Greedy algorithms

Huffman Encoding (Textbook Section 5.2)

Huffman Encoding

An encoding scheme used in, e.g., MP3 encoding

Huffman Encoding

An encoding scheme used in, e.g., MP3 encoding

Data: a string S of symbols over an alphabet Γ

Huffman Encoding

An encoding scheme used in, e.g., MP3 encoding

Data: a string S of symbols over an alphabet Γ
Goal: find a binary encoding e of Γ resulting in minimum encoded length of S

Huffman Encoding

An encoding scheme used in, e.g., MP3 encoding

Data: a string S of symbols over an alphabet Γ
Goal: find a binary encoding e of Γ resulting in minimum encoded length of S

Denote the encoded string by S_e

Huffman Encoding

$S = aba$

An encoding scheme used in, e.g., MP3 encoding

Data: a string S of symbols over an alphabet Γ

Goal: find a binary encoding e of Γ resulting in minimum encoded length of S

Denote the encoded string by S_e

$S_e = \underbrace{01100001}_{a}\underbrace{01100010}_{b}\underbrace{01100001}_{a}$

Example: ASCII encoding

e

a	01100001
b	01100010
$\vdots$	$\vdots$

Different encodings

Consider $\Gamma = \{a, b, c\}$

Different encodings

Consider $\Gamma = \{a, b, c\}$

Stats on S: a appears 45 times, b 16 times, and c twice

Different encodings

Consider $\Gamma = \{a, b, c\}$

Stats on S: a appears 45 times, b 16 times, and c twice

- Fixed-length encoding

Different encodings

Consider $\Gamma = \{a, b, c\}$

Stats on S: a appears 45 times, b 16 times, and c twice

- Fixed-length encoding

$$|S_{e_1}| = 45 \cdot 2 + 16 \cdot 2 + 2 \cdot 2 = 126$$

$$e_1 : \begin{array}{l} a \to 00 \\ b \to 01 \\ c \to 10 \end{array}$$

Different encodings

Consider $\Gamma = \{a, b, c\}$

Stats on S: a appears 45 times, b 16 times, and c twice

- Fixed-length encoding

$$e_1 : \begin{array}{l} a \to 00 \\ b \to 01 \\ c \to 10 \end{array} \qquad |S_{e_1}| = 45 \times 2 + 16 \times 2 + 2 \times 2 = 126$$

Different encodings

Consider $\Gamma = \{a, b, c\}$

Stats on S: a appears 45 times, b 16 times, and c twice

- Fixed-length encoding

$$e_1 : \begin{array}{l} a \to 00 \\ b \to 01 \\ c \to 10 \end{array} \qquad |S_{e_1}| = 45 \times 2 + 16 \times 2 + 2 \times 2 = 126$$

- Variable-length encoding

Different encodings

Consider $\Gamma = \{a, b, c\}$

Stats on S: a appears 45 times, b 16 times, and c twice

- Fixed-length encoding

$$e_1: \begin{array}{l} a \rightarrow 00 \\ b \rightarrow 01 \\ c \rightarrow 10 \end{array} \qquad |S_{e_1}| = 45 \times 2 + 16 \times 2 + 2 \times 2 = 126$$

- Variable-length encoding

$$e_2: \begin{array}{l} a \rightarrow 0 \\ b \rightarrow 10 \\ c \rightarrow 11 \end{array}$$

$|S_{e_2}| = 45 \cdot 1 + 16 \cdot 2 + 2 \cdot 2 =$

Different encodings

Consider $\Gamma = \{a, b, c\}$

Stats on S: a appears 45 times, b 16 times, and c twice

- Fixed-length encoding

$$e_1: \begin{array}{l} a \to 00 \\ b \to 01 \\ c \to 10 \end{array} \qquad |S_{e_1}| = 45 \times 2 + 16 \times 2 + 2 \times 2 = 126$$

- Variable-length encoding

$$e_2: \begin{array}{l} a \to 0 \\ b \to 10 \\ c \to 11 \end{array} \qquad |S_{e_2}| = 45 \times 1 + 16 \times 2 + 2 \times 2 = 81$$

Different encodings

Consider $\Gamma = \{a, b, c\}$

Stats on S: a appears 45 times, b 16 times, and c twice

- Fixed-length encoding

$$e_1 : \begin{array}{l} a \to 00 \\ b \to 01 \\ c \to 10 \end{array} \qquad |S_{e_1}| = 45 \times 2 + 16 \times 2 + 2 \times 2 = 126$$

- Variable-length encoding

$$e_2 : \begin{array}{l} a \to 0 \\ b \to 10 \\ c \to 11 \end{array} \qquad |S_{e_2}| = 45 \times 1 + 16 \times 2 + 2 \times 2 = 81$$

- Be careful!

