Problem 1. Find the best line to fit the data points

$$(a)\ (0,0),(1,3),(2,3),(5,6);\ \ (b)\ (0,5),(1,3),(2,3),(3,1).$$

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Homework 15

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Problem 2. Find the best parabola to fit the data points

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nial, and compare.

$y = F_3(t) = c_1 + c_2 \cos 2\pi t +$

RMSE.

Problem 5. Fit the data to the periodic models $F_3(t) = c_1 + c_2 \cos 2\pi t + c_3 \sin 2\pi t$ and $F_4(t) = c_1 + c_2 \cos 2\pi t + c_3 \sin 2\pi t + c_4 \cos 4\pi t$. Find the 2-norm errors $\|e\|_2$ and compare the fits of F_3 and F_4.

t	y
0	0
1/6	2
1/3	0
1/2	−1
2/3	1
5/6	1

Problem 5. Fit the data to the periodic models $F_3(t) = c_1 + c_2 \cos 2\pi t + c_3 \sin 2\pi t$ and $F_4(t) = c_1 + c_2 \cos 2\pi t + c_3 \sin 2\pi t + c_4 \cos 4\pi t$. Find the 2-norm errors $\|e\|_2$ and compare the fits of F_3 and F_4.

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5/6	1

Problem 1. Find the best line to fit the data points

(a) $(0,0),(1,3),(2,3),(5,6)$; (b) $(0,5),(1,3),(2,3),(3,1)$.

a)

$$\begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 2 & 1 \\ 5 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 3 \\ 6 \end{bmatrix}$$

A $\qquad$ b

$Y = Ax + b$

$$A^T = \begin{bmatrix} 0 & 1 & 2 & 5 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 2 & 5 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 2 & 1 \\ 5 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 & 1 & 2 & 5 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 3 \\ 3 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} 30 & 8 \\ 8 & 4 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 39 \\ 12 \end{bmatrix}$$

$a = \frac{15}{14}$

$b = \frac{6}{7}$

$y = \frac{15}{14}x + \frac{6}{7}$

b) $\begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \\ 3 \\ 1 \end{bmatrix}$

$\begin{bmatrix} 14 & 6 \\ 6 & 4 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 12 \\ 12 \end{bmatrix}$

$a = -\frac{6}{5}$ $b = \frac{24}{5}$

$y = -\frac{6}{5}x + \frac{24}{5}$

Problem 2. Find the best parabola to fit the data points

(a) $(0,0),(1,3),(2,3),(5,6)$; (b) $(0,5),(1,3),(2,3),(3,1)$.

a)
$$\begin{bmatrix} 0 & 0 \\ 1 & 3 \\ 2 & 3 \\ 5 & 6 \end{bmatrix}$$

$$y = a + bx + cx^2$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 5 \\ 0 & 1 & 4 & 25 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 5 & 25 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 5 \\ 0 & 1 & 4 & 25 \end{bmatrix} \begin{bmatrix} 0 \\ 3 \\ 3 \\ 6 \end{bmatrix}$$

$$a = \frac{63}{181} \qquad b = \frac{705}{362} \qquad c = \frac{-30}{181}$$

$$y = \frac{63}{181} + \frac{705}{362}x - \frac{30}{181}x^2$$

b) $\begin{bmatrix} 0 & 5 \\ 1 & 3 \\ 2 & 3 \\ 3 & 1 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{bmatrix}$ $\begin{bmatrix} 5 \\ 3 \\ 3 \\ 1 \end{bmatrix}$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \\ 0 & 1 & 4 & 9 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \\ 0 & 1 & 4 & 9 \end{bmatrix} \begin{bmatrix} 5 \\ 3 \\ 3 \\ 1 \end{bmatrix}$$

$$a = \frac{24}{5} \quad b = \frac{-6}{5} \quad c = 0$$

$$y = \frac{24}{5} - \frac{6}{5}x$$

Problem 3. Find the best degree 3 polynomial to fit the data points

$$(a)\ (0,0),(1,3),(2,3),(5,6);$$

$$\begin{bmatrix} 0 & 0 \\ 1 & 3 \\ 2 & 3 \\ 5 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 5 \\ 0 & 1 & 4 & 25 \\ 0 & 1 & 8 & 125 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & 8 \\ 1 & 5 & 25 & 125 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 5 \\ 0 & 1 & 4 & 25 \\ 0 & 1 & 8 & 125 \end{bmatrix} \begin{bmatrix} 0 \\ 3 \\ 3 \\ 6 \end{bmatrix}$$

$$y = \frac{26}{5}x - \frac{51}{20}x^2 + \frac{7}{20}x^3 \qquad a = 0 \quad b = \frac{26}{5} \quad c = \frac{-51}{20} \quad c = \frac{7}{20}$$

$$L_0(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} \quad ($$

$$= \frac{(x-1)(x-2)(x-5)}{-1 \quad -2 \quad -5}$$

$$= -\frac{1}{10}(x-1)(x-2)(x-5)$$

$$L_1(x) = \frac{1}{4}\, x(x-2)(x-5)$$

$$L_2(x) = -\frac{1}{6}\, x(x-1)(x-5)$$

$$L_3(x) = \frac{1}{60}\, x(x-1)(x-2)$$

$$Y = Y_0 L_0(x) + Y_1 L_1(x) + Y_2 L_2(x) + Y_3 L_3(x)$$

$$= 0 + 3 \cdot \frac{1}{4}\left(x(x-2)(x-5)\right) - 3 \cdot \frac{1}{6}\left(x(x-1)(x-5)\right)$$

$$+ 6\left(\frac{1}{60}\right)\left(x(x-1)(x-2)\right)$$

$$= \frac{x}{20}\left[7x^2 - 51x + 104\right]$$

$$y = \frac{26}{5}x - \frac{51}{20}x^2 + \frac{7}{20}x^3$$

Sum(

Problem 4. Fit data to the periodic model $y = F_3(t) = c_1 + c_2 \cos 2\pi t + c_3 \sin 2\pi t$. Find the 2 -norm error and the RMSE.

