Graphs

Directed Graphs and Undirected Graphs

A graph is usually denoted as $G=(V,E)$, where V represents vertices, and E represents edges. There are two types of graphs, directed ones and undirected ones (see Figures below).

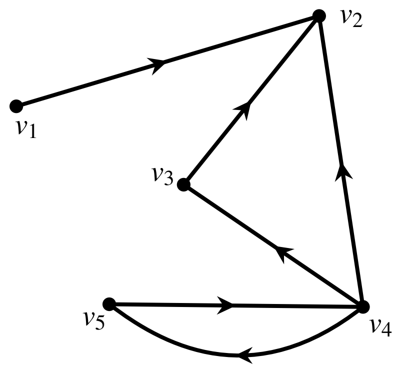


Figure 1: A directed graph $G=(V,E)$, where $V=\{v_1,v_2,v_3,v_4,v_5\}$, and $E=\{(v_1,v_2),(v_3,v_2),(v_4,v_3),(v_4,v_2),(v_4,v_5),(v_5,v_4)\}$.

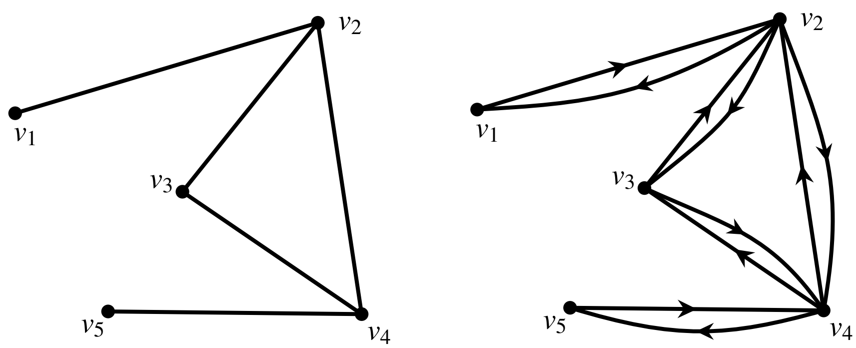


Figure 2: Left: a undirected graph $G=(V,E)$, where $V=\{v_1,v_2,v_3,v_4,v_5\}$, and $E=\{(v_1,v_2),(v_3,v_2),(v_4,v_3),(v_4,v_2),(v_4,v_5)\}$. Right: the corresponding directed graph of G in which each (undirected) edge is replaced with two edges connecting the same two vertices with opporsite directions.

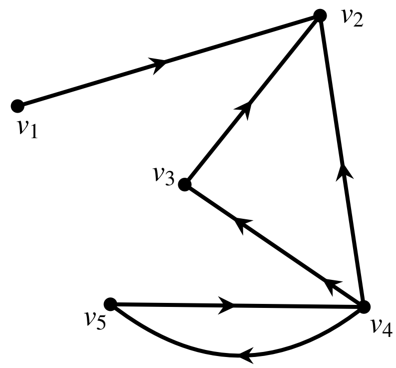
Notice that for an edge (v_i,v_j) in an undirected graph, the order of v_i and v_j are interchangable, i.e., $(v_i,v_j)=(v_j,v_i)$. In directed graph this is not the case.

An undirected graph $G=(V,E)$ can be transformed into a directed graph $G'=(V,E')$ by replacing an (undirected) edge $(v_i,v_j)\in E$ as two (directed) edges (v_i,v_j) and (v_j,v_i) in E'.

Representations of Graphs

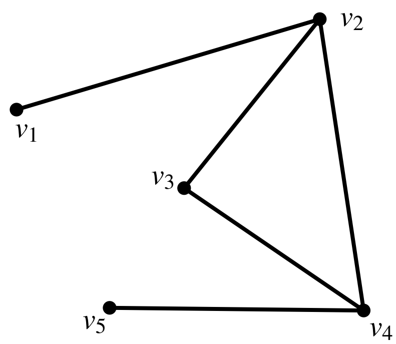
Adjacency matrix and adjacency list are two commonly-used data structures to represent a graph. Adjacency matrix uses a binary matrix M of size $|V|\times|V|$ to store a graph $G=(V,E)$: $M[i,j]=1$ if and only if

$(v_i, v_j) \in E$. For undirected graphs, M is symmetric, and the number of "1"s in M is exactly $2|E|$. For directed graphs, the number of "1"s in M is exactly $|E|$.



	v_1	v_2	v_3	v_4	v_5
v_1	0	1	0	0	0
v_2	0	0	0	0	0
v_3	0	1	0	0	0
v_4	0	1	1	0	1
v_5	0	0	0	1	0

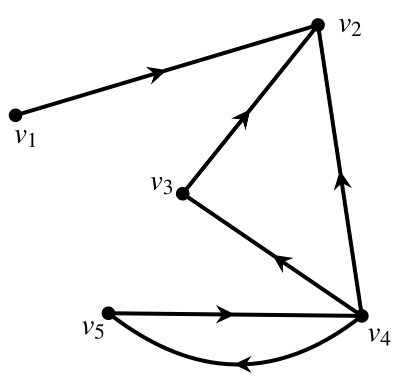
Figure 3: Adjacency matrix representation (directed graph.



	v_1	v_2	v_3	v_4	v_5
v_1	0	1	0	0	0
v_2	1	0	1	1	0
v_3	0	1	0	1	0
v_4	0	1	1	0	1
v_5	0	0	0	1	0

Figure 4: Adjacency matrix representation (undirected graph).

Adjacency list maintains a list/array A_i for each vertex $v_i \in V$, where A_i stores $\{v_j \in V \mid (v_i, v_j) \in E\}$, i.e., the "neighbors" of v_i. For undirected graph, $\sum_{v_i \in V} |A_i| = 2|E|$. For directed graph, $\sum_{v_i \in V} |A_i| = |E|$.



$v_1 \to (v_2)$

$v_2 \to ()$

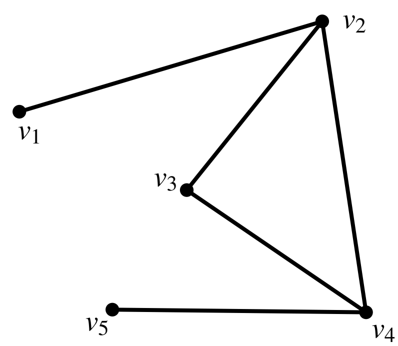
$v_3 \to (v_2)$

$v_4 \to (v_2,\ v_3,\ v_5)$

$v_5 \to (v_4)$

Figure 5: Adjacency list representation (directed graph).

Think: which one is better, adjacency matrix or adjacency list?



$v_1 \to (v_2)$

$v_2 \to (v_1,\ v_3,\ v_4)$

$v_3 \to (v_2,\ v_4)$

$v_4 \to (v_2,\ v_3,\ v_5)$

$v_5 \to (v_4)$

Figure 6: Adjacency list representation (undirected graph).