Data Analysis: Instructions

- **Do not touch envelopes until the start of the examination.**
- The data analysis examination lasts for 3 hours and is worth a total of 125 marks.
- There are **Answer Sheets** for carrying out detailed work and **Working Sheets** for rough work, which are already marked with your student code and question number.
- ***Use only the answer sheets for a particular question for your answer. Please write only on the printed side of the sheet. Do not use the reverse side.*** If you have written something on any sheet which you do not want to be evaluated, cross it out.
- Use as many mathematical expressions as you think may help the evaluator to better understand your solutions. The evaluator may not understand your language. If it is necessary to explain something in words, please use short phrases (if possible in English).
- You are not allowed to leave your work desk without permission. If you need any assistance (malfunctioning calculator, need to visit a restroom, etc.), please draw the attention of the supervisor.
- The beginning and end of the examination will be indicated by the supervisor. The remaining time will be displayed on a clock.
- At the end of the examination you must stop writing immediately. Put everything back in the envelope and leave it on the table.
- Once all envelopes are collected, your student guide will escort you out of the examination room.
- A list of constants and useful relations are included in the envelope.

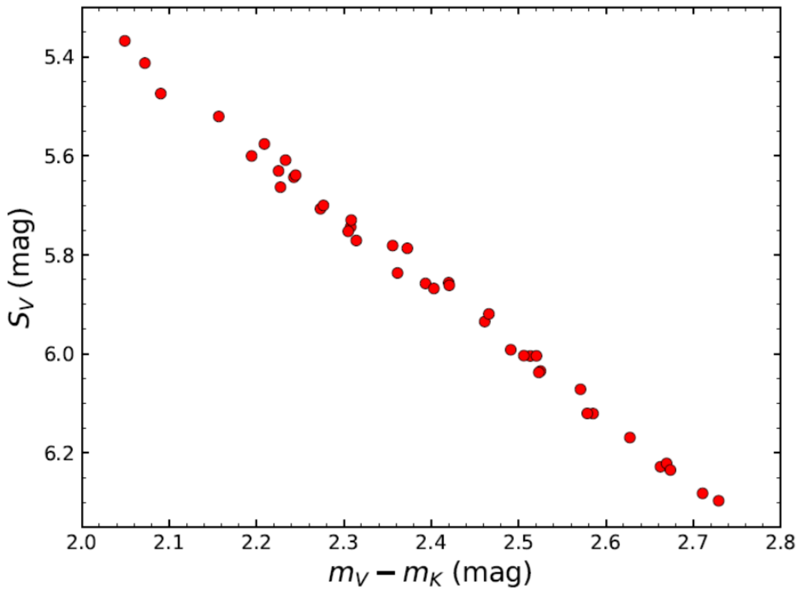
Data Analysis 1: ‘Distance to the Large Magellanic Cloud’

In 2019 an international collaboration led by Polish astronomers measured, with very high precision and accuracy, the distance to the Large Magellanic Cloud (LMC), a satellite galaxy of the Milky Way. In this way they set the zero point of the extragalactic distance scale, which allowed for a very precise measurement of the Hubble constant. Their method involved measuring the distances to 20 eclipsing binary stars in the LMC, using the concept of the surface brightness S_V of a star defined as:

$$S_V = m_V + 5\log_{10}\theta,$$

where m_V is the magnitude of a star in the optical V band and θ is the angular diameter of the star on the sky in milliarcseconds (mas).

The quantity S_V can be understood as the magnitude of a star with an angular diameter of 1 mas. An empirical relation has been established between S_V and the colour index $(m_V - m_K)$, where m_V and m_K are magnitudes in the V-band and infrared K-band. This is shown in the figure below for giant stars of spectral types G and K.



Using this relation, the distance to an eclipsing binary system can be determined by deriving the physical radii of the components (using photometry and spectroscopy), and comparing these with the angular diameters predicted by the $S_V - (m_V - m_K)$ relation.

