Project 2

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DSA 8420 - Adv. Mathematical Programming

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A company manufactures products A to G using two types of machines (P_1, P_2) and three raw materials (R_1, R_2, R_3). Relevant data are given below:

Item	Item input (in units) to make one unit of							Max. avail. per day
	A	B	C	D	E	F	G	
R_1 (in units)	0.10	0.30	0.20	0.10	0.20	0.10	0.20	500
R_2	0.20	0.10	0.40	0.20	0.20	0.30	0.40	750
R_3	0.20	0.10	0.10	0.20	0.10	0.20	0.30	350
P_1 time (mc. hrs.)	0.02	0.03	0.01	0.04	0.01	0.02	0.04	60
P_2	0.04		0.02	0.02	0.06	0.03	0.05	80
Req. daily output	≥ 200	≤ 800			≤ 400			
Profit ($/unit)	10	12	8	15	18	10	19	

By using AMPL to formulate and solve an LP of this product mix to maximize total daily profit, the

Primal Solution is:

Optimal Profit: $32487.50

Optimal Mix:
Units of product A: 200
Units of product B: 587.5
Units of product C: 787.5
Units of product D: 662.6
Units of product E: 400
Units of product F: 0
Units of product G: 0

Dual Solution:

Optimal Profit: $32487.50

Optimal Mix:
Units of product A: 200
Units of product B: 587.5
Units of product C: 787.5
Units of product D: 662.5
Units of product E: 400
Units of product F: 0
Units of product G: 0

From the primal model:

```
AMPL
ampl: commands project2-primal-solut
MINOS 5.51: optimal solution found.
4 iterations, objective 32487.5
Profit = 32487.5

a = 200
b = 587.5
c = 787.5
d = 662.5
e = 400
f = 0
g = 0

R1 = 12.5
R2 = 0
R3 = 41.25
P1 = 137.5
P2 = 0
A_Limit = -2.25
B_Limit = 0
E_Limit = 10

ampl:
```

```
# Project 2 - Original Model
# Ryan Standridge and Will Reddin
# reset;
# Solver: option solver minos;
#         option solver cplex;
# Model: model project2-original-solution.mod;
# Data: data project2.dat;

# Define set AVAIL (max availiable per day)

set AVAIL;
set PRICE;

# Max units/mc. hrs. per day available

param units {AVAIL};
param c_x {PRICE};

# Decision Variables

var a >= 0;
var b >= 0;
var c >= 0;
var d >= 0;
var e >= 0;
var f >= 0;
var g >= 0;

# Objective Function and Constraints

maximize Profit: c_x[1] * a + c_x[2] * b + c_x[3] * c + c_x[4] * d + c_x[5] * e + c_x[6] * f + c_x[7] * g;
subject to R1: 0.10 * a + 0.30 * b + 0.20 * c + 0.10 * d + 0.20 * e + 0.10 * f + 0.20 * g <= units[1];
subject to R2: 0.20 * a + 0.10 * b + 0.40 * c + 0.20 * d + 0.20 * e + 0.30 * f + 0.40 * g <= units[2];
subject to R3: 0.20 * a + 0.10 * b + 0.10 * c + 0.20 * d + 0.10 * e + 0.20 * f + 0.30 * g <= units[3];
subject to P1: 0.02 * a + 0.03 * b + 0.01 * c + 0.04 * d + 0.01 * e + 0.02 * f + 0.04 * g <= units[4];
subject to P2: 0.04 * a + 0.00 * b + 0.02 * c + 0.02 * d + 0.06 * e + 0.03 * f + 0.05 * g <= units[5];
subject to A_Limit: a >= 200;
subject to B_Limit: 0 <= b <= 800;
subject to E_Limit: 0 <= e <= 400;

# solve;

# optimal solution
# display Profit;

# values of the decision variables
# display a, b, c, d, e, f, g;

# values of final constraints (shadow price)
# display R1, R2, R3, P1, P2, A_Limit, B_Limit, E_Limit;
```

