解释

问题

Find the least squares polynomials of degrees 1,2, and 3 for the data in the following table. Compute the error E in each case. Graph the data and the polynomials.

x_i	1.0	1.1	1.3	1.5	1.9	2.1
y_i	1.84	1.96	2.21	2.45	2.94	3.18

解释

已验证

步骤1

步骤1/6

From the given data, we compute

i	1	2	3	4	5	6	$\sum_{i=1}^{6}$
x_i	1.0	1.1	1.3	1.5	1.9	2.1	8.9
x_i^2	1.0	1.21	1.69	2.25	3.61	4.41	14.17
x_i^3	1.0	1.331	2.197	3.375	6.859	9.261	24.023
x_i^4	1.0	1.464	2.856	5.062	13.032	19.448	42.863
x_i^5	1.0	1.611	3.713	7.594	24.761	40.841	79.519
x_i^6	1.0	1.772	4.827	11.391	47.046	85.766	151.801
y_i	1.84	1.96	2.21	2.45	2.94	3.18	14.58
$y_i x_i$	1.84	2.156	2.873	3.675	5.586	6.678	22.808
$y_i x_i^2$	1.84	2.372	3.735	5.512	10.613	14.024	38.096
$y_i x_i^3$	1.84	2.609	4.855	8.269	20.165	29.45	67.188

步骤2

Let $P_1(x) = a_0 + a_1 x$ be the linear least square polynomial. Then, multiplying by x_i and summing, we get

$$\sum_{i=1}^{6} y_i = \sum_{i=1}^{6} a_0 + a_1 \sum_{i=1}^{6} x_i$$
$$\sum_{i=1}^{6} x_i y_i = a_0 \sum_{i=1}^{6} x_i + a_1 \sum_{i=1}^{6} x_i^2$$

This gives

$$14.58 = 6a_0 + 8.9a_1$$
$$22.808 = 8.9a_0 + 14.17a_1$$

Upon solving this, we get $a_0 = 0.6208951, a_1 = 1.21962$. This gives us the linear least square polynomial

$$P_1(x) = 0.6208951 + 1.21962x$$

步骤3

Let $P_2(x) = a_0 + a_1 x + a_2 x^2$ be the quadratic least square polynomial. Then, multiplying by x_i and summing, we get

$$\sum_{i=1}^{6} y_i = \sum_{i=1}^{6} a_0 + a_1 \sum_{i=1}^{6} x_i + a_2 \sum_{i=1}^{6} x_i^2$$
$$\sum_{i=1}^{6} x_i y_i = a_0 \sum_{i=1}^{6} x_i + a_1 \sum_{i=1}^{6} x_i^2 + a_2 \sum_{i=1}^{6} x_i^3$$
$$\sum_{i=1}^{6} x_i^2 y_i = a_0 \sum_{i=1}^{6} x_i^2 + a_1 \sum_{i=1}^{6} x_i^3 + a_2 \sum_{i=1}^{6} x_i^4$$

This gives

$$6\,a_0 + 8.9\,a_1 + 14.17\,a_2 = 14.58$$
$$8.9\,a_0 + 14.17\,a_1 + 24.023\,a_2 = 22.808$$
$$14.17\,a_0 + 24.023\,a_1 + 42.8629\,a_2 = 38.0962$$

Upon solving this, we get $a_0 = 0.596581, a_1 = 1.25329$, and $a_2 = -0.0108534$. This gives us the quadratic least square polynomial

$$P_2(x) = 0.596581 + 1.25329x - 0.0108534x^2$$

步骤4

Let $P_3(x) = a_0 + a_1x + a_2x^2 + a_3x^3$ be the cubic least square polynomial. Then, multiplying by x_i and summing, we get

$$\sum_{i=1}^{6} y_i = \sum_{i=1}^{6} a_0 + a_1 \sum_{i=1}^{6} x_i + a_2 \sum_{i=1}^{6} x_i^2 + a_3 \sum_{i=1}^{6} x_i^3$$

$$\sum_{i=1}^{6} x_i y_i = a_0 \sum_{i=1}^{6} x_i + a_1 \sum_{i=1}^{6} x_i^2 + a_2 \sum_{i=1}^{6} x_i^3 + a_3 \sum_{i=1}^{6} x_i^4$$

