Math 486

Lesson 2 Homework

Due Tues, May 31 at 11:59 on Gradescope

Instructions

Please refer to the solution guidelines posted on Canvas under Course Essentials.

Exercise 1.

Suppose G is a two player game. The strategies w and z are both in Player 1's strategy set S_1 (the size of S_1 is unspecified and may be infinite). Write out the mathematical definition for each of the following statements:

(a) Player 1's strategy w is strictly dominated by Player 1's strategy z.

(b) Player 1's strategy w is weakly dominated by Player 1's strategy z.

Note: Writing out the mathematical definitions here means that the payoff notation should appear in your solutions and should be used correctly.

a) $\pi_1(z, s_2) > \pi_1(w, s_2)$ for all $s_2 \in S_2'$,

b) $\pi_1(z, s_2) \geq \pi(w, s_2)$ for all $s_2 \in S_2'$

and exist a $s_2 \in S_2'$ that $\pi_1(z, s_2) > \pi_1(w, s_2)$

Exercise 2.

The Hawk-Dove game is a two-player game used to analyze animal behavior in territorial conflicts among animals of the same species.

There are two strategies available to each player:

- hawk (h): fight aggressively until you prevail or are injured
- dove (d): bluff by displaying, but retreat if the other player begins to fight

We use the parameters v and w (both parameters are positive real numbers)

- v: the value of the territory or object being fought over ($v > 0$)
- w: the cost of getting injured in fighting ($w > 0$)

This leads to the payoff matrix shown. The factor of $\frac{1}{2}$ when both players use the same strategy indicates an expected payoff assuming each player has an equal chance of prevailing.

		Player 2 h	d
Player 1	h	$\frac{v-w}{2}, \frac{v-w}{2}$	$v, 0$
	d	$0, v$	$\frac{v}{2}, \frac{v}{2}$

Table 1: Hawk-Dove Payoff Matrix

Assume that w is some fixed positive value.

(a) Find all values of $v > 0$ such that the resulting game has a strategy that is strictly dominated by another strategy. Indicate which strategy is dominated.

(b) Find all values of $v > 0$ such that the resulting game has no dominated strategy (either strictly or weakly).

(c) Find all values of $v > 0$ such that the resulting game has a strategy that is weakly, but not strictly, dominated. Indicate which strategy is dominated.

Notes:

- This exercise is designed so that you have to think carefully about the meaning of "strictly dominated" and "weakly dominated" in the setting of a two player game and then apply that understanding.

  Note again that w is a fixed positive real number and v can be **any positive real number**. In particular, v need not be an integer or a rational number.

  Your solution should also demonstrate your reasoning. It is not enough to state the answer with no explanation or without some work that demonstrates your thinking.

a) strictly

		Player 2	
		h	d
Player 1	h	$\frac{v-w}{2}, \frac{v-w}{2}$	$v, 0$
	d	$0, v$	$\frac{v}{2}, \frac{v}{2}$

for player 1, let h strictly dominate d

so $\frac{V-W}{2} > 0$

$V > \frac{V}{2}$

So $\frac{V-W}{2} > 0$

$\boxed{V > W}$

b)

for player 1, no dominate strategy

V will have to be smaller then W

$V > \frac{V}{2}$

$\frac{V-W}{2} < 0$

$\boxed{V < W}$

c) weakly

for player 1, let h weakly dominatedly strategy W

V will have to be smaller then W

$V > \frac{V}{2}$

$\frac{V-W}{2} = 0$

$\boxed{V = W}$

Problem 1.

In the Lesson 2 lectures we consider a Stackelberg Duopoly where each firm has the same cost function. In this problem we consider a Stackelberg Duopoly where the firms have different costs.

- Firm 1 chooses a quantity $q_1 \in [0, \infty)$.
- Firm 2 observes q_1 and then chooses a quantity $q_2 \in [0, \infty)$.

The market price is given by

$$p = \begin{cases} 150 - q_1 - q_2, & \text{if } q_1 + q_2 \leq 150 \\ 0, & \text{if } q_1 + q_2 > 150 \end{cases}$$

and the cost functions for each firm are

$$c_1(q_1) = 9q_1, \qquad c_2(q_2) = 4q_2$$

The payoff function for each firm is the total profit (revenue minus cost).

(a) Determine the payoff function π_i for each firm.

(b) State the strategy set for each firm.

(c) Use backward induction to determine the rational outcome of the game.

$$p = \begin{cases} 150 - q_1 - q_2 & \text{if } q_1 + q_2 \leq 150 \\ 0 & \text{if } q_1 + q_2 > 150 \end{cases}$$

$$\pi_1 (q_1, q_2) = p q_1 - c q_1$$

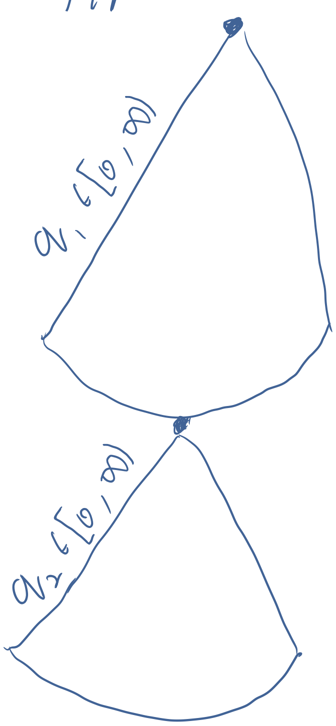
$$= \begin{cases} (150 - q_1 - q_2) q_1 - 9 q_1 & q_1 + q_2 \leq 150 \\ -12 q_1 & q_1 + q_2 > 150 \end{cases}$$

$$= \begin{cases} 141 q_1 - q_1^2 - q_1 q_2 & q_1 + q_2 \leq 150 \\ -12 q_1 & q_1 + q_2 > 150 \end{cases}$$

$$\pi_2(q_1, q_2) = \begin{cases} 150 q_2 - q_1 q_2 - q_2^2 - 4 q_2 & q_1 + q_2 \leq 150 \\ -4 q_2 & q_1 + q_2 \geq 150 \end{cases}$$

$$= \begin{cases} 146 q_2 - q_1 q_2 - q_2^2 & q_1 + q_2 \leq 150 \\ -4 q_2 & q_1 + q_2 \geq 150 \end{cases}$$