Different encodings

Consider $\Gamma = \{a, b, c\}$

Stats on S: a appears 45 times, b 16 times, and c twice

- Fixed-length encoding

$$e_1 : \begin{array}{l} a \to 00 \\ b \to 01 \\ c \to 10 \end{array} \qquad |S_{e_1}| = 45 \times 2 + 16 \times 2 + 2 \times 2 = 126$$

- Variable-length encoding

$$e_2 : \begin{array}{l} a \to 0 \\ b \to 10 \\ c \to 11 \end{array} \qquad |S_{e_2}| = 45 \times 1 + 16 \times 2 + 2 \times 2 = 81$$

- Be careful! $e_2 : \begin{array}{l} a \to 0 \\ b \to 1 \\ c \to 01 \end{array}$

Different encodings

Consider $\Gamma = \{a, b, c\}$

Stats on S: a appears 45 times, b 16 times, and c twice

- Fixed-length encoding

$$e_1 : \begin{array}{l} a \rightarrow 00 \\ b \rightarrow 01 \\ c \rightarrow 10 \end{array} \qquad |S_{e_1}| = 45 \times 2 + 16 \times 2 + 2 \times 2 = 126$$

- Variable-length encoding

$$e_2 : \begin{array}{l} a \rightarrow 0 \\ b \rightarrow 10 \\ c \rightarrow 11 \end{array} \qquad |S_{e_2}| = 45 \times 1 + 16 \times 2 + 2 \times 2 = 81$$

- Be careful! $e_2 : \begin{array}{l} a \rightarrow 0 \\ b \rightarrow 1 \\ c \rightarrow 01 \end{array}$ Decoding will lead to ambiguity

Prefix-free encoding

Consider the bad encoding e_2 :
$$\begin{aligned} a &\to 0 \\ b &\to 1 \\ c &\to 01 \end{aligned}$$

Prefix-free encoding

Consider the bad encoding e_2 :
$$
\begin{aligned}
a &\to 0\\
b &\to 1\\
c &\to 01
\end{aligned}
$$
How to decode $\overbrace{\underbrace{0}_{a}\underbrace{1}_{b}}^{c}0110$?

Prefix-free encoding

Consider the bad encoding $e_2 : \begin{array}{l} a \to 0 \\ b \to 1 \\ c \to 01 \end{array}$ How to decode 010110?

ababba?, *ccba*?, *abcba*?, or . . . ?

Prefix-free encoding

$$\text{Consider the bad encoding } e_2: \quad \begin{aligned} a &\to 0 \\ b &\to 1 \\ c &\to 01 \end{aligned} \qquad \text{How to decode 010110?}$$

ababba?, *ccba*?, *abcba*?, or ...?

To avoid ambiguity, we need the encoding to be prefix-free

Prefix-free encoding

Consider the bad encoding e_2 : $\begin{array}{l} a \to 0 \\ b \to 1 \\ c \to 01 \end{array}$ How to decode 010110?

ababba?, *ccba*?, *abcba*?, or ...?

To avoid ambiguity, we need the encoding to be prefix-free

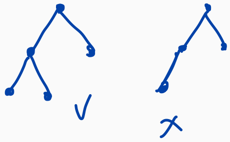
Definition

An encoding is **prefix-free** if no codeword is a prefix of any other codewords

Tree representation of a prefix-free encoding (I)

Definition

A **full binary tree** is a binary tree where each node is either a leaf or it has two children

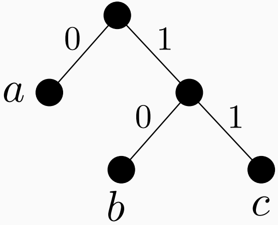


Tree representation of a prefix-free encoding (I)

Definition

A **full binary tree** is a binary tree where each node is either a leaf or it has two children

We use a full binary tree to represent a prefix-free encoding



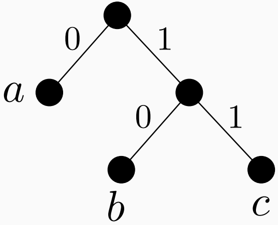
Tree representation of a prefix-free encoding (I)

Definition

A **full binary tree** is a binary tree where each node is either a leaf or it has two children

We use a full binary tree to represent a prefix-free encoding

- leaves are corresponding to symbols in Γ



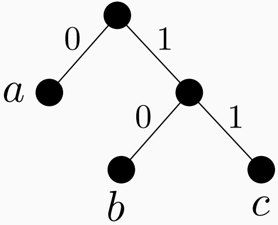
Tree representation of a prefix-free encoding (I)

Definition

A **full binary tree** is a binary tree where each node is either a leaf or it has two children

We use a full binary tree to represent a prefix-free encoding

- leaves are corresponding to symbols in Γ
- label edge to the left child with 0



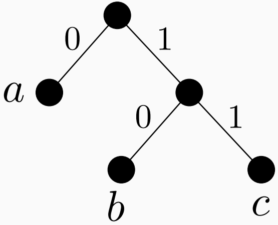
Tree representation of a prefix-free encoding (I)

Definition

A **full binary tree** is a binary tree where each node is either a leaf or it has two children

We use a full binary tree to represent a prefix-free encoding

- leaves are corresponding to symbols in Γ
- label edge to the left child with 0
- label edge to the right child with 1



Tree representation of a prefix-free encoding (I)

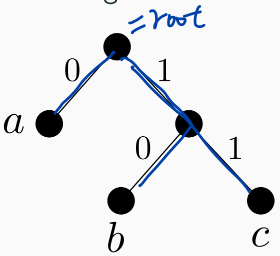
Definition

A **full binary tree** is a binary tree where each node is either a leaf or it has two children

