction. Given a known G_R and T_{recv} , the sensitivity expressed as the r.m.s. noise temperature, σ_T , of a radio telescope that observes a system temperature T_{sys} , in a frequency bin Δf (in Hz), is given by

$$\sigma_T = \frac{T_{\text{sys}}}{\sqrt{\Delta f \times t_{\text{int}}}},$$

where t_{int} is the integration time in seconds.

Equipment and software:


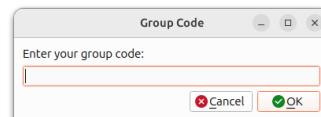
- A horn antenna radio telescope on an alt-azimuth mount. The azimuth can be measured with the protractor scale at the base of the mount. The altitude can be measured using the digital inclinometer as shown below.



- The telescope includes electronic units that amplify the signal, filter the desired frequency range, and output a spectrum.
- A laptop equipped with reading and displaying, data recording, calibrating and analyzing the output from the telescope.

Usage of the telescope and software:

- Your telescope set up is already aligned with North. Make sure that the zero degree on the horizontal dial coincides with "N" (north) mark on the table.
- Double click the icon - "Galactic Rotation Curve" on the laptop screen to start the program.



- Enter your group code and press "OK".
 - A folder with your group code will appear on the Desktop. You need to store all of your data files in this folder.

- The system will be powered up and you will see a white LED switch ON.
- The interface “Radio Astronomy Suite” will open.



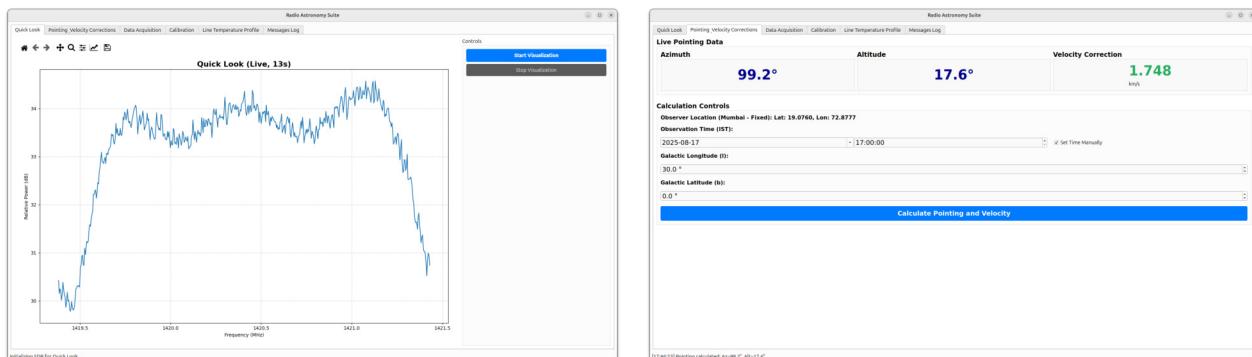
The “Radio Astronomy Suite” has following tabs:

- **Tab 1: Quick Look**

Tab 1 provides a quick system check to verify that the radio signal is being properly recorded. It runs a code that plots the received signal (relative power) on the Y-axis against the observed frequency on the X-axis.

1. Point the telescope first towards the sky and then towards the ground, and observe the resulting change in signal amplitude.
2. The code monitors the live signal spectrum for 60 s, but **does not** save the data.

Immediately inform the supervisor if the signal amplitude remains unchanged when the telescope pointing is varied.

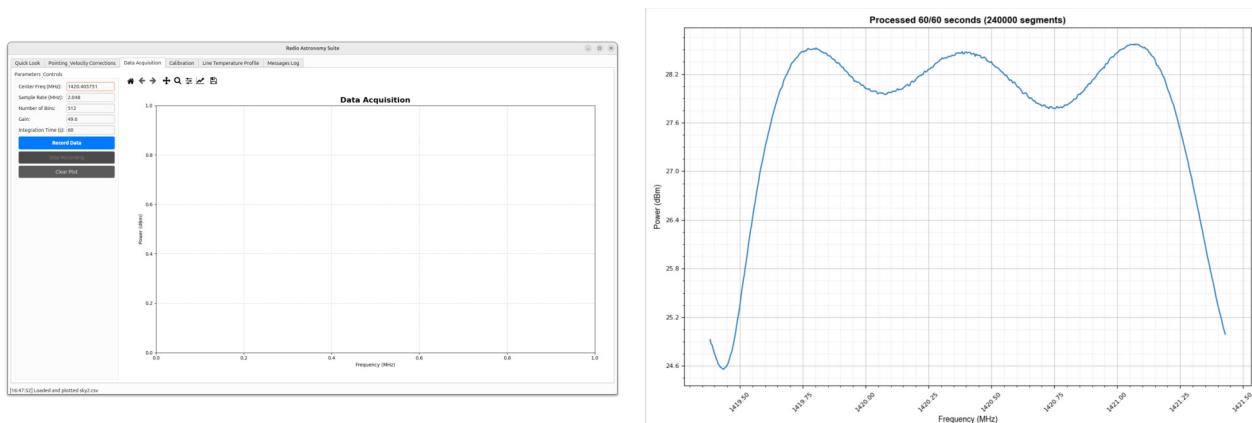


- **Tab 2: Pointing and Velocity Correction**

Tab 2 converts specified Galactic longitude and latitude into altitude and azimuth for the current date and time. Enter the desired Galactic coordinates to obtain the corresponding altitude and azimuth values, along with the velocity correction, v_{corr} .

- **Tab 3: Data Acquisition**

Tab 3 allows you to record the data. It has three buttons: (i) Record Data, (ii) Stop Recording, and (iii) Clear Plot.



