Quiz 2 (Section 2)

Started: Mar 3 at 4:53pm

Quiz Instructions

Question 1

1 pts

Let G^R be the reverse graph of directed graph G. Which one of the following is NOT true?

- ○ If G is a DAG then G^R is also a DAG.
- ● If u can reach v in G^R and v can reach u in G, then G^R is not a DAG.
- ○ The meta-graph of G has the same number of edges with the meta-graph of G^R.
- ○ The reverse graph of G^R is G.

Question 2 **1 pts**

How many connected components in the undirected graph below given by its adjacency matrix?

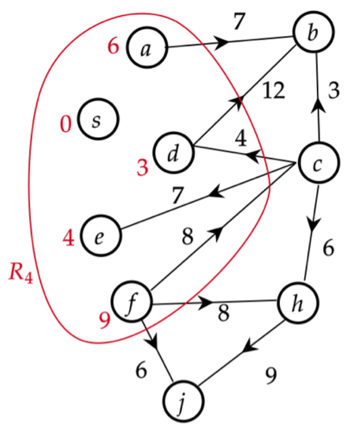
$$\begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

- ○ 2
- ○ 1
- ○ 4
- ◉ 3

Question 3

1 pts

We are in the middle of running Dijkstra’s algorithm on the graph given below starting from s: the first 4 vertices that are closest to s are marked as R_4; their distance from s is marked as red numbers. Which one will be the 5-th closest vertex from s?



- [x] vertex b
- [ ] vertex j
- [ ] vertex c
- [ ] vertex h

Question 4

1 pts

If G has n vertices, how many edges can G have at most assuming G is a DAG? Choose the most accurate number.

- [x] $n(n-1)/2$
- [ ] $n(n-1)$
- [ ] $n^2/2$
- [ ] n^2

Question 5

1 pts

Let G be a directed graph with positive edge length and let p be one shortest path from u to v. (A): If we increase the length of every edge by 2, then p is still one shortest path from u to v. (B): If we divide the length of every edge by 2, then p is still one shortest path from u to v.

- ○ (A) is true and (B) is true.
- ◉ (A) is false and (B) is true.
- ○ (A) is true and (B) is false.
- ○ (A) is false and (B) is false.

Question 6

1 pts

Suppose that after running DFS-with-timing on a directed graph G, the [pre, post] values for u and v are $[3, 10]$ and $[4, 5]$ respectively. Which one of the following is true?

- ○ u and v must be in the same connected component.
- ○ u and v must be not in the same connected component.
- ○ v can reach u
- ◉ u can reach v
- ○ none of the others is true.

Question 7

1 pts

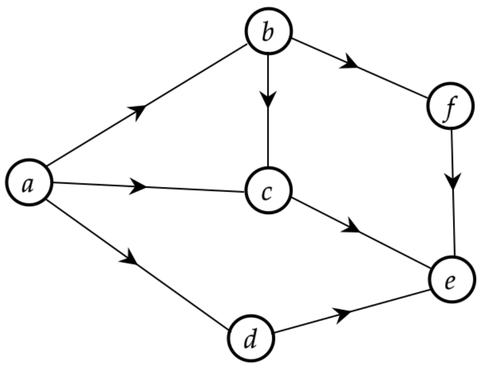
Assume we have a directed graph G. Would the algorithm below give us all connected components correctly? Step 1, run DFS with timing on G to get $postlist$; Step 2, run DFS on G_R with ordering of $postlist$.

- ○ False
- ◉ True

Question 8

1 pts

How many linearization does this graph have?



- 6
- 8
- 10
- 16

Question 9

1 pts

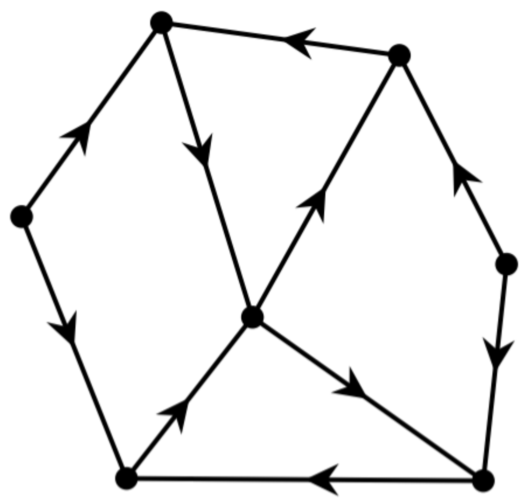
Let G be a directed graph possibly with positive edge length. Let $v_1 \to v_2 \to v_3 \to v_4$ be one shortest path from v_1 to v_4. Let $v_3 \to v_4 \to v_5 \to v_6$ be one shortest path from v_3 to v_6. Which one of the following is NOT true?

