Packet 2: Functions of Random Variables

Chap 5.5 Random Variables related with Normal distributions

Normal distribution (Gaussian distribution) is originally found by observing that mean of sample offen follows a special bell shaped distribution.

$X \sim N(\mu, \sigma^2)$, $E(X) = \mu$, $Var(X) = \sigma^2$, has p.d.f. and m.g.f.

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, M_X(t) = e^{\mu t + \frac{\sigma^2 t^2}{2}}$$

Theorem 5.5-1: If X_1, X_2, ... X_n are independent random variables with $X_i \sim N(\mu_i, \sigma_i^2)$, then $Y = \sum_{i=1}^n c_i X_i \sim N(\sum_{i=1}^n \mu_i, \sum_{i=1}^n \sigma_i^2)$.

Corollary 5.5-1: If X_1, X_2, ... X_n are independent random variables with $X_i \sim N(\mu, \sigma^2)$, then $\bar{X} \sim N(\mu, \sigma^2/n)$.

Theorem 5.5-2: If X_1, X_2, ... X_n are independent random variables with $X_i \sim N(\mu, \sigma^2)$, then the sample mean $\bar{X}$ and the sample variance $S^2 = \frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n-1}$ are independent,

$$\frac{S^2(n-1)}{\sigma^2} = \frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{\sigma^2} \sim \chi^2(n-1)$$

Theorem 5.5-3: Student's t distribution $T = \frac{Z}{\sqrt{U/r}} \sim t(r)$, where $Z \sim N(0,1)$ and $U \sim \chi^2(r)$. If X_1, X_2, ... X_n are independent random variables with $X_i \sim N(\mu, \sigma^2)$, then

$$T = \frac{\bar{X} - \mu}{S/\sqrt{n}} \sim t(n-1)$$

Chap 5.6 The Central Limit Theorem (CLT)

CLT tells us that, with sufficiently many i.i.d. samples collected, the sample mean $\bar{X}$ follows $N(\mu, \sigma^2/n)$ approximately, regardless the true distribution of X_i.