Clicking **Record Data** begins data acquisition for an integration time of 60 s in the direction the telescope is currently pointing. A dialog box will appear, prompting you to name and save the spectrum data to a file. If you wish to restart the measurement before any ongoing exposure completes, use the **Stop Recording** button. The **Clear Plot** button removes the displayed plot from the screen.

Precaution: Do not point the telescope between azimuth 0° and 40° (or between 240° and 360°) when the altitude is below 40° to prevent interfering signals from a mobile tower antenna located near the venue.

In order to measure the 21-cm HI emission from the Galactic disk at a given Galactic longitude, perform the following steps:

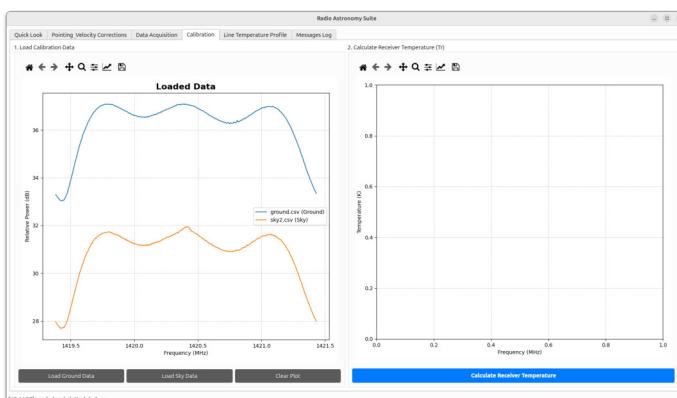
1. First use Tab 2 to calculate and note down the altitude, azimuth and v_{corr} for the Galactic longitude you wish to observe.
2. Then point the telescope to the desired location Galactic longitude “ ℓ ” in the sky and record the spectrum.
3. Save the spectrum with the file name $\ell.\text{csv}$.

Next, perform the following steps sequentially to obtain calibration measurements.

1. Point the telescope to the ‘ground’, record and save the spectrum with file name **ground.csv**.
2. Point to the ‘sky’, away from the Galactic plane, record and save the spectrum with file name **sky.csv**.

- **Tab 4: Calibration**

Tab 4 is used to perform calibration.



1. First, you must load the **ground** and **sky** data by clicking the corresponding buttons and selecting the appropriate files. Once loaded, the left-hand plot will display the relative power output for both the sky and ground.
2. Next, click on “**Calibrate gain and obtain T_{recv}** ”. This will generate a plot of T_{recv} as a function of frequency in the right-hand panel. You should see data consisting of noise fluctuations, along with a contaminating HI line (if any) within the telescope view while making the ‘sky’ calibration measurement.
3. **Click** on either side of the contaminating line to define a region to be masked (shown as a grey shaded area). The code will then fit a smooth curve to the rest of the data, and display the resulting frequency dependent T_{recv} as a green dashed line.

- **Tab 5. HI Line analysis**

This Tab applies the calibration obtained in Tab 4 to extract the HI line emission spectrum from the measurements taken in Tab 3 across different Galactic longitudes.

For each measurement at a given longitude, you will be performing the following steps:

1. Load the file $\ell.\text{csv}$.

2. Click on the button “Line temperature profile”, and a new Tab titled “HI Line Temperature” will open. You will see a plot of the HI line temperature after applying the gain and T_{recv} calibration. In addition, a baseline corresponding to sky background of 5 K has been subtracted off.
3. The most redshifted frequency (f_{obs}) belonging to the HI line can be estimated as the lowest frequency which has a temperature 5 K above the immediate baseline value. Identify f_{obs} for each measurement.

In the final step, in case you see a flat shoulder of about 5 K in the line temperature profile on the red side of the emission line, carry out the measurement for that longitude once again.

(G01) Uncovering Galactic dark matter using a radio telescope [150 marks]

(G01.1) We estimate the resolution of our radio telescope and compare it with the angular accuracy of its pointing devices.

(G01.1a) Measure the aperture dimensions of the radio telescope (longer length a , shorter length b) and express in metres. From this, estimate the resolution corresponding to each of the dimensions separately (θ_{res}^a and θ_{res}^b , respectively) in degrees. Tick (✓) the dimension which gives you the higher resolution. 7

(G01.1b) In order to compare with the above resolution, write down the least count (smallest measurable unit) of the protractor ($\theta_{\text{min}}^{\text{az}}$) and the digital inclinometer ($\theta_{\text{min}}^{\text{alt}}$). 2

(G01.2) Open the telescope operating software and verify using Tab 1 that the system is working. For each longitude,

- Use Tab 2 to enter a Galactic coordinate (longitude ℓ , latitude $b = 0$) and note the time, corresponding altitude, azimuth and v_{corr} in the Table in Summary Answer-sheet.
- Point to this altitude and azimuth and use Tab 3 to record the data ‘ $\ell.csv$ ’ (e.g. a file named ‘30.csv’ could contain data corresponding to longitude 30°).

Repeat this procedure for all the different longitudes from the table.

(G01.3) Use Tab 3 to record the calibration data ‘sky.csv’ and ‘ground.csv’ by pointing the telescope towards a region of the sky away from the Galactic plane, and then towards the ground, respectively. Add the altitude and azimuth you pointed to and the time when you carried out each of these measurements in the Summary Answersheet. 14

(G01.4) Perform the gain and temperature calibration using the procedure corresponding to Tab 4. 7

(G01.5) Now for each observation ‘ $\ell.csv$ ’, use Tab 5 to obtain the temperature spectrum at the 5 different longitudes ℓ .

