

Math456/CMPSC456 Practice Midterm Exam

Name:

1.(25 points) Let $f(x_1, x_2) = (x_1 - 2)^4 + (x_1 - 2)^2 x_2^2 + (x_2 + 1)^2$. Starting with $(1, 1)$, determine the approximation after one step of the Newton's iteration for the minimization problem, $\min f(x_1, x_2)$.

2.(25 points) Solve the least-square problem,

$$\begin{bmatrix} 2 & 0 \\ -1 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}.$$

Determine the square error.

2.(25 points) Solve the least-square problem,

$$\min \int_0^1 [\sqrt{x} - p_1(x)]^2 dx,$$

where $p_1(x)$ is a polynomial of degree at most 1.

3. (25 points) Apply the power method to the matrix,

$$A = \begin{bmatrix} 4 & -5 \\ 2 & -3 \end{bmatrix},$$

with initial vector,

$$\vec{x} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

Use $\|\bullet\|_\infty$ to normalize the approximate eigenvectors. Show the approximate eigenvalue and eigenvector after two steps. Compute the residual error at each step.