Section 1.4 Problems

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4.

An $n \times m$ multiplying any vector involves m^2n multiplications. If B is a $m \times p$, then there are m^2np.

6.

An example of matrices whose products are transposes:

```
A <- square(2, 3, 3, 4)
B <- square((-1)^(as.numeric(A)))

A %*% B
```

```
     [,1] [,2]
[1,]   -1    1
[2,]   -1    1
```

```
B %*% A
```

```
     [,1] [,2]
[1,]   -1   -1
[2,]    1    1
```

10.

True or false:

a. Columns 1 and 3 of B the same, columns 1 and 3 of A: true

```
square(1, 2, 3, 0, 0, 0, 7, 2, 8) %*% square(1, 1,
    1, 0, 0, 0, 1, 1, 1)
```

```
     [,1] [,2] [,3]
[1,]    8    0    8
[2,]    4    0    4
[3,]   11    0   11
```

b. Rows 1 and 3 of B the same, rows 1 and 3 of AB the same: true

```r
square(1, 0, 0, 0, 1, 0, 0, 0, 1) %*% square(1, 1,
    1, 0, 0, 0, 1, 1, 1, byrow = TRUE)
```

```
     [,1] [,2] [,3]
[1,]    1    1    1
[2,]    0    0    0
[3,]    1    1    1
```

c. Rows 1 and 3 of A the same, so are rows of AB.

```r
square(1, 2, 3, 1, 2, 3, 1, 2, 3) %*% c(0, 1, 1)
```

```
     [,1]
[1,]    2
[2,]    4
[3,]    6
```

d. $(AB)^2 = A^2B^2 : false$

```r
A <- square(1, 0, -1, 0)
B <- t(A)

(A %^% 2) %*% (B %^% 2)
```

```
     [,1] [,2]
[1,]    2    0
[2,]    0    0
```

```r
(A %*% B) %^% 2
```

```
     [,1] [,2]
[1,]    4    0
[2,]    0    0
```

13.

Examples of matrices.

a. $A^2 = -I$. Two 90-degree rotations, with matrix $\begin{bmatrix} 0 & - \\ 1 & 0 \end{bmatrix}$

b. $B^2 = 0$: $\begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}$

c.$CD = -DC$: $C = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, $D = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

d. $EF = 0$ $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ and $\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$

```r
A <- square(1, -1, 1, 1)
B <- square(1, 1, -1, 1)
A %*% B
```

```
     [,1] [,2]
[1,]    2    0
[2,]    0    2
```

```r
B %*% A %*% B
```

```
     [,1] [,2]
[1,]    2   -2
[2,]    2    2
```

16.

Since $(AB)x = A(BX)$, then the first column of AB must equal A times the first column of B. If it were something else, then $AB = C$ and

$$(AB)x = Cx \neq A(Bx)$$

18.

Each entry of AB = $\sqrt{n}$

39.

A is 3×5, B is 5×3, C is 5×1, D is 3×1. All entries are 1.

a. AB 3 by 3, all entries 5.

b. BA is 5×5, all entries 3.

c. ABD: 3×1, all entries 15.

d. DBA: undefined

e. $A(B + C)$: undefined

40.

How do you get:

a. col 3 of AB: each row of A's DP

b. row 1 of AB: row 1 of A with each column of B

c. $AB_{3,4}$ row 2 of A column 4 of B

d. $CDE_{1,1}$: row 1 of C by column 1 of D by column 1 of E.

41.

THe only matrices for which:

a. $BA = 4A$: $4I$

b. $BA = 1/4I$

c. BA has rows 1 and 3 of A reversed and row 2 unchanged:

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

d. All rows of BA are row 1 of A: first column of 1s, all others 0.

42.

True or false:

a. If A^2 is defined, A must be square.

b. If AB and BA are defined, both are square: false

c. The above, and AB and BA are square: true

d. If $AB = b$ then $A = I$: false, B might be the zero matrix.