Quiz 1 (for Section 2)

Started: Feb 4 at 11:52am

Quiz Instructions

Question 1

1 pts

Let $A[1 \cdots n]$ be an array with n distinct integers. Let A' be the sorted array of A in ascending order. We pick a number x from A uniformly at random. What is the probability for event $\{x \geq A'[n/3] \text{ and } x < A'[n/2]\}$?

- $2/3$
- $1/6$
- $1/2$
- $5/6$

Question 2

1 pts

We have 4 vertices of a sqare parallel to the x-axis and y-axis in 2D plane. What shape do the four dual lines make in the dual plane?

- must be a rectangle but not necessarily a square
- must be a sqaure
- none of the others
- must be a parallelogram but not necessarily a rectangle

Question 3

1 pts

Suppose S is an array with n distinct integers. Similar to the selection algorithm, we partition S into $n/21$ subarrays, each of which contains 21 numbers. Let x be the median of medians of the $n/21$ subarrays. How many numbers in S are guaranteed to be less than x?

- ○ $5n/21$
- ○ $13n/42$
- ● $11n/42$
- ○ $2n/7$

Question 4

1 pts

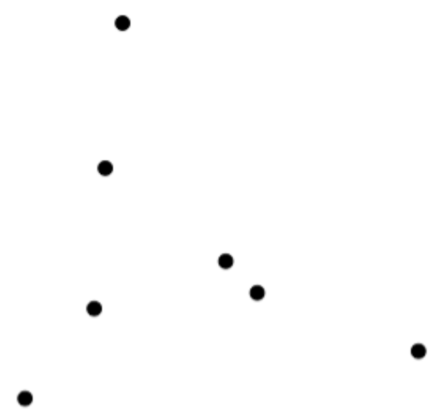
$n^{\sum_{k=1}^{n} 1/k} = \Omega((\log n)^{\log^2 n})$

- ○ True.
- ● False.

Question 5

1 pts

Consider running the Graham-Scan algorithm on the instance given below. How many pop operations will be executed?



- (selected) 4
- 2
- 3
- 5

Question 6

1 pts

For any two positive functions $f(n)$ and $g(n)$ over integers, we always have either $f = \Omega(g)$ or $g = \Omega(f)$.

- True.
- (selected) False.

Question 7 **1 pts**

Suppose you are given the following two algorithms: Algorithm A solves problems of size n by dividing them into 8 subproblems of size $n/4$ recursively solving each subproblem, and then combining the solutions in constant time; Algorithm B solves problems by dividing them into 2 subproblems of size $n/2$, recursively solving each subproblem, and then combining the solutions in linear time. Which of these algorithms has a faster running time?

- (•) B
- ( ) They have the same asymptotic running time
- ( ) A

Question 8 **1 pts**

You are given two sorted arrays $A[1 \cdots n]$ and $B[1 \cdots n]$ both in ascending order; all $2n$ numbers in them are distinct. How fast can you find the median of these $2n$ numbers?

- ( ) $\Theta(\log \log n)$
- ( ) $\Theta(n)$
- ( ) $\Theta(n \log n)$
- (•) $\Theta(\log n)$

Question 9 **1 pts**

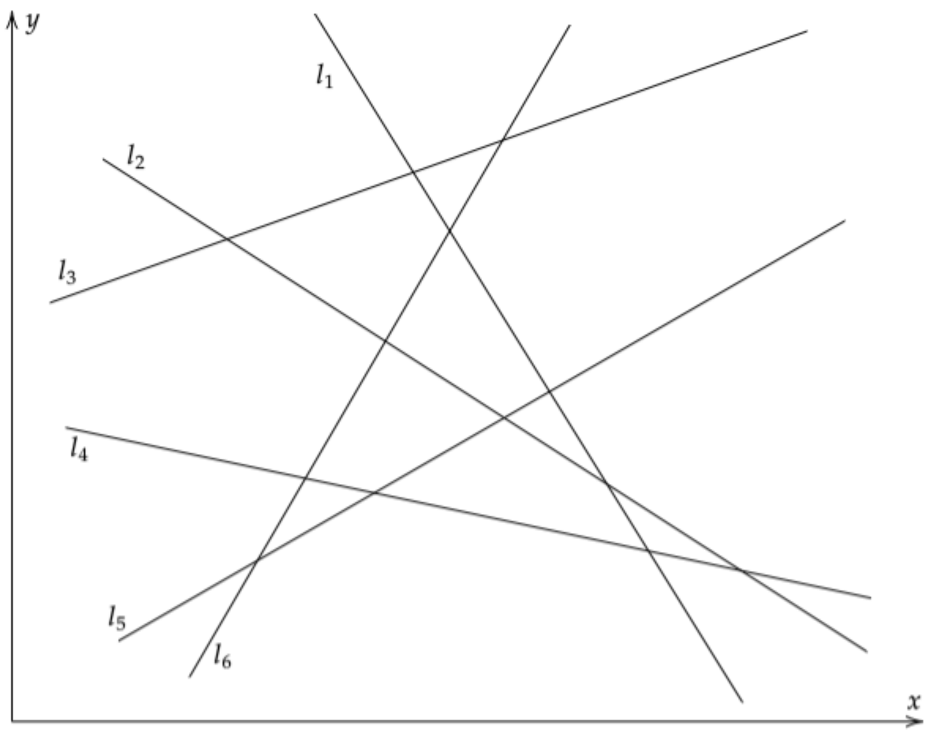
Which of the following is the asymptotic solution of recurrence $T(n) = n + T(n/3) + T(n/4) + T(n/5)$?

