Algorithm -> **Terminates** -> **Correctness** -> **Time Complexity**

f(n)=O(g(n)), if f(n)/g(n) <= C or lim∞ f'(n)/g'(n)=0
f(n)=o(g(n)), if f(n)/g(n) > C or lim∞ f'(n)/g'(n)=∞
f(n)=t(g(n)), if o(n)=t(n) or lim∞ f'(n)/g'(n)=0

logb(xy)=logb(x)+logb(y)
logb(x/y)=logb(x)-logb(y)
logb(x^n)=nlogb(x)
logb(x)=loga(x)/loga(b)
logb(1)=0
logb(b)=1
b^(logb(x))=x
arithmetic-sum = (n/2)*(2a + (n-1)d)
geometric-sum = (a*(1-r^n))/(1-r)

Topological (vertices connected left to right): O(|V|+|E|)
Djisktra (positive shortest path to all vertices): O(|V^2|)
Bellman-Ford (negative-non-cycles shortest path to all vertices): O(|V||E|)
Floyd-Warshall (negative-non-cycles shortest path for all pairs of vertices): O(|V|^3)
Prims/Kruskal (minimum spanning tree): O(|V^2|)
BFS (find all vertices accessible): O(|V|+|E|)
Heap O(logn)
SelfBalancingTree O(logn)
EdmondsKarp O(|E||f|), O(|V||E^2|)
Bipartite-Graph (vertices in disjoint sets with edges connecting only to other set)
Quicksort (divide different to merge):

DIVIDE AND CONQUER

T(n) = a·T(n/b) + f(n), *a = num-subproblems*, *b = size-subproblems*

critical: n^(logba)

f(n) <, t(n^(logba))

f(n) =, t(n^(logba)·log2)

f(n) > and a·f(n/b) <= c·f(n), t(f(n))

Binary-Search

If possible for n, possible larger values; monotonicity. Also have upper/lower bound

Algorithm

1. Recursively divide into two subarrays of approximately equal parts. Find distinct cards in the first k/2 and last k/2.

2. Merge the results of 2 subarrays by checking through each card one by one in both halves to get a subarray with distinct cards.

3. Base case: for n=1 students, a single collection contains distinct cards

Induction

For base case of n=1, we know a single collection has distinct cards.

Assume this works for n=k.

For n=k+1, by problem definition, merging two collections will always produce a collection of distinct cards.

So, merging two collections of n=k will produce a distinct collection at n=k+1.

GREEDY

Stays-Ahead

1. Let greedy solution be G=(g1, g2, g3, ..., gn) where gi represents a particular rod.

Let an alternative supposed optimal solution be O=(o1, o2, o3, ..., on)

2. Base case is welding the 2 shortest rods.

Welding g1 + g2 will yield the absolute shortest welded rod of absolute minimal cost of any 2 rods.

Therefore, welding g1 + g2 costs no more than welding o1 + o2

3. Assume that welding rods up to gk−1 costs no more than welding rods up to ok−1

4. As the cost of the resultant rod from gk−1 welds is no more than ok−1, gk−1 + gk cannot cost any

more than `ok−1 + ok`. As a result, since O is arbitrary, G must be optimal
ALTERNATE: If these rods were not present at this location in O, then they must appear closer to the centre of O.

Exchange

1. Let x be the activity that starts last overall
2. Consider an alternative schedule S that doesn't contain x
3. Let y be that activity in S that starts last
4. Since we know x is activity that starts last, x must start after y. y won't clash, meaning x won't clash
5. Therefore, can iteratively transform S into a new schedule S' that contains the activity that starts last

Contradiction

* Since deleting vertices from G, must first arrive at a graph containing H as a subgraph, i.e. must arrive at H' before H.
* Suppose alternative solution that deletes a smaller set of vertices `D=(v0, v1, ...)` at this point than our solution.

Therefore, there must exist at least one vertex vi in our solution not in D. For vi to not have been deleted, it
must not be adjacent to at least k vertices.

FLOW

Create a flow network with:
– Source vertex S and sink vertex T
– n children vertices R
– For each child i, an edge `(S, R)` with capacity 1

Correctness

For particular edge, flow conservation ensures that cannot recieve more ...
For particular vertex, capacity constraint ensures cannot hold more ...
Maximum flow is constrained by the total number of children, i.e. `|f| = n`

DYNAMIC PROGRAMMING

Subproblem:

Let `min(i, a)` be minimum time taken to travel to city `ci` arriving on animal `a`.
Let `animal(i)` be the animal that was used to arrive at city `ci` on a journey of

minimal time.

Recurrences:

`min(i, a) = minimum{min(i − 1, a1) + d(i)/v(a), if R(i − 1, a1, a)}`

`animal(i) = argmin{min(i,a)}` *a -> G,M,A,I,L*

Base Case:

Order:

Initialise 2D table `min[n][5]` bottom-up with an outer loop from `i = 1..n` and inner loop `j = 1..5`

Set all entries to ∞ to handle cases where no animal can arrive at a particular city.

To obtain list of animals, backtrack from `i = n..1`

Final Solution:

Minimal Amount of Time: `minimum{min(n, a)}`

Proof:

1. Base case
2. Assume that optimal determined by the optimal choice of between consecutive cities `ck` and `ck+1`.

Suppose an alternative optimal solution is where the choice between consecutive cities is not optimal.

