Math 486

Lesson 8 Homework

Due Tues, July 19 at 11:59 on Gradescope

Instructions

Please refer to the solution guidelines posted on Canvas under Course Essentials.

Exercise 1.

Consider the symmetric two-player game shown below. Let $\sigma = \left(\frac{11}{18}, \frac{5}{18}, \frac{2}{18}\right)$. In a previous homework we showed that (σ, σ) is a mixed strategy Nash equilibrium. Since both players use the same mixed strategy in a symmetric game, this is a symmetric mixed strategy Nash equilibrium.

		Player 2		
		a	b	c
Player 1	a	$2, 2$	$6, 3$	$7, 4$
	b	$3, 6$	$3, 3$	$9, 4$
	c	$4, 7$	$4, 9$	$1, 1$

(a) The population state σ is an evolutionarily stable state of the corresponding evolutionary game (I am not asking you to check this). Use Theorem 8.3 from the textbook to show that (σ, σ) is the only symmetric Nash equilibrium of the game.

(b) Explain why the result from part (a) implies that σ is the only evolutionarily stable state for the corresponding evolutionary game.

Remark: No explicit calculations are required for this exercise. This exercise is about understanding and applying the relevant definitions and theorem.

a) No strategy is dominated by other strategy, so all strategy is active.

let $\tau = (q_1, q_2 \cdots, q_n)$

$\pi_1(\sigma, \sigma) = \pi_1(s_1, \sigma) \longrightarrow \pi_1(\sigma, \sigma) = \pi(\tau, \sigma)$

σ is evolutionrality stable, So $\pi_1(\sigma, \tau) > \pi_1(\tau, \tau)$

So (τ, τ) is not NE.

$\pi_1(\sigma, \sigma) \geq \pi(\tau, \tau)$ will show (σ, σ) is Symmetric Mixed strategy NE,

So (σ, σ) is the only NE of the game

b) Since (τ, τ) is Not NE

$\pi_1(\tau, \tau) < \pi_1(b, \tau)$

for all $\tau \neq \sigma$, τ will not be evolutionarily stable, So σ is the only evolutionarily stable for the corresponding evolutionary game.

Exercise 2.

Consider the symmetric 2 player game shown below, with the three parameters w, x, y such that

$$0 < x < y < w$$

		Player 2: a	Player 2: b
Player 1	a	$w-x, w-x$	$w-y, w$
	b	$w, w-y$	$0, 0$

(a) Find any pure strategy Nash equilibria. You can use best responses to find and indicate pure strategy NE in the usual way.

(b) Which strategies used in a pure strategy Nash equilibrium found in part (a) (if any) correspond to evolutionarily stable states? Explain.

(c) Use Theorem 8.5 to find a symmetric mixed strategy Nash equilibrium, (σ, σ). Is σ an evolutionarily stable state of the corresponding evolutionary game? Explain. You can reference Theorem 8.5 in your explanation.

a)

		Player 2: a	Player 2: b
Player 1	a	$w-x, w-x$	$(\underline{w-y}, \underline{w})$
	b	$(\underline{w}, \underline{w-y})$	$0, 0$

2 pure strategy NE (b, a) & (a, b)

b) None, since 2 pure strategy NE (b, a) & (a, b) are not symmetric NE.

c) $\pi_1(a, \sigma) = \pi_1(b, \sigma)$

$\sigma = (p, 1-p)$ $\quad \tau = (q, 1-q)$

$p(w-x) + (1-p)(w-y) = wp$

$p = \frac{y-w}{y-w-x} \quad 1-p = \frac{-x}{y-w-x}$

$\sigma = \left(\frac{y-w}{y-w-x}, \frac{-x}{y-w-x}\right)$

(σ, σ) is a symeric NE

From theorem (σ, σ) is strict NE
and σ is evolutionarily stable state.

$$\pi_1(\sigma, \tau) > \pi_1(\tau, \tau)$$

$$\pi_1(\sigma, \tau - \pi_1(\tau, \tau) > 0$$

$$\pi_1(\sigma, \tau) - \pi_1(\tau, \tau) = -(p-q)^2[y-w-x]$$

$$y < w \qquad y-w-x < 0$$

So $\pi_1(\sigma, \tau) - \pi_1(\tau, \tau) = -(p-q)^2[y-w-x] > 0$

So σ is evolutionarily stable state.

Problem 1.