We use a full binary tree to represent a prefix-free encoding

- leaves are corresponding to symbols in Γ
- label edge to the left child with 0
- label edge to the right child with 1

To obtain the encoding, read edge labels from root to a symbol



a : 0

b: 10

c : 11

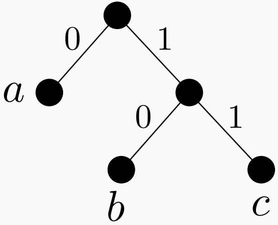
Tree representation of a prefix-free encoding (I)

Definition

A **full binary tree** is a binary tree where each node is either a leaf or it has two children

We use a full binary tree to represent a prefix-free encoding

- leaves are corresponding to symbols in Γ
- label edge to the left child with 0
- label edge to the right child with 1



To obtain the encoding, read edge labels from root to a symbol

$a \to 0$, $b \to 10$, $c \to 11$,

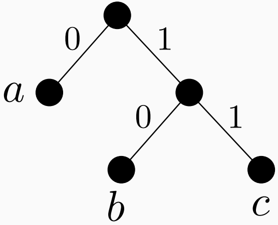
Tree representation of a prefix-free encoding (I)

Definition

A **full binary tree** is a binary tree where each node is either a leaf or it has two children

We use a full binary tree to represent a prefix-free encoding

- leaves are corresponding to symbols in Γ
- label edge to the left child with 0
- label edge to the right child with 1



To obtain the encoding, read edge labels from root to a symbol

$a \to 0$, $b \to 10$, $c \to 11$,

Depth of a leaf $\equiv$ length of its codeword

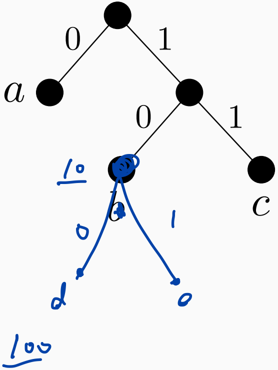
Tree representation of a prefix-free encoding (I)

Definition

A **full binary tree** is a binary tree where each node is either a leaf or it has two children

We use a full binary tree to represent a prefix-free encoding

- leaves are corresponding to symbols in Γ
- label edge to the left child with 0
- label edge to the right child with 1



To obtain the encoding, read edge labels from root to a symbol

$a \to 0,\ b \to 10,\ c \to 11,$

Depth of a leaf $\equiv$ length of its codeword

It guarantees to be prefix-free

Tree representation of a prefix-free encoding (II)

Let e be an encoding represented by a tree

Tree representation of a prefix-free encoding (II)

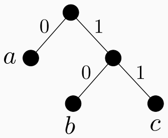
Let e be an encoding represented by a tree

For string S, let f_v be the symbol count in S for each $v \in \Gamma$

Tree representation of a prefix-free encoding (II)

Let e be an encoding represented by a tree

For string S, let f_v be the symbol count in S for each $v \in \Gamma$



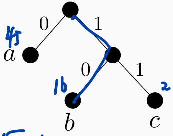
Tree representation of a prefix-free encoding (II)

Let e be an encoding represented by a tree

For string S, let f_v be the symbol count in S for each $v \in \Gamma$

the number of bits to represent v

$$|S_e| = \sum_{v \in \Gamma} f_v \cdot \mathrm{depth}(v)$$

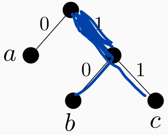


$|S_e| = 45 \cdot 1 + 16 \cdot 2 + 2 \cdot 2 = 81$

Let e be an encoding represented by a tree

For string S, let f_v be the symbol count in S for each $v \in \Gamma$

$$|S_e| = \sum_{v \in \Gamma} f_v \cdot \mathrm{depth}(v)$$



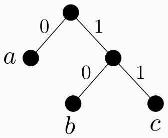
$$|S_e| = 45 \times 1 + 16 \times 2 + 2 \times 2 = 81$$

Tree representation of a prefix-free encoding (II)

Let e be an encoding represented by a tree

For string S, let f_v be the symbol count in S for each $v \in \Gamma$

$$|S_e| = \sum_{v \in \Gamma} f_v \cdot \mathrm{depth}(v)$$



$$|S_e| = 45 \times 1 + 16 \times 2 + 2 \times 2 = 81$$

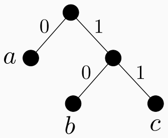
A useful re-write: label internal nodes with counts of descendants

Tree representation of a prefix-free encoding (II)

Let e be an encoding represented by a tree

For string S, let f_v be the symbol count in S for each $v \in \Gamma$

$$|S_e| = \sum_{v \in \Gamma} f_v \cdot \mathrm{depth}(v)$$



$$|S_e| = 45 \times 1 + 16 \times 2 + 2 \times 2 = 81$$

A useful re-write: label internal nodes with counts of descendants

For all non-root node v, define

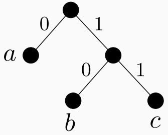
$\mathrm{cost}(v) :=$ sum of leaf node counts descending from v

Tree representation of a prefix-free encoding (II)

