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# Math 455 HW 12

**Total 16 points**

- The Honor Code is in effect for this HW. All work is to be your own.
- Be sure that your name is on the top of the page.

1. Solve $\begin{pmatrix} \theta & -1 & 1 \\ 1 & \theta & -1 \\ -1 & 1 & \theta \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} \theta \\ \theta \\ \theta \end{pmatrix}, x^0 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

| Stopping tolerance | # of iterations | | | |
|---|---|---|---|---|
| | Jacobi method | | Gauss-Seidel method | |
| $\theta$ | $\theta = 3$ | $\theta = 2$ | $\theta = 3$ | $\theta = 2$ |
| $10^{-5}$ | 1 | 1 | 16 | can't stop, too big |
| $10^{-10}$ | 1 | 1 | 31 | can't stop, too big |

2. Solve $\begin{pmatrix} 1+\delta & -1 & 0 \\ -1 & 2+\delta & -1 \\ 0 & -1 & 1+\delta \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix}, x^0 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

| $\delta$ | # of iterations with Stopping tolerance $10^{-5}$ | |
|---|---|---|
| | Jacobi method | Gauss-Seidel method |
| 1 | 21 | 11 |
| $10^{-1}$ | 158 | 67 |
| $10^{-2}$ | 1523 | 486 |
| $10^{-3}$ | 15165 | 3303 |
| 0 | can't stop, too big | 2 |

3. Solve $\begin{pmatrix} 10 & 1 & 1 \\ 1 & -10 & 1 \\ 1 & 1 & 10 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, x^0 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

| Stopping tolerance | # of iterations for SOR method | | | |
|---|---|---|---|---|
| | $\omega = 1.1$ | $\omega = 1.4$ | $\omega = 1.6$ | $\omega = 1.9$ |
| $10^{-5}$ | 8 | 19 | 38 | NaN |

| | | | | |
|---|---|---|---|---|
| $10^{-10}$ | 14 | 35 | 29 | NaN |

4. Solve $(R + \lambda I)x = b$, where $R$ is a randomized matrix, I is the identity matrix and b is a randomized vector. (Hint: R=rand(n); I=eye(n);b=rand(n,1);)

| Size n | # of iterations | | | |
|---|---|---|---|---|
| | Jacobi method | | Gauss Seidel method | |
| | $\lambda = 100$ | $\lambda = 10$ | $\lambda = 100$ | $\lambda = 10$ |
| 10 | 4 | 12 | 3 | 6 |
| 100 | 13 | NaN | 6 | 26 |
| 1000 | NaN | NaN | 24 | 702 |