(G01.5a) Determine the most redshifted frequency f_{obs} of the HI line (which has a temperature of 5 K above the baseline) corresponding to a given Galactic longitude (ℓ), and tabulate in the Summary Answersheet. 19

(G01.5b) Calculate $v_{\text{Earth}}^{\text{obs}}(\ell)$ and $v_{\text{LSR}}^{\text{max}}(\ell)$ and tabulate in the Summary Answersheet. 10

(G01.5c) Using values of $v_{\text{LSR}}^{\text{max}}$ at each of the observed Galactic longitudes, calculate the rotation velocity $v_{\text{rot}}(R)$ and the Galactic radius for the maximum redshifted emission for each of the 5 Galactic longitudes. Tabulate all these values in the Summary answersheet. 10

(G01.6) Plot the rotation velocity versus Galacto-centric radius on the graph-sheet provided as a part of Summary Answersheet and draw a smooth curve going through these points. 6

(G01.6a) Assuming a spherically symmetric mass distribution, estimate the enclosed mass within the corresponding radius of your observations using the formula:

$$M_{\text{encl}}(R) = \frac{v_{\text{rot}}^2 R}{G},$$

where R is galacto-centric radius, v_{rot} is the rotation velocity, and G is the gravitational constant. Express your answer in units of solar mass.

(G01.6b) The mass in ordinary baryonic matter enclosed at different Galacto-centric radii, R , of the Milky Way is shown with \odot symbols in the graph given in the answer-sheet. Plot $M_{\text{encl}}(R)$ from your measurements in the same graph-sheet, and draw two smooth physically correct curves, one for each set of measurements. 12

(G01.6c) Calculate the value of enclosed dark matter mass, $M_{\text{dm}}(R)$, and record it in the Table in the Summary answersheet. 10

(G01.7) Estimate the sensitivity of your observation per spectral bin in units of temperature in K given that the spectrum has a total of 512 bins spanning a total frequency range of 2.048 MHz.

9

(G01.8) Which of the following parameter(s) will improve if observations were made with a horn antenna of larger aperture dimensions. Tick against the correct option(s) in the Summary answersheet.

9

- A. Sensitivity σ_T for $T_{\text{ant}} = 5 \text{ K}$
- B. Angular resolution
- C. Estimation of v_{rot}
- D. Resolution in frequency



**Summary
Answersheet**
FRONT

Contestant Code | | | | - | | |

Q. No. | G01 | Page No. | SA-1

Final answers

| | | | | |
|----------|--|--|---|-------------|
| (G01.1a) | $a =$ $\theta_{\text{res}}^{\text{a}} =$ <input type="text" value="a"/> <input type="text"/> | m <input type="text" value="b"/> <input type="text"/> | $b =$ $\theta_{\text{res}}^{\text{b}} =$ | Marks (7.0) |
|----------|--|--|---|-------------|

| | | |
|----------|---|-------------|
| (G01.1b) | $\theta_{\text{min}}^{\text{alt}} =$ $\theta_{\text{min}}^{\text{az}} =$ | Marks (2.0) |
|----------|---|-------------|

| | | | | | | |
|---------|-----------------------|----------------|-----------------|--|------|--------------|
| (G01.2) | Galactic Longitude | Azimuth (°) | Altitude (°) | V_{corr} (km s ⁻¹) | Time | Marks (25.0) |
| | 30° | | | | | |
| | 40° | | | | | |
| | 50° | | | | | |
| | 60° | | | | | |
| | 70° | | | | | |

| | | | | | |
|---------|----------------------------|----------------|-----------------|------|--------------|
| (G01.3) | Calibration Measurement | Azimuth (°) | Altitude (°) | Time | Marks (14.0) |
| | Ground | | | | |
| | Sky | | | | |

| | | |
|---------|--|-------------|
| (G01.4) | Files corresponding to this subtask will be automatically saved in your group directory. | Marks (7.0) |
|---------|--|-------------|

Final answers

(G01.5a)

| Galactic Longitude | f_{obs} (MHz) |
|-----------------------|---------------------------|
| 30° | |
| 40° | |
| 50° | |
| 60° | |
| 70° | |

Marks (19.0)

(G01.5b)

| Galactic Longitude | $v_{\text{Earth}}^{\text{obs}}(\ell)$ (kms $^{-1}$) | $v_{\text{LSR}}^{\text{max}}(\ell)$ (kms $^{-1}$) |
|-----------------------|---|---|
| 30° | | |
| 40° | | |
| 50° | | |
| 60° | | |
| 70° | | |