t	y
0	1
1/4	3
1/2	2
3/4	0

$t = 0$

$c_1 + c_2 \cos 0 + c_3 \sin 0 = 1$

$1 = c_1 + c_2$

$t = \frac{1}{4}$

$c_1 + c_2 \cos 2\pi \frac{1}{4} + c_3 \sin 2\pi \frac{1}{4} = 3$

$c_1 + c_3 = 3$

$\vdots$

$$\begin{bmatrix} 1 & \cos 0 & \sin 0 \\ 1 & \cos \frac{\pi}{2} & \sin \frac{\pi}{2} \\ 1 & \cos \pi & \sin \pi \\ 1 & \cos \frac{3\pi}{2} & \sin \frac{3}{2}\pi \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 2 \\ 0 \end{bmatrix}$$

$c_1 = \frac{3}{2} \quad c_2 = -\frac{1}{2} \quad c_3 = \frac{3}{2}$

$y = \frac{3}{2} - \frac{1}{2} \cos 2\pi t + \frac{3}{2} \sin 2\pi t$

$$r = b - Ax = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \|r\|_2 = 0 \quad RMSE = \sqrt{\frac{SE}{n}} = 0$$

Problem 5. Fit the data to the periodic models $F_3(t) = c_1 + c_2 \cos 2\pi t + c_3 \sin 2\pi t$ and $F_4(t) = c_1 + c_2 \cos 2\pi t + c_3 \sin 2\pi t + c_4 \cos 4\pi t$. Find the 2-norm errors $\|e\|_2$ and compare the fits of F_3 and F_4.

t	y
0	0
1/6	2
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1/2	−1
2/3	1
5/6	1

$F_3(t) = C_1 + C_2 \cos 2\pi t + C_3 \sin 2\pi t$

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & \cos 2\pi\frac{1}{6} & \frac{\sqrt{3}}{2} \\ 1 & \cos 2\pi\frac{1}{3} & \frac{\sqrt{3}}{2} \\ 1 & \cos 2\pi\frac{1}{2} & 0 \\ 1 & \cos 2\pi\frac{2}{3} & -\frac{\sqrt{3}}{2} \\ 1 & \cos 2\pi\frac{5}{6} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 0 \\ -1 \\ 1 \\ 1 \end{bmatrix}$$

$C_1 = \frac{157}{350}$ $C_2 = \frac{108}{175}$ $C_3 = \frac{95\sqrt{3}}{175}$

$$e = \begin{bmatrix} 0 \\ 2 \\ 0 \\ -1 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 & 1 & 0 \\ 1 & \cos 2\pi\frac{1}{6} & \frac{\sqrt{3}}{2} \\ 1 & \cos 2\pi\frac{1}{3} & \frac{\sqrt{3}}{2} \\ 1 & \cos 2\pi\frac{1}{2} & 0 \\ 1 & \cos 2\pi\frac{2}{3} & -\frac{\sqrt{3}}{2} \\ 1 & \cos 2\pi\frac{5}{6} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix} = \begin{bmatrix} -\frac{371}{350} \\ \frac{204}{175} \\ -\frac{38}{175} \\ -\frac{141}{350} \\ \frac{164}{175} \\ \frac{2}{175} \end{bmatrix}$$

$\|e\|_2 = \sqrt{1.135 + 1.35 + 0.47 + 0.69 + 0.87 + 1.306 \times 10^{-4}}$

$= \frac{\sqrt{14154}}{20}$

$= 1.6996$

$RMSE_3 = \frac{1.6996}{\sqrt{6}} = 0.6958$

$$F_4(t) = C_1 + C_2\cos 2\pi t + C_3 \sin 2\pi t + C_4 \cos 4\pi t$$

$$\begin{matrix} & \cos 2\pi t & \sin 2\pi t & \cos 4\pi t \end{matrix}$$

$$\begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & \frac{1}{2} & \frac{\sqrt{3}}{2} & -0.5 \\ 1 & -\frac{1}{2} & \frac{\sqrt{3}}{2} & -0.5 \\ 1 & -1 & 0 & 1 \\ 1 & -\frac{1}{2} & -\frac{\sqrt{3}}{2} & -0.5 \\ 1 & \frac{1}{2} & -\frac{\sqrt{3}}{2} & -0.5 \end{bmatrix}$$

$$C_1 = \frac{79}{174} \qquad C_3 = \frac{4\sqrt{3}}{87}$$

$$C_2 = \frac{16}{24} \qquad C_4 = -\frac{83}{87}$$

$$e = \begin{bmatrix} 0 \\ 2 \\ 0 \\ -1 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & \frac{1}{2} & \frac{\sqrt{3}}{2} & -0.5 \\ 1 & -\frac{1}{2} & \frac{\sqrt{3}}{2} & -0.5 \\ 1 & -1 & 0 & 1 \\ 1 & -\frac{1}{2} & -\frac{\sqrt{3}}{2} & -0.5 \\ 1 & \frac{1}{2} & -\frac{\sqrt{3}}{2} & -0.5 \end{bmatrix} \cdot \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix}$$

$$\|e\|_2 = \frac{9\sqrt{58}}{58}$$

$$= 1.18176$$

$$RMSE_4 = \frac{1.18176}{\sqrt{m}} = \frac{3\sqrt{\frac{3}{29}}}{2}$$

$$= 0.482$$

$$RMSE_4 < RMSE_3$$

So F_4 is better fit