

HW13

by shi qiu

Gaussian elimination method and Gaussian elimination method with partial pivoting

```

1
2 - format long
3
4 - %clear all
5
6 - clc
7
8 - n=10;
9
10 - input_a=1;
11
12 - input_b=4;
13
14 - input_c=1;
15
16 - input_b_matrix=zeros(n,1);
17
18
19
20
21
22 - info=zeros(n);
23
24 - info(1,1)=input_b;
25
26 - info(1,2)=input_c;
27
28 - info(n,n-1)=input_a;
29
30 - info(n-1,n)=input_c;
31
32 - info(n,n)=input_b;
33
34 - input_b_matrix(1,1)=1;
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36 - input_b_matrix(n,1)=1;
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```

1
2     format long
3     %clear all
4     clc
5     n=10;
6     input_a=1;
7     input_b=4;
8     input_c=1;
9     input_b_matrix=zeros(n,1);
10
11     info=zeros(n);
12     info(1,1)=input_b;
13     info(1,2)=input_c;
14     info(n,n-1)=input_a;
15     info(n-1,n)=input_c;
16     info(n,n)=input_b;
17     input_b_matrix(1,1)=1;
18     input_b_matrix(n,1)=1;
19
20     %loop
21     for x=2:n-1
22         info(x,x)=input_b;
23         info(x,x-1)=input_a;
24         info(x,x+1)=input_c;
25         input_b_matrix(x,1)=1;
26     end
27
28
29
30     %info=[6 3 2; 6 4 3; 20 15 12];
31     %b=[6; 0; 0];
32     A=[info input_b_matrix];
33     for i=1:size(A,1)
34         for j=i+1:size(A,1)
35             key1=A(j,i)./A(i,i);
36             A(j,:)=A(j,:)-key1.*A(i,:);
37         end
38     end
39     x=zeros(1,size(info,2));
40     for i=size(A,1):-1:1
41         hg=sum(A(i,i+1:end-1).*(x(i+1:end)));
42         x(i)=(A(i,end)-hg)./A(i,i);
43     end
44
45     fprintf('Solution is x= %d \n ',x)

```

for $a=c=1, b=4$

$$n = 50$$
$$n = 250$$

$h = 10$

Solution is $x = 2.113246e-01$
Solution is $x = 1.547017e-01$
Solution is $x = 1.698686e-01$
Solution is $x = 1.658241e-01$
Solution is $x = 1.668352e-01$
Solution is $x = 1.668352e-01$
Solution is $x = 1.658241e-01$
Solution is $x = 1.698686e-01$
Solution is $x = 1.547017e-01$
Solution is $x = 2.113246e-01$
>>

[illegible][illegible]
$$X_0 = 1.66667e-01$$

```
Solution is x= 1.666667e-01
Solution is x= 1.666667e-01
Solution is x= 1.666667e-01
Solution is x= 1.666667e-01
Solution is x= 1.666666e-01
Solution is x= 1.666668e-01
Solution is x= 1.666663e-01
Solution is x= 1.666679e-01
Solution is x= 1.666622e-01
Solution is x= 1.666832e-01
Solution is x= 1.666050e-01
Solution is x= 1.668999e-01
Solution is x= 1.658075e-01
Solution is x= 1.698730e-01
Solution is x= 1.547005e-01
Solution is x= 2.113249e-01
```