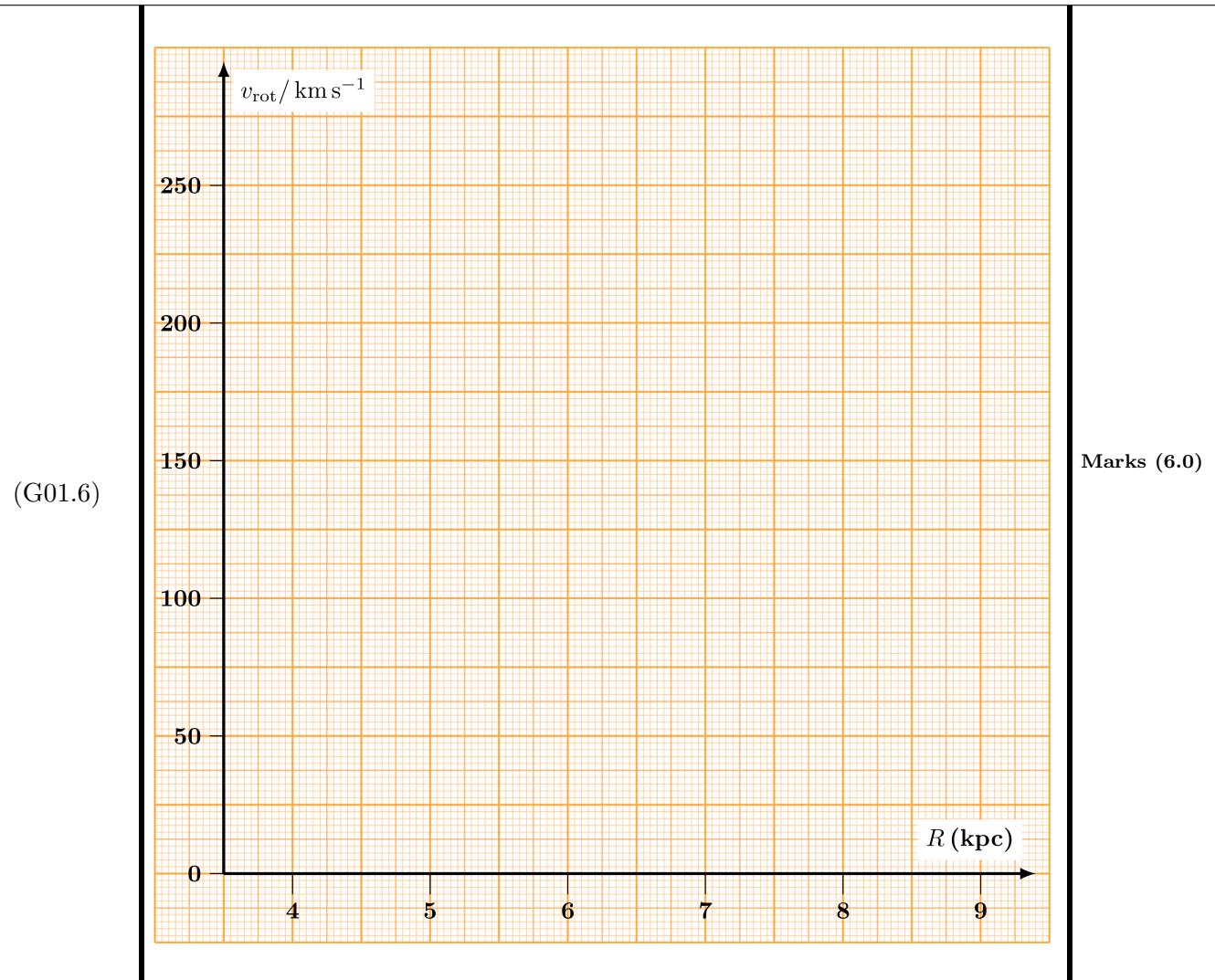
Marks (10.0)

(G01.5c)

| Galactic Longitude | $v_{\text{rot}}(R)$ (kms $^{-1}$) | Galactocentric radius R (kpc) |
|-----------------------|---------------------------------------|------------------------------------|
| 30° | | |
| 40° | | |
| 50° | | |
| 60° | | |
| 70° | | |

Marks (10.0)

