Stat/Math 415 Final Exam Practice Problems 2020

1. A company wants to determine whether the mean time required to complete a certain task differs among three levels of employee training. The company randomly selected 10 employees from each training level (Beginner, Intermediate, and Advanced), and data are summarized below:

Training Level	n	$\overline{X}$	S^2
Advanced	10	24.2	21.54
Intermediate	10	27.1	18.64
Beginner	10	30.2	17.76

Specify the appropriate null and alternative hypotheses, and use ANOVA to test your hypotheses at the 0.05 significance level. You should complete the ANOVA table below.

Source	S.S.	d.f.	M.S.	F-ratio
Treatment	180	3-1=2	180/2 = 90	90/19.3 = 4.66
Error	521.46	30-3=27	521.46/27=19.3	
Total	575.46	30-1=29		

H0: $\mu_{advanced} = \mu_{intermediate} = \mu_{beginner} = \mu$
H1: at least one group has a different mean

SS(E) = 9*21.54 + 9*18.64 + 9*17.76 = 521.46
Grand Mean = (24.2+27.1+30.2) / 3 = 27.17
SS(T) = 10 * (24.2 - 27.17)^2 + 10 * (27.1 - 27.17)^2 +10 * (30.2 - 27.17)^2 = 180
SS(To) = SS(E) + SS(T) = 521.46+180 = 701.46

F = 4.66 > 3.35 = F(2,27), so reject H0.

2. A brand of toy is sold in 5 types denoted by A_1, A_2, A_3, A_4, and A_5. Let pi be the probability that a randomly chosen toy is of type A_i, i = 1, 2, 3, 4, 5. At $\alpha = 0.01$, we wish to test H0: p_1 = 0.4, p_2 = 0.2, p_3 = 0.2, p_4 = 0.1, p_5 = 0.1 against H1: H0 is not valid, using a random sample of n = 580 toys whose types yielded the frequencies, A_1: 224, A_2: 119, A_3: 130, A_4: 48, and A_5: 59.

Sol: Q4 = 3.78 < chisq^2_0.01(4) = 13.28, do not reject H0

3. A random sample of n = 429 people were classified according to sex and according to whether they viewed the newspaper, television or radio as the most credible of all public media. The data are as follows:

	Newspaper	Television	Radio	Total
Male	92	108	19	
Female	97	81	32	
Total				

Use the chi-square statistic to test the hypothesis, at the 5% level of significance, that the variables "media credibility" and "sex" are independent, and provide the p-value of the test.

Sol: Q = 7.117 > chisq^2_0.05(2) = 5.991, reject H0

4. To compare the lifetimes of a type of light bulb made in two different factories, A and B, we test independent random samples of 10 bulbs from each factory. The data, in hundreds of hours, are:
   A : 5.6 4.6 6.8 4.9 6.1 5.3 4.5 5.8 5.4 4.7
   B : 7.2 8.1 5.1 7.3 6.9 7.8 5.9 6.7 6.5 7.1

   (a) We wish to test $H_0 : m_A = m_B$ vs. $H_1 : m_A \neq m_B$. Use the sign test to write down a critical region with level of significance 5%, approximately, and carry out the test and state your conclusion.

Median is (5.9+6.1)/2 = 6,
At factory A, 2 out of 10 are above median, we turn the problem into testing for
H0: proportion parameter = 0.5 v.s. H1: proportion parameter ≠ 0.5
Given binomial data, y=2 and n=10

   (b) We wish to test $H_0 : m_A = m_B$ vs. $H_1 : m_A \neq m_B$. Use the Wilcoxon statistic to write down a critical region with level of significance 5%, approximately, and carry out the test and state your conclusion.

Let Wilcoxon statistics, W, be the sum of the ranks of {y_1, …, y_n2}.
W = 5 + 10 + 12 + 13 + 15 + 16 + 17 + 18 + 19 + 20 = 145
Under H0, E(W) = n2(n1+n2+1)/2 = 10*21/2 = 105
Var(W) = n1 n2 (n1+n2+1)/12 = 10*10*21/12 = 175
|Z| = (W-105)/sqrt(175) = 3.02 > Z_{0.025} = 1.96
Reject H0 and believe A and B do not have the same median.

   (c) Use t-test for $H_0 : \mu_A = \mu_B$ vs. $H_1 : \mu_A \neq \mu_B$ with significance level 5%.

We need to assume equal variance between two factories, and use two sample t-test.