From the dual model:

```
AMPL
ampl: commands project2-dual-solutio
MINOS 5.51: optimal solution found.
7 iterations, objective 32487.5
Profit = 32487.5

a = 200
b = 587.5
c = 787.5
d = 662.5
e = 400
f = 0
g = -1.17253e-13

R1 = 12.5
R2 = 0
R3 = 41.25
P1 = 137.5
P2 = 0
A_Limit = -2.25
B_Limit = 0
E_Limit = 10

ampl:
```

```
# Project 2 - Question 1 Model
# Ryan Standridge and Will Reddin
# reset;
# Solver: option solver minos;
#         option solver cplex;
# Model: model project2-testing.mod;
# Data: data project2.dat;

# Define set AVAIL (max availiable per day)

set AVAIL;
set PRICE;

# Max units/mc. hrs. per day available

param units {AVAIL};
param c_x {PRICE};

# Decision Variables

var R1 >= 0;
var R2 >= 0;
var R3 >= 0;
var P1 >= 0;
var P2 >= 0;
var A_Limit <= 0;
var B_Limit >= 0;
var E_Limit >= 0;


# Objective Function and Constraints

minimize Profit: units[1] * R1 + units[2] * R2 + units[3] * R3 + units[4] * P1 + units[5] * P2 +
                        200 * A_Limit + 800 * B_Limit + 400 * E_Limit;
subject to a: 0.10 * R1 + 0.20 * R2 + 0.20 * R3 + 0.02 * P1 + 0.04 * P2 + A_Limit >= c_x[1];
subject to b: 0.30 * R1 + 0.10 * R2 + 0.10 * R3 + 0.03 * P1 + 0.00 * P2 + B_Limit >= c_x[2];
subject to c: 0.20 * R1 + 0.40 * R2 + 0.10 * R3 + 0.01 * P1 + 0.02 * P2 >= c_x[3];
subject to d: 0.10 * R1 + 0.20 * R2 + 0.20 * R3 + 0.04 * P1 + 0.02 * P2 >= c_x[4];
subject to e: 0.20 * R1 + 0.20 * R2 + 0.10 * R3 + 0.01 * P1 + 0.06 * P2 + E_Limit >= c_x[5];
subject to f: 0.10 * R1 + 0.30 * R2 + 0.20 * R3 + 0.02 * P1 + 0.03 * P2 >= c_x[6];
subject to g: 0.20 * R1 + 0.40 * R2 + 0.30 * R3 + 0.04 * P1 + 0.05 * P2 >= c_x[7];


# solve;

# optimal solution
# display Profit;

# values of the decision variables
# display a, b, c, d, e, f, g;

# values of final constraints (shadow price)
```

1. **Are the marginal values (shadow prices, relative profits) of the various items well defined and meaningful? What might make you suspect they might not be? If they are, what are their values and interpretation?**

   Answer:

   The marginal values of the various items are well defined and meaningful. Resulting from the original AMPL model and solution, we are able to determine the following:

Shadow Prices:		
	R_1: 12.5	P_1: 137.5
	R_2: 0	P_2: 0
	R_3: 41.25	

   The interpretation of these shadow prices are that as we increase the maximum available units of R_1 per day by one unit, the total profit will increase \$12.50. Also, if we increase the maximum available units per day of R_2, the total profit is unchanged. Similar to R_1, as we increase the maximum available units per day of R_3, the total profit will increase by \$41.25 for each unit added. Next, as we can see in the results, for each hour added to the maximum available hours for machine P_1, the total profit will increase \$137.50 and for each hour added to the maximum available hours for machine P_2, the total profit is unchanged.

2. **Is it worth increasing the supply of R_1 beyond the present 500 units/day? The current supplier for R_1 is unable to supply any more than the current amount. The procurement manager has identified a new supplier for R_1, but that supplier's price is \$15/unit higher than the current supplier's. Should additional supplies of R_1 be ordered from this new supplier?**

   Answer:

   By searching for breakpoints in AMPL by solving the problem repeatedly with different parameter values for the R_1 available units/day, we were able to determine that it is worth increasing the supply of R_1 beyond the present 500 units per day since the shadow price is $12.50. This means that as the company increases the supply of R_1, their total profit increases by $12.50. Also, it is good to note that the once R_1 is increased above 585 max. units per day, the shadow price changes to $3.33 and then once R_1 is increased to 615, the shadow price changes to 0. This is worth noting since the company can see that the most they can increase the maximum available units/day for R_1 while continuing to increase their profit is to 615 units which will achieve a profit $33650.

With the current supplier being unable to supply any more than the current amount and the new supplier charging $15/unit higher than the current supplier, the company should NOT order additional supplies of R_1 from the new supplier. Knowing that the company is only able to increase their profit by $12.50 when additional supplies of R_1 are added, their profit will decrease by $2.50 ($12.50 - $15) per additional unit of R_1, if they choose to order additional supplies of R_1 from the new supplier.

