Quiz 2 (Section 1)

Started: Mar 3 at 4:48pm

Quiz Instructions

Question 1

1 pts

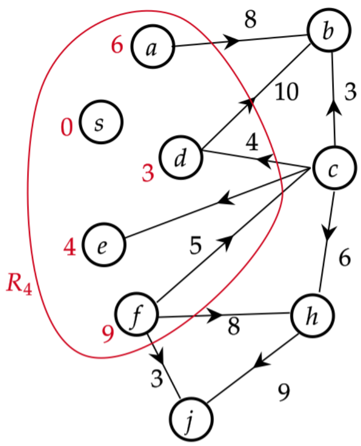
If v_1, v_2 are in the same connected component C_1 and v_3, v_4 are in the same connected component C_2. Which of the following is an impossible combination for the postlist?

- ○ (v_1, v_3, v_4, v_2)
- ◉ (v_1, v_3, v_2, v_4)
- ○ (v_1, v_2, v_3, v_4)
- ○ (v_3, v_1, v_2, v_4)

Question 2

1 pts

We are in the middle of running Dijkstra's algorithm on the graph given below starting from s: the first 4 vertices that are closest to s are marked as R_4. Which one will be the 5-th closest vertex from s?

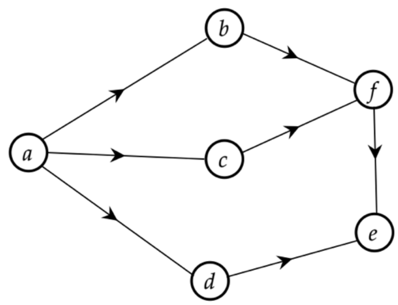


- ○ vertex h
- ○ vertex b
- ● vertex j
- ○ vertex c

Question 3

1 pts

How many linearization does this graph have?



- ○ 10
- ○ 6
- ○ 16
- ● 8

Question 4

1 pts

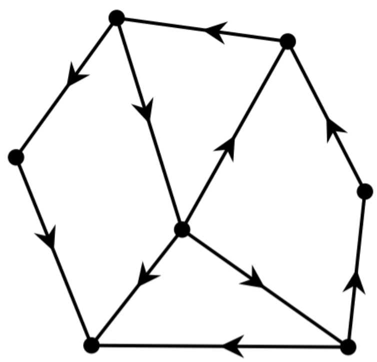
Let G^R be the reverse graph of directed graph G. Which one of the following is NOT true?

- ○ If G is a DAG then G^R is also a DAG.
- ○ The reverse graph of G^R is G.
- ● The meta-graph of G is also the meta-graph of G^R.
- ○ If u can reach v in G^R and u can reach v in G, then G^R is not a DAG.

Question 5

1 pts

How many vertices are in the meta-graph of the following graph?



- ○ 2
- ○ 1
- ◉ 3
- ○ 4

Question 6

1 pts

Let G be a directed graph with positive edge length and let p be one shortest path from u to v. (A). If we increase the length of every edge by 2, then p is still one shortest path from u to v. (B). If we multiply the length of every edge by 2, then p is still one shortest path from u to v.

- ○ (A) is false and (B) is false.
- ○ (A) is true and (B) is false.
- ○ (A) is true and (B) is true.
- ◉ (A) is false and (B) is true.

Question 7

1 pts

Assume we have a directed graph G. Would the algorithm below give us all connected components correctly? Step 1, run DFS with timing on G to get postlist; Step 2, run DFS on G_R with ordering of above postlist.

- ○ False
- ◉ True

Question 8

1 pts

Let G be a directed graph possibly with negative edge length but without negative cycle. Let $v_1 \to v_2 \to v_3 \to v_4 \to v_5$ be one shortest path from v_1 to v_2. Which one of the following is NOT true?

- ◉ $distance(v_1, v_5) \geq distance(v_2, v_4)$.
- ○ $distance(v_1, v_3) + distance(v_3, v_5) = distance(v_1, v_5)$.
- ○ The path $v_1 \to v_2 \to v_3$ is one shortest path from v_1 to v_3.
- ○ The path $v_2 \to v_3 \to v_4$ is one shortest path from v_2 to v_4.

Question 9

1 pts

Let s be a source vertex and let t be a sink of of a DAG G. Then there must exist a linearization of G in which s is the first vertex and t is the last vertex.