Let e be an encoding represented by a tree

For string S, let f_v be the symbol count in S for each $v \in \Gamma$

$$|S_e| = \sum_{v \in \Gamma} f_v \cdot \mathrm{depth}(v)$$

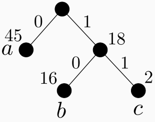


$$|S_e| = 45 \times 1 + 16 \times 2 + 2 \times 2 = 81$$

A useful re-write: label internal nodes with counts of descendants

For all non-root node v, define

$\mathrm{cost}(v) :=$ sum of leaf node counts descending from v

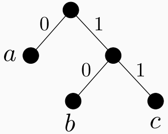


Tree representation of a prefix-free encoding (II)

Let e be an encoding represented by a tree

For string S, let f_v be the symbol count in S for each $v \in \Gamma$

$$|S_e| = \sum_{v\in\Gamma} f_v \cdot \mathrm{depth}(v)$$

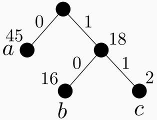


$$|S_e| = 45 \times 1 + 16 \times 2 + 2 \times 2 = 81$$

A useful re-write: label internal nodes with counts of descendants

For all non-root node v, define

$\mathrm{cost}(v) :=$ sum of leaf node counts descending from v



$$|S_e| = \sum_{v\in \underbrace{T-\{\mathrm{root}\}}_{\text{all nodes except for the root.}}} \mathrm{cost}(v)$$

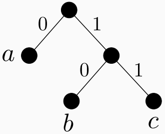
$|S_e| = 45+16+2+18$

Tree representation of a prefix-free encoding (II)

Let e be an encoding represented by a tree

For string S, let f_v be the symbol count in S for each $v \in \Gamma$

$$|S_e| = \sum_{v \in \Gamma} f_v \cdot \mathrm{depth}(v)$$



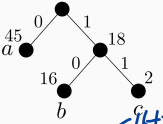
$$|S_e| = 45 \times 1 + 16 \times 2 + 2 \times 2 = 81$$

A useful re-write: label internal nodes with counts of descendants

For all non-root node v, define

$\mathrm{cost}(v) :=$ sum of leaf node counts descending from v

$$|S_e| = \sum_{v \in T - \{\mathrm{root}\}} \mathrm{cost}(v)$$



$=16+2$

$$|S_e| = 45 + 16 + 2 + 18 = 81$$

Constructing the prefix-free encoding tree

Idea: put more frequent symbols at smaller depth

Constructing the prefix-free encoding tree

Idea: put more frequent symbols at smaller depth

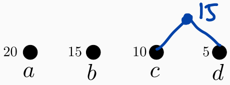
Greedy approach: continually merge least frequent symbols/nodes until you have a full binary tree encoding all symbols

Constructing the prefix-free encoding tree – examples

- $a : 20, b : 15, c : 10, d : 5$

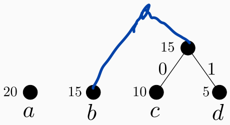
Constructing the prefix-free encoding tree – examples

- $a : 20, b : 15, c : 10, d : 5$



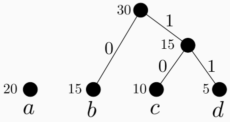
Constructing the prefix-free encoding tree – examples

- $a : 20, b : 15, c : 10, d : 5$



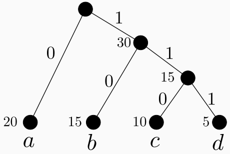
Constructing the prefix-free encoding tree – examples

- $a : 20, b : 15, c : 10, d : 5$



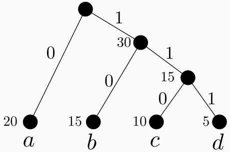
Constructing the prefix-free encoding tree – examples

