Section 4.3 Problems

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Remember, scalars $a, b, c, \ldots$ apply after every computation is finished when calculating a transformation.

% Standard custom LaTeX commands

% 1: term 1 % 2: subscript 1 % 3: term 2 % 4: subscript 2 % 5. operation

12.

$$T(M) = M \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

13.

$$T(M) = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 2 & 0 & 2 & 0 \\ 0 & 2 & 0 & 2 \end{bmatrix}$$

14.

A matrix transformation with an ugly alternate basis.

$$\begin{bmatrix} 0 & 0 \end{bmatrix}$$

15.

$$T(x + iy) = x - iy$$
$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

16.

$$T(x + iy) = x - iy$$

$$T(x) = x$$
$$T(iy) = -iy$$

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

18.

$$T(z) = (2 + 3i)z$$
$$= 2x + 3ix + 2iy + 3i^2 y$$
$$= 2x + 3ix - iy$$
$$B = \begin{bmatrix} 1 & 0 \\ 3 & -1 \end{bmatrix}$$

Isomorphism.

19.

$$T(z) = (p + iq)z$$
$$= px + iqx + piy + i^2 qy$$
$$= px + iqx + piy - qy$$
$$B = \begin{bmatrix} p & -q \\ q & p \end{bmatrix}$$

Isomorphism.

21.

$$T(f(t)) = f' - 3f$$
$$= b + 2ct - 3a - 3bt - 3ct^2$$
$$B = \begin{bmatrix} -3 & 1 & 0 \\ 0 & -3 & 2 \\ 0 & 0 & -3 \end{bmatrix}$$

Isomorphic.

22.

$$T(f(t)) = f'' + 4f'$$
$$T(1) = a(0 + 4(0)) = 0$$
$$T(t) = b(0 + 2) = 2b$$
$$T(t^2) = c(2 + 4(1 + 2t)) = 6c + 8ct$$

$$\begin{bmatrix} 0 & 2 & 1 \\ 0 & 0 & 8 \\ 0 & 0 & 0 \end{bmatrix}$$

23.

$$T(f(t)) = \quad f(3)P_2 \to P_2$$

$$B = \begin{bmatrix} 1 & 3 & 9 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Correct.

24.

$$T(f(t)) = f(3$$

and basis $(1, (t-3), (t-3)^2)$

$$T(1) = 1$$
$$T((t-3) = b(3-3) = 0$$
$$T((t-3)^2) = (3-3)^2 = 0$$

This represents the zero of a function.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

25.

$$T(f(t)) = f(-t)$$
$$= a - bt + ct^2$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Isomorphic

26.

$$T(f(t) = f(2t)$$

$$T(1) = a(1) = a$$
$$T(t) = b(2t) = 2bt$$
$$T(t^2) = c((2t)^2) = 4ct^2$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

27.

$$T(f(t)) = f(2t - 1)$$
$$f(2t - 1) = a + 2b - 2 + c(2t - 1)^2$$
$$= a + 2bt - 2b + c(4t^2 - 4t + 1)$$
$$= a + 2bt - 2b + c(4t^2 - 4t + 1)$$
$$B = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -4 \\ 0 & 0 & 4 \end{bmatrix}$$

Isomporphism.

28.

With basis $(1, t - 1, t - 1)^2$

$$T(f(t)) = f(2t - 1)$$
$$T(f(1)) = a$$
$$T(f(2t - 1)) = b(2t - 1 - 1) = 2b(t - 1)$$
$$T(f(t - 1)^2) = c(2(t - 1)^2 - 1) = c(2(t^2 - 2t)) = 2c(t - 1)^2$$

Change of basis makes this a lot easier to interpret.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

29.

$$T(f(t)) = \int_0^2 f(t)dt$$

First get the transformed bases:

$$T(1) = a(t1) = at$$
$$T(t) = b(t^2/2) = bt^2/2$$
$$T(t^2) = c(t^3/3)$$

Then compute the integral:

$$\int_0^2 f(t)dt = 8$$

$$\begin{bmatrix} 2 & 2 & 8/3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

30.

This joy:

$$T(f(t)) = \frac{f(t+h) - f(t)}{h}$$

, where h is nonzero

$$T(1) = a\frac{1-1}{h} = 0$$

$$T(t) = b(\frac{t+h-t}{h}) = b$$

$$T(t^2) = c(\frac{(t^2 + h^2 + 2ht - t^2)}{h}) = c(h + 2t)$$

And the matrix:

$$\begin{bmatrix} 0 & 1 & h+2 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

33.

$$T(f(t)) = f(1) + f'(1)(t-1)$$

$$T(1) = (a + 0) = a$$
$$T(t) = b(1 + 1(t-1) = bt$$
$$T(t^2) = c(1 + 2(t-1)) = c(2t - 1) = 2ct - c$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

39.

$$T(M) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} M - M \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

I think this is

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Some transformations in the space $\sin(t) + \cos(t)$. ## 48.

$$T(f) = f'$$

$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

49.

$$T(f) = f'' + 2f' + 3f$$

$$T(\cos(t)) = 2\cos t - 2\sin t$$
$$T(\sin(t)) = 2\cos t - \sin t$$

$$\begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix}$$

50.

$$T(f) = f'' + bf' + cf$$

$$T(\cos t) = (-\cos t) + a(-\sin t) + b(\cos t)$$
$$= (b-1)\cos t - a\sin t$$
$$T(\sin(t)) = -\sin t + a\cos t + b\sin t$$
$$= a\cos t + (b-1)\sin t$$

implying the matrix

$$\begin{bmatrix} b-1 & -a \\ a & b-1 \end{bmatrix}$$

Not isomorphic if $b = 1$ and $a = 0$ or $(b-1)^2 + a^2 = 0$

51.

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

52.

$$T(f(t)) = f(t - \pi/4)$$

$$\frac{1}{\sqrt{2}} \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}$$

53.

The general rotation matrix

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$