Consider the following two player symmetric game:

		Hunter 2 a	Hunter 2 b
Hunter 1	a	4,4	0,4
	b	4,0	1,1

(a) Find any pure strategy Nash equilibria. You can use best responses to find and indicate pure strategy NE in the usual way.

(b) Let $\tau = (p, 1-p)$ denote a mixed strategy. Use Theorem 8.1 from the textbook and τ to decide whether either of the pure strategies used in the Nash equilibria from part (a) corresponds to an evolutionarily stable state (show that each pure strategy either satisfies or fails to satisfy the conditions of Theorem 8.1).

Remark: In part (b) you want to consider each pure strategy and then show that the pure strategy either satisfies or fails to satisfy the conditions of Theorem 8.1. Note that to satisfy the conditions, the conditions must hold for all τ, while if the conditions are not satisfied, it is sufficient to find one example τ where one of the conditions fails.

a) pure strategy NE (a,a) (b,b)

$\pi_1(a,a) \geq \pi_1(\tau,a)$ so $\pi_1(a,\tau) \geq \pi_1(\tau,\tau)$

$4 \geq 4$ $\pi_1(a,b)=0 < \pi_1(b,b)=1$

b) $\tau=(p,1-p)$ $\pi_1(a,a) \geq \pi_1(a,\tau)$

$4 \geq 0+4(1-p)$

$4 \geq 4-4p$

So (a,a) is evolutionarily stable state.

$\pi_1(b,b) \geq \pi_1(b,\tau)$

$1 \geq 0+1-p$

$1 \geq 1-p$

So (b,b) is evolutionarily stable state.

Problem 2.

In previous assignments we have considered the hawk-dove game. Here we consider a variation where each player has three strategies. In this variation we assume that players are fighting over valuable territory and one of the players has arrived in the territory first (with equal probability).

There are three strategies available to each player:

- hawk (h): fight aggressively until you prevail or are injured
- dove (d): bluff by displaying, but retreat if the other player begins to fight
- owl (o):
  - if you are in the territory first, play h (defend your territory)
  - if the other player is in the territory first, play d (dove) (you cede the territory to the other player if they fight for it).

We are using the following parameters:

- v: the value of the territory you are fighting over
- w the cost of getting injured in fighting
- We will assume that $v < w$: the cost (if injured) is greater than the benefit (if victorious in the fight).

Our assumptions lead to the follwing payoff matrix:

		Player 2		
		h	d	o
	h	$\left(\frac{v-w}{2}, \frac{v-w}{2}\right)$	$(v, 0)$	$\left(\frac{3v-w}{4}, \frac{v-w}{4}\right)$
Player 1	d	$(0, v)$	$\left(\frac{v}{2}, \frac{v}{2}\right)$	$\left(\frac{v}{4}, \frac{3v}{4}\right)$
	o	$\left(\frac{v-w}{4}, \frac{3v-w}{4}\right)$	$\left(\frac{3v}{4}, \frac{v}{4}\right)$	$\left(\frac{v}{2}, \frac{v}{2}\right)$

(a) Show how the payoffs for the strategy profile (d, o) were calculated.

(b) Use best responses to find all pure strategy Nash equilibria.

(c) For each Nash equilibrium determine whether the equilibrium strategy is an evolutionarily stable state of the corresponding evolutionary game.

$V < W$

a) $\pi_1(d,o) = \frac{1}{2} \cdot \frac{V}{2} = \frac{V}{4}$

$\pi_2(d,o) = \frac{1}{2} \cdot \frac{V}{2} + \frac{1}{2} \cdot V = \frac{3}{4} V$

$\pi(d,o) = \frac{1}{2}\pi(d,d) + \frac{1}{2}\pi(d,h) = \left(\frac{V}{4}, \frac{3}{4}V\right)$

b) (d,h), (h,d) & (o,o) are pure strategy NE.

c (d,h) & (h,d) are not symmetric NE, so not evolutionarily stable stage,

$\pi_1(0,0) = \frac{V}{2}$

$$\pi_1(\tau,0) = \begin{cases} \pi_1(h,o) = \frac{3V-W}{4} < \frac{V}{2} \\ \pi_1(d,o) = \frac{V}{4} < \frac{V}{2} \end{cases}$$

so $\pi_1(0,0) > \pi_1(\tau,0)$

So O is an evolutionarizily stable stage.