UCSD MAS WES268A - Lab 5 Report

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1 Part 1: Coding Gain for Linear Block Codes

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$$test$$

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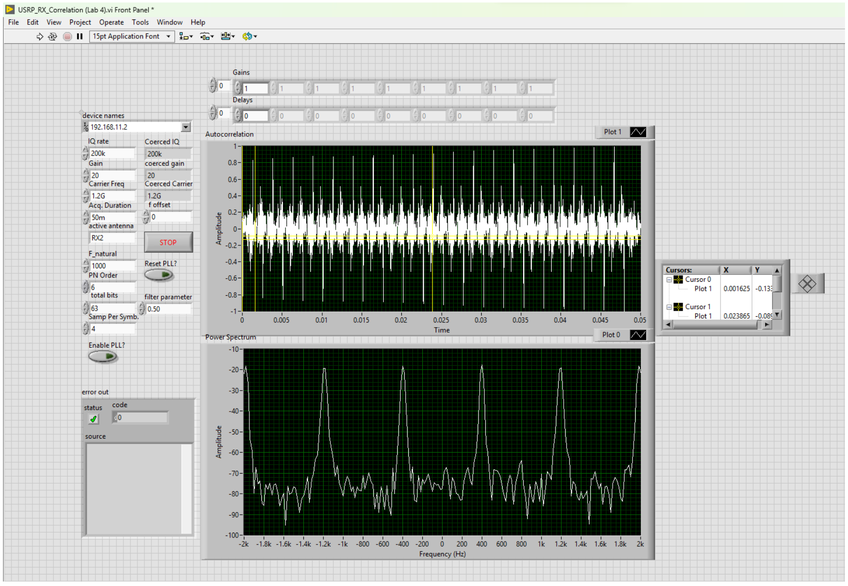


Figure 1: 1.1.1.5/6 - Frequency at DC Component and PN sequence spectrum

Now, for values $A = 0.5$ and $T = 4$, find a $\hat{g}(n)$.

Answer. Given our previous equalization filter derivation from earlier:

$$g(n) = \delta(n) - A\delta(n-T) + A^2\delta(n-2T) - A^3\delta(n-3T) + \ldots$$

$$g(n) = \delta(n) - 0.5\delta(n-4) + 0.5^2\delta(n-8) - 0.5^3\delta(n-12) + \ldots$$

Again, we truncate as before:

$$\hat{g}(n) = \delta(n) - 0.5\delta(n-4) + 0.25\delta(n-8) - 0.125\delta(n-12)$$

1.0.1

Based on your observations from this lab, describe the relationship between invertibility of a channel $H(z)$ in respect to its poles and/or zero locations. In other words, what kind of channels $h[n]$ are invertible (i.e. does a causal equalizer filter $g[n]$ exist?) and non-invertible?

Answer. The channel $H(z)$ is invertible if none of the zeros of $H(z)$ lie inside the unit circle. This is because the equalizer $G(z)$ is given by

$$G(z) = 1/H(z)$$

and if $H(z)$ has zeros inside the unit circle, then $G(z)$ will have poles inside the unit circle, making it non-causal and unstable.

1.0.2

a) Suppose that we wish to estimate the channel using a particular training sequence called the "WES" sequence. All you know about this code is that it has very good auto-correlation properties

$\Sigma w(k)w^*(n+k) = \delta(n)$ *and the following quasi-periodic property:*

$$w(n) = \begin{cases} w(n+mL) & 0 \le m \le 9, \\ 0 & \textit{otherwise} \end{cases}$$

Effectively, the "WES" sequence is a length L sequence that repeats itself 10 times. Now, assuming the input to your receiver is the "WES" sequence $w(n)$ with a frequency offset of f_0

$$x(n) = w(n)e^{j2\pi f_0 n}$$

Find the cross-correlation between the received sequence $x(n)$ and the "WES" sequence $w(n)$, denoted $r_{xw}(n)$

$$r_{xw}(n) = \sum_k x(n+k)w^*(k)$$

for values of $n = \{0, L, ..., 9L\}$, you may assume that the term A is a constant where

$$A = \sum_k |w(k)|^2 e^{j2\pi f_0 k}$$

Answer. Assume the receiver input is

$$x(n) = w(n)e^{j2\pi f_0 n},$$

and the WES code satisfies the quasi-periodic property over the 10 repeats,

$$w(n+mL) = w(n), \quad m = 0, \dots, 9.$$

With a single length-L reference in the correlator,

$$r_{xw}(n) = \sum_{k=0}^{L-1} x(n+k)w^*(k) = \sum_{k=0}^{L-1} w(n+k)e^{j2\pi f_0(n+k)}w^*(k).$$

