CMPSC 465
Data Structures and Algorithms
Spring 2022

Instructor: Chunhao Wang

Greedy algorithms

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Huffman Encoding (Textbook Section 5.2)

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Example: ASCII encoding $\begin{array}{cc} a & 01100001 \\ b & 01100010 \\ \vdots & \vdots \end{array}$

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Definition

An encoding is **prefix-free** if no codeword is a prefix of any other codewords

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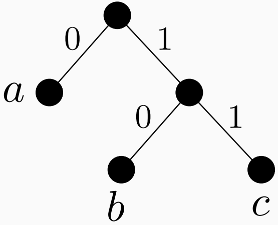

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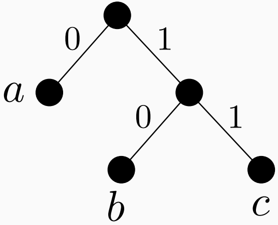

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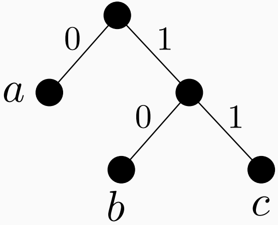

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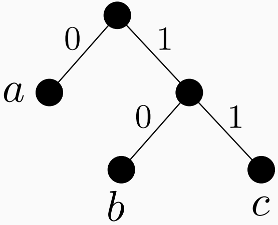

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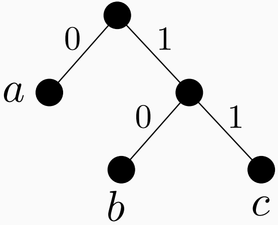

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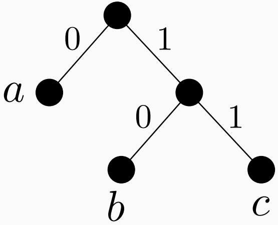

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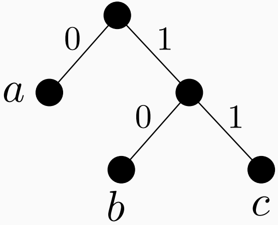

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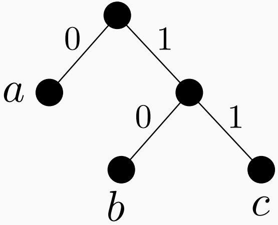

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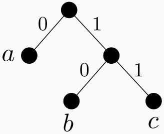

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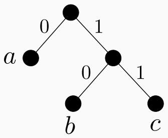

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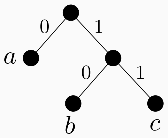

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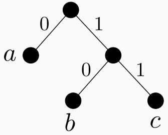

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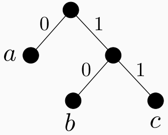

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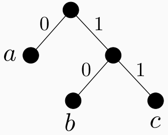

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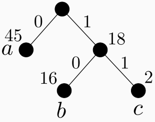

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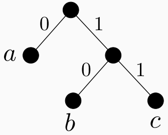

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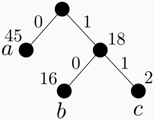

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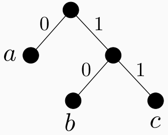

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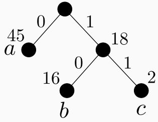

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Constructing the prefix-free encoding tree

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Greedy approach: continually merge least frequent symbols/nodes until you have a full binary tree encoding all symbols

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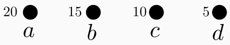

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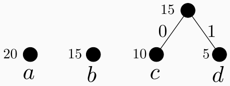

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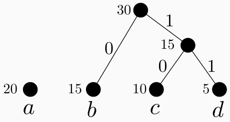

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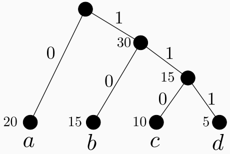

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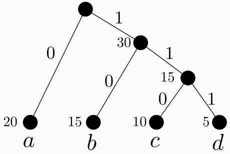

