Quiz 1 (for Section 1)

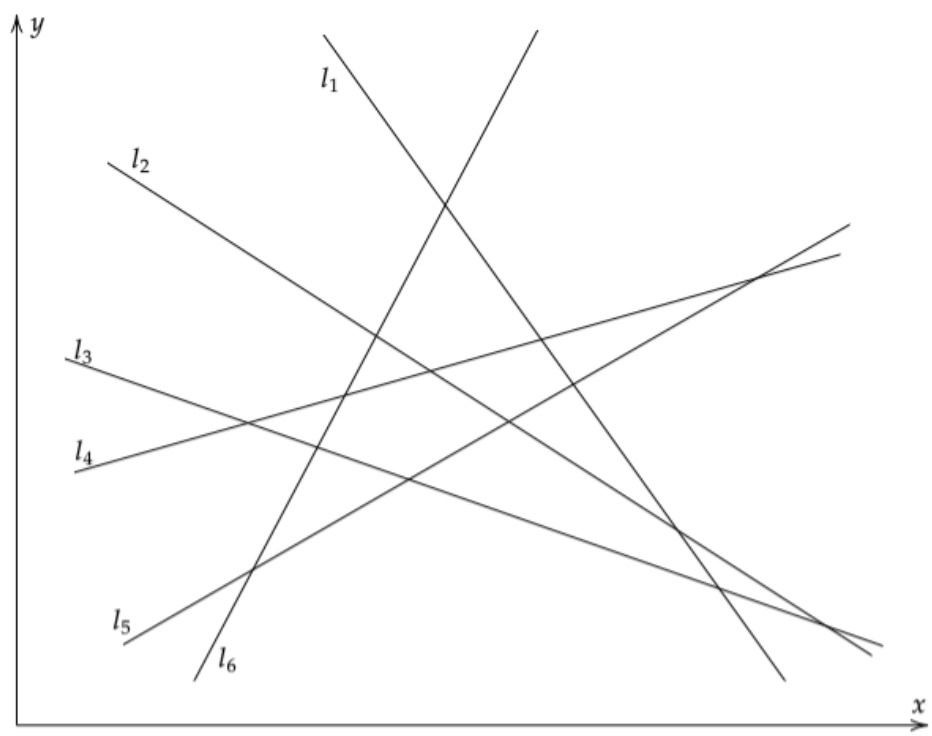
Started: Feb 4 at 11:49am

Quiz Instructions

Question 1

1 pts

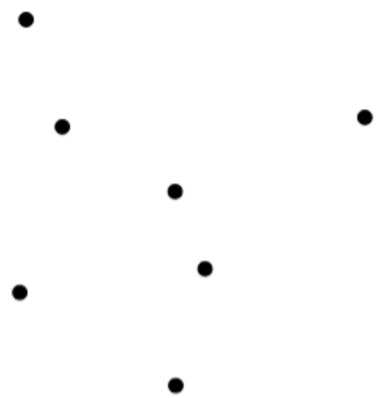
Consider the dual points of the lines given below: how many vertices will be on the convex hull of these dual points?



- ○ 3
- ○ 5
- ○ 6
- ◉ 4

Question 2 **1 pts**

Consider running the Graham-Scan algorithm on the instance given below. How many pop operations will be executed?



- ○ 5
- ○ 4
- ○ 2
- ● 3

Question 3 **1 pts**

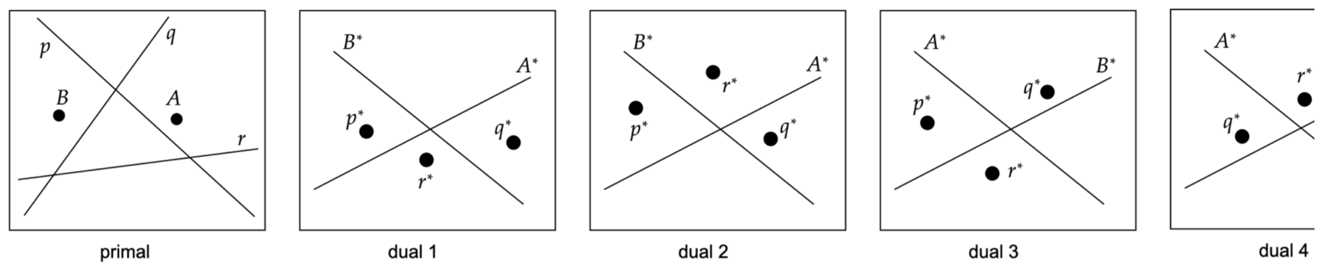
Which of the following is the asymptotic solution of recurrence $T(n) = n + T(n/3) + T(n/4)$?

- ● n
- ○ $n \cdot \log n$
- ○ n^3
- ○ n^4

Question 4

1 pts

Which one of the following shows the correct dual points and dual lines for the lines and points in the primal plane?



- ○ dual 4
- ○ dual 3
- ○ dual 1
- ◉ dual 2

Question 5

1 pts

Let $A[1\cdots n]$ be an array with n distinct integers. Let A' be the sorted array of A in ascending order. We pick a number x from A uniformly at random. What is the probability for event $\{x \leq A'[n/3] \text{ or } x > A'[n/2]\}$?

- ◉ $5/6$
- ○ $1/2$
- ○ $3/5$
- ○ $2/3$

Question 6

1 pts

Suppose you are given the following two algorithms: Algorithm A solves problems of size n by dividing them into 9 subproblems of size $n/3$ recursively solving each subproblem, and then combining the solutions in constant time; Algorithm B solves problems by dividing them into 2 subproblems of half the size, recursively solving each subproblem, and then combining the solutions in linear time. Which of these algorithms has a faster running time?

