20210826 Math 455

0.1 Exercise

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1. Rewrite the following polynomials in nested form. Evaluate with and without nested form at $x = 1/3$.

(a) $P(x) = 6x^4 + x^3 + 5x^2 + x + 1$

(b) $P(x) = -3x^4 + 4x^3 + 5x^2 - 5x + 1$

(c) $P(x) = 2x^4 + x^3 - x^2 + 1$

(a) $P(x) = 1+x+5x^2+x^3+6x^4$ $\quad x = \frac{1}{3}$

$= 1+x(1+x(5+x(1+6x)))$

$= 1+\frac{1}{3}(1+\frac{1}{3}(5+\frac{1}{3}(1+\frac{6}{3})))$

$= 1+\frac{1}{3}(1+\frac{6}{3})$

$= 1+1$

$= 2$

(b) $P(x) = -3x^4+4x^3+5x^2-5x+1$ $\quad x = \frac{1}{3}$

$= 1-5x+5x^2+4x^3-3x^4$

$= 1+x(-5+x(5+x(4-3x)))$

$= 1+\frac{1}{3}(-5+\frac{1}{3}(5+\frac{1}{3}(4-\frac{3}{3})))$

$= 1+\frac{1}{3}(-5+2)$

$= 1+-1$

$= 0$

(c) $P(x) = 2x^4+x^3-x^2+1$ $\quad x = \frac{1}{3}$

$= 1-x^2+x^3+2x^4$

$= 1+x^2(-1+x(1+2x))$

$= 1 + \frac{1}{3} \cdot \frac{1}{3}\left(-1 + \frac{1}{3}\left(1 + \frac{2}{3}\right)\right)$

$= 1 + \frac{1}{9}\left(-1 + \frac{5}{9}\right)$

$= 1 + \frac{-4}{81}$

$= \frac{77}{81}$

3. Evaluate $P(x) = x^6 - 4x^4 + 2x^2 + 1$ at $x = 1/2$ by considering $P(x)$ as a polynomial in x^2 and using nested multiplication.

$P(x) = x^6 - 4x^4 + 2x^2 + 1$ $\qquad x = \frac{1}{2}$

$= 1 + 2x^2 - 4x^4 + x^6$ $\qquad x^2 = \frac{1}{4}$

$= 1 + x^2(2 + x^2(-4 + x^2))$

$= 1 + \frac{1}{4}\left(2 + \frac{1}{4}\left(-4 + \frac{1}{4}\right)\right)$

$= 1 + \frac{1}{4}\left(2 + \frac{-15}{16}\right)$

$= \frac{64 + 17}{64}$

$= \frac{81}{64}$

6. Explain how to evaluate the polynomial for a given input x, using as few operations as possible. How many multiplications and how many additions are required?

(a) $P(x) = a_0 + a_5x^5 + a_{10}x^{10} + a_{15}x^{15}$

(b) $P(x) = a_7x^7 + a_{12}x^{12} + a_{17}x^{17} + a_{22}x^{22} + a_{27}x^{27}$.

(a) need to calculate $x^5 = x^2 \cdot x^3$ $\qquad x \cdot x = x^2$, $x^2 \cdot x = x^3$

$P(x) = a_0 + x^5(a_5 + x^5(a_{10} + a_{15}x^5))$

6 multi & 3 add

(b) $P(x) = x^7(a_7 + x^5(a_{12} + x^5(a_{17} + x^5(a_{22} + a_{27}x^5))))$

$x \cdot x = x^2$

$x^1 \cdot x^2 = x^4$

$x^4 \cdot x^1 = x^5$

$x^5 \cdot x^2 = x^7$

4+5=9 muti

&

4 add