STAT/MATH 415 HW#5

November 3, 2017

EXERCISES

7.4.3 A company packages powdered soap in "6-pound" boxes. The sample mean and standard deviation of the soap in these boxes are currently 6.09 pounds and 0.02 pound respectively. If the mean fill can be lowered by 0.01 pound, $14,000 would be saved per year. Adjustments were made in the filling equipment, but it can be assumed that the standard deviation remains unchanged.

a) How large a sample is needed so that the maximum error of the estimate of the new μ is $\varepsilon = 0.001$ with 90% confidence?

Answer: $\alpha = 0.10,\ \sigma = 0.02,\ \varepsilon = 0.001$

$$n = {Z_{0.05}}^2 \frac{\sigma^2}{\varepsilon^2} = 1.645^2 \frac{0.02^2}{0.001^2} = 1082.41$$

Thus at least 1083 samples are needed.

b) A random sample of size $n = 1219$ yielded $\bar{x} = 6.048$ and $s = 0.022$. Calculate a 90% confidence interval for μ.

Answer: $\alpha = 0.10,\ t_{0.05(1218)} = 1.645$

$$\bar{X} \pm t_{0.05(1218)} \frac{s}{\sqrt{n}} = 6.048 \pm 1.645 \frac{0.022}{\sqrt{1219}} = 6.048 \pm 0.001$$

The 90% confidence interval for μ is [6.047, 6.049]

c) Estimate the savings per year with these new adjustments.

Answer: Let X denote the mean weight before, Y denote the mean weight after adjustments.

$$E(X) = 6.09 \qquad E(Y) = 6.048 \qquad E(X - Y) = 0.042 \qquad \text{Saving} = \frac{0.042}{0.01} 14000 = 58800$$

d) Estimate the proportion of boxes that will now weigh less than 6 pounds.

Answer: Since $Y \sim N(6.048, 0.022^2)$

$$\hat{p} = P(Y < 6) = P(Z < \frac{6 - 6.048}{0.022}) = P(Z < -2.18) = 1 - \Phi(2.18) = 0.0146$$

7.4.7 For a public opinion poll for a close presidential election, let p denote the proportion of voters who favor candidate A. How large a sample should be taken if we want the maximum error of the estimate of p to be equal to

a) 0.03 with 95% confidence?

Answer: $\alpha = 0.05,\ \varepsilon = 0.03$, thus we need 1068 samples.

$$n = {Z_{0.025}}^2 \times \frac{\hat{p}(1 - \hat{p})}{\varepsilon^2} \geq 0.5(1 - 0.5) \frac{1.96^2}{0.03^2} = 1067.1$$

b) 0.02 with 95% confidence?

Answer: $\alpha = 0.05,\ \varepsilon = 0.02$. Similarly, we need 2401 samples.

$$n = {Z_{0.025}}^2 \times \frac{\hat{p}(1-\hat{p})}{\varepsilon^2} \geq 0.5(1-0.5)\frac{1.96^2}{0.02^2} = 2401$$

c) 0.03 with 90% confidence?

Answer: $\alpha = 0.10,\ \varepsilon = 0.03$, so we need at least 752 samples.

$$n = {Z_{0.05}}^2 \times \frac{\hat{p}(1-\hat{p})}{\varepsilon^2} \geq 0.5(1-0.5)\frac{1.645^2}{0.03^2} = 751.7$$

7.4.8 Some college professors and students examined 137 Canadian geese for patent schistosome in the year they hatched. Of these 137 birds, 54 were infected. The professors and students were interested in estimating p, the proportion of infected birds of this type. For future studies, determine the sample size n so that the estimate of p is within $\varepsilon = 0.04$ of the unknown p with 90% confidence.

Answer: $\alpha = 0.10,\ \varepsilon = 0.04,\ \hat{p} = {}^{54}/_{137}$

$$n = {Z_{0.05}}^2 \times \frac{\hat{p}(1-\hat{p})}{\varepsilon^2} = \frac{1.645^2}{0.04^2} \times \frac{54}{137} \times \frac{83}{137} = 403.87$$

The sample size at least needs to be 404.

