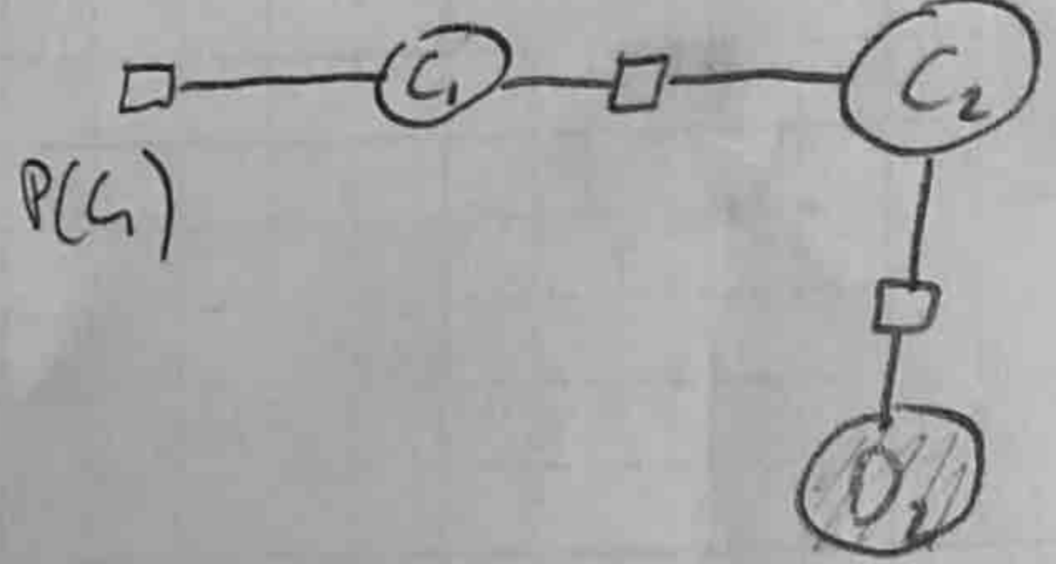
1a) $P(C_2=1 \mid D_2=0)$




$$P(C_2=1 \mid D_2=0) \propto P(d_2=0 \mid c_2=1)\left(\sum_{c_1} p(c_1)\, p(c_2=1 \mid c_1)\right)$$

$$P(d_2=0 \mid c_2=1)\left(p(1)\, p(c_2=1 \mid c_1=1) + p(0)\, p(c_2=1 \mid c_1\right.$$

$$\propto \eta(.5(\varepsilon)+.5(1-\varepsilon))$$

$$\propto \eta$$

$$P(C_2=0 \mid D_2=0) \propto P(D_2=0 \mid C_2=0)\left(\sum_{c_1} p(c_1)\, p(c_2=0 \mid c_1)\right)$$

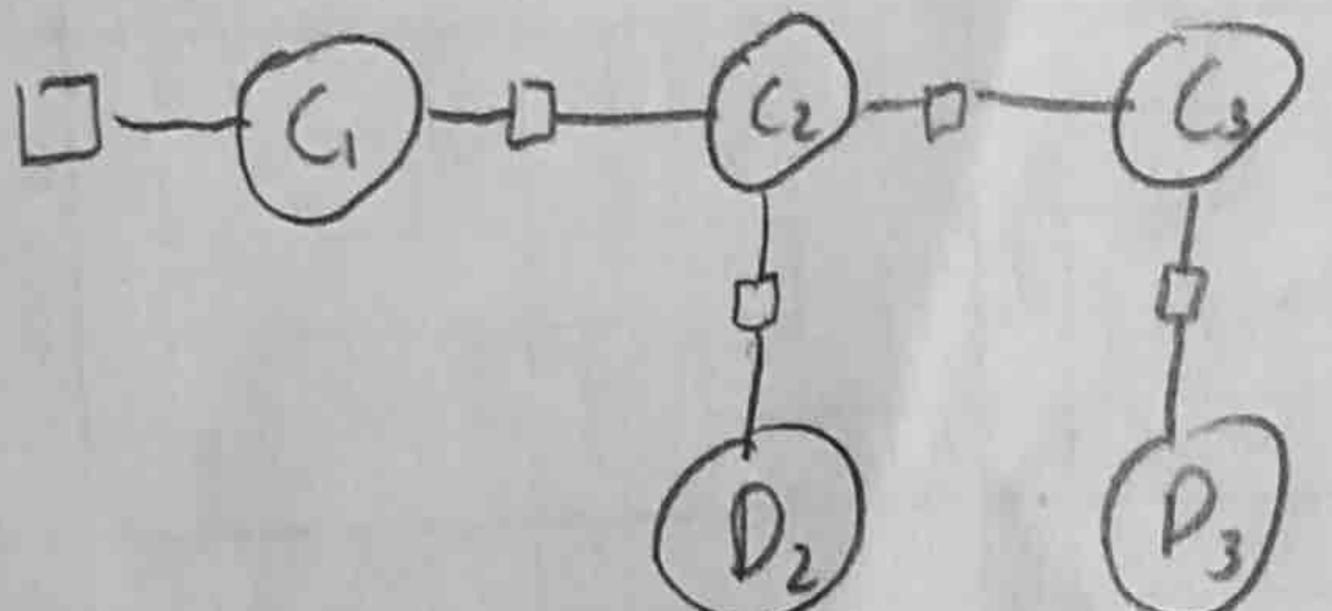
$$\propto (1-\eta)(.5(\varepsilon)+.5(1-\varepsilon))$$

