Notes

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##4.

If Q_1 and Q_2 are both orthogonal, so is Q_1Q_2.

$$
\begin{aligned}
&(Q_1Q_2)(Q_1Q_2)^T = I \\
&Q_1Q_2Q_2^TQ_1^T = I \\
&Q_1Q_1^T = I \\
&I = I
\end{aligned}
$$

##5.

Show the reflection matrix $R = 2P - I$ is orthogonal. I proved in the last chapter that $R^2 = I$, and since it is symmetric, $R^TR = I$, making it orthogonal as well.

8.

Example showing that projections onto non-orthogonal vectors sum to more than b:

```
b <- 1:2

b - (tcrossprod(c(1, 0)) %*% b) + (tcrossprod(c(1, 2))) %*% b
```

```
     [,1]
[1,]    5
[2,]   12
```

9.

If q_1 and q_2 are orthogonal, the closest combination to q_3 is $0q_1 + 0q_2$

10.

Given GRaham-Schmidt inputs a and b, $a = ||a||u_1||$, and $b = ||a||u_1 + u_1 \cdot b$

11.

Upper triangular orthogonals must be diagonal. Since $Q^2 = I$, each diagonal element Q_{ii} must be ± 1.

18.

$$
\begin{aligned}
A &= QR \\
P &= QR((QR)^T QR)^{-1}(QR)^T \\
&= QR(R^T Q^T QR)^{-1} R^T Q^T \\
&= QR(R^T R)^{-1} R^T Q^T \\
&= QQ^T
\end{aligned}
$$

If Q is square, then the projection is the identity.

31.

a. If Q is orthogonal, Q^{-1} is as well by definition.

b. True. For a 3×2, $Qx = x_1 q_1 + x_2 q_2$. Since the columns are unit length, the sum of squares doesn't change.

32.

$$x_1 + x_2 + x_3 - x_4 = 0$$

All solutions

$$\begin{bmatrix} -1 & -1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Non-solutions: $\begin{bmatrix} -1 \\ -1 \\ -1 \\ 1 \end{bmatrix}$

If $b = (1, 1, 1, 1)$, then its projection $S^{\perp}$ is '$rtcrossprod(1/4 * c(-1, -1, -1, 1))$, leaving $\begin{bmatrix} 7/8 \\ 7/8 \\ 7/8 \\ 9/8 \end{bmatrix}$ in S.