Greedy algorithms

Huffman Encoding (Textbook Section 5.2)

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$S = aba$

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$S_e = 01100001|01100010|01100001|$

Example: ASCII encoding

e=

a	01100001
b	01100010
$\vdots$	$\vdots$

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$$e_2: \begin{array}{l} a \to 0 \\ b \to 10 \\ c \to 11 \end{array}$$

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- Be careful! $e_2 : \begin{array}{l} a \to 0 \\ b \to 1 \\ c \to 01 \end{array}$ Decoding will lead to ambiguity

Prefix-free encoding

$$\text{Consider the bad encoding } e_2 : \quad \begin{aligned} a &\to 0 \\ b &\to 1 \\ c &\to 01 \end{aligned}$$

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ababba?, *ccba*?, *abcba*?, or . . . ?

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Definition

An encoding is **prefix-free** if no codeword is a prefix of any other codewords

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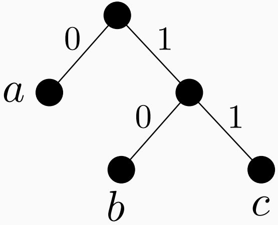

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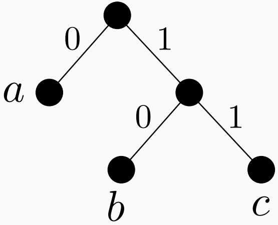

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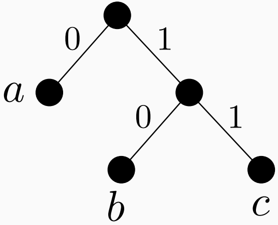

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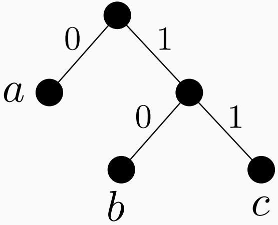

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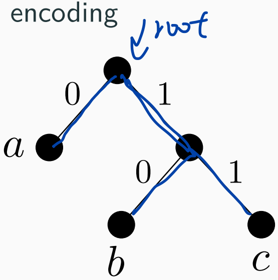

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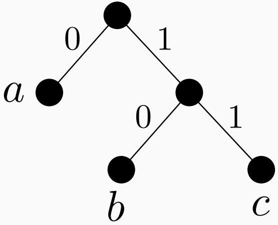

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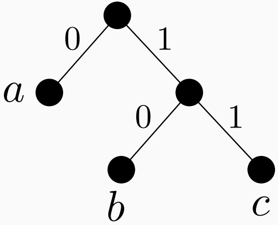

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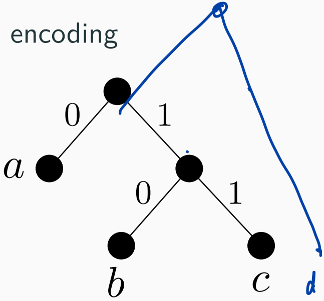

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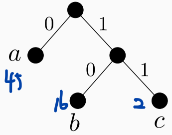

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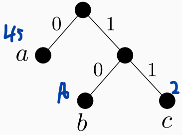

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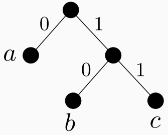

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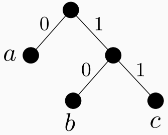

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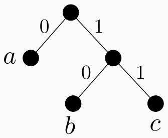

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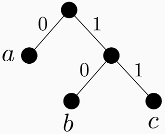

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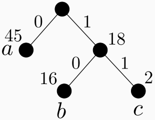

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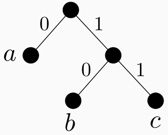

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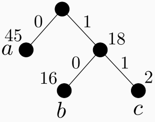

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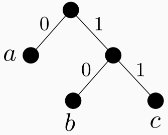

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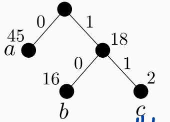

$=16+2$

$$|S_e| = \sum_{v \in \boxed{T - \{\mathrm{root}\}}} \mathrm{cost}(v)$$

T: tree

all the non-root nodes

$$|S_e| = 45 + 16 + 2 + 18 = 81$$

Constructing the prefix-free encoding tree

Idea: put more frequent symbols at smaller depth

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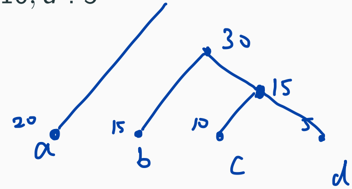
Greedy approach: continually merge least frequent symbols/nodes until you have a full binary tree encoding all symbols

Constructing the prefix-free encoding tree – examples

- $a : 20, b : 15, c : 10, d : 5$



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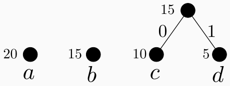

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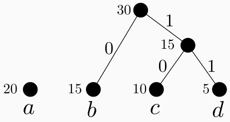

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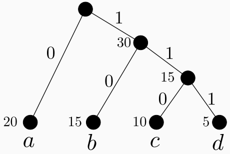