- $distance(v_1, v_3) + distance(v_3, v_6) = distance(v_1, v_6)$.
- The path $v_3 \to v_4 \to v_5$ is one shortest path from v_3 to v_5.
- $distance(v_1, v_4) \geq distance(v_2, v_3)$.
- The path $v_2 \to v_3 \to v_4$ is one shortest path from v_2 to v_4.

Question 10

1 pts

How many vertices are in the meta-graph of the following graph?



- ○ 1
- ◉ 3
- ○ 4
- ○ 2

Quiz saved at 4:55pm

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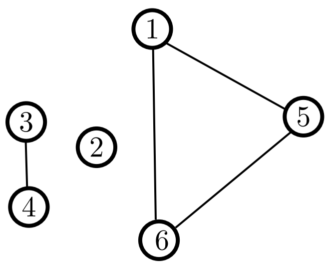
Quiz 2 (Section 2)

1. (1 pts.) In fact, u can reach v in G^R is equivalent to that v can reach u in G. But this does not guarantee that there must be a cycle in G. Other three statements are all true.

2. (1 pts.) The graph is given below, which has 3 connected components.



3. (1 pts.) With respect to R_4, we have $dist[b] = \min\{6+7, 3+12\} = 13$, $dist[c] = 9+8 = 17$, $dist[h] = 9+8 = 17$, $dist[j] = 9+6 = 15$. So vertex b is the 5-th closest vertex from s.

4. (1 pts.) The first vertex can have at most $n-1$ edges to connect with all other vertices. The second vertex can have at most $n-2$ edge to connect with all other vertices besides the first vertex. Following this way of building G, the second last vertex can have at most 1 edge to connect with the last vertex, and the last vertex can't have any edge. As a result, G can have at most overall $\sum_{i=1}^{n-1} i = n \cdot (n-1)/2$ edges.

5. (1 pts.) Statement (A) is false. The key is that the number of edges in shortest paths may be different. Counter-example: $p = \{e_1 = 1, e_2 = 1\}$ is the shortest path from u to v, and we have another path $p' = \{e_3 = 3\}$ from u to v. If we increase the length of every edge by 2, the length of p becomes 6, the length of p' becomes 5, then p is no longer the shortest path from u to v.

Statement (B) is true. The key is that the length of *every* path is doubled. So their relationship remains.

6. (1 pts.) The $[4, 5]$ for v is contained in interval $[3, 10]$ for u. In other words, v is explored within exploring u. Therefore, u can reach v. Other statements are all false.

7. (1 pts.) We know that G and G_R have the same collection of connected components. The given algorithm is to find connected component of G_R.

8. (1 pts.) Notice that there is a path $a \to b \to f \to e$. So the relative positions of these 4 vertices are fixed. Vertex c can be either between b and f, or between f and e; in either case, which gives a list of 5 vertices, d can be placed in any of the 4 spaces in between. So, the total number of distinct linearization is 8.

9. (1 pts.) (a), This statement is false: we have that $v_1 \to v_2 \to v_3$ is one shortest path from v_1 to v_3 and that $v_3 \to v_4 \to v_5 \to v_6$ is one shortest path from v_3 to v_6, but these two does not imply that $v_1 \to v_2 \to v_3 \to v_4 \to v_5 \to v_6$ is one shortest path from v_1 to v_6.

(b), According to the optimal substructure property, since $v_3 \to v_4 \to v_5 \to v_6$ is one shortest path from v_3 to v_6, we have that $v_3 \to v_4 \to v_5$ is one shortest path from v_3 to v_5.

(c), The statement that $distance(v_1, v_4) \geq distance(v_2, v_3)$ is also correct, as all edge lengths are positive.

(d), According to the optimal substructure property, since $v_1 \to v_2 \to v_3 \to v_4$ is one shortest path from v_1 to v_4, we know that $v_2 \to v_3 \to v_4$ is one shortest path from v_2 to v_4.

10. (1 pts.) Three connected components in the graph, corresponding to three vertices in the meta-graph.