Final answers

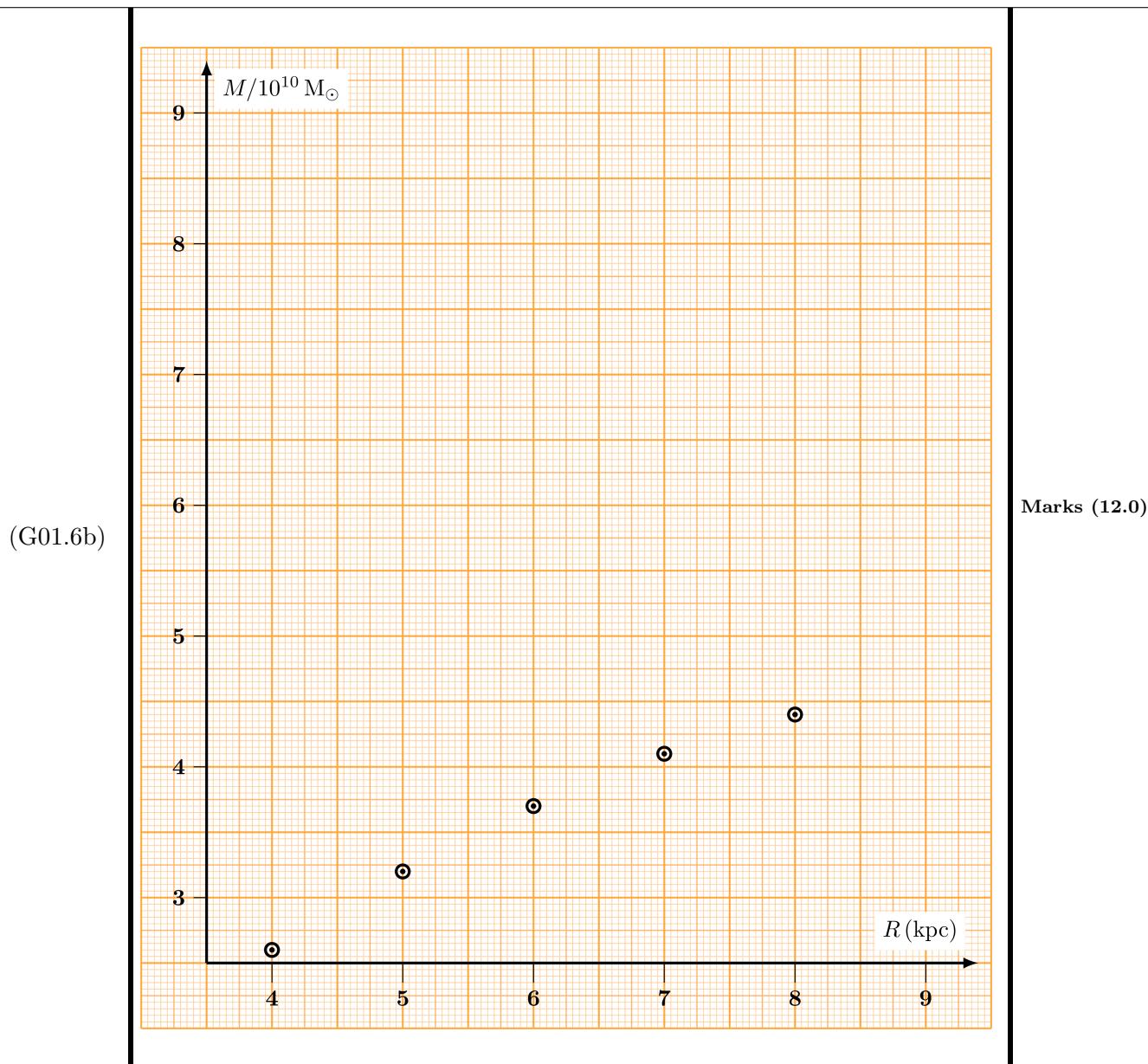


(G01.6a)

| Galactocentric Radius R (kpc) | M_{enc} (in M_{\odot}) |
|------------------------------------|---------------------------------------|
| | |
| | |
| | |
| | |
| | |

Marks (10.0)

Final answers



(G01.6c) Marks (10.0)

| Galactocentric Radius R (kpc) | | | M_{dm} (in M_{\odot}) |
|------------------------------------|--|--|--|
| | | | |
| | | | |
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| | | | |
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**Summary
Answersheet**
FRONT

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| Contestant Code | | | | | | | |
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| Q. No. | G01 | Page No. | SA-5 |
|--------|-----|----------|------|

Final answers

| | | |
|---------|--|-------------|
| (G01.7) | Sensitivity = <input type="text" value="K"/> | Marks (9.0) |
|---------|--|-------------|

| | | | | | |
|---------|------------------------|------------------------|------------------------|------------------------|-------------|
| (G01.8) | A <input type="text"/> | B <input type="text"/> | C <input type="text"/> | D <input type="text"/> | Marks (9.0) |
|---------|------------------------|------------------------|------------------------|------------------------|-------------|

Total:

End of Summary Answersheet



Summary Answersheet

BACK

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|-----------------|--|--|--|--|--|--|--|
| Contestant Code | | | | | | | |
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| Q. No. | G01 | Page No. | SA-6 |
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DO NOT WRITE

(G01) Uncovering Galactic dark matter using a radio telescope [150 marks]

(G01.1) We estimate the resolution of our radio telescope and compare it with the angular accuracy of its pointing devices.

(G01.1a) Measure the aperture dimensions of the radio telescope (longer length a , shorter length b) and express in metres. From this, estimate the resolution corresponding to each of the dimensions separately (θ_{res}^a and θ_{res}^b , respectively) in degrees. Tick (✓) the dimension which gives you the higher resolution. 7

Solution:
1. Given Parameters

- Frequency (f): 1420.40575 MHz or 1.42040575×10^9 Hz
- Horn Dimensions:
 - (a): 56.5 cm or 0.565 m
 - (b): 41.5 cm or 0.415 m
- Speed of light (c): 299,792,458 m/s

2. Calculate the Wavelength (λ)

$$\lambda = \frac{c}{f}$$

$$\lambda = \frac{299,792,458 \text{ m/s}}{1,420,405,750 \text{ Hz}} \approx 0.2111 \text{ meters}$$

3. Calculate the Angular Resolution (HPBW)
Resolution along Larger Dimension (θ_a)

This resolution is associated with the larger dimension, a .

$$\theta_a \approx \frac{180}{\pi} \left(\frac{\lambda}{a} \right)$$

$$\theta_a \approx \frac{180}{\pi} \left(\frac{0.2111 \text{ m}}{0.565 \text{ m}} \right) \approx 21.4^\circ$$

Resolution along Smaller dimension (θ_b)

This resolution is associated with the smaller dimension, b .

$$\theta_b \approx \frac{180}{\pi} \left(\frac{\lambda}{b} \right)$$

$$\theta_b \approx \frac{180}{3.14} \times \left(\frac{0.2111 \text{ m}}{0.415 \text{ m}} \right) \approx 29.2^\circ$$

| | |
|--|-----|
| Measurement in two directions (1+1) | 2.0 |
| Estimating wavelength | 1.0 |
| Writing the formula for resolution | 1.0 |
| Computing the correct value in both directions (1 + 1) | 2.0 |
| Choosing the dimension which gives the smaller of the two angles | 1.0 |

(G01.1b) In order to compare with the above resolution, write down the least count 2

(smallest measurable unit) of the protractor ($\theta_{\min}^{\text{az}}$) and the digital inclinometer ($\theta_{\min}^{\text{alt}}$).

Solution:

$$\theta_{\min}^{\text{alt}} = 0.01^\circ \text{ and } \theta_{\min}^{\text{az}} = 1^\circ$$

- (G01.2) Open the telescope operating software and verify using Tab 1 that the system is working. For each longitude,

25

- Use Tab 2 to enter a Galactic coordinate (longitude ℓ , latitude $b = 0$) and note the time, corresponding altitude, azimuth and v_{corr} in the Table in Summary Answer-sheet.
- Point to this altitude and azimuth and use Tab 3 to record the data ‘ $\ell.csv$ ’ (e.g. a file named ‘30.csv’ could contain data corresponding to longitude 30°).