- ○ False
- ◉ True

Question 10

1 pts

How many connected components in the graph of the adjacency matrix below?

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

- (selected) 3
- 4
- 1
- 2

Quiz saved at 4:49pm

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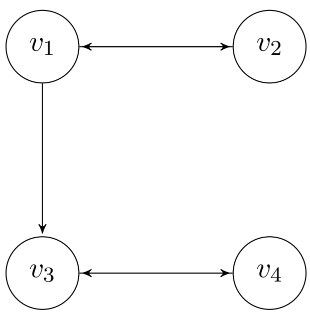
CMPSC 465 | Data Structures & Algorithms
Spring 2022 | Chunhao Wang and Mingfu Shao

Quiz 2 (Section 1)

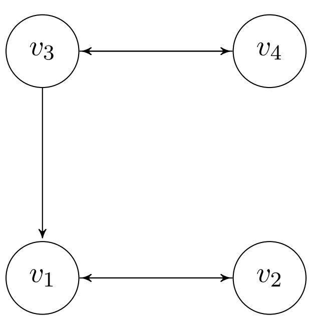
1. (1 pts.) There are 3 possibilities: C_1 can reach C_2, or C_2 can reach C_1, or neither of them. For the third case, the two components are independent, so the post values will be either $(v_1/v_2, v_3/v_4)$ or $(v_3/v_4, v_1/v_2)$. This case includes (c). Specifically, if we run DFS in the order of (v_3, v_4, v_1, v_2) on the graph below, we can get the postlist (v_1, v_2, v_3, v_4).

For the first case, we know that the vertex with largest post value must be in C_1; statements (a) and (b) fall in this case. Running DFS on the graph given below starting from v_1, and when exploring v_1 it visits v_2 first, gives postlist (v_1, v_3, v_4, v_2).



Statement (b) is not possible.

For the second case, we know that the vertex with largest post value must be in C_2; statement (d) falls in this case. Running DFS on the graph given below starting from v_3, and when exploring v_3 it visits v_4 first, gives postlist (v_3, v_1, v_2, v_4).



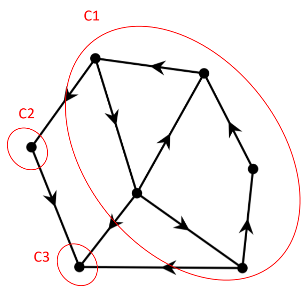
2. (1 pts.) With respect to R_4, we have $dist[b] = \min\{6+8, 3+10\} = 13$, $dist[c] = 9+5 = 14$, $dist[h] = 9+8 = 17$, $dist[j] = 9+3 = 12$. So vertex j is the 5-th closest vertex from s.

3. (1 pts.) Notice that there is a path $a \to b \to f \to e$. So the relative positions of these 4 vertices are fixed.

Vertex c can be either between a and b, or between b and f; in either case, which gives a list of 5 vertices, d can be placed in any of the 4 spaces in between. So, the total number of distinct linearization is 8.

4. (1 pts.) Statement (c) is not correct: the two meta-graphs have the same set of vertices but the direction of all edges are opposite.

5. (1 pts.) Three connected components in the graph, corresponding to three vertices in the meta-graph.



6. (1 pts.) Statement (A) is false. The key is that the number of edges in shortest paths may be different. Counter-example: $p = \{e_1 = 1, e_2 = 1\}$ is the shortest path from u to v, and we have another path $p' = \{e_3 = 3\}$ from u to v. If we increase the length of every edge by 2, the length of p becomes 6, the length of p' becomes 5, then p is no longer the shortest path from u to v.

Statement (B) is true. The key is that the length of *every* path is halved. So their relationship remains.

7. (1 pts.) We know that G and G_R have the same collection of connected components. The given algorithm is to find connected component of G_R.

8. (1 pts.) Statement (a) is false, as edge length may be negative. All other three statements are direct consequence of the optimal substructure property.

9. (1 pts.) Let X be a linearization of G. We can always modify X by moving s to the front and moving t to the end. The resulting ordering will be also a linearization of G. (The proof is to check for every edge (u, v), u will also be before v in the new ordering.)

10. (1 pts.) The corresponding graph is as the following. So, the number of connected components is 3.