CMPSC 465 Spring 2022	Data Structures & Algorithms Chunhao Wang and Mingfu Shao	Final Part 2

Instructions:

- Please log into https://psu.zoom.us/j/93567164986?pwd=L3lXWTVQL28vTExldkc4V2VYZitmQT09 using your PSU credential.
- If you have a question during the exam, you may ask the Instructor privately via Zoom chat.
- Instructor will announce any major corrections vocally over Zoom.
- Write your solutions by hand. Typed solutions will not be accepted. You may handwrite on a tablet as well.
- **At 8:40pm, you must put your pens down. You have until 8:55 to upload your solutions to Gradescope.**
- You must use a scanning app and not just take pictures.

1. **(15+15=30 pts.)** Let $G = (V, E)$ be a graph whose edge weights are positive. A minimum spanning tree of G is $T = (V, E')$; assume G and T are given as adjacency lists. Now, the weight of a particular edge $e^* \in E$ is modified from $w(e^*)$ to a new value $\hat{w}(e^*)$. We wish to quickly update the minimum spanning tree T to reflect this change without recomputing the entire tree from scratch. Consider the following two cases. In each case, give a description of an algorithm for updating T, a proof of correctness, and a runtime analysis for the algorithm. Note that for some of the cases these may be quite brief.

   (a) $e \notin E'$ and $\hat{w}(e) > w(e)$

   (b) $e \notin E'$ and $\hat{w}(e) < w(e)$

2. **(5+2.5+10+5+5+2.5=30 pts.)** You own a food company and want to distribute food trucks along East College Avenue in front of Penn State main campus. There are n blocks in total along East College Avenue and your marketing team reported that if you place a food truck at block i, the expected revenue is $r_i > 0$. The city of State College has a regulation saying that each company can place at most one food truck at each block. In addition, if you placed a food truck at block i, then you are not allowed to place another food truck two blocks to its East or two blocks to its West. The goal is to construct a dynamic programming algorithm to figure out the locations of your food trucks that maximize total revenue. Answer the following questions.

   (a) Define the subproblem as a one-dimensional table.

   (b) Based on the subproblem defined above, what is the maximum total revenue?

   (c) Give a recurrence for computing $R(i)$.

   (d) Give the base cases.

   (e) Define another table that helps you recover the optimal locations.

   (f) What is the running time of this dynamic programming algorithm? Justify briefly.

1. (15+15=30 pts.) Let $G = (V, E)$ be a graph whose edge weights are positive. A minimum spanning tree of G is $T = (V, E')$; assume G and T are given as adjacency lists. Now, the weight of a particular edge $e^* \in E$ is modified from $w(e^*)$ to a new value $\hat{w}(e^*)$. We wish to quickly update the minimum spanning tree T to reflect this change without recomputing the entire tree from scratch. Consider the following two cases. In each case, give a description of an algorithm for updating T, a proof of correctness, and a runtime analysis for the algorithm. Note that for some of the cases these may be quite brief.

(a) $e \notin E'$ and $\hat{w}(e^*) > w(e^*)$

(b) $e \notin E'$ and $\hat{w}(e) < w(e)$

It will still be mst under this circumstance, no change.
So no time analysis.

W(e) is in the tree, so it will leave a cut,
in linear time, we can find lightest edge
across cut, as we include it, we have a
MST

by using cut property, we can check if
it contradict MST

it will be linear time

2. (5+2.5+10+5+5+2.5=30 pts.) You own a food company and want to distribute food trucks along East College Avenue in front of Penn State main campus. There are n blocks in total along East College Avenue and your marketing team reported that if you place a food truck at block i, the expected revenue is $r_i > 0$. The city of State College has a regulation saying that each company can place at most one food truck at each block. In addition, if you placed a food truck at block i, then you are not allowed to place another food truck two blocks to its East or two blocks to its West. The goal is to construct a dynamic programming algorithm to figure out the locations of your food trucks that maximize total revenue. Answer the following questions.

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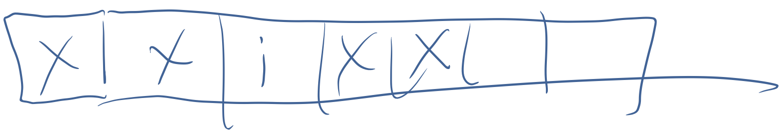

a) it will be a n long list, each list-value will be it's revenue and if n is taken, n-1, n-2, n+1, n+2 can not be distibuted.

b) for this list, we use "task scheduling" We sort the revenue by downward, and check if it is possible, so maxinzed revenue can be gain.

c) ① sort block list by revenue with more revenue on top

② for 1 to n

mark the block and add value to sum if no ±2 block has been marked

if no such block exist, break

end

③ sum will be R(i)

d) base case will be add 1 to n for every 5 block e)

block	6	3	1	7	0	...	...	...	
marked?	✓	✓			✓				
revenue	10	8	7	5	4				

0	1	2	3	4	5	6	7	
4			8			10		...

f) time will be $O(n^2)$

since in loop, we will have to go through every block once.