Stat/Math 415 Midterm 2 Practice Problems

The exam covers Confidence Interval and Hypothesis Test for proportion and mean parameters, Sample Size, Regression

You may use a calculator, a hard copy of formula sheet provided on Canvas, *Useful Formula.pdf*, and one page of notes (double sided). You may not use any book or share a calculator with another student without permission of the instructor.

To earn partial credit, explain your arguments carefully and show all your work. If you need extra paper then please write on the back of each page.

Midterm exam #2 will have 4 questions.

1. To determine the percentage of customer satisfaction for a certain product, we will randomly select some customers who purchased the product and ask for their satisfaction (for simplicity, assume their answers are either yes or no). Suppose we need the estimate to be within ε=0.05 of the true percentage of satisfaction with 90% confidence, how many customers we need to survey?

   Textbook example 7.4-3

2. A university official is interested in testing whether the proportions of males and females at the university are the same. To test her hypothesis, she takes a random sample of 100 students — 47 are males and 53 are females.

a) Using statistical notation to specify the null and alternative hypotheses.
b) Test the null hypothesis at $\alpha = 0.05$ significance level. You need to find an appropriate test statistic, identify the critical region, and then make the decision.
c) Calculate the p-value of the test statistic.

Sol: (a) H0 p=0.5, H1 p!=0.5
(b) Z = -0.6, critical region |Z|>= 1.96, do not reject
(c) p-value = P(|Z|>0.6) = 2P(Z>0.6) = 2 x 0.274 = 0.548

3. The mean number of angina attacks among patients is around 1.3 per week. It is hoped that a new drug will reduce this figure. A random sample of 20 patients who are using the new drug has a mean of 0.8 attacks per week with a standard deviation of 0.83 attacks per week.

a) Using statistical notation to specify the null and alternative hypotheses.

b) Test the null hypothesis at $\alpha = 0.01$ significance level.

c) A one-sided upper bound for μ is $[0, \bar{x} + t_\alpha(n-1)\frac{S}{\sqrt{n}}]$. Illustrate how to use this confidence interval to make a decision for the above hypothesis test.

Sol: (a) H0: mu=1.3 v.s. H1: mu<1.3
(b) one sample, one side, T-test, T = -2.694 < -t0.01(19) = -2.539, reject H0
(c) [0, 1.271] does not cover 1.3, so we believe the number of attacks are reduced.

4. Let $X_1, ..., X_n$ be an i.i.d. random sample from $N(\mu, 100)$. Suppose we decide to reject $H_0 : \mu = 80$ in favor of $H_1 : \mu > 80$ if $\bar{x} \geq 83$, where $\bar{x}$ is the sample mean with sample size *n* = 25.

a) What is the probability that we will make a type I error for the test?

b) If the true value of μ is 85, what is the probability that we will make a type II error for the test?

Sol: (a) 0.0668
(b) 0.1587

5. The ACT composite score and the first year college GPA are given in the following table for 9 students.

ACT:	23	29	15	23	28	27	23	26	20
GPA:	2.40	3.10	2.51	2.98	3.93	1.91	1.69	2.88	2.60

a) Calculate the least square regression line for these data.

b) Find a 95% CI for α, β

c) Suppose a new student has ACT score 17, calculate his/her expected GPA score. Find the 95% CI for his/her expected GPA and then find the 95% prediction interval for his/her GPA.

Sol: (a) y = 2.666 + 0.052 (x – 23.78) = 1.430 + 0.052 x
(b) alpha: [2.141, 3.192], beta: [-0.076, 0.180]
(c) 2.314 is the expected GPA