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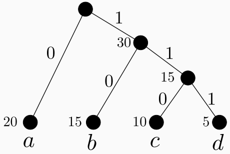

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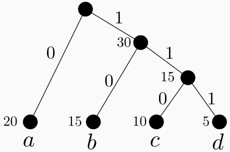

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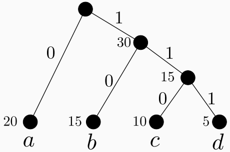

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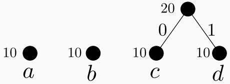

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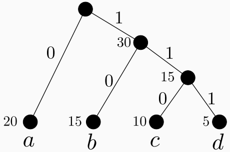

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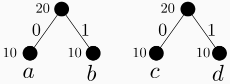

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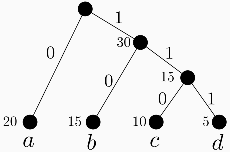

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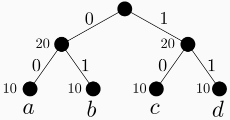

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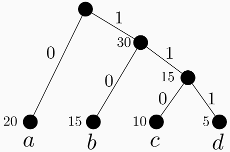

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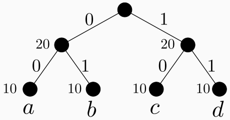

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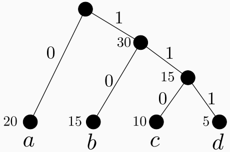
Constructing the prefix-free encoding tree – examples

- $a : 20, b : 15, c : 10, d : 5$



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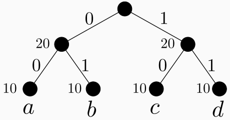

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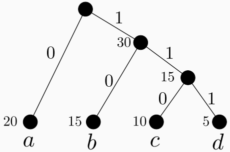

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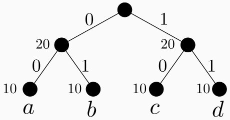

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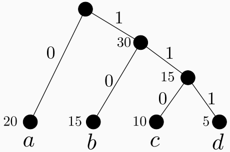

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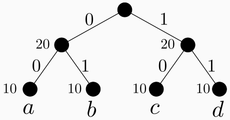

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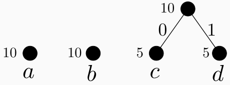

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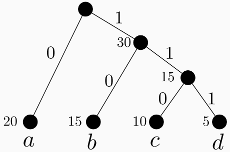

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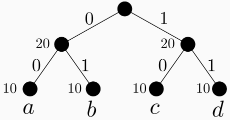

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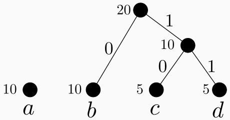

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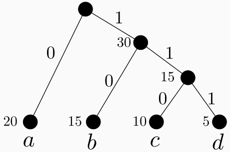

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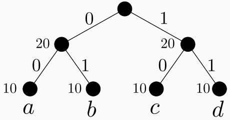

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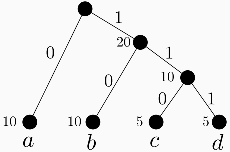

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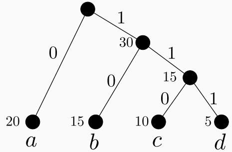

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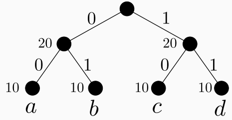

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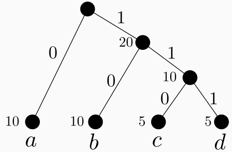

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Proof sketch

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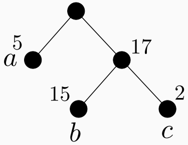

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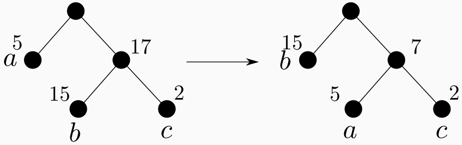

