Math 486

Lesson 4 Homework

Due Tues, June 21 at 11:59 on Gradescope

Instructions

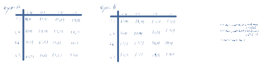
Please refer to the solution guidelines posted on Canvas under Course Essentials.

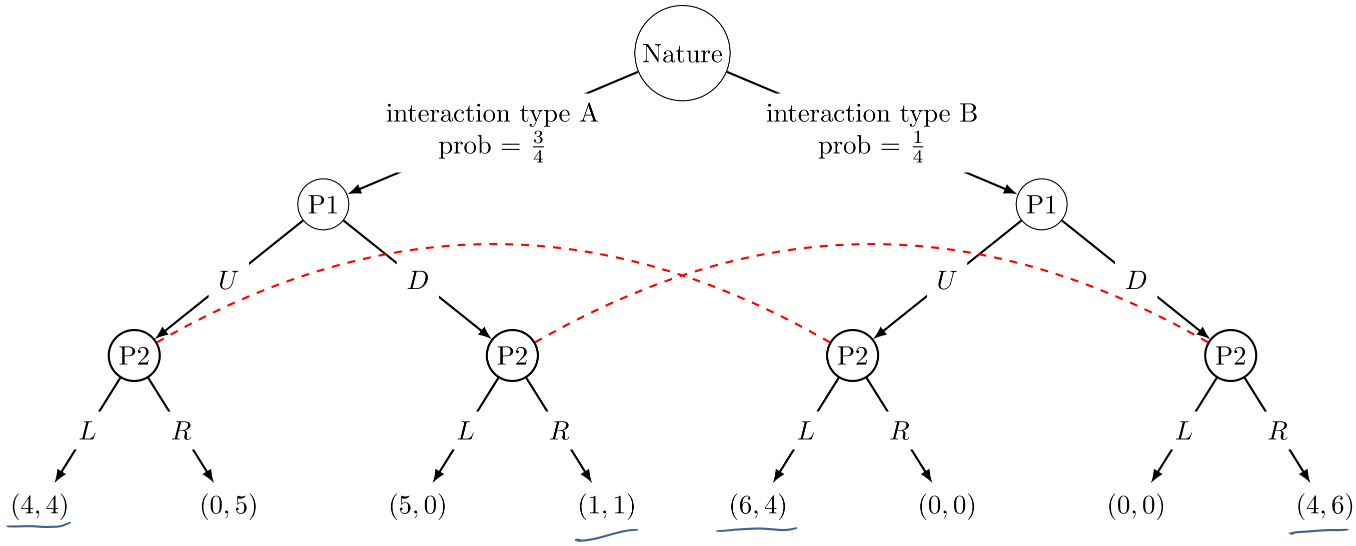
Exercise 1.

Consider a game with incomplete information where there are two players, $P1$ and $P2$. There are two possible interactions that could occur between the players, type A and type B.

- The interaction will by of type A with probability $\frac{3}{4}$ and of type B with probability $\frac{1}{4}$.
- Player 1 knows the type, but player 2 does not.
- Player 1 observes whether the interaction will be type A or type B and chooses to move either up (U) or down (D).
- Player 2 observes Player 1's move (but not the interaction type) and chooses to move either left (L) or right (R).
- Refer to the extensive form game shown below, which includes the payoffs.
- Each player has four strategies:

$$S_1 = \{UU, UD, DU, DD\}, \qquad S_2 = \{LL, LR, RL, RR\}$$





(a) Describe Player 1's strategy DU in words.

(b) Describe Player 2's strategy LR in words.

(c) What are the payoffs for the strategy profile (DU, LR) when the interaction is of type A? type B?

a) for player 1, in type A, if he choose D, expected value will be bigger than U, and in type B, expected value of U will be bigger then D

So P1 will choose D in type A and U in type B

b) for player 2, he do not know the type, so

if P1 choose U, e for L will be 4 and e for R will be $\frac{15}{4}$, L will be prefered

if P1 choose D, e for L will be 0 and e for R will be $\frac{9}{4}$, R will be prefered

P2 will choose L if P1 choose U and R if P1 choose D

C. in type A, $\pi(DU, LR) = (1,1)$ in type B, $\pi(DU, LR) = (6,4)$

However in a), if using backward-induction, player 1 will choose U in both type.