3. **The production manager has identified an arrangement by which 20 hours/day of either P_1- or P_2-time can be made available at a cost of $150/day. Is it worth accepting this arrangement?**

<u>*Answer:*</u>

By searching for breakpoints in AMPL by solving the problem repeatedly with different parameter values of P_1, we are able to determine that the maximum available P_1 per day can be increased by 5.67 (to 65.67) units/day while continuing to increase profit. Once the max. available P_1 per day is increased above 65.67 units/day, the shadow price becomes zero showing that the extra 14.33 of the 20 hours/day on machine P_1 will not change the total profit. Since, the company can increase P_1 by 5.67 units while continuing to increase their profit by the shadow price of $137.50 per unit, it is clear that the total profit will be $32487.5 + ($5.67 *$137.5) - $150

= \$33117.13. As we can see, the company will make an additional profit of \$629.63, making the arrangement of the additional 20 hours/day for machine P_1 worth accepting.

Next, because machine P_2 has an original shadow price of 0 and does not change when adding 20 hours/day to the maximum available hours/day, the additional hours will not have an impact on the total profit. Thus, the company will spend \$150 on the additional 20 hours/day for machine P_2 while not making any profits making the arrangement not worth accepting.

4. **The sales manager would like to know the relative contributions to the various products to the company's total profit. What are they?**

<u>*Answer:*</u>

The relative contributions of the various products to the company's total profit are:

Product A: (\$10/unit * 200 units) = \$2000 which is approximately 6.16% of the total profit.

Product B: (\$12/unit * 587.5 units) = \$7050 which is approximately 21.7% of the total profit.

Product C: (\$8/unit * 787.5 units) = \$6300 which is approximately 19.39% of the total profit.

Product D: (\$15/unit * 662.5 units) = \$9937.5 which is approx. 30.59% of the total profit.

Product E: (\$18/unit * 400 units) = \$7200 which is approximately 22.16% of the total profit.

Product F: (\$10/unit * 0 units) = \$0 which is 0% of the total profit.

Product G: (\$19/unit * 0 units) = \$0 which is 0% of the total profit.

5. **The sales manager believes that product C is priced too low for a good image. This manager claims that if the selling price of C were increased by \$2/unit, the demand for it would be 600 units/day. What is the effect of this change?**

<u>*Answer:*</u>

If the selling price of C were increased by \$2/unit and the demand for it is 600 units/day, the profit would increase to \$33500 with the new optimal mix as shown in the result below:

Original Solution is:			**New Solution**:		
Optimal Profit:		\$32487.50	*Optimal Profit*:		\$33500
Optimal Mix:	Units of product A:	200	*Optimal Mix*:	Units of product A:	200
	Units of product B:	587.5		Units of product B:	672.727
	Units of product C:	787.5		Units of product C:	600
	Units of product D:	662.6		Units of product D:	509.091
	Units of product E:	400		Units of product E:	400
	Units of product F:	0		Units of product F:	0
	Units of product G:	0		Units of product G:	136.364

As we can see, the amount of units of product B and G made will increase from the original optimal mix to the new optimal mix. Also, the amount of units of product C and D produced will decrease. Lastly, the amount of units of product A, E, and F produced will remain unchanged from the original optimal mix.

Also note that the shadow prices of the machines and materials will change to:

Shadow Prices:	R_1: 10	P_1: 200
	R_2: 0	P_2: 0
	R_3: 30	

6. **The production manager claims that the manufacturing process for G can be changed so that its need for P_1-time goes down by 50% without affecting quality, demand, or selling price. What will be the effect of this change on the optimum product mix and total profit?**

<u>*Answer:*</u>

When the manufacturing process for G is changed so that the P_1 time goes down by 50% (without affecting quality, demand, or selling price), the total profit will slightly increase to $32700 with the new optimal mix shown in the result below:

Original Solution is:			**New Solution**:		
Optimal Profit:		$32487.50	*Optimal Profit*:		$32700
Optimal Mix:	Units of product A:	200	*Optimal Mix*:	Units of product A:	200
	Units of product B:	587.5		Units of product B:	800
	Units of product C:	787.5		Units of product C:	381.818
	Units of product D:	662.6		Units of product D:	527.273
	Units of product E:	400		Units of product E:	400
	Units of product F:	0		Units of product F:	0
	Units of product G:	0		Units of product G:	154.545

As we can see, the amount of units of product B and G made will increase from the original optimal mix to the new optimal mix. Also, the amount of units of product C and D produced will decrease. Lastly, the amount of units of product A, E, and F produced will remain the unchanged from the original optimal mix.