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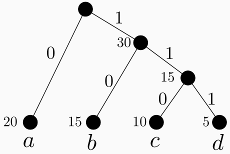

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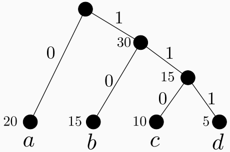

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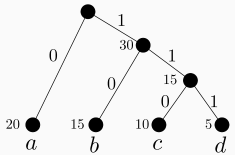

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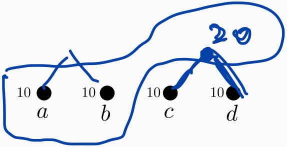

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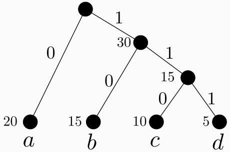

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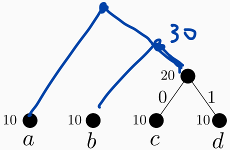

$|S_e| = 10+10+10+10+20+30 = 90$

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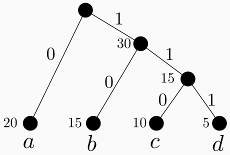

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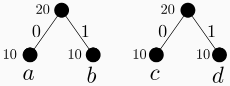

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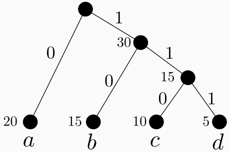

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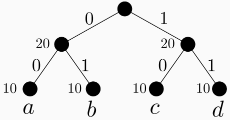
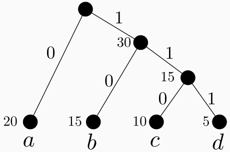


$|S_e| = 10+10+10+10+20+20 = 80$

Constructing the prefix-free encoding tree – examples

- $a : 20, b : 15, c : 10, d : 5$



$a \to 0$

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$c \to 110$

$c \to 111$

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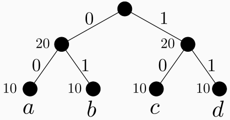

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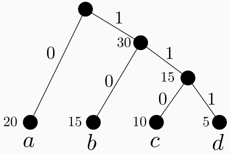

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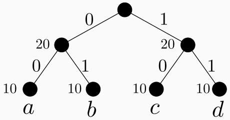

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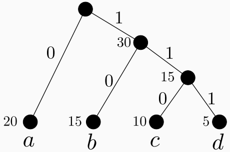

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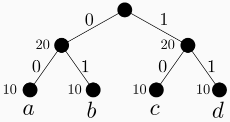

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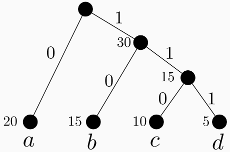

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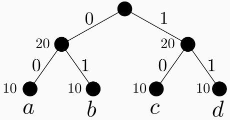

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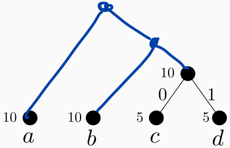

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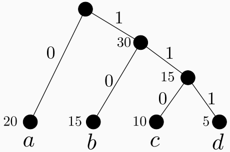

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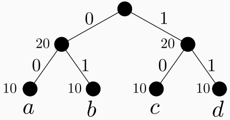

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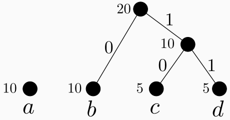

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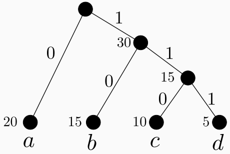

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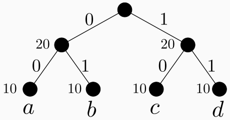

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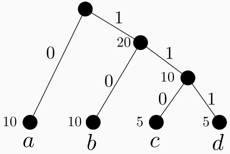

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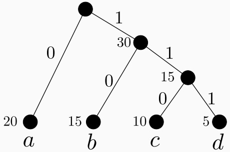

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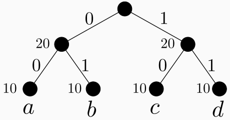

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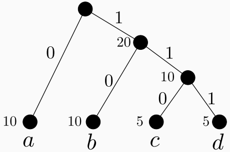

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Proof of optimality (I)

Proof sketch

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- Greedy choice property:

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Every optimal solution ~~that~~ has two lowest frequent symbols as leaves connected to an internal node of greatest depth

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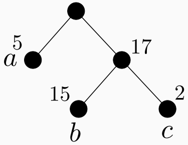

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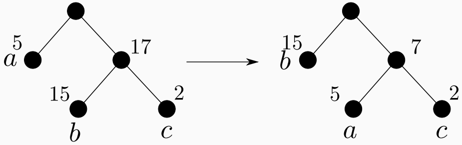

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- Optimal substructure
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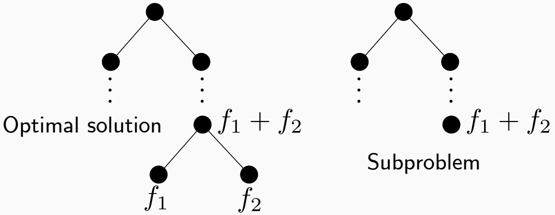

