Notes

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1.

If 0 is an eigenvalue, $\det A = 0$ because the determinant is the product of an eigenvalue.

2.

True, that is the characteristic polynomial.

3.

True.

4.

True.

5.

False; a repeat eigenvalue may correspond to a single eigenvector.

6.

True, diagonalization requires an eigenbasis.

7.

True. The eigenvlues of a diaonal matrix are just the diagonal, so $\lambda_n x_n = \lambda_n e_n$

8.

True.

$$
\begin{aligned}
Ax &= \lambda x \\
A^3 x &= A^2 \lambda x \\
&= A^2 A x \\
&= A^3 x
\end{aligned}
$$

9.

False; an odd-rank skew-symmetric has 0 as an eigenvalue, thought he others are complex

10.

True.

$$
\begin{aligned}
A^2 &= -I \\
Av^2 &= -v \\
\lambda^2 v &= -v
\end{aligned}
$$

But odd-ranked matrices can't have purely imaginary eignvalues, so False.

11.

True, since eignvalues sum to the trace.

12.

True; that means multiplication always respects vector length.

13.

True. Rotation matrices are always skew-symmetric, so they have unitary complex eignvectors.

14.

True; that is simply the dimension of the kernel, since $A - 0I = A$.

15.

True. All similar matrices arediagonalizable,a dn teh

16.

17.

False.

$$
A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}
$$

18.

19.

20.

21.

False. Consider

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

22.

23.

24.

25.

26.

27.

28.

29.

30.

31.

True. Distinct eigenvalues always correspond to independent eigenvectors.

32.

33.

False. Orhtogonal matrices can be diagonalizable yet cannot ahve more than 2 distinct eigenvalues.

34.

35.

False. Consider

$$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

and its transpose. ## 36.

37.

False.

$$\begin{aligned}
&\lambda_v v = Av \\
&\lambda_w w = Aw \\
&A(v+w) = \lambda_v v + \lambda_w w
\end{aligned}$$

38.

39.

40.

41.

42.

43.

True, from the definition of similarity.

44.

45.

46.

47.

True by definition.

48.

True. The number of 0 eigenvalues is $n -$ rank, so each distinct eigenvalue adds a dimension of rank.

49.

True. The image of A is just a single vector if it has rank 1, so that vector satisfiies $Ax = \lambda x$.

50.

True. Then the one nonzero eigenvalue has geometric multiplicity 1 (corersponding to the image of A) and the $n - 1$ 0 eigenvalues have the geomtric multiplicity of the kernel's dimesnion, $n - 1$

51.

52.

53.

True. If $\lambda x = Ax = 0$, it is in the kernel; otherwise it is in the image, since any scalar of v still qualifies as an eigenvector.

54.

True. Symmetric matrices guarantee orthogononal and thus distinct - eigenvectors.

55.

True. $Au = \lambda u = 4u$.

56.

True. This eigenvector corresponds to the subspace u the matrix is projecting into, since $Pu = u$.

57.

58.