Homework 14

Problem 1. Consider the polynomial interpolation for the following data points

x	0	2	3	4
y	7	11	28	63

Problem 2. The polynomial $p(x) = x^4 - x^3 + x^2 - x + 1$ has the values shown.

x	-2	-1	0	1	2	3
p(x)	31	5	1	1	11	61

Find a polynomial $q(x)$ that takes these values (you don't need expand it):

x	-2	-1	0	1	2	3
q(x)	31	5	1	1	11	30

(Hint: This can be done with little work. Try the Lagrange form.)

Problem 3. Let $P_3(x)$ be the interpolating polynomial for the data $(0,0)$, $(0.5,y)$, $(1,3)$ and $(2,2)$. Find y if the coefficient of x^3 in $P_3(x)$ is 6.

Problem 4. Find a, b and c such that $s(x)$ is a cubic spline, where

$$S(x) = \begin{cases} s_0(x) = 3(x-1) + 2(x-1)^2 - (x-1)^3 & 1 \le x \le 2 \\ s_1(x) = a + b(x-2) + c(x-2)^2 + (x-2)^3 & 2 \le x \le 3 \end{cases}$$

Problem 1. Consider the polynomial interpolation for the following data points

x	0	2	3	4
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$$p(x) = \frac{(x-2)(x-3)(x-4)}{(0-2)(0-3)(0-4)} \cdot 7 + \frac{(x-0)(x-3)(x-4)}{(2-0)(2-3)(2-4)} \cdot 11$$

$$+ \frac{(x-0)(x-2)(x-4)}{(3-0)(3-2)(3-4)} \cdot 28 + \frac{(x-0)(x-2)(x-3)}{(4-0)(4-2)(4-3)} \cdot 63$$

$$= x^3 - 2x + 7$$

Problem 2. The polynomial $p(x) = x^4 - x^3 + x^2 - x + 1$ has the values shown.

x	-2	-1	0	1	2	3
p(x)	31	5	1	1	11	61

Find a polynomial $q(x)$ that takes these values (you don't need expand it):

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q(x)	31	5	1	1	11	30

(Hint: This can be done with little work. Try the Lagrange form.)

$$q(x) = p(x) + C(x+2)(x+1)(x)(x-1)(x-2)$$

$$q(3) = p(3) + C \cdot 120$$

$$q(3) - p(3) = 30 - 61$$

$$C = \frac{-31}{120}$$

$$q(x) = p(x) - \frac{31}{120}(x+2)(x+1)(x)(x-1)(x-2)$$

$$= x^4 - x^3 + x^2 - x + 1 - \frac{31}{120}(x+2)(x+1)(x)(x-1)(x-2)$$

Problem 3. Let $P_3(x)$ be the interpolating polynomial for the data $(0,0)$, $(0.5,y)$, $(1,3)$ and $(2,2)$. Find y if the coefficient of x^3 in $P_3(x)$ is 6.

x	0	0.5	1	2
$f(x)$	0	y	3	2

$$P_3(x) = \frac{(x-0)(x-1)(x-2)}{(0.5-0)(0.5-1)(0.5-2)} \cdot y + \frac{(x-0)(x-0.5)(x-2)}{(1-0)(1-0.5)(1-2)} \cdot 3$$
$$+ \frac{(x-0)(x-0.5)(x-1)}{(2-0)(2-0.5)(2-1)} \cdot 2$$

$$6 = \frac{8y-16}{3}$$

$$y = 4.25$$

Problem 4. Find a, b and c such that $s(x)$ is a cubic spline, where

$$S(x) = \begin{cases} s_0(x) = 3(x-1) + 2(x-1)^2 - (x-1)^3 & 1 \leq x \leq 2 \\ s_1(x) = a + b(x-2) + c(x-2)^2 + (x-2)^3 & 2 \leq x \leq 3 \end{cases}$$

$$S_0(x) = 3x - 3 + 2(x^2 - 2x + 1) - (x+1)(x^2+2x+1)$$

$$= 3x - 3 + 2x^2 - 4x + 2 - x^3 + 3x^2 - 3x + 1$$

$$= -x^3 + 5x^2 - 4x$$

$S_0(2) = 4$ $\quad S_1(2) = a$ $\quad$ so $\quad a = 4$

$S_0'(x) = -3x^2 + 10x - 4$ $\qquad S_0''(2) = -2$

$S_0'(2) = -3 \cdot 4 + 10 \cdot 2 - 4$ $\qquad S_1''(x) = 2C$

$= 4$

$S_1'(2) = b + 2C(x-2) + 3(x-2)^2$

$$\begin{cases} a = 4 \\ b = 4 \\ C = -1 \end{cases}$$