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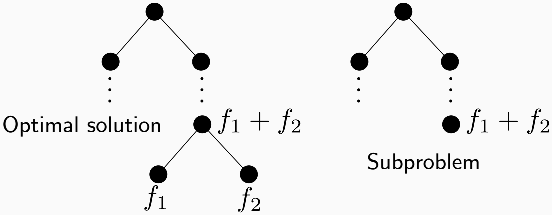

Thus, the greedy solution will lead to the global optimal solution

Pseudocode

def HUFFMAN(f): // $f : f[1], \ldots, f[n]$; Γ has n symbols

    T: empty tree;

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def HUFFMAN(f): // f : f[1], ..., f[n]; Γ has n symbols
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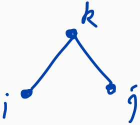

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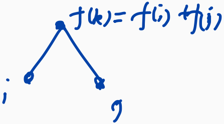

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Binary heap: insert $O(\log n)$, extract_min: $O(\log n)$

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7          i := extract_min(H);
8          j := extract_min(H);
9          Creat a node k in T with children i and j;
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Lines 4-5: $O(n \log n)$;

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Binary heap: insert $O(\log n)$, extract_min: $O(\log n)$

Lines 4-5: $O(n \log n)$; Lines 6-11: $O(n \log n)$

Total cost: $O(n \log n)$

More about the pseudocode

Question: why $2n - 1$ in line 6?

More about the pseudocode

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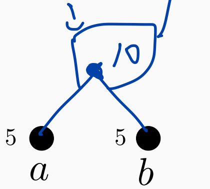
Answer: if a full binary tree has n leaves, then it has $2n-1$ total nodes

More examples

$a : 5, a : 5, c : 7, d : 10, e : 15, f : 17$

More examples

$\cancel{a : 5, b : 5}, c : 7, d : 10, e : 15, f : 17$



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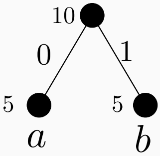

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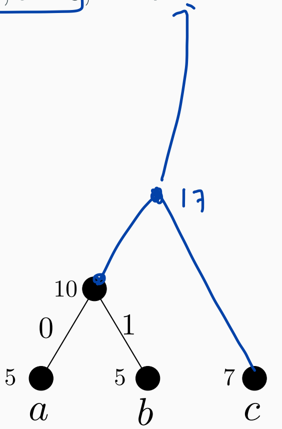
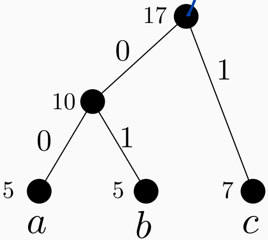


More examples

17

~~*a* : 5, *a* : 5, *c* : 7,~~ *d* : 10, *e* : 15, *f* : 17 17



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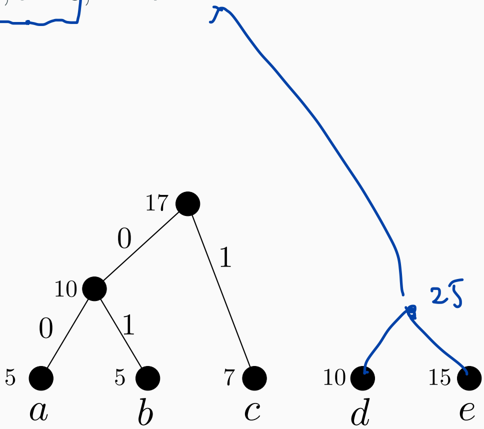

More examples

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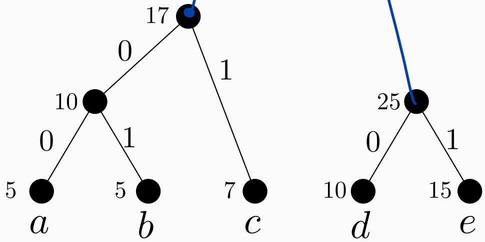
17

25



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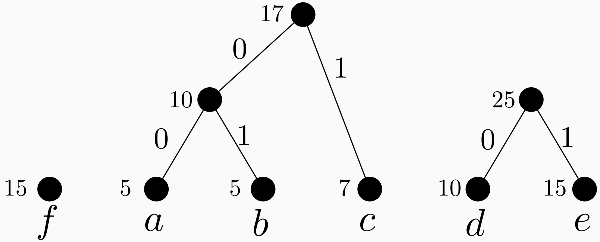

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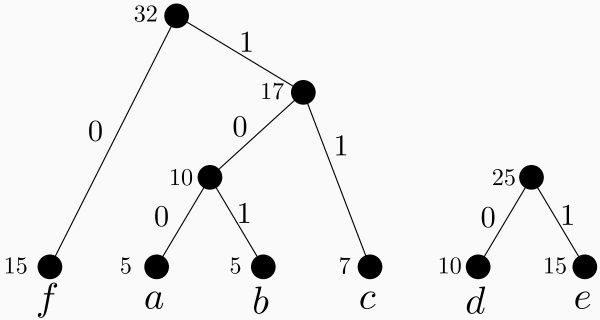

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