Section 4.3 Problems

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August 26, 2021

Remember, scalars $a, b, c, \ldots$ apply after every computation is finished when calculating a transformation.

% Standard custom LaTeX commands

% 1: term 1 % 2: subscript 1 % 3: term 2 % 4: subscript 2 % 5. operation

12.

$$T(M) = M \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

13.

$$T(M) = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 2 & 0 & 2 & 0 \\ 0 & 2 & 0 & 2 \end{bmatrix}$$

14.

A matrix transformation with an ugly alternate basis.

$$\begin{bmatrix} 0 & 0 \end{bmatrix}$$

15.

$$T(x + iy) = x - iy$$
$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

16.

$$T(x+iy) = x - iy$$

$$\begin{aligned} T(x) &= x \\ T(iy) &= -iy \end{aligned}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

18.

$$\begin{aligned} &T(z) = (2+3i)z \\ &= 2x + 3ix + 2iy + 3i^2y \\ &= 2x + 3ix - iy \\ &B = \begin{bmatrix} 1 & 0 \\ 3 & -1 \end{bmatrix} \end{aligned}$$

Isomorphism.

19.

$$\begin{aligned} &T(z) = (p+iq)z \\ &= px + iqx + piy + i^2qy \\ &= px + iqx + piy - qy \\ &B = \begin{bmatrix} p & -q \\ q & p \end{bmatrix} \end{aligned}$$

Isomorphism.

21.

$$\begin{aligned} &T(f(t)) = f' - 3f \\ &= b + 2ct - 3a - 3bt - 3ct^2 \\ &B = \begin{bmatrix} -3 & 1 & 0 \\ 0 & -3 & 2 \\ 0 & 0 & -3 \end{bmatrix} \end{aligned}$$

Isomorphic.

22.

$$T(f(t)) = f'' + 4f'$$

$$\begin{aligned} &T(1) = a(0 + 4(0)) = 0 \\ &T(t) = b(0+2) = 2b \\ &T(t^2) = c(2 + 4(1+2t)) = 6c + 8ct \end{aligned}$$

$$\begin{bmatrix} 0 & 2 & 1 \\ 0 & 0 & 8 \\ 0 & 0 & 0 \end{bmatrix}$$

23.

$$\begin{aligned} T(f(t)) = & \quad f(3) P_2 \to P_2 \\ B = & \begin{bmatrix} 1 & 3 & 9 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{aligned}$$

Correct.

24.

$$T(f(t)) = f(3$$

and basis $(1, (t-3), (t-3)^2)$

$$\begin{aligned} & T(1) = 1 \\ & T((t-3) = b(3-3) = 0 \\ & T((t-3)^2) = (3-3)^2 = 0 \end{aligned}$$

This represents the zero of a function.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

25.

$$\begin{aligned} T(f(t)) &= f(-t) \\ &= a - bt + ct^2 \\ & \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

Isomorphic

26.

$$T(f(t) = f(2t)$$

$$\begin{aligned} & T(1) = a(1) = a \\ & T(t) = b(2t) = 2bt \\ & T(t^2) = c((2t)^2) = 4ct^2 \end{aligned}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

27.

$$
\begin{aligned}
&T(f(t)) = f(2t-1) \\
&f(2t-1) = a + 2b - 2 + c(2t-1)^2 \\
&= a + 2bt - 2b + c(4t^2 - 4t + 1) \\
&= a + 2bt - 2b + c(4t^2 - 4t + 1) \\
&B = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -4 \\ 0 & 0 & 4 \end{bmatrix}
\end{aligned}
$$

Isomporphism.

28.

With basis $(1, t-1, t-1)^2$

$$
\begin{aligned}
&T(f(t)) = f(2t-1) \\
&T(f(1)) = a \\
&T(f(2t-1)) = b(2t-1-1) = 2b(t-1) \\
&T(f(t-1)^2) = c(2(t-1)^2 - 1) = c(2(t^2 - 2t)) = 2c(t-1)^2
\end{aligned}
$$

Change of basis makes this a lot easier to interpret.

$$
\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}
$$

29.

$$
T(f(t)) = \int_0^2 f(t)dt
$$

First get the transformed bases:

$$
\begin{aligned}
&T(1) = a(t1) = at \\
&T(t) = b(t^2/2) = bt^2/2 \\
&T(t^2) = c(t^3/3)
\end{aligned}
$$

Then compute the integral:

$$
\int_0^2 f(t)dt = 8
$$

$$
\begin{bmatrix} 2 & 2 & 8/3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}
$$

30.

This joy:

$$T(f(t) = \frac{f(t+h) - f(t)}{h}$$

, where h is nonzero

$$\begin{aligned}
T(1) &= a\frac{1-1}{h} = 0 \\
T(t) &= b(\frac{t+h-t}{h}) = b \\
T(t^2) &= c(\frac{(t^2+h^2+2ht-t^2}{h}) = c(h+2t)
\end{aligned}$$

And the matrix:

$$\begin{bmatrix} 0 & 1 & h+2 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

33.

$$T(f(t)) = f(1) + f'(1)(t-1)$$

$$\begin{aligned}
&T(1) = (a+0) = a \\
&T(t) = b(1+1(t-1) = bt \\
&T(t^2) = c(1+2(t-1)) = c(2t-1) = 2ct - c
\end{aligned}$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

39.

$$T(M) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} M - M \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

I think this is

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Some transformations in the space $\sin(t) + \cos(t)$. ## 48.

$$T(f) = f'$$

$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

49.

$$T(f) = f“ + 2f‘ + 3f$$

$$T(\cos(t)) = 2\cos t - 2\sin t$$
$$T(\sin(t)) = 2\cos t - \sin t$$

$$\begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix}$$

50.

$$T(f) = f'' + bf' + cf$$

$$\begin{aligned} &T(\cos t) = (-\cos t) + a(-\sin t) + b(\cos t) \\ &= (b-1)\cos t - a\sin t \\ &T(\sin(t)) = -\sin t + a\cos t + b\sin t \\ &= a\cos t + (b-1)\sin t \end{aligned}$$

implying the matrix

$$\begin{bmatrix} b-1 & -a \\ a & b-1 \end{bmatrix}$$

Not isomorphic if $b = 1$ and $a = 0$ or $(b-1)^2 + a^2 = 0$

51.

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

52.

$$T(f(t)) = f(t - \pi/4)$$

$$\frac{1}{\sqrt{2}} \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}$$

53.

The general rotation matrix

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$