- ( ) $n \cdot \log n$
- ( ) n^3
- (•) n
- ( ) n^2

Question 10

1 pts

Consider the dual points of the lines given below: how many vertices will be on the convex hull of these dual points?



- ○ 6
- ○ 4
- ◉ 5
- ○ 3

Quiz saved at 11:53am

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CMPSC 465 | Data Structures & Algorithms
Spring 2022 | Chunhao Wang and Mingfu Shao

Quiz 1 (Section 2)

1. **(1 pts.)** Since x is drawn uniformly at random, we have $P(\{x \leq A'[n/3] \text{ and } x > A'[n/2]\})$ is equal to $(n/2 - n/3)/n = 1/6$.

2. **(1 pts.)** Let $(a, b), (a, d), (c, b), (c, d)$ be 4 vertices of a rectangle. Their duals are $y = ax - b, y = ax - d, y = cx - b, y = cx - d$. The first and third lines meet at $(0, -b)$ and the other two meet at $(0, -d)$. From each point, there are two lines with slopes a and c which generate a parallelogram.

3. **(1 pts.)** We have at least $n/42$ medians is less than x if x is the median of medians of the $n/21$ subarrays. For the corresponding $n/42$ subarrays, we have at least 11 numbers (including median itself) in each subarray that are less than x. So we have $11/42$ numbers in S are guaranteed to be less than x.

4. **(1 pts.)** First, we observe that $\sum_{k=1}^{n} 1/k = \Theta(\log n)$. So the left hand side is $n^{c \log n}$ for some constant c. On the other hands, the right hand side is

$$(\log n)^{\log^2 n} = (\log n)^{(\log n)\cdot(\log n)} \tag{1}$$

$$= ((\log n)^{\log n})^{\log n} = (n^{\log\log n})^{\log n} \tag{2}$$

Since the exponents of two sides have the same order, if we compare the bases of them, we can figure out that n is much smaller than $n^{\log\log n}$ so it is false.

5. **(1 pts.)** Since there are 4 points inside the convex hull, it means there will be 4 pop operations.

6. **(1 pts.)** Counterexample: Define $f(n) = n$ if n is even and $f(n) = 1$ otherwise. And $g(n) = n$ if n is odd and $g(n) = 1$ otherwise. Then, either $f = \Omega(g(n))$ or $g = \Omega(f(n))$ is not true.

7. **(1 pts.)**

   - Algorithm A time complexity: $T(n) = 8T(\frac{n}{4}) + \Theta(1) = \Theta(n^{\log_4 8}) = \Theta(n^1.5)$
   - Algorithm B time complexity: $T(n) = 2T(\frac{n}{2}) + \Theta(n) = \Theta(n \log n)$

   Thus, algorithm B has a faster running time.

8. **(1 pts.)** This is the Q5 of Assignment 03.

9. **(1 pts.)** $c_1 = 1/3$, $c_2 = 1/4$ and $c_3 = 1/5$, we have $T(n) = \Theta(n)$ if $c_1 + c_2 + c_3 < 1$. This generalize the conclusion introduced in class but it is straightforward.

10. **(1 pts.)** There are two lines $\{l_1, l_3, l_6\}$ in the upper envelop and therefore $\{l_1^*, l_3^*, l_6^*\}$ form the lower hull of the dual points; there are four lines $\{l_6, l_5, l_4, l_1\}$ in the lower envelop and therefore $\{l_6^*, l_5^*, l_4^*, l_1^*\}$ form the upper hull of the dual points. Notice that l_1^* and l_6* appear in both, so the convex hull of dual points consists of 5 vertices: $\{l_6^*, l_5^*, l_4^*, l_3^*, l_1^*\}$.