Firm



$\pi_1(q_1, q_2)$

$\pi_2(q_1, q_2)$

$$\frac{\partial \pi}{\partial q_2} = 146 - q_1 - 2q_2 = 0$$

$$q_2 = \frac{146 - q_1}{2}$$

$$\frac{\partial \pi}{\partial q_1} = 141 - 2q_1 - q_2 = 0$$

$$141 - 2q_1 - \frac{146 - q_1}{2} = 0$$

$$282 - 4q_1 - 146 + q_1 = 0$$

$$3q_1 = 136$$

$$q_1 = \frac{136}{3} \qquad q_2 = \frac{151}{3}$$

Problem 2.

A large retailer purchases a particular model of computer tablet from a manufacturer to sell in its stores. We will model this as a two player game involving the manufacturer and the retailer. The approach is very similar to the Stackelberg Duopoly game.

- Suppose that it costs the manufacturer \$100 to manufacture each tablet. The manufacturer will choose a wholesale price, w, for each tablet with $w \in [100, \infty)$.
- The retailer observes the wholesale price and chooses a quantity $q \in [0, \infty)$ to purchase from the manufacturer.
- The retailer wishes to sell every tablet. We assume the retailer faces a market price p,

$$p = \begin{cases} 500 - 0.1q, & \text{if } 0 \leq q \leq 5000, \\ 0, \text{ if } q > 5000 \end{cases}$$

For each player, the payoff is the total profit (revenue minus cost).

(a) Write the payoff function π_i for each player.

(b) State the strategy set for each player.

(c) Use backward induction to determine the rational outcome of the game.

(d) Suppose instead that the manufacturer is able to sell directly to consumers without the retailer. The manufacturer chooses a quantity q to manufacture and has total profit

$$pq - 100q$$

Notice that the manufacturer now faces the market price p (the same market price that applied to the retailer in the original problem). Find the optimal choice for q for the manufacturer.

(e) Compare the manufacturer's profit when selling directly to the consumer with the total profit for both the manufacturer and retailer in the previous steps. This is an example of double marginalization.

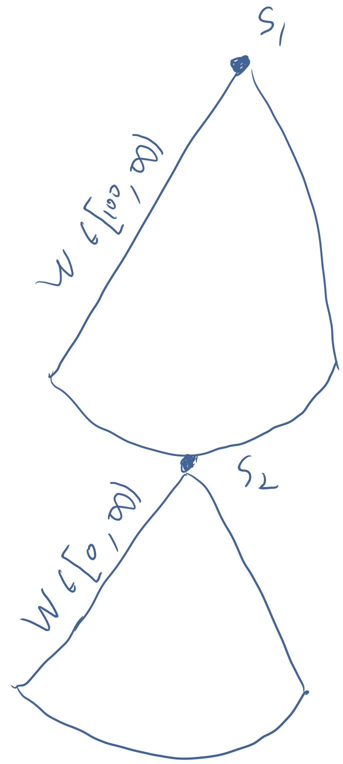
$\pi_1 (w, q) = wq - 100q$ $\qquad$ $[100, \infty)$

$$\pi_1 \begin{cases} (500 - 0.1q) \cdot q - wq & \text{if } 0 \leq q \leq 5000 \\ -wq & \text{if } q > 5000 \end{cases}$$

$$\pi_1 \begin{cases} 500q - 0.1q^2 - wq & \text{if } 0 \leq q \leq 5000 \\ -wq & \text{if } q > 5000 \end{cases}$$

$S_1 = [100, \infty)$

$S_2 = \{ f\ [100, \infty) \to [0, \infty) \}$



$\pi_1(w, q)$

$\pi_2(w, q)$

$\frac{\partial \pi_2}{\partial q} = 500 - 0.2q - w = 0$

$q = \frac{500 - w}{0.2}$

$q = 2500 - 5w$

$\pi_1(w, q) = wq - 100q$

$= w(2500 - 5w) - 100(2500 - 5w)$

$= 2500w - 5w^2 - 250000 + 500w$

$= -5w^2 + 3000w - 250000$

$\frac{\partial \pi}{\partial w} = -10w + 3000 = 0$

$10w = 3000$

$w = 300$

$q = 2500 - 5(100)$

$= 2000$

d $\pi_1 = pq - 100q$

$= (500 - 0.1q)q - 100q$

$= 500q - 0.1q^2 - 100q$

$= -0.1q^2 + 400q$

$\frac{\partial \pi_1}{\partial q} = -0.2q + 400 = 0$

$q = 2000$

e $p_1 = 500 - 0.1 \cdot 2000 = 300$

$\pi_1 = 300 \cdot 2000 - 100 \cdot 2000 = 400000$

$p_2 = 500 - 0.1 \cdot 1000 = 400$

$\pi_1 = 300 \cdot 1000 - 100 \cdot 1000 = 200000$

$+$

$\pi_2 = 500 \cdot 1000 - 300 \cdot 1000 = 100000$

$= 300000 < 400000$