- $a : 20, b : 15, c : 10, d : 5$



Constructing the prefix-free encoding tree – examples

- a : 20, b : 15, c : 10, d : 5



$a \to 0$

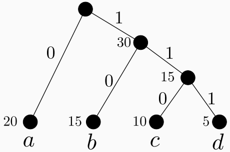
$b \to 10$

$c \to 110$

$d\ \cancel{c} \to 111$

Constructing the prefix-free encoding tree – examples

- $a: 20, b: 15, c: 10, d: 5$



$a \to 0$

$b \to 10$

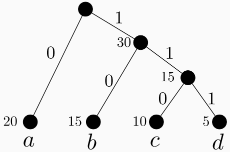
$c \to 110$

$c \to 111$

- $a: 10, b: 10, c: 10, d: 10$

Constructing the prefix-free encoding tree – examples

- $a : 20, b : 15, c : 10, d : 5$



$a \to 0$

$b \to 10$

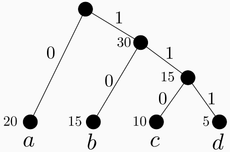
$c \to 110$

$c \to 111$

- $a : 10, b : 10, c : 10, d : 10$

Constructing the prefix-free encoding tree – examples

- $a : 20, b : 15, c : 10, d : 5$



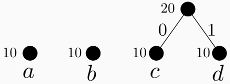
$a \rightarrow 0$

$b \rightarrow 10$

$c \rightarrow 110$

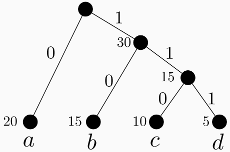
$c \rightarrow 111$

- $a : 10, b : 10, c : 10, d : 10$



Constructing the prefix-free encoding tree – examples

- $a : 20, b : 15, c : 10, d : 5$



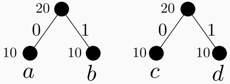
$a \to 0$

$b \to 10$

$c \to 110$

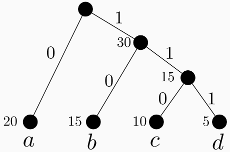
$c \to 111$

- $a : 10, b : 10, c : 10, d : 10$



Constructing the prefix-free encoding tree – examples

- $a : 20, b : 15, c : 10, d : 5$



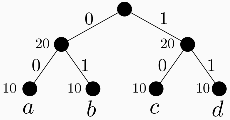
$a \to 0$

$b \to 10$

$c \to 110$

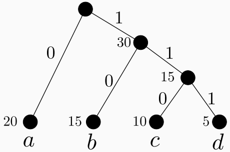
$c \to 111$

- $a : 10, b : 10, c : 10, d : 10$



Constructing the prefix-free encoding tree – examples

- $a : 20, b : 15, c : 10, d : 5$



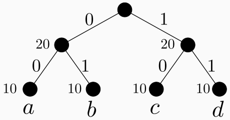
$a \to 0$

$b \to 10$

$c \to 110$

$c \to 111$

- $a : 10, b : 10, c : 10, d : 10$



$a \to 00$

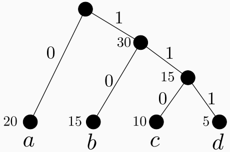
$b \to 01$

$c \to 10$

$d\ \cancel{c} \to 11$

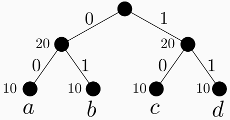
Constructing the prefix-free encoding tree – examples