Exercise 2

Consider the following normal formal game:

		Player 2		
		$\frac{1}{7}$ a	b $\frac{2}{7}$	$\frac{4}{7}$ c
Player 1	$\frac{2}{3}$ x	$2, 4$	$2, -1$	$6, 4$
	$\frac{1}{3}$ y	$1, 2$	$3, 5$	$5, 2$

Define the mixed strategies $\sigma_1 = \left(\frac{2}{3}, \frac{1}{3}\right)$ and $\sigma_2 = \left(\frac{1}{7}, \frac{2}{7}, \frac{4}{7}\right)$.

Compute $\pi_1(\sigma_1, \sigma_2)$ and $\pi_2(\sigma_1, \sigma_2)$.

$$\pi_1(\sigma_1, \sigma_2) = \sum_{all(s_1, s_2)} p_{s_1} p_{s_2} \pi_1(s_1, s_2)$$

$$= \left(\tfrac{2}{3}\right)\left(\tfrac{1}{7}\right)\cdot 2 + \left(\tfrac{2}{3}\right)\left(\tfrac{2}{7}\right)\cdot 2 + \left(\tfrac{2}{3}\right)\left(\tfrac{4}{7}\right)\cdot 6$$

$$+ \left(\tfrac{1}{3}\right)\left(\tfrac{1}{7}\right)\cdot 1 + \left(\tfrac{1}{3}\right)\left(\tfrac{2}{7}\right)\cdot 3 + \left(\tfrac{1}{3}\right)\left(\tfrac{4}{7}\right)\cdot 5$$

$$= \frac{87}{21} = \frac{29}{7}$$

$$\pi_2(\sigma_1, \sigma_2) = \frac{8 + (-4) + 32}{(3 \cdot 7)} = \frac{56}{21} = \frac{8}{3}$$

$$+ \frac{2 + 10 + 8}{21}$$

Problem 1.

Create the Bayesian Normal form game for Exercise 1 and determine any Bayesian Nash equilibria.

type A

P1

P2	UU	UD	DU	DD
LL	(4,4)	(4,4)	(5,0)	(5,0)
LR	(4,4)	(4,4)	(5,0)	(5,0)
RL	(0,5)	(0,5)	(1,1)	(1,1)
RR	(0,5)	(0,5)	(1,1)	(1,1)

Type B

P1

P2	UU	UD	DU	DD
LL	(6,4)	(0,0)	(6,4)	(0,0)
LR	(0,0)	(4,6)	(0,0)	(4,6)
RL	(6,4)	(0,0)	(6,4)	(0,0)
RR	(0,0)	(4,6)	(0,0)	(4,6)

P1

P2	UU	UD	DU	DD
LL	$(\frac{9}{2}, \underline{4})$	$(3,3)$	$(\underline{\frac{21}{4}}, 1)$	$(\frac{15}{4}, 0)$
LR	$(3,3)$	$(4, \frac{9}{2})$	$(\frac{15}{4}, 0)$	$(\underline{\frac{19}{4}}, \frac{3}{2})$
RL	$(\frac{3}{2}, \frac{15}{4})$	$(0, \frac{15}{4})$	$(\underline{\frac{9}{4}}, \underline{\frac{7}{4}})$	$(\frac{3}{4}, \frac{1}{4})$
RR	$(0, \frac{15}{4})$	$(1, \underline{\frac{21}{4}})$	$(\frac{3}{4}, \frac{1}{4})$	$(\underline{\frac{7}{4}}, \underline{\frac{9}{4}})$

$$\frac{3}{4}(4,4) + \frac{1}{4}(6,4)$$
$$= \left(3 + \frac{6}{4}, 3 + 1\right)$$
$$= \left(\frac{9}{2}, 4\right)$$

2 Nash Equilibria

$\left(\frac{9}{4}, \frac{7}{4}\right)$ $\left(\frac{7}{4}, \frac{9}{4}\right)$

Problem 2.

In this problem we consider a simple poker game. There are two players and only two cards in the deck, a high card (H) and a low card (L). The deck is shuffled and one of the two cards is dealt to player 1. Player 2 does not receive a card.

Player 1 observes their card and then chooses whether to bid (B) or fold (F). If Player 1 folds, then the game ends with player 1 getting a payoff of -2 and Player 2 getting a payoff of 2. If Player 1 bids, then Player 2 must decide whether to bid or fold. When Player 2 makes this decision, they know that Player 1 has bid, but they do not know Player 1's card. The game ends after Player 2's action.

If Player 2 folds, then Player 1 gets 2 and Player 2 gets -2. If Player 2 bids, then the payoff depends on Player 1's card:

- If Player 1 holds the high card, then Player 1 gets 4 and Player 2 gets -4.
- If Player 1 holds the low card, then Player 1 gets -4 and Player 2 gets 4.

