HW3 revision

score: 17/20

- 2 Quesgions.
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P1 4/5

P2 3/5

Math 486

Lesson 3 Homework

Due Tues, June 7 at 11:59 on Gradescope

Instructions

Please refer to the solution guidelines posted on Canvas under Course Essentials.

Exercise 1.

In this exercise, we consider a three player game in normal form where each player has two strategies A and B (for each player i, $S_i = \{A, B\}$).

We can represent such a game using two payoff matrices, one for when player 3 plays A, and the second for when player 3 plays B. These payoff matrices are shown below. Payoffs are listed in the usual order (Player 1, Player 2, Player 3).

Analyze this game using iterated elimination of strictly dominated strategies. Be sure to state the order in which strategies are eliminated. If you find a dominant strategy equilibrium, state what it is.

Player 3 plays A		**Player 2** A	B
Player 1	A	$20, 10, 15$	$5, 30, 20$
	B	$10, 15, 10$	$0, 10, 30$

Player 3 plays B		**Player 2** A	B
Player 1	A	$20, 20, 10$	$25, 15, 15$
	B	$15, 20, 5$	$30, 25, 20$

for player 3, B is strictly dominated by A, so eliminate player 3 plays A.

$$\begin{pmatrix} 15 & 20 \\ 10 & 30 \end{pmatrix} > \begin{pmatrix} 10 & 15 \\ 5 & 20 \end{pmatrix}$$

Player 3 plays A		**Player 2** A	B
Player 1	A	$20, \cancel{10}, 15$	$5, 30, 20$
	B	~~$10, 15, 10$~~	~~$0, 10, 30$~~

for player 1, B is strictly dominated by A

$(20 \quad 5) > (10 \quad 0)$

last, for player 2, A is dominated by B, so $(5, 30, 20)$ is left as dominant strategy equilibrium.

① player 3 → ② player 1 → ③ player 2

$30 > 10$

X means either A or B will not effect.

for player 3, $\pi_3(X, X, A) > \pi_3(X, X, B)$

for player 1, $\pi_1(A, X, A) > \pi_1(B, X, A)$

for player 2, $\pi_2(A, B, A) > \pi_2(A, A, A)$

So strategy profiles that will work is (A, B, A)

Problem 2.

Two competing banks each plan to open a branch along the main avenue of a particular city. The avenue has 99 blocks and each bank can open their new branch in any one of the 99 blocks. It is possible that they could each choose the same block.

We will make the following assumptions:

- Each of the 99 blocks is home to exactly 100 residents and each resident will become a customer of one of the two bank branches.
- The 99 blocks can be represented as regions along a line:

1	2	3	4	5	6	$\cdots$	99

- Each resident will become a customer of the bank that is closest to the block where they live.
- If a particular block is equidistant from both banks, we will assume half the residents from that block become customers of one bank, and half become customers of the second bank.

  Example: If Bank A is in block 20 and Bank B is in block 60, then all the residents from blocks 1–39 will become customers of bank A, all of the residents from block 41–99 will become customers of bank B, and the residents of block 40 will be divided between the two banks. Bank A will have 3950 new customers and Bank B will have 5950 new customers.
- We assume that the payoff to each bank is the total number of new customers.
- The players A and B will simultaneously choose where to build their respective banks. Neither knows in advance where the other is choosing to locate.
- Each player has the strategy set $S_i = \{1, 2, \ldots, 99\}$, denoting their location choices.

Remark: In solving this problem, please **do not write out** a 99×99 matrix game.

(a) Argue that for A the strategy $s_A = 2$ is not dominated by $s_A = 3$. Recalling the definition of a dominated strategy, you need to show that there is at least one choice for player B such that player A does better by choosing $s_A = 2$ instead of $s_A = 3$.

Remark: It doesn't matter that we have chosen A. The problem is symmetric and the same result will hold for B.

(b) Argue that for either player, $s_i = 1$ is strictly dominated by $s_i = 2$ and $s_i = 99$ is strictly dominated by $s_i = 98$.