- $a : 20, b : 15, c : 10, d : 5$



$a \to 0$
$b \to 10$
$c \to 110$
$c \to 111$

- $a : 10, b : 10, c : 10, d : 10$

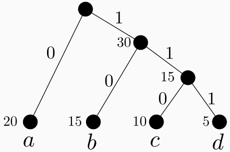


$a \to 00$
$b \to 01$
$c \to 10$
$c \to 11$

- $a : 10, b : 10, c : 5, d : 5$

Constructing the prefix-free encoding tree – examples

- $a : 20, b : 15, c : 10, d : 5$



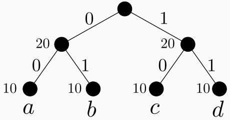
$a \to 0$

$b \to 10$

$c \to 110$

$c \to 111$

- $a : 10, b : 10, c : 10, d : 10$



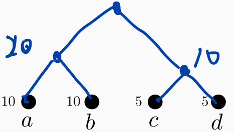
$a \to 00$

$b \to 01$

$c \to 10$

$c \to 11$

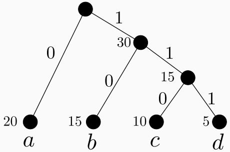
- $a : 10, b : 10, c : 5, d : 5$



$|S_e| = 10+10+5+5$
$+20+10 = 60$

Constructing the prefix-free encoding tree – examples

- $a : 20, b : 15, c : 10, d : 5$



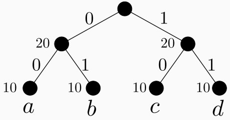
$a \to 0$

$b \to 10$

$c \to 110$

$c \to 111$

- $a : 10, b : 10, c : 10, d : 10$



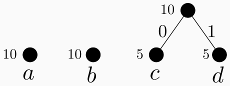
$a \to 00$

$b \to 01$

$c \to 10$

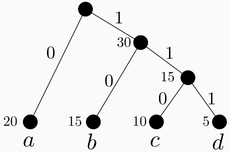
$c \to 11$

- $a : 10, b : 10, c : 5, d : 5$



Constructing the prefix-free encoding tree – examples

- $a : 20, b : 15, c : 10, d : 5$



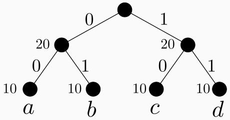
$a \to 0$

$b \to 10$

$c \to 110$

$c \to 111$

- $a : 10, b : 10, c : 10, d : 10$



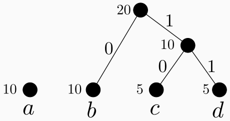
$a \to 00$

$b \to 01$

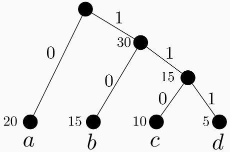
$c \to 10$

$c \to 11$

- $a : 10, b : 10, c : 5, d : 5$



- $a : 20, b : 15, c : 10, d : 5$



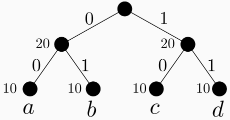
$a \to 0$

$b \to 10$

$c \to 110$

$c \to 111$

- $a : 10, b : 10, c : 10, d : 10$



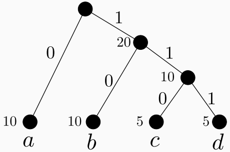
$a \to 00$

$b \to 01$

$c \to 10$

$c \to 11$

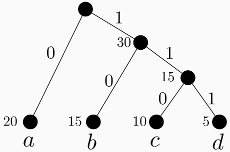
- $a : 10, b : 10, c : 5, d : 5$



$|S_e| = 10+10+5+5+10+20 = 60$

Constructing the prefix-free encoding tree – examples

- a : 20, b : 15, c : 10, d : 5



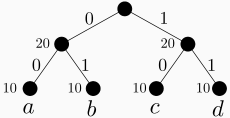
$a \rightarrow 0$

$b \rightarrow 10$

$c \rightarrow 110$

d $\cancel{c} \rightarrow 111$

- a : 10, b : 10, c : 10, d : 10



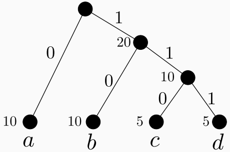
$a \rightarrow 00$

$b \rightarrow 01$

$c \rightarrow 10$

d $\cancel{c} \rightarrow 11$

- a : 10, b : 10, c : 5, d : 5



$a \rightarrow 0$

$b \rightarrow 10$

$c \rightarrow 110$

d $\cancel{c} \rightarrow 111$

Proof of optimality (I)

Proof sketch

Proof of optimality (I)

Proof sketch

- Greedy choice property:

Proof of optimality (I)

Proof sketch

- Greedy choice property:

Claim

Every optimal solution has two lowest frequent symbols as leaves connected to an internal node of greatest depth

Proof of optimality (I)

Proof sketch

- Greedy choice property:

Claim

Every optimal solution has two lowest frequent symbols as leaves connected to an internal node of greatest depth

Proof. (exchange argument).

Proof of optimality (I)

Proof sketch

- Greedy choice property:

Claim

Every optimal solution has two lowest frequent symbols as leaves connected to an internal node of greatest depth

Proof. (exchange argument).

Suppose we have a tree T with two lowest frequent symbols not as deep as possible.

Proof of optimality (I)

Proof sketch

- Greedy choice property:

Claim

Every optimal solution has two lowest frequent symbols as leaves connected to an internal node of greatest depth

Proof. (exchange argument).

Suppose we have a tree T with two lowest frequent symbols not as deep as possible. Then at least one has a smaller depth.

Proof sketch

- Greedy choice property:

Claim

Every optimal solution has two lowest frequent symbols as leaves connected to an internal node of greatest depth

Proof. (exchange argument).

Suppose we have a tree T with two lowest frequent symbols not as deep as possible. Then at least one has a smaller depth. Switch it with one of the deepest nodes that is more frequent.

Proof of optimality (I)

Proof sketch

- Greedy choice property:

Claim

Every optimal solution has two lowest frequent symbols as leaves connected to an internal node of greatest depth

Proof. (exchange argument).

Suppose we have a tree T with two lowest frequent symbols not as deep as possible. Then at least one has a smaller depth. Switch it with one of the deepest nodes that is more frequent.

This improves the encoding length. Thus T is not optimal $\square$

Proof of optimality (I)

Proof sketch

- Greedy choice property:

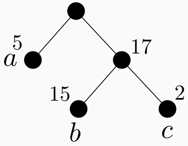
Claim

Every optimal solution has two lowest frequent symbols as leaves connected to an internal node of greatest depth

Proof. (exchange argument).

Suppose we have a tree T with two lowest frequent symbols not as deep as possible. Then at least one has a smaller depth. Switch it with one of the deepest nodes that is more frequent.

This improves the encoding length. Thus T is not optimal □



Proof of optimality (I)

Proof sketch

- Greedy choice property:

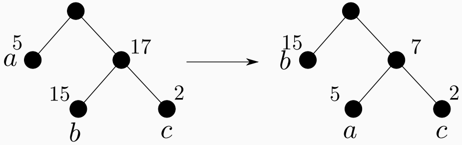
Claim

Every optimal solution has two lowest frequent symbols as leaves connected to an internal node of greatest depth

Proof. (exchange argument).

Suppose we have a tree T with two lowest frequent symbols not as deep as possible. Then at least one has a smaller depth. Switch it with one of the deepest nodes that is more frequent.

This improves the encoding length. Thus T is not optimal □



Proof of optimality (II)

- Optimal substructure
  Consider a global optimal solution for $(f_1, f_2, \ldots, f_n)$ with f_1, f_2 the least frequencies.

Proof of optimality (II)

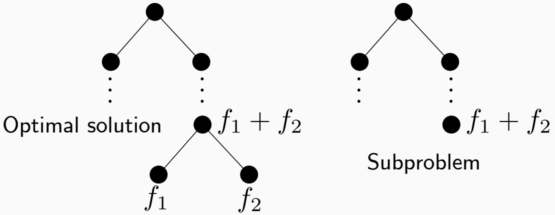
- Optimal substructure
  Consider a global optimal solution for $(f_1, f_2, \ldots, f_n)$ with f_1, f_2 the least frequencies.
  Our subproblem is: find an optimal solution for $(f_1 + f_2, f_3, \ldots, f_n)$

Proof of optimality (II)

- Optimal substructure

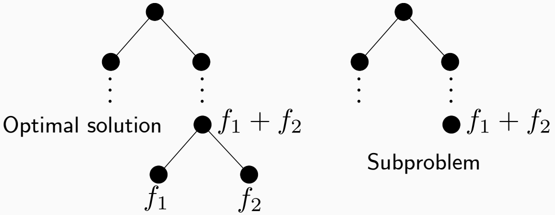
  Consider a global optimal solution for $(f_1, f_2, \ldots, f_n)$ with f_1, f_2 the least frequencies.