they both have the same
output three conditions

```
MATLAB R2021a - academic use
主 页  绘图  APP  编辑器  发布  视图
新建  打开  保存  查找文件  转至  注释  插入  断点  运行  运行并前进  运行并计时
文件  导航  编辑  断点  运行

D:\OneDrive - The Pennsylvania State University\PSU\MATH 455\20211211 project13
D:\OneDrive - The Pennsylvania State University\PSU\MATH 455\20211211 project13\IterScript.m
IterScript.m  SOR.m  Jacobi.m  Gauss_seidel.m  M_20211211_Gaussian_elimination_method_self.m  M_20211211_partial_pivoting_method_self.m

1  options.MaxIter=1000;
2  options.Tol=1e-5;
3
4  clc
5  n=10;
6  input_a=1;
7  input_b=4;
8  input_c=1;
9
10
11
12  input_b_matrix=ones(n,1);
13  info=zeros(n);
14  x0=zeros(n,1);
15  info(1,1)=input_b;
16  info(1,2)=input_c;
17  info(n,n-1)=input_a;
18  info(n-1,n)=input_c;
19  info(n,n)=input_b;
20
21
22  %loop
23  for x=2:n-1
24      info(x,x)=input_b;
25      info(x,x-1)=input_a;
26      info(x,x+1)=input_c;
27  end
28
29  info;
30  input_b_matrix;
31  x0;
32
33
34  %x=Jacobi(C,c,x0,options)
35
36  %x=Gauss_seidel(C,c,x0,options)
37
38  %omega=1/2;
39  %x=SOR(A,b,omega,x0,options)
40
41  %omega=1.2;
42  %x=SOR(A,b,omega,x0,options)
43
44  %omega=1.5;
45  %x=SOR(A,b,omega,x0,options)
46  %D=[10 1 1:-10 1:1 10];
47  %d=[1:2:3];
48  %omega=1.1;
49  %x=SOR(D,d,omega,x0,options)
50  %omega=1.4;
51  %x=SOR(D,d,omega,x0,options)
52  %Lam=10;
53  %n=100;
54  %A=rand(n)+Lam*eye(n);
55  %b=rand(n,1);
56  fprintf('Jacobi');
57  x=Jacobi(info,input_b_matrix,x0,options)
58  fprintf('Gauss_seidel');
59  x=Gauss_seidel(info,input_b_matrix,x0,options)
60  fprintf('SOR');
61  % omega_vec = 0.01:0.01:1.99;
62  % min_iter = 999;
63  % for w = omega_vec
64  %     iter = SOR(info,input_b_matrix,w,x0,options);
65  %     if iter < min_iter
66  %         min_iter = w;
67  %     end
68  % end
69  % min_iter
70  w=1
71  x=SOR(info,input_b_matrix,w,x0,options);
72
73
74
75  %x=SOR
76  %x=Gauss_seidel(A,b,x0,options);
77
```

} tol

} a,b,c

} matrix generate

old stuff

3 methods

← omega

by comparison, G-s and SOR both only have to only do 9 iterations

output

```
Jacobi1-th: Residual is 1.4577
2-th: Residual is 0.69034
3-th: Residual is 0.32924
4-th: Residual is 0.15751
5-th: Residual is 0.075467
6-th: Residual is 0.036182
7-th: Residual is 0.017353
8-th: Residual is 0.0083237
9-th: Residual is 0.0039929
10-th: Residual is 0.0019155
11-th: Residual is 0.00091893
12-th: Residual is 0.00044084
13-th: Residual is 0.00021149
14-th: Residual is 0.00010146
15-th: Residual is 4.8675e-05
16-th: Residual is 2.3352e-05
17-th: Residual is 1.1203e-05

x =

    0.211324792937376
    0.154702164116316
    0.169869155040942
    0.165824805735610
    0.166835982876364
    0.166835982876364
    0.165824805735610
    0.169869155040942
    0.154702164116316
    0.211324792937376

Gauss_seidel1-th: Residual is 0.5968
2-th: Residual is 0.11327
3-th: Residual is 0.021547
4-th: Residual is 0.0042455
5-th: Residual is 0.00089928
6-th: Residual is 0.00021759
7-th: Residual is 6.0006e-05
8-th: Residual is 1.8044e-05
9-th: Residual is 5.636e-06

x =

    0.211324965464883
    0.154700622340897
    0.169870014044136
    0.165822729417414
    0.166836140762825
    0.166834622399506
    0.165824349483486
    0.169868431488059
    0.154701762413253
    0.211324559396687
```

```
SOR
w =

    1

1-th: Residual is 0.5968
2-th: Residual is 0.11327
3-th: Residual is 0.021547
4-th: Residual is 0.0042455
5-th: Residual is 0.00089928
6-th: Residual is 0.00021759
7-th: Residual is 6.0006e-05
8-th: Residual is 1.8044e-05
9-th: Residual is 5.636e-06
```

for $n=50$ and $n=250$, $\omega=1$,

both G-S and SOR still
have the same 9 iterations.

for $a=c=-1$ $b=2$

Gaussian elimination method and Gaussian elimination method with partial pivoting

```
Solution is x= 5.000000e+00
Solution is x= 9.000000e+00
Solution is x= 1.200000e+01
Solution is x= 14
Solution is x= 15
Solution is x= 1.500000e+01
Solution is x= 14
Solution is x= 12
Solution is x= 9
Solution is x= 5
```