Factor out the n-dependent exponential:

$$r_{xw}(n) = e^{j2\pi f_0 n} \sum_{k=0}^{L-1} w(n+k)w^*(k)e^{j2\pi f_0 k}.$$

Evaluate at $n = mL$ and use $w(mL+k) = w(k)$:

$$\begin{aligned} r_{xw}(mL) &= e^{j2\pi f_0 mL} \sum_{k=0}^{L-1} w(k)w^*(k)e^{j2\pi f_0 k} \\ &= e^{j2\pi f_0 mL} \sum_{k=0}^{L-1} |w(k)|^2 e^{j2\pi f_0 k}. \end{aligned}$$

Define the constant (over one repetition)

$$A = \sum_{k=0}^{L-1} |w(k)|^2 e^{j2\pi f_0 k}.$$

Thus,

$$r_{xw}(mL) = Ae^{j2\pi f_0 mL}, \quad m = 0, \dots, 9.$$

b) Using your calculation of $r(0)$, and the fact that $|w(k)|^2 = 1$, comment on the effect that frequency offset has on the magnitude of the cross-correlation.
Answer. From part 3A,

$$r_{xw}(0) = A = \sum_{k=0}^{L-1} |w(k)|^2 e^{j2\pi f_0 k}.$$

With $|w(k)|^2 = 1$ over one length-L block,

$$r_{xw}(0) = \sum_{k=0}^{L-1} e^{j2\pi f_0 k} = e^{j\pi f_0 (L-1)} \frac{\sin(\pi f_0 L)}{\sin(\pi f_0)}.$$

Hence

$$|r_{xw}(0)| = \left| \frac{\sin(\pi f_0 L)}{\sin(\pi f_0)} \right|.$$

The magnitude is maximized at $f_0 = 0$ (equal to L for this normalization) and decreases as $|f_0|$ increases due to phasor cancellation. The correlation peak magnitude has nulls at

$$f_0 = \pm\frac{1}{L}, \pm\frac{2}{L}, \ldots,$$

which is consistent with the sinusoidal-like envelope behavior observed when a nonzero frequency offset is present.

c) Explain how you would calculate the frequency offset from your calculated values of $r_{xw}(mL)$.
Answer. From part 3A,

$$r_{xw}(mL) = Ae^{j2\pi f_0 mL}.$$

Thus the phase of the correlation samples is linear in m. Using the ratio of adjacent peaks,

$$\frac{r_{xw}((m+1)L)}{r_{xw}(mL)} = e^{j2\pi f_0 L},$$

an estimator is

$$\widehat{f_0} = \frac{1}{2\pi L} \angle\big(r_{xw}((m+1)L)\, r_{xw}^*(mL)\big),$$

and in practice this can be averaged over $m = 0, \ldots, 8$ to reduce noise sensitivity.

d) What is the maximum frequency offset f_{max} that can be tracked using the cross-correlation approach and the "WES" code $w(n)$ of length L (Hint: Think of the nyquist sampling theorem).
Answer. The sequence of correlation peaks

$$r_{xw}(mL) = Ae^{j2\pi f_0 mL}$$

samples a complex exponential once per length-L block. In the index m, the normalized frequency is $f_0 L$ cycles per sample of m. To avoid aliasing,

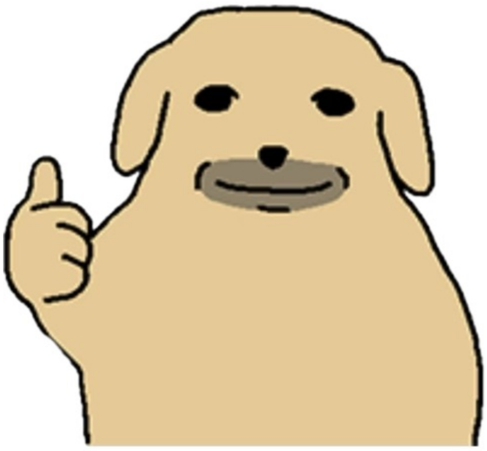
$$|f_0 L| < \frac{1}{2} \quad \Rightarrow \quad |f_0| < \frac{1}{2L}.$$

Thus the maximum unambiguous offset is

$$f_{\max} = \frac{1}{2L} \text{ cycles/sample.}$$

or

$$f_{\max,\mathrm{Hz}} = \frac{f_s}{2L}.$$



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