$$\sum_{i=1}^{6} x_i^2 y_i = a_0 \sum_{i=1}^{6} x_i^2 + a_1 \sum_{i=1}^{6} x_i^3 + a_2 \sum_{i=1}^{6} x_i^4 + a_3 \sum_{i=1}^{6} x_i^5$$

$$\sum_{i=1}^{6} x_i^3 y_i = a_0 \sum_{i=1}^{6} x_i^3 + a_1 \sum_{i=1}^{6} x_i^4 + a_2 \sum_{i=1}^{6} x_i^5 + a_3 \sum_{i=1}^{6} x_i^6$$

This gives

$$\begin{aligned} 6\,a_0 + 8.9\,a_1 + 14.17\,a_2 + 24.023\,a_3 &= 14.58, \\ 8.9a_0 + 14.17\,a_1 + 24.023\,a_2 + 42.863\,a_3 &= 22.808, \\ 14.17a_0 + 24.023\,a_1 + 42.863\,a_2 + 79.52\,a_3 &= 38.096 \\ 24.023\,a_0 + 42.8629\,a_1 + 79.52\,a_2 + 151.801\,a_3 &= 67.19 \end{aligned}$$

Upon solving this, we get $a_0 = 0.629019, a_1 = 1.18501, a_2 = 0.0353325, a_3 = -0.0100472$. This gives us the cubic least square polynomial

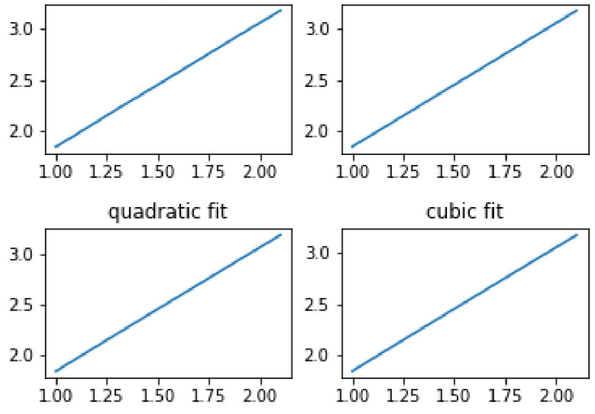
$$P_3(x) = 0.629019 + 1.18501x + 0.0353325x^2 - 0.0100472x^3$$

步骤5

For error calculation, we get

i	1	2	3	4	5	6	Total
x_i	1.0	1.1	1.3	1.5	1.9	2.1	
y_i	1.84	1.96	2.21	2.45	2.94	3.18	
$P_1(x_i)$	1.841	1.962	2.206	2.45	2.938	3.182	
$y_i - P_1(x_i)$	-0.001	-0.002	0.004	0.0	0.002	-0.002	
$(y_i - P_1(x_i))^2$	1e-06	4e-06	1.6e-05	0.0	4e-06	4e-06	2.9e-05
$P_2(x_i)$	1.839	1.962	2.208	2.452	2.939	3.181	
$y_i - P_2(x_i)$	0.001	-0.002	0.002	-0.002	0.001	-0.001	
$(y_i - P_2(x_i))^2$	1e-06	4e-06	4e-06	4e-06	1e-06	1e-06	1.5e-05
$P_3(x_i)$	1.839	1.962	2.207	2.452	2.939	3.18	
$y_i - P_3(x_i)$	0.001	-0.002	0.003	-0.002	0.001	0.0	
$(y_i - P_3(x_i))^2$	1e-06	4e-06	9e-06	4e-06	1e-06	0.0	1.9e-05

Plotting, we get the following graph.



结果

步骤6/6

We get,

$$P_1(x) = 0.6208951 + 1.21962x \text{ with } E = 2.9 \times 10^{-5},$$

$$P_2(x) = 0.596581 + 1.25329x - 0.0108534x^2 \text{ with } E = 1.5 \times 10^{-5}, \text{ and}$$
$$P_3(x) = 0.62902 + 1.185x + 0.0353x^2 - 0.01005x^3 \text{ with } E = 1.9 \times 10^{-5}.$$

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