In this case, the total travel time could be reduced by changing the animals between these cities.

For example, if this solution has animals `a1, a2` there exists animals `a3, a4` such that smaller.

This contradicts assumption that this alternative solution is optimal.

Therefore, optimal solution is built up from optimal solutions to its subproblems.

3. Explaining recurrence, considering minimum of all allowable combinations between `ci−1, ci` the optimal choice for `ci−1, ci` will be made.

STRINGS

Rabin-Karp O(n), O(mn)

'm' as could have false positive at every point with 'm' comparisons to verify

1. alphabet `{"a": 0, "b": 1, "c": 2}` compute all particular length substrings `"abc" = 012`
2. with example prime of 5, compute horner's rule hash `0121 % 5 = 5`

3. compare polynomial rolling hash with horner hash and traverse if equal

Knuth-Morris-Pratt O(n+m)

For substring “abc”, 4 states in finite-automaton which is traversed by character `(0, a, ab, abc)`

LINEAR

maximise `P = 5x1 + 3x2 + 4x3`

: `x1 + 2x2 + x3 <= 6`

: `3x1 + x2 + x3 <= 4`

minimise `P* = 6y1 + 4y2`

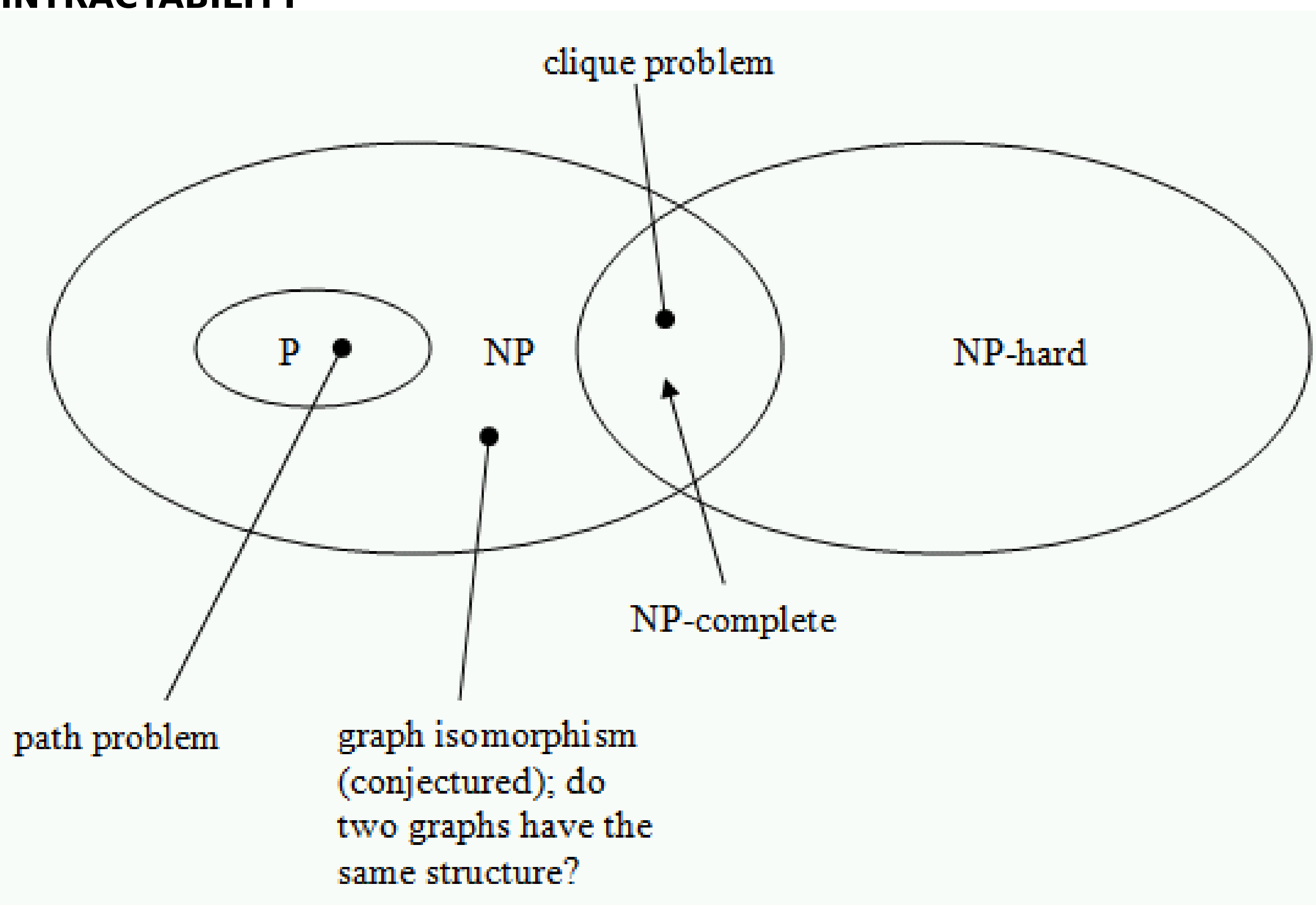
: `y1 + 3y2 >= 5`

: `2y1 + y2 >= 3`

: `y1 + y2 >= 4`

unconstrained `x -> R`; `x = x' - x''`

INTRACTABILITY



Polynomial-time reduction is one-way