Also note that the shadow prices of the machines and materials will change to:

Shadow Prices:	R_1:	10	P_1:	100
	R_2:	0	P_2:	0
	R_3:	50		

7. **The production manager believes that by changing specifications, it should be possible to make product B with 33.3% less of R_1, and this would have no effect on**

the saleability of this product. What will be the effect of this change on the optimum product mix and total profit?

Answer:

When the production for product B is made with 33.3% less of R_1, the total profit will slightly increase to $33465.40 with the new optimal mix shown in the result below:

Original Solution is:

Optimal Profit: $32487.50

Optimal Mix:

Units of product A: 200

Units of product B: 587.5

Units of product C: 787.5

Units of product D: 662.6

Units of product E: 400

Units of product F: 0

Units of product G: 0

New Solution:

Optimal Profit: $33465.40

Optimal Mix:

Units of product A: 200

Units of product B: 783.072

Units of product C: 983.072

Units of product D: 466.928

Units of product E: 400

Units of product F: 0

Units of product G: 0

As we can see, the amount of units of product B and C made will increase from the original optimal mix to the new optimal mix. Also, the amount of units of product D produced will decrease. Lastly, the amount of units of product A, E, F, and G produced will remain unchanged from the original optimal mix.

Also note that the shadow prices of the machines and materials will change to:

Shadow Prices:

R_1: 16.6611

R_2: 0

R_3: 26.6861

P_1: 199.917

P_2: 0

8. The company's research division has formulated a new product, H, which they believe can yield a profit of $8–10/unit made. The input requirements to make one unit of this product will be as follows.

Item	R_1	R_2	R_3	P_1-time	$P-2$-time
Input	0.10	0.20	0.10	0.02	0.02

Is this product worth further consideration?

Answer:

By searching for breakpoints in AMPL by solving the problem repeatedly with different parameter values for the profit of $8-10/unit made, we were able to find that when the profit per unit made for product H is less than $8.125 (approx. $8.13), there is no change to the optimal profit and there are zero products of product H produced. Thus product H is not worth further consideration when the profit per unit made for product H is less than $8.125 (approx. $8.13). However, when the profit per unit is $8.125 (approx. $8.13) or greater, then product H is worth further consideration since the optimal profit begins to increase and product H begins to be produced. When the profit per unit made of product H is $8.13, the total profit will be $32496.30 and the new optimal mix and shadow prices are as follows:

New Optimal Mix:	Units of product A:	200	Units of product E:	400
	Units of product B:	366.667	Units of product F:	0
	Units of product C:	566.667	Units of product G:	0
	Units of product D:	0	Units of product H:	1766.67
Shadow Prices:	R_1:	12.4667	P_1:	137.667
	R_2:	0	P_2:	0
	R_3:	41.3		

One thing to note is as the price per unit made of product H increases from $8.13 to $10, the total profit continues to increase while maintaining the same optimal mix. Thus when the profit per unit made of product H is $10, the total profit will be $35800 and the new optimal mix and shadow prices are as follows:

New Optimal Mix:	Units of product A:	200	Units of product E:	400
	Units of product B:	366.667	Units of product F:	0
	Units of product C:	566.667	Units of product G:	0
	Units of product D:	0	Units of product H:	1766.67
Shadow Prices:	R_1:	0	P_1:	200
	R_2:	0	P_2:	0
	R_3:	60		

9. The sales manager feels that the selling price/unit of product F can be increased by $2 without affecting the demand for it. Would this lead to any changes in the optimum production plan? What is the effect of this change on the total profit?

Answer:

If the selling price/unit of product F is increased by $2 without affecting the demand for it, then there will be no changes in the optimum production plan and the total profit will remain the same as shown below:

Original Solution is:			**New Solution with increased price of F**:		
Optimal Profit:	$32487.50		*Optimal Profit*:	$32487.50	
Optimal Mix:	Units of product A:	200	*Optimal Mix*:	Units of product A:	200
	Units of product B:	587.5		Units of product B:	587.5
	Units of product C:	787.5		Units of product C:	787.5
	Units of product D:	662.6		Units of product D:	662.6

Units of product E: 400	Units of product E: 400
Units of product F: 0	Units of product F: 0
Units of product G: 0	Units of product G: 0

Also, note that the shadow prices remain unchanged from the original solution when the selling price/unit of product F is increased by $2 without affecting the demand for it.