Repeat this procedure for all the different longitudes from the table.

Solution:

Date: 24th July 2025

| Galactic Longitude | Azimuth ($^\circ$) | Altitude ($^\circ$) | V_{corr} kms $^{-1}$ | Time |
|--------------------|----------------------|-----------------------|------------------------|----------|
| 30° | 119.7° | 50.5° | -7.484 | 21:01:11 |
| 40° | 104.2° | 51.3° | -11.802 | 21:03:31 |
| 50° | 88.9° | 50.0° | -15.761 | 21:05:33 |
| 60° | 75.0° | 46.9° | -19.240 | 21:08:16 |
| 70° | 61.8° | 50.1° | -22.078 | 21:47:57 |

| | |
|--|------|
| Azimuth for 5 longitudes | 5.0 |
| Altitude for 5 longitudes | 5.0 |
| v_{corr} for 5 longitudes | 5.0 |
| if sign of v_{corr} is not maintained (-0.5 for each longitude) | -2.5 |
| Time for 5 longitudes | 5.0 |
| Correct source profile with central HI line feature above two side peaks (1.0 \times each longitude) | 5.0 |

- (G01.3) Use Tab 3 to record the calibration data ‘sky.csv’ and ‘ground.csv’ by pointing the telescope towards a region of the sky away from the Galactic plane, and then towards the ground, respectively. Add the altitude and azimuth you pointed to and the time when you carried out each of these measurements in the Summary Answersheet.

14

Solution:

| | |
|---|-----|
| Altitude for sky away from galactic plane | 2.0 |
| Azimuth for sky away from galactic plane | 2.0 |
| Time for sky | 1.0 |
| Time for ground | 1.0 |
| Quality of ground data | 3.0 |
| Quality of sky data | 3.0 |

| | |
|--|-----|
| Difference between ground and sky data is $> 3\text{dB}$ | 2.0 |
|--|-----|

- (G01.4) Perform the gain and temperature calibration using the procedure corresponding to Tab 4.

7

Solution:

| | |
|--|-----|
| Region chosen for masking the contaminating line | 3.0 |
|--|-----|

| | |
|--------------------|-----|
| Value of T_{sys} | 2.0 |
|--------------------|-----|

| | |
|--------------|-----|
| Correct gain | 2.0 |
|--------------|-----|

- (G01.5) Now for each observation ' $\ell.csv$ ', use Tab 5 to obtain the temperature spectrum at the 5 different longitudes ℓ .

- (G01.5a) Determine the most redshifted frequency f_{obs} of the HI line (which has a temperature of 5 K above the baseline) corresponding to a given Galactic longitude (ℓ), and tabulate in the Summary Answersheet.

19

Solution:

| Galactic Longitude | f_{obs} [MHz] |
|--------------------|-----------------|
| 30° | 1420.0207 |
| 40° | 1420.0288 |
| 50° | 1420.1096 |
| 60° | 1420.1503 |
| 70° | 1420.2479 |

| | |
|---|------|
| Value of f_{obs} at 5K for 5 longitudes | 10.0 |
|---|------|

| | |
|---|-----|
| Bonus for completing all 5 measurements | 4.0 |
|---|-----|

| | |
|---|-----|
| Baseline coinciding with 0 in the line temperature curve for 5 longitudes | 5.0 |
|---|-----|

- (G01.5b) Calculate $v_{Earth}^{obs}(\ell)$ and $v_{LSR}^{\max}(\ell)$ and tabulate in the Summary Answersheet.

10

Solution:

| Galactic Longitude | $v_{Earth}^{obs}(\ell)$ [kms^{-1}] | $v_{LSR}^{\max}(\ell)$ [kms^{-1}] |
|--------------------|---|--|
| 30° | 81.325 | 73.841 |
| 40° | 79.615 | 67.813 |
| 50° | 62.549 | 46.788 |
| 60° | 53.953 | 34.713 |
| 70° | 33.339 | 11.261 |

| | |
|---|-----|
| Value of $v_{Earth}^{obs}(\ell)$ for 5 longitudes | 5.0 |
|---|-----|

| | |
|--|-----|
| Value of $v_{LSR}^{\max}(\ell)$ for 5 longitudes | 5.0 |
|--|-----|

10

- (G01.5c) Using values of v_{LSR}^{\max} at each of the observed Galactic longitudes, calculate the rotation velocity $v_{\text{rot}}(R)$ and the Galactic radius for the maximum redshifted emission for each of the 5 Galactic longitudes. Tabulate all these values in the Summary answersheet.

Solution:

