Chapter 4 True or False

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1.

True.

2.

True, $T(f) = 3f - 4f'$ is linear and infinite-dimensional.

##3.

True.

$$\begin{bmatrix} a & 0 \\ b & c \end{bmatrix} + \begin{bmatrix} d & 0 \\ e & f \end{bmatrix} = \begin{bmatrix} a+d & 0 \\ b+e & c+f \end{bmatrix}$$

$$k \begin{bmatrix} a & 0 \\ b & c \end{bmatrix} = \begin{bmatrix} ka & 0 \\ kb & kc \end{bmatrix}$$

##4.

True, the kernel is a subspace of the domain, as with matrices.

5.

False. $R^{2\times 3}$ has 6 independent elements and so is 6-dimensional.

6.

True, by definition of the basis.

7.

True; they both have a basis of $(1, t)$, where t is some coefficient.

8.

True. Since the transformation is $P_4 \to P_4$, the zero kernel implies it is one-to-one as well as onto.

9.

True. They must intersect at least in the zero element, and the zero element qualifies as a subspace.

10.

False. The kernel could easily be larger.

11.

True; there is no other way to account for the constant, degree-1, and degree-2 terms of a member of P_3.

12.

True.

13.

False. It has kernel $f'' = f$.

14.

False. Counterexample:

$$T(P) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

15.

False. They can be exactly the same, as in one homework problem where they were both $\sin x$, $\cos x$

16.

True. The transpose is a linear transformation.

17.

True.

18.

True.

19.

False.

$$\det(3I + 5I) = \det(8I) = 64$$
$$\det(3I) + \det(5I) = 9 + 25 = 36$$

21.

True; that is the rank of its matrix.

23.

True.

24.

True. Inverse is $M = S^{-1}BS^{-1}$.

25.

26.

True, I think.

27.

True; we know it is onto, but also must be one-to-one if it is $P \to P$.

28.

True; those three elements are independent and span the space.

29.

False.

$$kT(f(t)) = kf(4t - 3)$$
$$T(kf(t)) = f(k(4t - 3))$$

30.

True. Inverse is $\frac{g+3}{4}$

31.

False. P_n is a finite-dimensional subspace of the infinite-dimensional P.

33.

False. Consider $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$

36.

False. It has rank $n-1$.

37.

True; this matrix is invertible.

38.

True; any subspace with dimension 10 will do.

39.

True. If they are orthogonal complements, then their union is $R^{2\times 2}$, which is a subspace of itself.

40.

True; it is possible to represent every element in a low-dimensional subspace with some combination of a higher-dimensional subspace.

41.

True, for the same reason a triangular matrix already has full rank. Basis member k is absent in members $[1, k-1]$, so no linear dependence is possible.

42.

False. the polynomial derivative zeroes out the constant term, making it noninvertible.

43.

True. It is impossible to represent every member of a space of dimension $ n>p$ with a subspace of dimension p.

44.

True; it is just the zero kernel!

45.

True. $T(f) = f$ for at least the zero element, which is itself a subspace.

50.

False. A 2×2 matrix is invertible only with at least two nonzero elements, and some members of the subspace will have only one dimension.

51.

True.

55.

True.

60.

False. Two distinct three-dimensional subspaces of the five-dimensional P_4 is impossible.