Chapter 2.4 Problems

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1.

$v_4 = 3v_1 - v_2 + 4v_3$

2.

4.

3.

In case 1, $v_1 = 0v_2$. In the seond, $v_2 = \frac{b}{a}v_1$. In the third, $v_3 = \frac{c-be/d}{a}v_1 + e/dv_2$

4.

Evident form the fact $0/c = 0$ for nonzero c and $0 \pm 0 = 0$.

5.

6.

The matrix has RREF

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

7.

The matrix

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

reduces to the identity.

8.

9.

a.

The biggest possible dimension of a set of vectors in R^3 is 3.

b.

Scalars of each other.

c.

$0v = 0$ for all vectors v. ### 10.

Kernel basis

$$\begin{bmatrix} -2 & 3 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

11.

a.

Line.

b.

Plane

c.

Plane

d.

Not a subspace.

12.

$Ax = b$, $x^T A = c$, false; the zero vector is always in the row space.

13.

All 2, as it turns out.

14.

15.

16.

They do.

17.

The same way: a row of zeroes.

18.

19.

n, span, greater than or equal

20.

a.

$\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

b.

$\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$

c.

$$\begin{bmatrix} 0 & 0 \\ 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

d.

For the kernel,

$$\begin{bmatrix} -1 & 0 & -1 \\ 0 & -1 & - \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

21.

For the column, $\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$, or a similar basis of opposite-sign pairs. FOr the row space, the columns of the transpose themselves or any scalars of them.

22.

a.

Might not.

b.

Are not.

c.

Might not.

d.

Might not.

23.

Then rank is n, and at most m and at least 1 if they span R^m. If they are a basis, there are m linearly independent vectors.

24.

25.

26.

Both true, from the definitions of orthogonal complements.

27.

For both the kernel is.

$$\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

28.

a.

False, if $m > n$ and rank is n

b.

False, consider $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

c.

True.

d.

False, some vectors may be redundant.

29.

At least one is nonzero.

30.

31.

Say W has dimension 1 and none of the vectors are a multiple of it.

32.

3, 0, 16.

33.

$\mathbf{W}^{\perp}$ has dimension 2, which is less than the dimension of $\mathbf{V}$. Therefor the dimension of vectors in the second subspcae but not the first is at least 1.

34.

Assume $W^{\perp} \in V$. That is not a basis for V, leaving at least a one-dimensional subspace in V not in the kernel and therefore in W. and exactly k vectors spanning ensures no redundancy.

35.

a.

False; it has zero or one solutions

b.

True.

36.

6 in each case.

37.

The first has dimension n the latter 2 $n(n-1)/2$

38.

a.

All constant functions

b.

$f(x) = 3x + c$ ### c.

39.

40.

41.

Respectively constants, linear functions, quadratics.

42.

$1 - x, 1 - x^2$

43.

Solving for the vector in $R^{3\times3}$

```
I <- list(list(1, 0, 0), list(c(0, 1, 0)), list(c(0, 0, 1)))
perms <- sapply(list(c(1, 3, 2), c(2, 1, 3), c(3, 2, 1), c(2, 3, 1), c(3, 1, 2)), \(x) unlist(I[x]))
solve(t(perms) %*% perms) %*% t(perms) %*% unlist(I)
```

```
     [,1]
[1,]    1
[2,]    1
[3,]    1
[4,]   -1
[5,]   -1
```

44.

45.

If the expanded matrix is invertible, then b is not a linear combination of the preceding columns, meaning no solution exists. But if the expanded matrix is singular, then b is such a linear combination, making the equation solvable.