The table below gives the parameters of three detached eclipsing binary stars. R_1 and R_2 are the radii of each component, V_{1+2} and K_{1+2} are the total brightness in magnitudes of the binary in the V- and K-bands, and L_2/L_1 is the luminosity ratio of the components in each band.

source ID	R_1 $[R_\odot]$	R_2 $[R_\odot]$	V_{1+2} [mag]	K_{1+2} [mag]	L_2/L_1 (V)	L_2/L_1 (K)
OGLE LMC-ECL-03160	17.03	37.42	16.73	14.10	2.80	4.23
OGLE LMC-ECL-10567	24.60	36.64	16.15	13.83	1.41	1.99
OGLE LMC-ECL-18365	37.30	15.94	16.27	14.01	0.206	0.188

Apply the method outlined above to the three eclipsing binary systems and calculate the distance to the LMC in kiloparsecs. Estimate the total error of the result. Assume that the fitting of the $S_V - (m_V - m_K)$ relation contributes to a bias of up to 0.8% in all measurements simultaneously.

(Total: 50 points)

Hint: in your calculations keep at least three significant figures and two decimal places. Assume that interstellar extinction is negligible and that the angular size of the LMC is small.

Data Analysis 2: 'Isolated black hole'

In 2022, two independent groups reported the discovery of an isolated black hole based on observations of the gravitational microlensing event OGLE-2011-BLG-0462. In this problem, we will analyze data from the Hubble Space Telescope to reproduce their findings.

Gravitational microlensing occurs when the light of a distant star (the 'source') is bent and magnified by the gravitational field of an intervening object (the 'lens'). The characteristic angular scale of gravitational microlensing events, called the angular Einstein radius $\theta_{\rm E}$, depends on the mass M and distance D_ℓ from the Earth to the lens:

$$\theta_{\rm E} = \sqrt{\frac{4GM}{c^2}\frac{D_s - D_\ell}{D_s D_\ell}},$$

where D_s is the distance to the source star. For typical microlensing events observed in the Milky Way, the source stars are in the Galactic bulge, near the Galactic center, so $D_s \approx 8$ kpc.

(a) Calculate the angular Einstein radius in milliarcseconds (mas) for an example lens of 1 $M_\odot$ located at a distance of 1 kpc. (2 points)

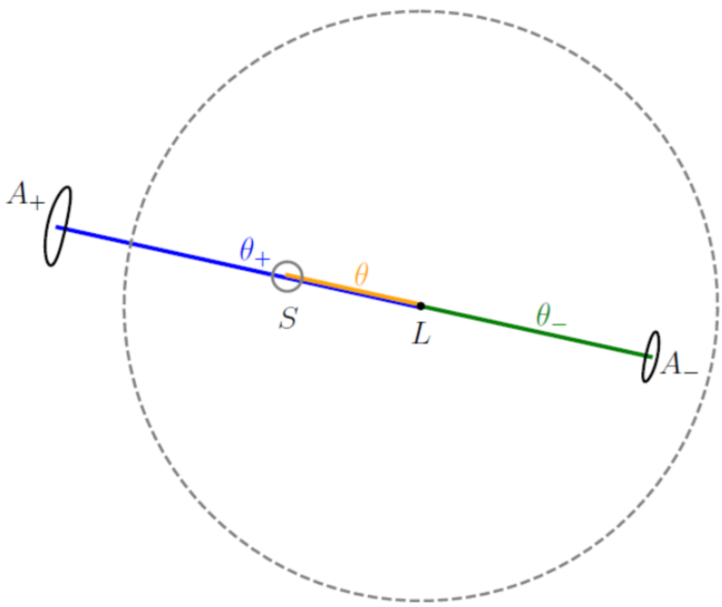
Suppose that at time t the lens and the source are separated by an angle $\theta \equiv u(t)\theta_{\rm E}$ on the sky. Two images of the source are created on a line through the positions of the source and the lens, at angular distances θ_+ and θ_- from the lens given by:

$$\theta_\pm = \frac{1}{2}\left(u \pm \sqrt{u^2+4}\right)\theta_{\rm E}.$$

These two images are magnified, relative to the unlensed brightness of the source. The absolute magnification of the images is:

$$A_\pm = \frac{1}{2}\left(\frac{u^2+2}{u\sqrt{u^2+4}} \pm 1\right).$$

The image below shows the geometry of the event. The position of the lens is marked as L, the unlensed position of the source is marked as S, while A_+ and A mark the positions of the two images of the source. The dashed circle has a radius of one Einstein radius.