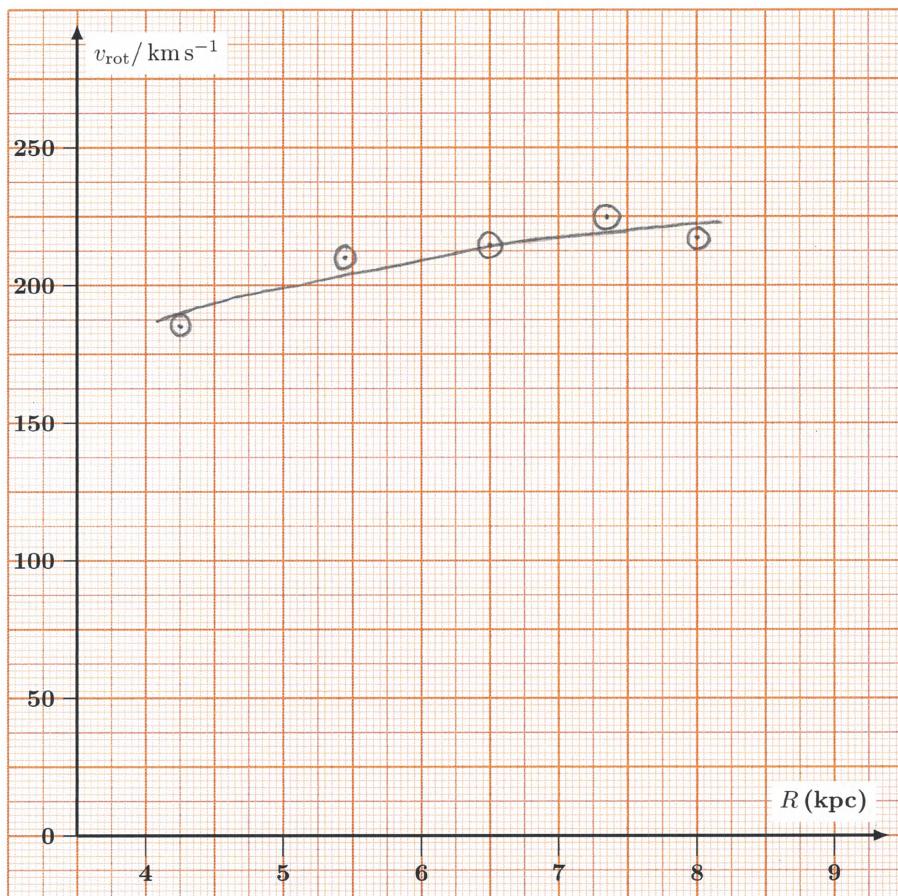
| Galactic Longitude | $v_{\text{rot}}(R) / 10^3 \text{ ms}^{-1}$ | Galacto-centric Radius, R kpc |
|--------------------|--|---------------------------------|
| 30° | 183.791 | 4.25 |
| 40° | 209.166 | 5.46 |
| 50° | 215.255 | 6.51 |
| 60° | 225.180 | 7.36 |
| 70° | 217.947 | 7.99 |

| | |
|--|-----|
| Value of v_{rot} for 5 longitudes | 5.0 |
|--|-----|

| | |
|-------------------------------|-----|
| Value of R for 5 longitudes | 5.0 |
|-------------------------------|-----|

- (G01.6) Plot the rotation velocity versus Galacto-centric radius on the graph-sheet provided as a part of Summary Answersheet and draw a smooth curve going through these points.

6

Solution:


| | |
|--------------------------------|-----|
| Plotting of points | 4.0 |
| all 5 points correctly plotted | 4.0 |
| 4 points correctly plotted | 3.0 |
| 3 points correctly plotted | 2.0 |
| 2 points correctly plotted | 1.0 |
| less than 2 points plotted | 0.0 |
| Best fit/Smooth Curve | 2.0 |

- (G01.6a) Assuming a spherically symmetric mass distribution, estimate the enclosed mass within the corresponding radius of your observations using the formula: 10

$$M_{\text{encl}}(R) = \frac{v_{\text{rot}}^2 R}{G},$$

where R is galacto-centric radius, v_{rot} is the rotation velocity, and G is the gravitational constant. Express your answer in units of solar mass.

Solution:

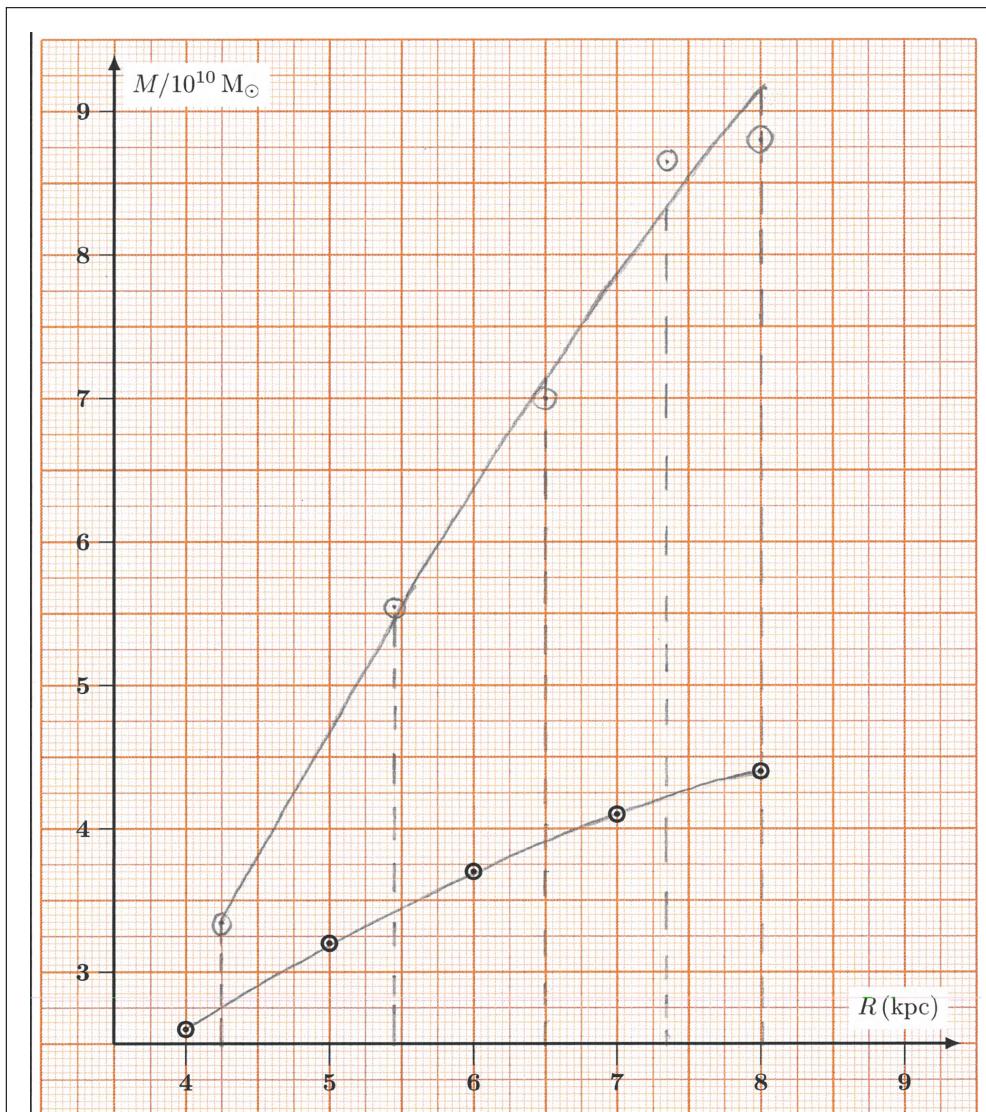
| Galactic longitude | Galacto-centric radius [kpc] | $M_{\text{encl}} [10^{10} M_{\odot}]$ |
|--------------------|------------------------------|---------------------------------------|
| 30° | 4.25 | 3.34 |
| 40° | 5.46 | 5.55 |
| 50° | 6.51 | 7.01 |
| 60° | 7.36 | 8.67 |
| 70° | 7.98 | 8.82 |