$$\propto 1-\eta$$

<u>Proportion:</u>

$$P(c_2=1 \mid D_2=0) = \frac{\eta}{\eta+1-\eta} = \eta$$

1b)




$$\left[\sum_{c_1} P(c_1)\,P(c_2=1|c_1)\right] \cdot P(D_2=0|C_2=1) \cdot \left[\sum_{c_3} P(c_3|c_2=1)\,P(D_3=1|c_3)\right]$$

$$f_1(c_1) = \sum_{c_1} p(c_1)\,P(c_2=1|c_1) \qquad f_2 = \sum_{c_3} P(c_3|c_2=1)\,P(D_3=1|c_3)$$

$$P(C_2=1|D_2=0,D_3=1) \propto f_1(c_2)\,f_2(c_2)\,P(D_2=0|C_2=1)$$

for $C_2=1$

$$f_1(1) = .5(\varepsilon+1-\varepsilon) = .5$$

$$f_2(1) = (1-\varepsilon)(1-\eta)+(\varepsilon\eta) = 1-\varepsilon-\eta+2\varepsilon\eta$$

$$\propto f_1(c_2)\,f_2(c_2)\,P(D=0|C_2=1)$$

$$\propto .5(1-\varepsilon-\eta+2\varepsilon\eta)\,\eta$$

for $C_2=0$

$$f_1(0) = .5(\varepsilon+1-\varepsilon) = .5$$

$$f_2(0) = (1-\varepsilon)\eta+(1-\eta)\varepsilon = \eta-2\varepsilon\eta+\varepsilon$$

$$\propto f_1(c_2)\,f_2(c_2)\,P(D=0|C_2=0)$$

$$(1-\eta)(.5)(\eta+\varepsilon-2\varepsilon\eta)$$

Proportion $\rightarrow$

$$P(C_2=1|D_2=0,D_3=1) = \boxed{\frac{.5(1-\varepsilon-\eta+2\varepsilon\eta)\,\eta}{.5\left[\eta(1-\varepsilon-\eta+2\varepsilon\eta)+(1-\eta)(\eta+\varepsilon-2\varepsilon\eta)\right]}}$$

1c) i) $\varepsilon = .1$ $\eta = .2$

$$\boxed{\begin{aligned} P(C_2=1 \mid D_2=0) &= 0.2 = \eta \\ P(C_2=1 \mid D_2=0, D_3=1) &= .4157 \end{aligned}}$$

ii) $D_3$ being 1 made it much more likely that the cars position was 1 @ $C_2$, despite $D_2$ being 0. This is due to $\varepsilon$ being small, meaning the car is unlikely to change position, even more unlikely to change position then the sensor is to get the reading wrong. Since $D_3 = 1$, it makes it more likely $C_2 = 1$ as well since its very unlikely the car would change position.

iii) You could set $\varepsilon = .5$, meaning car has equal probability of being in either position @ any observation time. This would make any future or previous sensor readings irrelevant and useless, only information could be gained from current sensor reading.

5a) $K=2 \quad T=1$

$P(C_{11}, C_{12} \mid E_1 = e_1)$ ] bayes

$$\propto \underbrace{p(E_1 \mid C_{11}, C_{12})}_{\text{must solve for this}} \underbrace{p(C_{11})\, p(C_{12})}_{\text{we know this}}$$

$$\propto p(e_1, e_{2} \mid C_{11}, C_{12})$$

- we don't know which e goes w/ each c
- take all permutations and add

$$\propto p(D_{11} = e_1 \mid C_{11})\, p(D_{12} = e_2 \mid C_{12}) \leftarrow \text{perm } 1$$

$$+$$

$$p(D_{11} = e_1 \mid C_{12})\, p(D_{12} = e_2 \mid C_{11}) \leftarrow \text{perm } 2$$

$$p(C_{11}, C_{12} \mid E_1 = e_1) = P_N(e_1; \|a_1 - C_{11}\|, \sigma^2) P_N(e_2; \|a_1 - C_{12}\|, \sigma^2)$$

