Discrete Distributions

Bernoulli

$0 < p < 1$

$$f(x) = p^x(1-p)^{1-x}, \qquad x = 0, 1$$
$$M(t) = 1 - p + pe^t, \qquad -\infty < t < \infty$$
$$\mu = p, \qquad \sigma^2 = p(1-p)$$

Binomial

$b(n, p)$

$0 < p < 1$

$$f(x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}, \qquad x = 0, 1, 2, \ldots, n$$
$$M(t) = (1 - p + pe^t)^n, \qquad -\infty < t < \infty$$
$$\mu = np, \qquad \sigma^2 = np(1-p)$$

Geometric

$0 < p < 1$

$$f(x) = (1-p)^{x-1}p, \qquad x = 1, 2, 3, \ldots$$
$$M(t) = \frac{pe^t}{1-(1-p)e^t}, \qquad t < -\ln(1-p)$$
$$\mu = \frac{1}{p}, \qquad \sigma^2 = \frac{1-p}{p^2}$$

Hypergeometric

$N_1 > 0, \;\; N_2 > 0$

$N = N_1 + N_2$

$$f(x) = \frac{\binom{N_1}{x}\binom{N_2}{n-x}}{\binom{N}{n}}, \qquad x \le n,\; x \le N_1,\; n - x \le N_2$$
$$\mu = n\left(\frac{N_1}{N}\right), \qquad \sigma^2 = n\left(\frac{N_1}{N}\right)\left(\frac{N_2}{N}\right)\left(\frac{N-n}{N-1}\right)$$

Negative Binomial

$0 < p < 1$

$r = 1, 2, 3, \ldots$

$$f(x) = \binom{x-1}{r-1} p^r (1-p)^{x-r}, \qquad x = r, r+1, r+2, \ldots$$
$$M(t) = \frac{(pe^t)^r}{[1-(1-p)e^t]^r}, \qquad t < -\ln(1-p)$$
$$\mu = r\left(\frac{1}{p}\right), \qquad \sigma^2 = \frac{r(1-p)}{p^2}$$

Poisson

$\lambda > 0$

$$f(x) = \frac{\lambda^x e^{-\lambda}}{x!}, \qquad x = 0, 1, 2, \ldots$$
$$M(t) = e^{\lambda(e^t - 1)}, \qquad -\infty < t < \infty$$
$$\mu = \lambda, \qquad \sigma^2 = \lambda$$

Uniform

$m > 0$

$$f(x) = \frac{1}{m}, \qquad x = 1, 2, \ldots, m$$
$$\mu = \frac{m+1}{2}, \qquad \sigma^2 = \frac{m^2-1}{12}$$

Continuous Distributions

Beta
$\alpha > 0$
$\beta > 0$

$$f(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\, x^{\alpha-1}(1-x)^{\beta-1}, \qquad 0 < x < 1$$

$$\mu = \frac{\alpha}{\alpha+\beta}, \qquad \sigma^2 = \frac{\alpha\beta}{(\alpha+\beta+1)(\alpha+\beta)^2}$$

Chi-square
$\chi^2(r)$
$r = 1, 2, \ldots$

$$f(x) = \frac{1}{\Gamma(r/2)2^{r/2}}\, x^{r/2-1}e^{-x/2}, \qquad 0 < x < \infty$$

$$M(t) = \frac{1}{(1-2t)^{r/2}}, \qquad t < \frac{1}{2}$$

$$\mu = r, \qquad \sigma^2 = 2r$$

Exponential
$\theta > 0$

$$f(x) = \frac{1}{\theta}\, e^{-x/\theta}, \qquad 0 \le x < \infty$$

$$M(t) = \frac{1}{1-\theta t}, \qquad t < \frac{1}{\theta}$$

$$\mu = \theta, \qquad \sigma^2 = \theta^2$$

Gamma
$\alpha > 0$
$\theta > 0$

$$f(x) = \frac{1}{\Gamma(\alpha)\theta^{\alpha}}\, x^{\alpha-1}e^{-x/\theta}, \qquad 0 < x < \infty$$

$$M(t) = \frac{1}{(1-\theta t)^{\alpha}}, \qquad t < \frac{1}{\theta}$$

$$\mu = \alpha\theta, \qquad \sigma^2 = \alpha\theta^2$$

Normal
$N(\mu, \sigma^2)$
$-\infty < \mu < \infty$
$\sigma > 0$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-(x-\mu)^2/2\sigma^2}, \qquad -\infty < x < \infty$$