6.5.3 The midterm and final exam scores of 10 students in a statistics course are tabulated as shown.

Midterm	Final	Midterm	Final
70	87	67	73
74	79	70	83
80	88	64	79
84	98	74	91
80	96	82	94

a) Calculate the least squares regression line for these data.

Answer: Let X_i denote midterm score, Y_i denote final score.

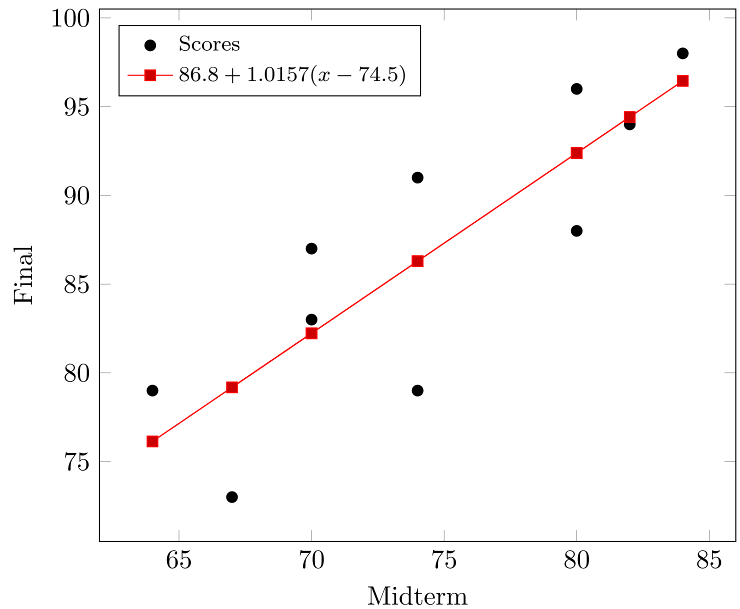
$$\alpha = \bar{y} = \frac{\sum Y_i}{10} = 86.8$$

$$\beta = \frac{\sum x_i y_i - n\bar{x}\bar{y}}{\sum {x_i}^2 - n\bar{x}^2} = \frac{65087 - 10 \times 74.5 \times 86.8}{55917 - 10(74.5)^2} = \frac{421}{414.5} = 1.0157$$

Thus the regression line is $\hat{y}_i = 86.8 + 1.0157(x_i - 74.5)$

b) Plot the points and the least squares regression line on the same graph

Answer: Here shows the points (x = midterm, y = final) and the regression line



c) Find the value of $\hat{\sigma^2}$

Answer: $\hat{\varepsilon_i} = y_i - \hat{y_i},\ i = 1, ..., 10$

$$\hat{\sigma^2} = \frac{1}{10}\sum_{i=1}^{10}\hat{\varepsilon_i}^2 = 18.00$$

6.5.5 A student was interested in how the horsepower and weight of a car affected the time that it takes the car to go from 0 to 60 mph. The following table gives, for each of 14 cars, the horsepower, the time in seconds to go from 0 to 60 mph, and the weight in pounds:

Horsepower	0-60	Weight	Horsepower	0-60	Weight
230	8.1	3516	282	6.2	3627
225	7.8	3690	300	6.4	3892
375	4.7	2976	220	7.7	3377
322	6.6	4215	250	7.0	3625
190	8.4	3761	315	5.3	3230
150	8.4	2940	200	6.2	2657
178	7.2	2818	300	5.5	3518

a) Calculate the least squares regression line for "0-60" versus horsepower.

