20210829 P7-8 2, 3, 6, 8

2. Find the binary representation of the base 10 numbers. (a) 1/8 (b) 7/8 (c) 35/16 (d) 31/64

(a) $\frac{1}{8}_{10}$ → binary

0	$\frac{1}{8}$	×2
0	$\frac{1}{4}$	×2
0	$\frac{1}{2}$	×2
1	1	

0.001_2

(b) 7/8 → binary

0	$\frac{7}{8}$	×2
1	$\frac{3}{4}$	
1	$\frac{1}{2}$	
	1	

0.111_2

(c) 35/16 → bin

2	$\frac{3}{16}$	×2
0	$\frac{3}{8}$	
0	$\frac{3}{4}$	
1	$\frac{1}{2}$	
	1	

10.0011_2

(d) 31/64 → bin

0	$\frac{31}{64}$	×2
0	$\frac{31}{32}$	
1	$\frac{15}{16}$	
1	$\frac{7}{8}$	
1	$\frac{3}{4}$	
1	$\frac{1}{2}$	
	1	

0.011111_2

3. Convert the following base 10 numbers to binary. Use overbar notation for nonterminating binary numbers. (a) 10.5 (b) 1/3 (c) 5/7 (d) 12.8 (e) 55.4 (f) 0.1

(a) $10.5 \rightarrow$ bin

$10 \rightarrow$ bin

$2 \mid 10$	0
$2 \mid 5$	1
$2 \mid 2$	0
$2 \mid 1$	1
0	

1010_2

0.5	×2	0.1_2
1		

$10.5_{10} \rightarrow 1010.1_2$

(b) $\frac{1}{3} \rightarrow$ bin

$0\ \frac{1}{3}$	×2
$0\ \frac{2}{3}$	
$1\ \frac{1}{3}$	
$0\ \frac{2}{3}$	
$1\ \frac{1}{3}$	

$0.0\dot{1}_2$

(c) $\frac{5}{7} \rightarrow$ bin

$0\ \frac{5}{7}$	×2
$1\ \frac{3}{7}$	
$0\ \frac{6}{7}$	
$1\ \frac{5}{7}$	
$1\ \frac{3}{7}$	

$0.\overline{101}_2$

(d) $12.8 \rightarrow$ bin

$2 \mid 12$	0
$2 \mid 6$	0
$2 \mid 3$	1
$2 \mid 1$	1
0	

1100_2

0.8	×2	$0.\overline{1100}_2$
1.6		
1.2		
0.4		
0.8		
1.6		
1.2		

$1100.\overline{1100}_2$

(e) 55.4 → bin

2 | 55	1
2 | 27	1
2 | 13	1
2 | 6	0
2 | 3	1
2 | 1	1
0	

110111_2

0.4	×2
0.8	
1.6	
1.2	
0.4	
0.8	
1.6	

$0.\overline{0110}_2$

$110111.\overline{0110}_2$

(f) 0.1 → bin

0.1	×2
0.2	
0.4	
0.8	
1.6	
1.2	
0.4	
0.8	
1.6	
1.2	

$0.0\overline{0011}_2$

6. Find the first 15 bits in the binary representation of e.

2.718281828459045235 36

0.7183	×2
1.4366	
0.8732	
1.7464	
1.4928	
0.9856	

e_{10}

$= 10.101101111100_2$

1.9712	x2
1.9424	
1.8848	
1.7696	
1.5392	
1.0784	
0.1568	
0.3136	
0.6272	
1.2544	

8. Convert the following binary numbers to base 10: (a) 11011 (b) 110111.001 (c) $111.\overline{001}$ (d) $1010.\overline{01}$ (e) $10111.\overline{10101}$ (f) $1111.010\overline{001}$

(a) $1\times2^0+1\times2^1+0\times2^2+1\times2^3+1\times2^4$

$= 1+2+8+16$

$= 27_{10}$

(b) $1\times2^5+1\times2^4+0\times2^3+1\times2^2+1\times2^1+1\times2^0+0\times2^{-1}+0\times2^{-2}$

$+1\times2^{-3}$

$= 55+\frac{1}{8}$

$= 55.125_{10}$

(c) $111.\overline{001}$

$111_2 \to 7_{10}$

$0.\overline{001}_2 \times 2^3$

$0\times2^2+0\times2^1+1\times2^0=1$

$\frac{1}{2^3-1}=\frac{1}{7}_{10}$

$7+\frac{1}{7}=\frac{50}{7}_{10}$

(d) $1010.\overline{01}$

$1010_2=10_{10}$

0.01×2^2

$0\times2^1+1\times2^0=1$

$10+\frac{1}{3}=\frac{31}{3}$

(e) $10111.\overline{10101}$

$10111_2=23_{10}$

$2x=1.\overline{0101}$

(f) $1111.010\overline{001}$

$1111_2=15_{10}$

$0.010\overline{001}\times2^3$

$z = 0.\overline{0101}$

$0\times2^3+1\times2^2+0\times2^1+1\times2^0 = 5$

$\frac{5}{2^4-1} = \frac{1}{3}$

$2x = 1+\frac{1}{3}$

$x = \frac{2}{3}$

$23+\frac{2}{3} = \frac{71}{3}_{10}$

$= 10.\overline{001}$

$z = 0.\overline{001} \quad \times 2^3$

$\frac{1}{7}$

$8x = \frac{15}{7}$

$x = \frac{15}{56}$

$15+x = \frac{840+15}{56} = \frac{855}{56}$