$$M(t) = e^{\mu t + \sigma^2 t^2/2}, \qquad -\infty < t < \infty$$

$$E(X) = \mu, \qquad \mathrm{Var}(X) = \sigma^2$$

Uniform
$U(a, b)$
$-\infty < a < b < \infty$

$$f(x) = \frac{1}{b-a}, \qquad a \le x \le b$$

$$M(t) = \frac{e^{tb} - e^{ta}}{t(b-a)}, \qquad t \ne 0; \qquad M(0) = 1$$

$$\mu = \frac{a+b}{2}, \qquad \sigma^2 = \frac{(b-a)^2}{12}$$

Confidence Intervals

Parameter	Assumptions	Endpoints
μ	$N(\mu, \sigma^2)$ or n large, σ^2 known	$\overline{x} \pm z_{\alpha/2} \dfrac{\sigma}{\sqrt{n}}$
μ	$N(\mu, \sigma^2)$ σ^2 unknown	$\overline{x} \pm t_{\alpha/2}(n-1) \dfrac{s}{\sqrt{n}}$
$\mu_X - \mu_Y$	$N(\mu_X, \sigma_X^2)$ $N(\mu_Y, \sigma_Y^2)$ σ_X^2, σ_Y^2 known	$\overline{x} - \overline{y} \pm z_{\alpha/2}\sqrt{\dfrac{\sigma_X^2}{n} + \dfrac{\sigma_Y^2}{m}}$
$\mu_X - \mu_Y$	Variances unknown, large samples	$\overline{x} - \overline{y} \pm z_{\alpha/2}\sqrt{\dfrac{s_x^2}{n} + \dfrac{s_y^2}{m}}$
$\mu_X - \mu_Y$	$N(\mu_X, \sigma_X^2)$ $N(\mu_Y, \sigma_Y^2)$ $\sigma_X^2 = \sigma_Y^2$, unknown	$\overline{x} - \overline{y} \pm t_{\alpha/2}(n+m-2) s_p \sqrt{\dfrac{1}{n} + \dfrac{1}{m}}$, $s_p = \sqrt{\dfrac{(n-1)s_x^2 + (m-1)s_y^2}{n+m-2}}$
$\mu_D = \mu_X - \mu_Y$	X and Y normal, but dependent	$\overline{d} \pm t_{\alpha/2}(n-1) \dfrac{s_d}{\sqrt{n}}$
p	$b(n, p)$ n is large	$\dfrac{y}{n} \pm z_{\alpha/2}\sqrt{\dfrac{(y/n)[1-(y/n)]}{n}}$
$p_1 - p_2$	$b(n_1, p_1)$ $b(n_2, p_2)$	$\dfrac{y_1}{n_1} - \dfrac{y_2}{n_2} \pm z_{\alpha/2}\sqrt{\dfrac{\widehat{p_1}(1-\widehat{p_1})}{n_1} + \dfrac{\widehat{p_2}(1-\widehat{p_2})}{n_2}}$, $\widehat{p_1} = y_1/n_1$, $\widehat{p_2} = y_2/n_2$

Tests of Hypotheses

Hypotheses	Assumptions	Critical Region
H_0: $\mu = \mu_0$ H_1: $\mu > \mu_0$	$N(\mu, \sigma^2)$ or n large, σ^2 known	$z = \dfrac{\overline{x} - \mu_0}{\sigma/\sqrt{n}} \ge z_\alpha$
H_0: $\mu = \mu_0$ H_1: $\mu > \mu_0$	$N(\mu, \sigma^2)$ σ^2 unknown	$t = \dfrac{\overline{x} - \mu_0}{s/\sqrt{n}} \ge t_\alpha(n-1)$
H_0: $\mu_X - \mu_Y = 0$ H_1: $\mu_X - \mu_Y > 0$	$N(\mu_X, \sigma_X^2)$ $N(\mu_Y, \sigma_Y^2)$ σ_X^2, σ_Y^2 known	$z = \dfrac{\overline{x} - \overline{y} - 0}{\sqrt{(\sigma_X^2/n) + (\sigma_Y^2/m)}} \ge z_\alpha$
H_0: $\mu_X - \mu_Y = 0$ H_1: $\mu_X - \mu_Y > 0$	Variances unknown, large samples	$z = \dfrac{\overline{x} - \overline{y} - 0}{\sqrt{(s_x^2/n) + (s_y^2/m)}} \ge z_\alpha$
H_0: $\mu_X - \mu_Y = 0$ H_1: $\mu_X - \mu_Y > 0$	$N(\mu_X, \sigma_X^2)$ $N(\mu_Y, \sigma_Y^2)$ $\sigma_X^2 = \sigma_Y^2$, unknown	$t = \dfrac{\overline{x} - \overline{y} - 0}{s_p\sqrt{(1/n) + (1/m)}} \ge t_\alpha(n+m-2)$ $s_p = \sqrt{\dfrac{(n-1)s_x^2 + (m-1)s_y^2}{n+m-2}}$
H_0: $\mu_D = \mu_X - \mu_Y = 0$ H_1: $\mu_D = \mu_X - \mu_Y > 0$	X and Y normal, but dependent	$t = \dfrac{\overline{d} - 0}{s_d/\sqrt{n}} \ge t_\alpha(n-1)$
H_0: $p = p_0$ H_1: $p > p_0$	$b(n, p)$ n is large	$z = \dfrac{(y/n) - p_0}{\sqrt{p_0(1-p_0)/n}} \ge z_\alpha$
H_0: $p_1 - p_2 = 0$ H_1: $p_1 - p_2 > 0$	$b(n_1, p_1)$ $b(n_2, p_2)$	$z = \dfrac{(y_1/n_1) - (y_2/n_2) - 0}{\sqrt{\left(\dfrac{y_1+y_2}{n_1+n_2}\right)\left(1 - \dfrac{y_1+y_2}{n_1+n_2}\right)\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}} \ge z_\alpha$