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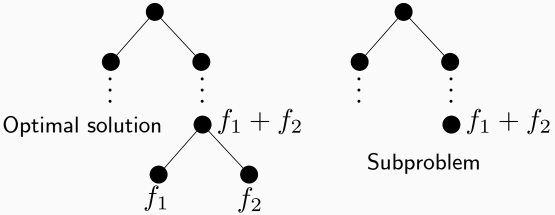

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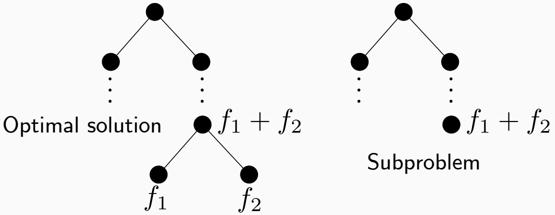

Thus, the greedy solution will lead to the global optimal solution

Pseudocode

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def HUFFMAN(f): // f : f[1], ..., f[n]; Γ has n symbols
    T: empty tree;
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Binary heap: insert $O(\log n)$, extract_min: $O(\log n)$

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8          j := extract_min(H);
9          Creat a node k in T with children i and j;
10         f[k] := f[i] + f[j];
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Lines 4-5: $O(n \log n)$;

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Lines 4-5: $O(n \log n)$; Lines 6-11: $O(n \log n)$

Total cost: $O(n \log n)$

More about the pseudocode

Question: why $2n-1$ in line 6?

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Answer: if a full binary tree has n leaves, then it has $2n - 1$ total nodes

More examples

$a : 5, b : 5, c : 7, d : 10, e : 15, f : 17$

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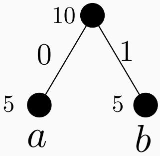

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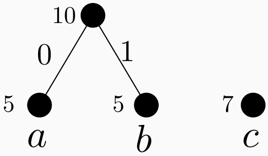

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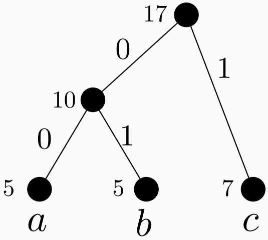

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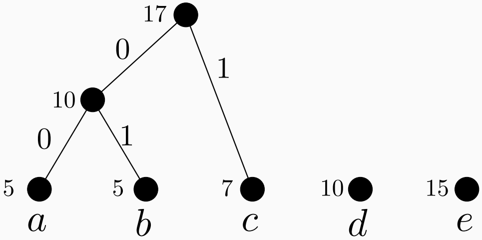

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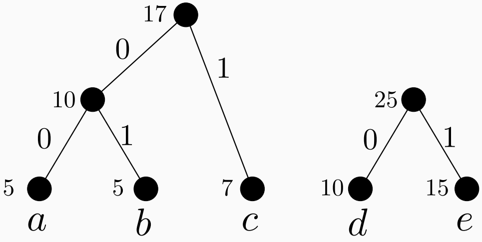

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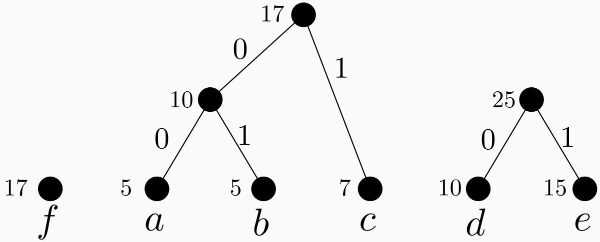

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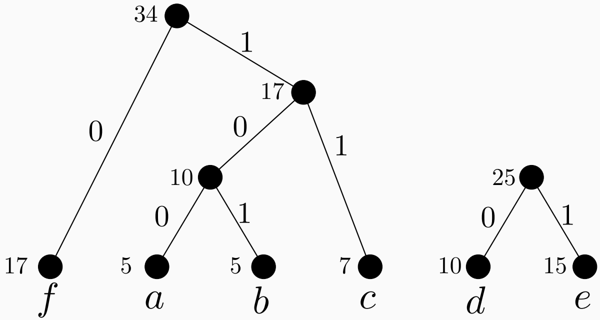

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