Remark: Make the argument for either A or B. Because of the symmetry of the game, there is no need to show the argument for both players. A straight-forward way to make the argument that $s_i = 1$ is strictly dominated by $s_i = 2$ is to consider 3 separate cases. Show that $s_i = 2$ is better for player i than $s_i = 1$:

(i) when $s_j = 1$,

(ii) when $s_j = 2$,

(iii) and when $s_j > 2$.

Here i stands for one player (either A or B) and j stands for the other player.

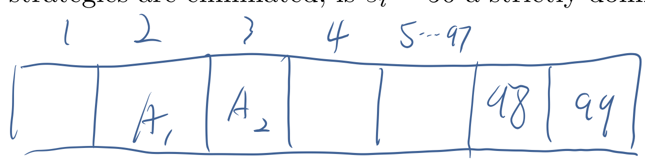
(c) Eliminate the strictly dominated strategies 1 and 99. Briefly explain why, in this reduced game, $s_i = 2$ is strictly dominated by $s_i = 3$ and $s_i = 98$ is strictly dominated by $s_i = 97$.

(d) We can continue this process of iterated elimination of strictly dominated strategies, eventually arriving at the reduced game where $S_i = \{49, 50, 51\}$. Check that in this reduced game, $s_i = 49$ and $s_i = 51$ are both strictly dominated by $s_i = 50$. Note that although we have eliminated all other locations as strategy choices for each bank, we have not eliminated the locations themselves. All of those customers still exist!

Remark: I am not asking you to show each step of this iterated elimination process. The reasoning for each step is identical to the argument that shows $s_i = 1$ is strictly dominated by $s_i = 2$ in the original game.

(e) Based on the results from part (d), is $(50, 50)$ a dominant strategy equilibrium? Briefly explain.

(f) In the original game, before any strategies are eliminated, is $s_i = 50$ a strictly dominant strategy for either player? Explain.



if $S_A = 2$
and $S_B = 1$

payoff will be $(9800, 100)$

and if $S_A = 3$
$S_B = 1$

payoff will be $(9750, 150)$

$9750 < 9800$

so $S_A = 2$ is not strictly dominated by $S_A = 3$

b)

$S_B = 1$

$\pi_A(1,1) < \pi_A(2,1)$

$4950 < 9800$

$S_B = 2$

$\pi_A(1,2) < \pi_A(2,2)$

$100 < 4950$

$S_B \geq 3$

$\pi_A(1, S_B) < \pi_A(2, S_B)$

So $S_B = 1$ is strictly dominated by $S_B = 2$

With same reason, $S_B = 99$ is strictly dominated by $S_B = 98$

as we do for $S_B = 2$, we can show that $S_B = 2$ is strictly dominated by $S_B = 3$ and etc.

$\pi_A(2, S_B) < \pi_A(3, S_B)$

d

	$s_j=49$	$s_j=50$	$s_j=51$
$s_i=49$	4950, 4950	4900, 5000	4950, 4950
$s_i=50$	5000, 4900	4950, 4950	5000, 4900
$s_i=51$	4950, 4950	4900, 5000	4950, 4950

So $s_i=49, 51$ is strictly dominated by $s_i=50$

e

Yes, (50, 50) is a dominate strategy equilibrium.

Since after elimation, $s_i=50$ will be left over.

f No, $s_i=50$ will not be a strictly dominated strategy, since if $s_j=1$, and $s_i=2$ will be a better payoff

Exercise 2.

Use iterated elimination of **weakly dominated strategies** to reduce the following games to smaller games in which iterated elimination of dominated strategies can no longer be used. State the order in which you eliminate strategies, and, for each strategy that you eliminate, state which remaining strategy dominates it. If you find a dominant strategy equilibrium, state what it is.