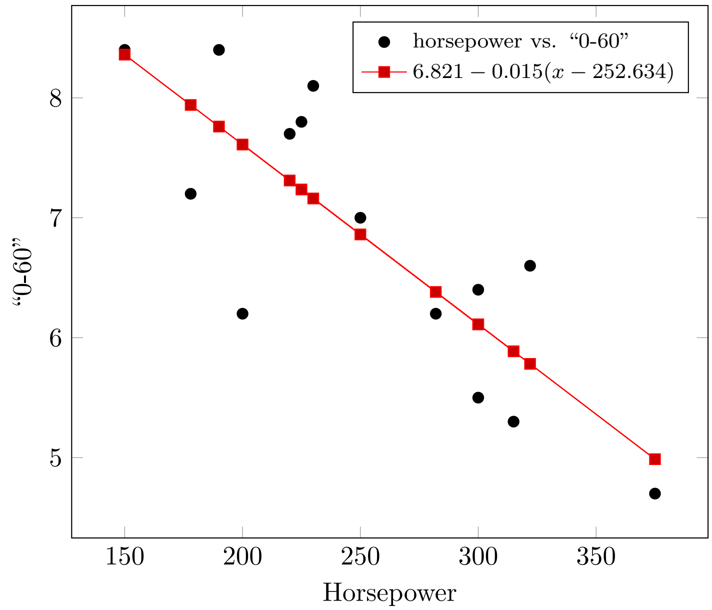
Answer: Let X_i denote horsepower, Y_i denote acceleration time.

$$\hat{\alpha} = \bar{y} = \frac{\sum Y_i}{14} = 6.821$$

$$\hat{\beta} = \frac{\sum x_i y_i - 14\bar{x}\bar{y}}{\sum {x_i}^2 - 14\bar{x}^2} = \frac{23315.2 - 24127.393}{947767 - 893597.786} = -0.015$$

Thus the regression line for "0-60" vs. horsepower is $\hat{y_i} = 6.821 - 0.015(x_i - 252.634)$

b) Plot the points and the least squares regression line on the same graph.



c) Calculate the least squares regression line for "0-60" versus weight.

Answer: Let X_i denote weight, Y_i denote acceleration time.

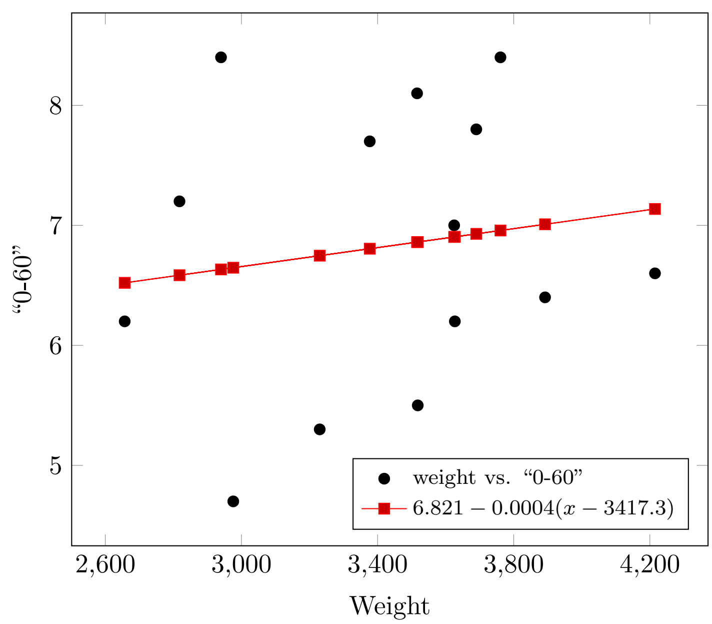
$$\hat{\alpha} = \bar{y} = \frac{\sum Y_i}{14} = 6.821$$

$$\hat{\beta} = \frac{\sum x_i y_i - 14\bar{x}\bar{y}}{\sum {x_i}^2 - 14\bar{x}^2} = \frac{327361.3 - 326350.7857}{166047422 - 163489783.1} = 0.000395$$

Correction

Thus the regression line for "0-60" vs. weight is $\hat{y}_i = 6.821 + 0.000395(x_i - 3417.2857)$

d) Plot the points and the least squares regression line on the same graph.



e) Which of the two variables, horsepower or weight, has the most effect on the "0-60" time?

Answer: The 95% C.I. for the slope of horsepower is $[-0.0216, -0.00843]$ and that of weight is $[-0.001243, 0.0002033]$. By taking absolute value, we have two range $[0.00843, 0.0216]$ and $[0, 0.002033]$, respectively. we can conclude that horsepower ($\beta = -0.015$) has more effect on "0-60" time than weight ($\beta = 0.000395$).