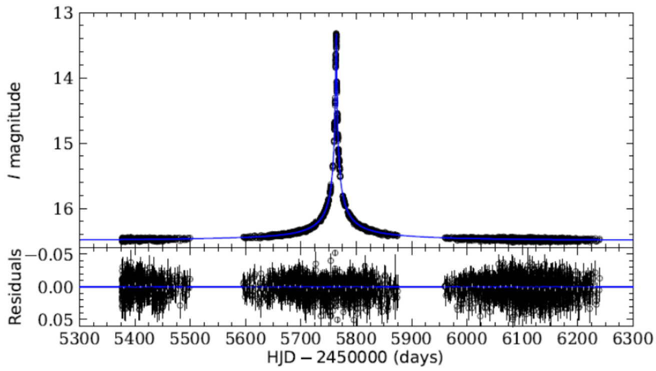


(b) Current telescopes cannot normally resolve this pair of images, but only measure the position of the image centroid, i.e. the brightness-weighted mean of the positions of the two images. Derive an expression for the angular separation θ_c of the image centroid relative to the lens as a function of u and $\theta_{\rm E}$. (8 points)

(c) Derive an expression for the source deflection $\Delta\theta$, i.e. the difference between the location of the centroid and the unlensed position of the source, as a function of u and $\theta_{\rm E}$. What is the source deflection when the lens and the source are nearly perfectly aligned ($u \approx 0$)? (4 points)

The source and lens are moving relative to each other in the sky. Thus, both the total magnification of the images and the position of the centroid changes with time, resulting in observable photometric and astrometric microlensing effects. For now, we assume that the source-lens relative motion is rectilinear.

The plot below shows the light curve of the gravitational microlensing event OGLE-2011-BLG- 0462, discovered by the OGLE sky survey led by astronomers from the University of Warsaw. The solid line shows the best-fitting light curve model. The Einstein timescale of the event, i.e. the time needed for the source to move by one angular Einstein radius relative to the lens, was $t_{\mathrm{E}} = 247$ days. The event peaked on 21 July 2011 (HJD = 2455763). The minimal separation between the lens and the source was $u_0 \approx 0$.



The table below shows the measured positions of the source star against the background objects in the East and North directions based on images from the Hubble Space Telescope.

HJD	E position (mas)	N position (mas)
2455765.2	2.58 ± 0.13	7.29 ± 0.16
2455865.7	2.32 ± 0.12	5.44 ± 0.24
2456179.7	0.46 ± 0.14	1.62 ± 0.08
2456195.8	0.88 ± 0.36	1.56 ± 0.77
2456426.2	-1.02 ± 0.21	-0.94 ± 0.12
2456587.7	-2.04 ± 0.07	-1.88 ± 0.40
2456956.6	-4.54 ± 0.25	-5.16 ± 0.29
2457995.2	-11.14 ± 0.12	-15.14 ± 0.17

(d) Plot the measured positions of the source star against the background objects in the East and North directions as a function of time. (10 points)

(e) The observed motion of the source star is the sum of two effects: rectilinear proper motion of the source and astrometric microlensing effects. Calculate the proper motion (in mas/year) of the source in the East and North directions and its uncertainty. (8 points)

(f) After subtracting the effects of proper motion from the data, calculate and plot the total resultant astrometric deflection as a function of u. Neglect the uncertainty of the proper motion determination. (20 points)

(g) Analyse the data to determine the angular Einstein radius θ_{E} of the event and its uncertainty. (Hint: it may be helpful to linearise the expression for $\Delta\theta$). (16 points)

(h) For long-timescale events such as OGLE-2011-BLG-0462, the rectilinear approximation of the relative lens-source proper motion is not strictly true and the orbital motion of the Earth has to be taken into account. This allows measurement of a dimensionless quantity called the microlensing parallax, defined as $\pi_{\mathrm{E}} = (\pi_l - \pi_s)/\theta_{\mathrm{E}}$, where π_l and π_s are parallaxes of the lens and the source, respectively.

For this event $\pi_{\mathrm{E}} = 0.095 \pm 0.009$. Rearrange the expression for θ_{E} given earlier to calculate the mass of the lens in solar masses and its uncertainty. (7 points)

(Total: 75 points)