Game (a)		Player 2		
		x	y	z
Player 1	a	$10, 30$	$5, 25$	$25, 50$
	b	$-20, 20$	$20, 20$	$20, 30$
	c	$15, 40$	$20, 40$	$10, 30$

Game (b)		Player 2		
		x	y	z
Player 1	a	$1, 4$	$0, 6$	$-1, 5$
	b	$3, 3$	$-1, 3$	$1, 2$
	c	$3, 1$	$0, 2$	$2, 1$

Game (a)		Player 2		
		x	y	z
Player 1	a	$10, 30$	$\cancel{5, 25}$	$25, 50$
	b	$\cancel{-20, 20}$	$\cancel{20, 20}$	$\cancel{20, 30}$
	c	$15, 40$	$\cancel{20, 40}$	$10, 30$

① for player 2, y is weakly dominated by x

② for player 1, b is strictly dominated by a

player 1		x	z
	a	10, 30	25, 50
	c	15, 40	10, 30

Game (b)		Player 2		
		x	y	z
Player 1	a	$\cancel{1, 4}$	$\cancel{0, 6}$	$\cancel{-1, 5}$
	b	$\cancel{3, 3}$	$\cancel{-1, 3}$	$\cancel{1, 2}$
	c	$\cancel{3, 1}$	$0, 2$	$\cancel{2, 1}$

① for player 2, x is weakly dominated by y

② for player 1, a is strictly dominated by c

③ for player 2, z is weakly dominated by y

④ for player 1, b is dominated by a.

after elimation, player 1

	player 2
	Y
c	0, 2

(0, 2)

Problem 1.

Prior to the British agricultural revolution in the 17th century, most agriculture in Britain occurred on common land.

Consider a common pasture where N individuals graze their cows. The pasture can productively support kN cows. If the total number of cows exceeds kN, the pasture becomes degraded supporting fewer cows.

Suppose that each individual currently grazes k cows. We will assume that each individual has two strategies available

- The responsible strategy (R): continue grazing k cows.
- The irresponsible strategy (I): graze $k+1$ cows.

Each cow in the pasture above the kN limit imposes a cost $c > 0$ to the community in terms of degradation. This cost is shared equally by all individual. Thus, for every cow above kN in the pasture, each individual experiences a cost of $\frac{c}{N}$.

Each individual also realizes a profit p for each of their cows that graze in the pasture.

Assume $\frac{c}{N} < p < c$. That is, the cost of grazing an extra cow (to the community as a whole) is greater than the profit from the cow, but each individual's share of the cost is less than the profit.

(a) Let $\pi_i(s_i, m)$ denote the payoff to player i when m *other players* have chosen to graze $k+1$ cows ($m \in \{0, 1, \ldots, N-1\}$). Write out the payoff to player i for each of their own strategy choices, (R and I).

$m \uparrow \quad m \cdot (k+1) \quad \frac{mc}{N}$

(b) Prove that R is strictly dominated by I.

(c) If every player uses their dominant strategy I, what is the payoff to each player? How does this compare with the payoff if every player instead plays their dominated strategy, R?

(d) How does this game compare with the prisoner's dilemma game?

$$\pi_i(s_i, m) = \begin{cases} \boxed{kp - \frac{mc}{N},\ (k+1)p - \frac{mc}{N}} & \text{if } i \text{ chosing } R \\ \boxed{(k+1)p - \frac{(m+1)c}{N},\ (k+1)p - \frac{(m+1)c}{N}} & \text{if } i \text{ chosing } I \end{cases}$$

since $\frac{c}{N} < p < c$

$m < N$

$$kp = (k+1)p - p \qquad \frac{mc}{N} = \frac{(m+1)c}{N} - \frac{c}{N}$$

$$(k+1)p - kp = p \quad > \quad \frac{(m+1)c}{N} - \frac{mc}{N} = \frac{c}{N}$$

$$\frac{c}{N} < p$$

$$\text{So } (k+1)p - \frac{(m+1)c}{N} > kp - \frac{mc}{N}$$

So R is strictly dominated by I

$(m = n-1)$

if every player choose I, pay off will be $(k+1)p - c$

and since $kp + p - c < kp$ so they will have less

$(p < c)$

profit then orginal kp profit.

it is similar to prisoner's dilemma game,

Defect is I and Cooperate is R

player \ other player	Coop/R	Def/I
Coop/R	0, 0	loss, gain
Def/I	gain, loss	everyone lose

there exist an outcome is better for every player, but if everyone follow strategy, always prefered lead to a worse result for every one.