  Our subproblem is: find an optimal solution for $(f_1 + f_2, f_3, \ldots, f_n)$



Proof of optimality (II)

- Optimal substructure
  Consider a global optimal solution for $(f_1, f_2, \ldots, f_n)$ with f_1, f_2 the least frequencies.
  Our subproblem is: find an optimal solution for $(f_1 + f_2, f_3, \ldots, f_n)$



Thus, the greedy solution will lead to the global optimal solution

Pseudocode

```
def HUFFMAN(f): // f : f[1], ..., f[n]; Γ has n symbols
    T: empty tree;
```

Pseudocode

def HUFFMAN(f): // $f: f[1], \ldots, f[n]$; Γ has n symbols

    T: empty tree;

    H: priority queue ordered by f;

Pseudocode

```
def HUFFMAN(f): // f : f[1], ..., f[n]; Γ has n symbols
    T: empty tree;
    H: priority queue ordered by f;
    for i := 1 in n:
```

Pseudocode

def HUFFMAN(f): // $f: f[1], \ldots, f[n]$; Γ has n symbols

  T: empty tree;

  H: priority queue ordered by f;

  for $i := 1$ **in** n:

    insert(H, i); $(i, f[i]) \to H$

Pseudocode

```
def HUFFMAN(f): // f : f[1], ..., f[n]; Γ has n symbols
    T: empty tree;
    H: priority queue ordered by f;
    for i := 1 in n:
        insert(H, i);
    for k := n + 1 in 2n − 1:
```

Pseudocode

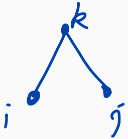
def HUFFMAN(f): // $f : f[1], \ldots, f[n]$; Γ has n symbols
    T: empty tree;
    H: priority queue ordered by f;
    for $i := 1$ **in** n:
        $\mathrm{insert}(H, i)$;
    for $k := n + 1$ **in** $2n - 1$:
        $i := \mathrm{extract\_min}(H)$;

Pseudocode

```
def HUFFMAN(f): // f : f[1], ..., f[n]; Γ has n symbols
    T: empty tree;
    H: priority queue ordered by f;
    for i := 1 in n:
        insert(H, i);
    for k := n + 1 in 2n − 1:
        i := extract_min(H);
        j := extract_min(H);
```

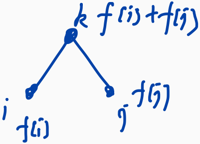
Pseudocode

```
def HUFFMAN(f): // f : f[1], ..., f[n]; Γ has n symbols
    T: empty tree;
    H: priority queue ordered by f;
    for i := 1 in n:
        insert(H, i);
    for k := n + 1 in 2n − 1:
        i := extract_min(H);
        j := extract_min(H);
        Creat a node k in T with children i and j;
```



Pseudocode

```
def HUFFMAN(f): // f : f[1], ..., f[n]; Γ has n symbols
    T: empty tree;
    H: priority queue ordered by f;
    for i := 1 in n:
        insert(H, i);
    for k := n + 1 in 2n − 1:
        i := extract_min(H);
        j := extract_min(H);
        Creat a node k in T with children i and j;
        f[k] := f[i] + f[j];
```



Pseudocode

```
def HUFFMAN(f): // f : f[1], ..., f[n]; Γ has n symbols
    T: empty tree;
    H: priority queue ordered by f;
    for i := 1 in n:
        insert(H, i);
    for k := n + 1 in 2n − 1:
        i := extract_min(H);
        j := extract_min(H);
        Creat a node k in T with children i and j;
        f[k] := f[i] + f[j];
        insert(H, k);
```

Pseudocode

def $\textsc{Huffman}(f)$**:** // $f : f[1], \ldots, f[n]$; Γ has n symbols

T: empty tree;

H: priority queue ordered by f;

for $i := 1$ **in** n**:**

insert(H, i); $\to O(\log n)$ $O(n\log n)$

for $k := n + 1$ **in** $2n - 1$**:**

$i :=$ extract_min(H); $\to O(\log n)$

$j :=$ extract_min(H); $\to$

Creat a node k in T with children i and j;

$f[k] := f[i] + f[j]$;

insert(H, k); $\to O(\log n)$

$O(n \log n)$

Binary heap: insert $O(\log n)$, extract_min: $O(\log n)$

Pseudocode

```
1  def HUFFMAN(f): // f : f[1], ..., f[n]; Γ has n symbols
2      T: empty tree;
3      H: priority queue ordered by f;
4      for i := 1 in n:
5          insert(H, i);
6      for k := n + 1 in 2n − 1:
7          i := extract_min(H);
8          j := extract_min(H);
9          Creat a node k in T with children i and j;
10         f[k] := f[i] + f[j];
11         insert(H, k);
```

Binary heap: insert $O(\log n)$, extract_min: $O(\log n)$

Lines 4-5: $O(n \log n)$;