$n=10$

```
Solution is x= 2.500000e+01
Solution is x= 4.900000e+01
Solution is x= 7.200000e+01
Solution is x= 9.400000e+01
Solution is x= 1.150000e+02
Solution is x= 1.350000e+02
Solution is x= 1.540000e+02
Solution is x= 1.720000e+02
Solution is x= 1.890000e+02
Solution is x= 2.050000e+02
Solution is x= 2.200000e+02
Solution is x= 2.340000e+02
Solution is x= 2.470000e+02
Solution is x= 2.590000e+02
Solution is x= 2.700000e+02
Solution is x= 2.800000e+02
Solution is x= 2.890000e+02
Solution is x= 2.970000e+02
Solution is x= 3.040000e+02
Solution is x= 3.100000e+02
Solution is x= 3.150000e+02
Solution is x= 3.190000e+02
Solution is x= 3.220000e+02
Solution is x= 3.240000e+02
Solution is x= 3.250000e+02
Solution is x= 3.250000e+02
Solution is x= 3.240000e+02
Solution is x= 3.220000e+02
Solution is x= 3.190000e+02
Solution is x= 3.150000e+02
Solution is x= 3.100000e+02
Solution is x= 3.040000e+02
Solution is x= 2.970000e+02
Solution is x= 2.890000e+02
Solution is x= 2.800000e+02
Solution is x= 2.700000e+02
Solution is x= 2.590000e+02
Solution is x= 2.470000e+02
Solution is x= 2.340000e+02
Solution is x= 2.200000e+02
Solution is x= 2.050000e+02
Solution is x= 1.890000e+02
Solution is x= 1.720000e+02
Solution is x= 1.540000e+02
Solution is x= 1.350000e+02
Solution is x= 1.150000e+02
Solution is x= 9.400000e+01
Solution is x= 7.200000e+01
Solution is x= 4.900000e+01
Solution is x= 2.500000e+01
```

$n=50$

```
Solution is x= 1.250000e+02
Solution is x= 2.490000e+02
Solution is x= 3.720000e+02
Solution is x= 4.940000e+02
Solution is x= 6.150000e+02
Solution is x= 7.350000e+02
Solution is x= 8.540000e+02
Solution is x= 9.720000e+02
Solution is x= 1.089000e+03
Solution is x= 1.205000e+03
Solution is x= 1.320000e+03
Solution is x= 1.434000e+03
Solution is x= 1.547000e+03
Solution is x= 1.659000e+03
Solution is x= 1.770000e+03
Solution is x= 1.880000e+03
Solution is x= 1.989000e+03
Solution is x= 2.097000e+03
Solution is x= 2.204000e+03
Solution is x= 2.310000e+03
Solution is x= 2.415000e+03
```

```
Solution is x= 1.880000e+03
Solution is x= 1.770000e+03
Solution is x= 1.659000e+03
Solution is x= 1.547000e+03
Solution is x= 1.434000e+03
Solution is x= 1.320000e+03
Solution is x= 1.205000e+03
Solution is x= 1.089000e+03
Solution is x= 9.720000e+02
Solution is x= 8.540000e+02
Solution is x= 7.350000e+02
Solution is x= 6.150000e+02
Solution is x= 4.940000e+02
Solution is x= 3.720000e+02
Solution is x= 2.490000e+02
Solution is x= 1.250000e+02
```

$n=250$

1.59
1.89
1.98

$$n = 10$$

$$\text{Jacobi} = 289 \text{ th}$$

$$\text{Gauss seidel} = 154 \text{ th}$$

$$\text{SOR} = 28 \text{ th}$$

$$n = 50$$

$$\text{Jacobi} = 6681 \text{ th}$$

$$\text{Gauss seidel} = 3524 \text{ th}$$

$$\text{SOR} = 145 \text{ th}$$

$$n = 250$$

$$\text{Jacobi} = 172060 \text{ th}$$

$$\text{Gauss seidel} = 90455 \text{ th}$$

$$\text{SOR} = 841 \text{ th}$$

$$a=b=c=1$$

```
Solution is x= NaN  
Solution is x= NaN  
Solution is x= NaN  
Solution is x= NaN  
Solution is x= NaN  
Solution is x= NaN  
Solution is x= NaN  
Solution is x= NaN  
Solution is x= NaN  
Solution is x= NaN  
fx >>
```

$n=10$ and same with $n=50$ & $n=250$
 $x = NaN$

for

$n=10, 50$ & 250

both Jacobi, Gauss seidel and SOR can not figure out