影像處理

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(a) Show that

$$f(ax) <=> \frac{1}{a}\, F(\frac{u}{a})$$

$$f(ax) => \frac{1}{a}\, F(\frac{u}{a})$$

$$f(ax)$$

$$= \int_{-\infty}^{\infty} f(ax)\exp(-j2\pi ux)\, dx$$

$$= \int_{-\infty}^{\infty} f(y)\exp\left(-j2\pi u\ \frac{y}{a}\right) \frac{1}{a} dy$$ (let y = ax dy= a dx)

$$= \frac{1}{a}\int_{-\infty}^{\infty} f(y)\exp\left(-j2\pi\ \frac{u}{a}\ y\right)\ dy$$

$$= \frac{1}{a}\ F(\frac{u}{a})$$

$$f(ax) <= \ \frac{1}{a}\, F(\frac{u}{a})$$

$$\frac{1}{a}\, F(\frac{u}{a})$$

$$= \frac{1}{a}\int_{-\infty}^{\infty} f\left(\frac{u}{a}\right)\exp(-j2\pi\ ux)\ \ dx$$

$$= \int_{-\infty}^{\infty} \frac{1}{a} F(y)\exp(-j2\pi ayx)a\ \ dx$$

$$= \frac{1}{a}\int_{-\infty}^{\infty} F(y)\exp(-j2\pi\ y\ ax)\ \ dx$$

$$= f(ax)$$

Because $f(ax) <= \frac{1}{a} F(\frac{u}{a})$ and $f(ax) => \frac{1}{a} F(\frac{u}{a})$ are both hold , so that the

$$f(ax) <=> \frac{1}{a} F(\frac{u}{a}) \text{ will exist.}$$

(b)

Show that $f(x - x_0) <=> F(u)\exp(-j2\pi u x_0)$

$$f(x - x_0) => F(u)\exp(-j2\pi u x_0)$$
$$f(x - x_0)$$
$$= \int_{-\infty}^{\infty} f(x - x_0)\exp(-j2\pi ux)\, dx$$
$$= \int_{-\infty}^{\infty} f(y)\exp(-j2\pi u(y + x_0)))\, dy \quad let\ y = x - x_0\ dy = dx$$
$$= \int_{-\infty}^{\infty} f(y)e^{-j2\pi u(y+x_0)} dy = \int_{-\infty}^{\infty} f(y)e^{-j2\pi uy} \cdot e^{-j2\pi u x_0}\, dy$$
$$= F(n) \cdot e^{-j2\pi u x_0} = F(n)\exp(-j2\pi u x_0)$$

$$f(x - x_0) <= F(u)\exp(-j2\pi u x_0)$$
$$= \int_{-\infty}^{\infty} F(u)\exp(-j2\pi u x_0)\exp(j2\pi ux)\, dx$$
$$= \int_{-\infty}^{\infty} F(u)e^{j2\pi u(x-x_0)} dx = f(x - x_0)$$

Because the $f(x - x_0) <= F(u)\exp(-j2\pi u x_0)$ and

$f(x - x_0) => F(u)\exp(-j2\pi u x_0)$ both hold ,so the

$f(x - x_0) <=> F(u)\exp(-j2\pi u x_0)$ will be holds too.