Project 1

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We let T, our transition matrix be equal to

$$\begin{bmatrix} 0 & 0 & 35 & 25 & 10 & 5 & 0 & 0 \\ 0.03 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.6 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.6 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.2 & 0 \end{bmatrix}$$

And we let the column vector of our fish population at time $v_0 =$

$$\begin{bmatrix} 50000 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

By the given definition, $v_1 = Tv_0$ and subsequently $v_2 = Tv_1 \ldots v_i = Tv_{i-1}$. By updating the vectors through matrix multiplication we eventually arrive at vectors (written in a singular matrix as to preserve space, where $col_1 = v_0, col_2 = v_1$ and so on.)

$$\begin{bmatrix} 50000 & 0 & 0 & 31500 & 13500 & 2700 & 20385 & 17010 & 7047 \\ 0 & 1500 & 0 & 0 & 945 & 405 & 81 & 611.55 & 510.3 \\ 0 & 0 & 900 & 0 & 0 & 567 & 243 & 48.6 & 366.93 \\ 0 & 0 & 0 & 540 & 0 & 0 & 340.2 & 145.8 & 29.16 \\ 0 & 0 & 0 & 0 & 270 & 0 & 0 & 170.1 & 72.9 \\ 0 & 0 & 0 & 0 & 0 & 108 & 0 & 0 & 68.04 \\ 0 & 0 & 0 & 0 & 0 & 0 & 32.4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 6.48 & 0 \end{bmatrix}$$

Relating back to our original matrix, T, we observe that the numbers along the diagonal are how many original fish remain over the 8 year period. For instance, the $T_{5,4} = 0.5$ would indicate that only 50% of the original fish survived from the previous year. The pattern would be similar for all diagonal numbers. The integers across $T_{1,3:6}$ would indicate the rate at which fish are breeding and thus producing new fish.
As a definition for the general term (T_{ij}) would show the age of the fish present in the lake at a given time. Relating back to $T_{,4} = v_3$ we would have 540 three year old fish and 31,500 one year old fish.
Looking into the future, we can observe the $v_2 = Tv_1 = T^2v_0$ so it would follow that $v_{40} = T^{40}v_0$

Below we have the vectors (once again bound into a matrix to preserve space) for $col_1 = v_0, col_2 = v_{40}, col_3 = v_{50}, col_4 = v_{100}, col_5 = v_{250}$

$$\begin{bmatrix} 50000 & 9593 & 8643 & 5125 & 1068 \\ 0 & 291 & 262 & 155 & 32 \\ 0 & 176 & 159 & 94 & 20 \\ 0 & 107 & 96 & 57 & 12 \\ 0 & 54 & 49 & 29 & 6 \\ 0 & 22 & 20 & 12 & 2 \\ 0 & 7 & 6 & 4 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

To simulate how small changes in entries in T effects prediction, we rerun the simulation and add +2.5% to the nonzero element in row three yielding:

$$\begin{bmatrix} 0 & 0 & 35 & 25 & 10 & 5 & 0 & 0 \\ 0.03 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.625 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.6 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 50000 & 15468 & 15701 & 16888 & 21011 \\ 0 & 464 & 470 & 506 & 629 \\ 0 & 289 & 294 & 316 & 393 \\ 0 & 173 & 176 & 189 & 235 \\ 0 & 87 & 88 & 94 & 117 \\ 0 & 35 & 35 & 38 & 47 \\ 0 & 10 & 11 & 11 & 14 \\ 0 & 2 & 2 & 2 & 3 \end{bmatrix}$$

Let's do the same by subtracting 4% from the nonzero element in row seven yielding:

$$\begin{bmatrix} 0 & 0 & 35 & 25 & 10 & 5 & 0 & 0 \\ 0.03 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.6 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.6 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.26 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 50000 & 9593 & 8643 & 5125 & 1068 \\ 0 & 291 & 262 & 155 & 32 \\ 0 & 176 & 159 & 94 & 20 \\ 0 & 107 & 96 & 57 & 12 \\ 0 & 54 & 49 & 29 & 6 \\ 0 & 22 & 20 & 12 & 2 \\ 0 & 6 & 5 & 3 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

Repeat by changing the nonzero element in row 2 with +4%

$$\begin{bmatrix} 0 & 0 & 35 & 25 & 10 & 5 & 0 & 0 \\ 0.07 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.6 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.6 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 50000 & 222440563 & 2464582907 & 405218800258125 & 1.80037486221461e+30 \\ 0 & 12279659 & 135600070 & 22308480542878 & 9.91158611782225e+28 \\ 0 & 5825896 & 64088704 & 10526969597362 & 4.67709926572088e+28 \\ 0 & 2725186 & 30223549 & 4967485261063 & 2.2070390431328e+28 \\ 0 & 1072281 & 11865773 & 1953390329752 & 8.67885032505252e+27 \\ 0 & 341228 & 3741584 & 614513248251 & 2.7302622742096e+27 \\ 0 & 79597 & 882857 & 144988740135 & 6.44180836756775e+26 \\ 0 & 12453 & 138411 & 22805876043 & 1.01325784550568e+26 \end{bmatrix}$$

Now let's combine all of these changes and observe the results:

$$\begin{bmatrix} 0 & 0 & 35 & 25 & 10 & 5 & 0 & 0 \\ 0.07 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.625 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.6 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.26 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 50000 & 363486560 & 4552815082 & 1380966761390112 & 3.85302259893255e+31 \\ 0 & 19816873 & 247403640 & 75100706686704 & 2.09537685036857e+30 \\ 0 & 9681726 & 120335070 & 36465869161766 & 1.0174303865909e+30 \\ 0 & 4472056 & 56063468 & 16998088582478 & 4.74262187119887e+29 \\ 0 & 1737036 & 21736951 & 6602872576932 & 1.84226053772413e+29 \\ 0 & 546629 & 6771463 & 2051897094725 & 5.72497488693552e+28 \\ 0 & 109153 & 1368226 & 414467968936 & 1.15640371761672e+28 \\ 0 & 16846 & 211818 & 64399693744 & 1.79680943790209e+27 \end{bmatrix}$$

It appears that the earlier the inaccuracy, the worse the overall outcome accuracy compared to the original as we see the largest significant change when we add +4% survival rate on the first year of life among the fish population.