- ○ A
- ○ They have the same asymptotic running time
- ● B

Question 7

1 pts

Which of the following is the asymptotic solution of recurrence $T(n) = \log^2 n + T(\sqrt{n})$?

- ○ $\Theta(\log^2 n \cdot \sqrt{n})$
- ○ $\Theta(\sqrt{n})$
- ● $\Theta(\log^2 n)$
- ○ $\Theta(\log n)$

Question 8

1 pts

Suppose we have two sorted arrays $A = [a_1, a_2, a_3]$ and $B = [b_1, b_2, b_3]$, both in ascending order. Which of the following can't be a possible outcome of running merge-two-sorted-arrays(A, B)?

- ● None of the others
- ○ $[b_1, a_1, a_2, b_2, b_3, a_3]$
- ○ $[b_1, b_2, b_3, a_1, a_2, a_3]$
- ○ $[a_1, b_1, a_2, b_2, a_3, b_3]$

Question 9

1 pts

$n^{\log n} = O(2^{\log^2 n})$

- ○ False.
- ● True.

Question 10

1 pts

Suppose S is an array with n distinct integers. Similar to the selection algorithm, we partition S into $n/13$ subarrays, each of which contains 13 numbers. Let x be the median of medians of the $n/13$ subarrays. How many numbers in S are guaranteed to be less than x?

- ○ $5n/26$
- ○ $3n/13$
- ● $7n/26$
- ○ $4n/13$

Quiz saved at 11:51am Submit Quiz

CMPSC 465 Data Structures & Algorithms
Spring 2022 Chunhao Wang and Mingfu Shao

Quiz 1 (Section 1)

1. **(1 pts.)** There are two lines $\{l_1, l_6\}$ in the upper envelop and therefore $\{l_1^*, l_6^*\}$ form the lower hull of the dual points; there are four lines $\{l_6, l_5, l_3, l_1\}$ in the lower envelop and therefore $\{l_6^*, l_5^*, l_3^*, l_1^*\}$ form the upper hull of the dual points. Notice that l_1^* and l_6* appear in both, so the convex hull of dual points consists of 4 vertices: $\{l_6^*, l_5^*, l_3^*, l_1^*\}$.

2. **(1 pts.)** Since there are 3 points inside the convex hull, it means there will be 3 pop operations.

3. **(1 pts.)** $c_1 = 1/3$ and $c_2 = 1/4$, we have $T(n) = \Theta(n)$ if $c_1 + c_2 < 1$.

4. **(1 pts.)** A couple of things we may use to pick the right solution: (1), the slopes of p, r, and q are increasing, so in the dual plane, from left to right we should have p^*, r^*, and q^*, which removes dual-4. (2), in primal plane, A and B are above line r, so in the dual plane r^* should be above A^* and B^*, which removes dual-1 and dual-3. (3), B is above q and r in the primal plane, so q^* and r^* should be above B^* which removes dual 1 and dual 3. (4), A is located to the right of B, which means that the x-coordinate of A is bigger than the one of B. Thus, the slope of A^* is also bigger than the one of B^*, which removes dual-3 and dual-4.

5. **(1 pts.)** Since x is drawn uniformly at random, we have $P(\{x \leq A'[n/3]$ or $x > A'[n/2]\})$ is equal to $(n/3 + n/2)/n = 5/6$.

6. **(1 pts.)**

   - Algorithm A time complexity: $T(n) = 9T(n/3) + \Theta(1) = \Theta(n^2)$
   - Algorithm B time complexity: $T(n) = 2T(n/2) + \Theta(n) = \Theta(n \log n)$

   Thus, algorithm B has a faster running time.

7. **(1 pts.)**

$$
\begin{aligned}
T(n) &= \log^2 n + T(n^{1/2}) = \log^2 n + \log^2(n^{1/2}) + T(n^{1/4}) && (1)\\
&= \log^2 n + \frac{1}{4}\log^2 n + \log^2(n^{1/4}) + T(n^{1/8}) = ... && (2)\\
&= \log^2 n + \frac{1}{4}\log^2 n + \frac{1}{16}\log^2 n + ... = \Theta(\log^2 n) && (3)
\end{aligned}
$$

8. **(1 pts.)** The only relationships we know regarding the elements in A and B are $a_1 \leq a_2 \leq a_3$ and $b_1 \leq b_2 \leq b_3$. As we know merge-two-sorted-arrays (A, B) outputs an sorted array of elements in A and B in ascending order, so the only type of impossible sequence is a larger element comes in front of a smaller element. However, with the known relationships, we can't be certain any element is larger than other elements. So, any sequence of these 6 elements can be a possible outcome.

9. **(1 pts.)** Let $f(n) = n^{\log n}$ and $g(n) = 2^{\log^2 n}$. Suppose we take a logarithm on $f(n)$ and $g(n)$, we have $\log(f(n)) = \log n \times \log n$, $\log(g(n)) = \log 2 \times \log^2 n$. So we have $\log(f(n)) = O(\log(g(n)))$. Then we can use the formal definition of Big-O to prove that we have $f(n)) = O((g(n))$ if $\log(f(n)) = O(\log(g(n)))$.

10. (1 pts.) We have at least $n/26$ medians is less than x if x is the median of medians of the $n/13$ subarrays. For the corresponding $n/26$ subarrays, we have at least 7 numbers (including median itself) in each subarray that are less than x. So we have $7/26$ numbers in S are guaranteed to be less than x.