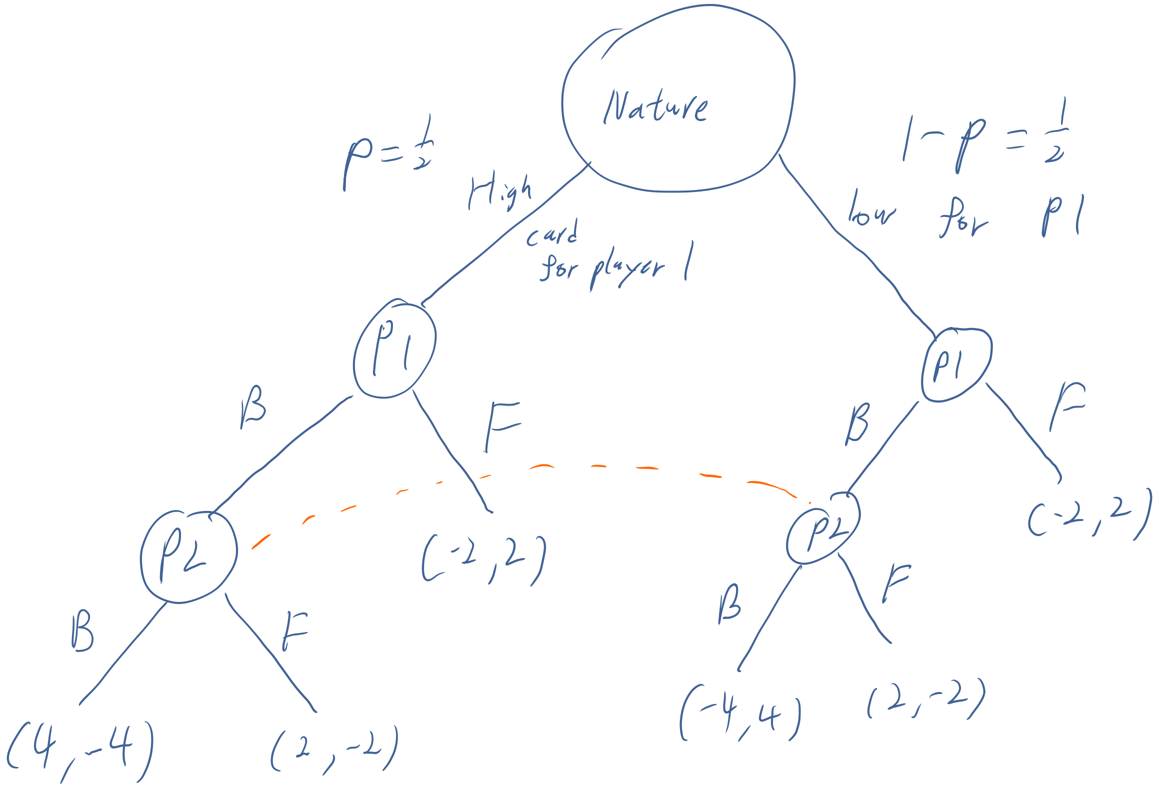
(a) Draw the extensive form game, including a node for Nature that determines which card Player 1 gets. Note that Player 1 has an equal chance of getting either card.

(b) Construct the Bayesian normal form for this game.

(c) Explain why there are no Bayesian Nash Equilibria for this game. We will soon extend the definition of a Nash equilibrium to include **mixed strategies**. We will then revisit this game and see that there is a **mixed strategy Nash equilibrium**.

Remark: When you create the Bayesian Normal Form Game, please keep to our standard convention where "Player 1" is listed as the "row player" and "Player 2" is listed as the column player, with the payoffs listed as $(P1, P2)$ in the payoff matrix.

Certainly this isn't the only way to do this, but keeping to the convention is an easy way to avoid mistakes in your analysis. I include this remark based on my experience of the types of errors made by students in previous semesters on similar problems.



High for P1

P1 \ P2	B	F
B	(4, -4)	(2, -2)
F	(-2, 2)	(-2, 2)

low for P1

P1 \ P2	B	F
B	(-4, 4)	(2, -2)
F	(-2, 2)	(-2, 2)

P1 \ P2	B	F
B	(0, 0)	(2, -2)
F	(-2, 2)	(-2, 2)

$$4 \cdot \frac{1}{2} + -4 \cdot \frac{1}{2} = 0$$

there will not be a BNE Because neither of them can get a best response