Pseudocode

```
1  def HUFFMAN(f): // f : f[1], ..., f[n]; Γ has n symbols
2      T: empty tree;
3      H: priority queue ordered by f;
4      for i := 1 in n:
5          insert(H, i);
6      for k := n + 1 in 2n − 1:
7          i := extract_min(H);
8          j := extract_min(H);
9          Creat a node k in T with children i and j;
10         f[k] := f[i] + f[j];
11         insert(H, k);
```

Binary heap: insert $O(\log n)$, extract_min: $O(\log n)$

Lines 4-5: $O(n \log n)$; Lines 6-11: $O(n \log n)$

Pseudocode

```
1  def Huffman(f): // f : f[1], ..., f[n]; Γ has n symbols
2      T: empty tree;
3      H: priority queue ordered by f;
4      for i := 1 in n:
5          insert(H, i);
6      for k := n + 1 in 2n − 1:
7          i := extract_min(H);
8          j := extract_min(H);
9          Creat a node k in T with children i and j;
10         f[k] := f[i] + f[j];
11         insert(H, k);
```

Binary heap: insert $O(\log n)$, extract_min: $O(\log n)$

Lines 4-5: $O(n \log n)$; Lines 6-11: $O(n \log n)$

Total cost: $O(n \log n)$

More about the pseudocode

Question: why $2n - 1$ in line 6?

More about the pseudocode

Question: why $2n-1$ in line 6?

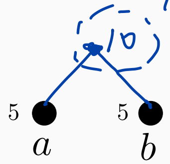
Answer: if a full binary tree has n leaves, then it has $2n-1$ total nodes

More examples

$a : 5, b : 5, c : 7, d : 10, e : 15, f : 17$

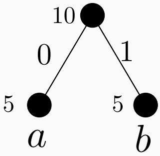
More examples

$a : 5, b : 5, c : 7, d : 10, e : 15, f : 17$



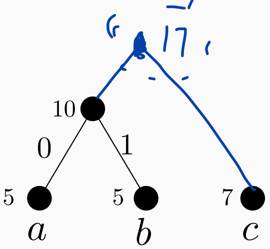
More examples

$a : 5, b : 5, c : 7, d : 10, e : 15, f : 17$

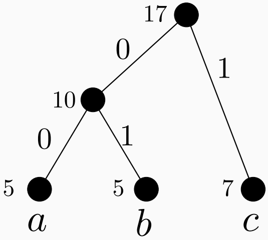


More examples

~~$a : 5, b : 5, c : 7$~~, $d : 10, e : 15, f : 17$

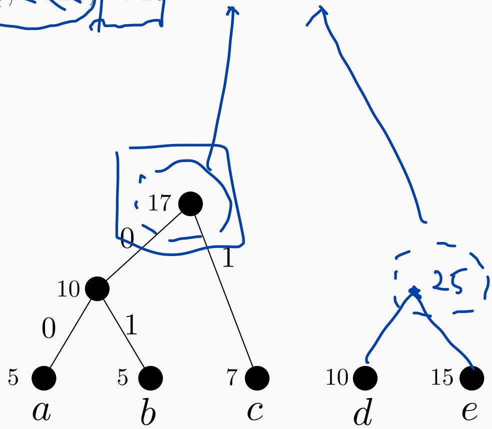


$a : 5, b : 5, c : 7, d : 10, e : 15, f : 17$



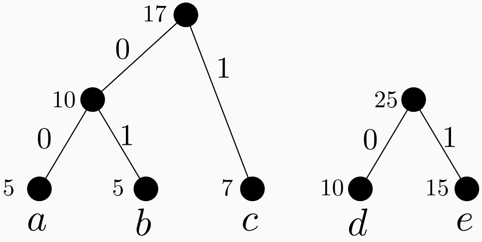
More examples

$a : 5, b : 5, c : 7, d : 10, e : 15, f : 17$



More examples

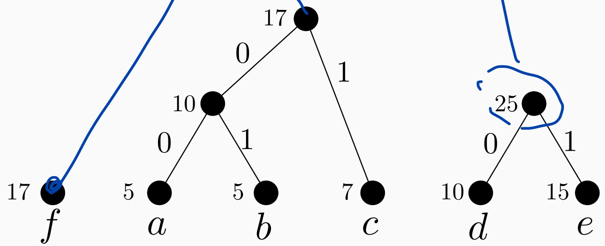
$a : 5, b : 5, c : 7, d : 10, e : 15, f : 17$



More examples

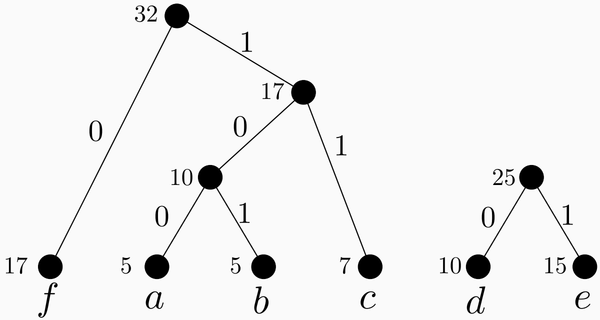
$a:5, b:5, c:7, d:10, e:15, f:17$

34



More examples

$a : 5, b : 5, c : 7, d : 10, e : 15, f : 17$



More examples

$a : 5, b : 5, c : 7, d : 10, e : 15, f : 17$