# Math456/CMPSC456 Homework 4

Due Feb 9, 2022

**1.** (15 points) For the weight function $w(x) = e^{-x}$ on the interval $[0, +\infty)$, determine the orthogonal polynomials $\phi_0(x), \phi_1(x)$, and $\phi_2(x)$.

**2.** (15 points) Computer project: find a least squares cubic polynomial approximation of $f(x) = \ln(x + 2)$ on the interval $[-1, 1]$,

$$\min \int_{-1}^{1} [f(x) - p_3(x)]^2 dx.$$

You may use composite 3-point Gaussian quadrature (or composite Trapezoid method) for the integrals. Express the polynomials using the Legendre polynomials and show the coefficients.

**3.** (10 points) Computer project: find a least square polynomial approximation of $f(x) = H(x)$ (the heaviside function) on the interval $[-1, 1]$,

$$\min \int_{-1}^{1} [f(x) - p_n(x)]^2 dx.$$

Use the recurrence relation to generate the orthogonal polynomials. Use the composite Gaussian quadrature (or composite Trapezoid method) to approximate the integrals. Show the graphs of approximating polynomials for $n = 10, 20, 30$. Compare them to the graph of $f(x)$.

**1.** (15 points) For the weight function $w(x) = e^{-x}$ on the interval $[0, +\infty)$, determine the orthogonal polynomials $\phi_0(x), \phi_1(x)$, and $\phi_2(x)$.

$\phi_0(x) = 1$

$\phi_1(x) = x + a$

$\phi_2(x) = x^2 + bx + c$

$$(\phi_0, \phi_1) = \int_0^\infty 1 \cdot (x+a) e^{-x} dx = 0$$

$$a + 1 = 0$$

$$a = -1$$

$$(\phi_0, \phi_2) = \int_0^\infty 1 (x^2 + bx + c) e^{-x} dx = 0$$

$$b + c + 2 = 0$$

$$(\phi_1, \phi_2) = \int_0^\infty e^{-x} (2-a)(x^2 + bx + c) dx = 0$$

$$(a+1)c + (a+2)b + 2a + b = 0$$

$$a = -1 \quad b = -4 \quad c = 2$$

$$\phi_0(x) = 1, \quad \phi_1(x) = x - 1, \quad \phi_2(x) = x^2 - 4x + 2$$

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%syms P3(a0,l0,a1,l1,a2,l2,a3,l3)
%P3(x1,x2)= x1.^2-x2.^2+2.*x2;

%a0=1.295836866

x=-1:0.01:1;
y1=log(x+2);
y2=0.6479*x.^3-0.5281223505*x.^2-0.0937411683*x+0.020022718;

plot(x,y1,x,y2);
```

2.