CSE 461: Programming Languages Concepts

Prof. G. Tan
Spring 2022

Homework 5
Due on Apr 13th at 6pm in Canvas.

Submission: Please submit your homework via Canvas. It's okay if you submit a scanned version of your on-paper answers, but please make sure your scanned version is legible.

1. (5 points) Answer the following questions based on the lambda-calculus term $(\lambda x.\ \lambda y.\ y\ x)\ (\lambda z.\ y)$.

   (a) (2 point) Calculate its free variables using the `FV` function we discussed in class. Show the steps. Note that "$y\ x$" stands for a function application calling y with argument x.

   (b) (2 point) Use lambda calculus reduction to reduce the term to its normal form. Begin by renaming bound variables and show every step.

   (c) (1 point) Describe what would go wrong if you did not rename bound variables.

2. (4 points) Reduce the following lambda-calculus terms to the normal form. Show all intermediate steps, with one beta reduction at a time. In the reduction, if necessary, assume that you are supplied with extra rules that allow you to reduce the addition of two natural numbers into the corresponding result.

   (a) $(\lambda f.\ \lambda x.\ f\ (f\ x))\ (\lambda y.\ 2 + y)\ 1$.

   (b) $(\lambda z.\ z)\ (\lambda y.\ y\ y)\ (\lambda x.\ x\ a)$.

3. (4 points) In class, we discussed the encoding of Church numerals, the successor function, and the addition function:

   $\text{SUCC} = \lambda n.\ \lambda f.\ \lambda x.\ f\ (n\ f\ x)$
   $\text{ADD} = \lambda n1.\ \lambda n2.\ n1\ \text{SUCC}\ n2$

   Reduce term "ADD 2 1" to the normal form; start by expanding the definition of ADD and show every step of reduction.