$$+$$

$$P_N(e_2; \|a_1 - C_{12}\|, \sigma^2) P_N(e_1; \|a_1 - C_{12}\|, \sigma^2)$$

5b) $P(c_{i1})$ same for all $i$

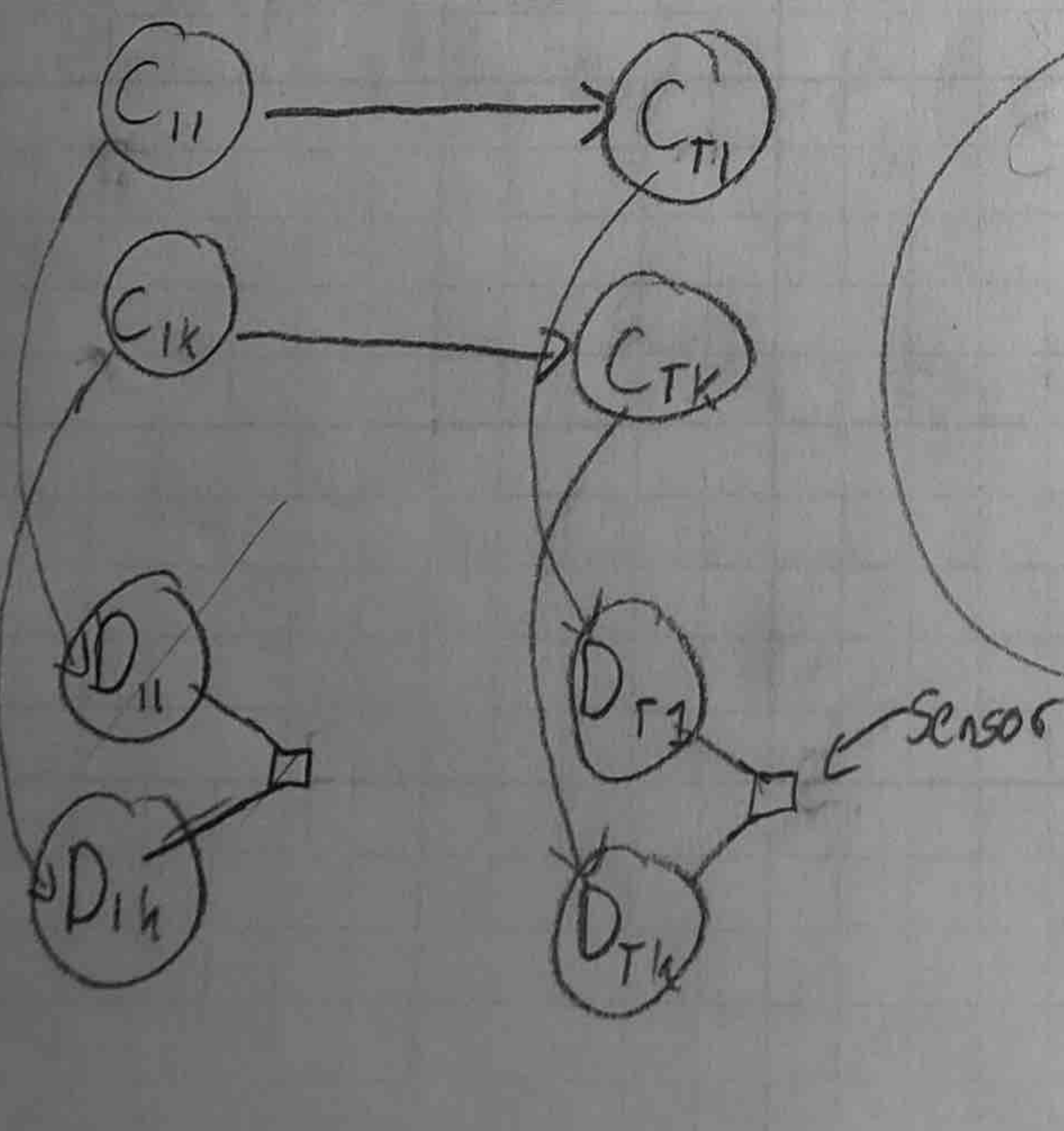
- Cars $\in (c_{11}, \ldots, c_{1k})$ for $k$ # of cars.

Since all locations for a car on grid have equal probability, maximum value of $P(C_{11} \ldots \mid E_1 = e_1)$ will be reached by at least one formation of cars. With that one formation, we can permutabe assignments of the vehicles, which results in $k!$ permutations. We can still do permutations and reach $P_{max}$ since $E_1$ is sensor reading, but is not assigned to any one particular car.

5c) Tree width

$$P(C_{11} = c_{11}, \ldots \mid E_1 = e_1, \ldots E_t = e_t)$$

Bayesian Network:




For current time step $T$, can marginalize out readings from previous step, tree width is $K$ for time step $t = T$.

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d) Algorithm w/ wrap around

$P(C_1 \dots C_n \mid E_1 \dots E_n)$ w/ E having wrap around capability.

- check $c_1$ w/ all $e_n \in E$, use beam search to to find those w/ highest probability, $c_1 \leftarrow e_x$ where $x \in [0, K]$
- if $c_1 \leftarrow e_x$ then $c_2 \leftarrow e_{x+1}$ ... due to wrap around, can compute remaining probabilities/ weights.

- Compute for each time step T

$O(nK \cdot T)$ for n width beam search, T time steps.