Chapter 5 True or False

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\newcommand{\meq}[1]{\begin{split}#1\end{split}}
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1.

False. A linear transformation that preserves length but not angle is not orthogonal.

2.

True: assuming invertibility, the inverse of the transpose is the transpose of the inverse.

3.

Assuming A is square, A^2 is also orthogonal because $A_j \cdot A_j = 1$ and $A_j \cdot A_k = 0$ by definition.

4.

False. $(AB)^T = B^T A^T$ The $2,1$ element in the first case is row 2 of A times column 1 of B, in the second column 1 of A times row 2 of B.

5.

True.

6.

7.

False. A symmetric matrix may have zeroes on the diagonal, like

$$\begin{bmatrix} ,0 & 0 & 1 \\ ,0 & 0 & 0 \\ ,1 & 0 & 0, \end{bmatrix}$$

8.

True. If $AA^T = I$, then $A^{-1} = A^T$, which is true only of orthogonal matrices.

9.

False. It is $(x \cdot u)u$

10.

True.

11.

False. This is true only of parallel vectors.For two opposite-signed vectors, $||x + y||^2 = 0$.

12.

True. The determinant of R is the same as that of A, and R is identical for the transpose, since $Q^T = Q^{-1}$

13.

True. For orthogonal matrices, $A^T = A^{-1}$, and the inverse is also orthogonal.

14.

False.

$$\begin{aligned} AB &= A^T B^T \\ &= (BA)^T \end{aligned}$$

15.

16.

False. If A is not square, the projection is $A(A^T A)^{-1} A^T$. If it is square, the projection is I.

17.

True. If B is symmetric, so is B^2, since $B^2 = B^T B^T = B^2$.

18.

True.

$$\begin{aligned} A^T B^T &= B^T A^T \\ BA &= AB \end{aligned}$$

19.

False. The subspaces partition R^5 completely, and dimensions can only be integers, so if $V^\perp$ has an odd dimension $V^\parallel$ has an even one and vice versa.

20.

True. This is just a restatement of the QR decomposition.

21.

False. Since the columns are all unit length, no scaling of volume occurs. But the matrix may have negative orientation, in which case $\det(A) = -1$.

22.

True.

23.

True. If this were false, the vector would have greater than unit length.

24.

True. By definition, a basis consists of linearly independent vectors, which can always be orthonormalized.

25.

False. the columns are not unit length.

26.

True.

$$x = x^\parallel + x^\perp$$
$$x - x^\parallel = x^\perp$$

By definition, this vector is orthogonal to the subspace.

27.

False. consider:

$$A = \begin{bmatrix} -1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \quad B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

28.

True. Since A is symmetric, the orthogonal complement of the image is the kernel of the transpose, which is the same as A's kernel since $A^T = A$. So those vectors must be orthogonal.

29.

True; proved in an earlier problem.

30.

False. This is true of any symmetric matrix, since $A^2 = A^2$.

31.

32.

True. A scaling matrix would not leave some vectors with lengths unchanged.

33.

True. If $A^{-1} = A$, then $A_j \cdot A_i = 0$ and $A_j \cdot A_j = 1$.

34.

True. If the entries are all positive, then they must be in the positive-signed quadrant, which encloses angles of less than 90 degrees in R^2 and less in higher dimensions.

35.

True. $\ker(B^T) = (B^T)^{\perp} = im(B^T)$

36.

True. A^T is always symmetric.

37.

38.

True. $\ker(B^T) = B^{\perp}$, so the image doesn't lose dimension.

39.

40.

True. This is true only if the matrix is symmetric, which implies $\ker(A^T) = \ker(A)$

41.

42.

True. This is a restatement of Cauchy-Swarz.

43.

True.

44.

True.

45.

True.

46.

True. This is the definition of the eigenvalue.

47.

False. $R^{2\times 2}$ has dimension 4. But any orthogonal matrix spans R^2, so those basis elements would not be linearly independent and therefore not a valid basis.

48.

49.

50.

True. Projection matrices are always symmetric, which guarantees orthogonal eigenvectors.