For 5 longitudes

10.0

- (G01.6b) The mass in ordinary baryonic matter enclosed at different Galacto-centric radii, R , of the Milky Way is shown with \odot symbols in the graph given in the answer-sheet. Plot $M_{\text{encl}}(R)$ from your measurements in the same graph-sheet, and draw two smooth physically correct curves, one for each set of measurements. 12

Solution:



| | |
|--|-----|
| all 5 points of M_{encl} correctly plotted | 5.0 |
| Physically correct smooth curves (monotonic 2 × 1.5, non-intersecting 2.0) | 5.0 |
| Interpolating M^{bary} at the location of the measurements. | 2.0 |

(G01.6c) Calculate the value of enclosed dark matter mass, $M_{\text{dm}}(R)$, and record it in the Table in the Summary answersheet. 10

Solution:

| R /kpc | M_{encl} / $10^{10} M_{\odot}$ | M_{bary} / $10^{10} M_{\odot}$ | M_{dm} / $10^{10} M_{\odot}$ |
|----------|---|---|---------------------------------------|
| 4.0 | 2.8 | 2.6 | 0.2 |
| 5.0 | 4.7 | 3.2 | 1.5 |
| 6.0 | 6.6 | 3.7 | 2.9 |
| 7.0 | 8.1 | 4.1 | 4.0 |
| 8.0 | 8.6 | 4.4 | 4.2 |

| | |
|--|-----|
| Reading M_{encl} from the graph for given five R | 5.0 |
| Calculating M_{dm} for the given five R | 5.0 |

- (G01.7) Estimate the sensitivity of your observation per spectral bin in units of temperature in K given that the spectrum has a total of 512 bins spanning a total frequency range of 2.048 MHz. 9

Solution:
1. Calculate Channel Bandwidth (Δf)

Determine the frequency range, or bandwidth, covered by a single "bin" or channel in the antenna.

- **Formula:** $\Delta f = \frac{\text{Total Frequency Range}}{\text{Number of Bins}}$

- **Given Data:**

- Total Frequency Range = 2.048 MHz or 2,048,000 Hz
- Number of Bins = 512

- **Calculation:**

$$\Delta f = \frac{2,048,000 \text{ Hz}}{512} = 4,000 \text{ Hz} \text{ (or } 4 \text{ kHz)}$$

2. Calculate Sensitivity (σ_T)

Next, we use the radiometer equation with the channel bandwidth, integration time and the system temperature.

- **Formula:**

$$\sigma_T = \frac{T_{sys}}{\sqrt{\Delta f \cdot t_{int}}}$$

- **Given Data:**

- $T_{sys} = 115 \text{ K}$ (Measured T_{recv} + assumed T_{ant} of 5 K)
- $\Delta f = 4,000 \text{ Hz}$
- Integration Time (τ) = 60 s

- **Calculation:**

Sensitivity

$$= \sigma_T = \frac{115 \text{ K}}{\sqrt{4000 \text{ Hz} \times 60 \text{ s}}} = \frac{115 \text{ K}}{\sqrt{240,000}} \approx 0.235 \text{ K}$$

| | |
|---|-----|
| Determination of T_{sys} from calibration tab, using T_{recv} | 4.0 |
| Calculation of Δf | 2.0 |
| Using correct t_{int} | 1.0 |
| Calculation of RMS noise accurate to two significant digits (i.e., sensitivity) with unit | 2.0 |

(G01.8) Which of the following parameter(s) will improve if observations were made with a horn antenna of larger aperture dimensions. Tick against the correct option(s) in the Summary answersheet.

- A. Sensitivity σ_T for $T_{\text{ant}} = 5 \text{ K}$
- B. Angular resolution
- C. Estimation of v_{rot}
- D. Resolution in frequency

Solution:

Correct Options:

- B. Angular resolution
- C. Estimation of v_{rot}

| | |
|--|------|
| Each correct option chosen | 3.0 |
| Each incorrect option chosen | -2.0 |
| Bonus marks if only correct options are ticked | 3.0 |