Math456/CMPSC456 Homework 3

Due Feb 2 2022

1. (10 points) Fit the data $(t, y) = \{(0, 1), (1/4, 3), (1/2, 2), (3/4, 0)\}$ to the periodic model $y = c_1 + c_2 \cos 2\pi t + c_3 \sin 2\pi t$. Find the square error.

2. (10 points) Find the QR decomposition of the following matrix using the Gram-Schmidt method,

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 9 \\ 1 & 9 \\ 1 & 1 \end{bmatrix}.$$

3. (20 points)

Find the least squares polynomials of degrees 1, 2, and 3 for the data in the following table. Compute the error E in each case. Graph the data and the polynomials.

x_i	1.0	1.1	1.3	1.5	1.9	2.1
y_i	1.84	1.96	2.21	2.45	2.94	3.18

.

The error here refers to the square error.

1. (10 points) Fit the data $(t, y) = \{(0, 1), (1/4, 3), (1/2, 2), (3/4, 0)\}$ to the periodic model $y = c_1 + c_2 \cos 2\pi t + c_3 \sin 2\pi t$. Find the square error.

t	y
0	1
$\frac{1}{4}$	3
$\frac{1}{2}$	2
$\frac{3}{4}$	0

$y = c_1 + c_2 \cos 2\pi t + c_3 \sin 2\pi t$

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 2 \\ 0 \end{bmatrix}$$

A $\qquad$ $\vec{b}$

$$A^T = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix}$$

$$A^T A = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$A^T b = \begin{bmatrix} 6 \\ -1 \\ 3 \end{bmatrix}$$

$A^T A x = A^T b$

$$\begin{bmatrix} 4 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 6 \\ -1 \\ 3 \end{bmatrix}$$

$c_1 = \frac{3}{2}$

$c_2 = -\frac{1}{2}$

$c_3 = \frac{3}{2}$

$\hat{y} = \frac{3}{2} - \frac{1}{2}\cos 2\pi t + \frac{3}{2}\sin 2\pi t$

Square error

$$= \sqrt{\frac{1}{4}\sum_{i=1}^{4} (\hat{y} - y)}$$

$= 0$

2. (10 points) Find the QR decomposition of the following matrix using the Gram-Schmidt method,

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 9 \\ 1 & 9 \\ 1 & 1 \end{bmatrix}.$$

a_1 a_2

$\|a_1\| = 2$

$\|a_2\| = 12.81$

$q_1 = \frac{1}{\|a_1\|} \cdot a_1 = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$

$q_2' = a_2 - q_1^T \cdot a_2 \cdot q_1 = \begin{bmatrix} 1 \\ 9 \\ 9 \\ 1 \end{bmatrix} - \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 1 \\ 9 \\ 9 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \end{bmatrix} = \begin{bmatrix} -4 \\ 4 \\ 4 \\ -4 \end{bmatrix}$

$\|q_2'\| = \sqrt{16 \cdot 4} = 8$

$q_2 = \frac{1}{\|q_2'\|} q_2' = \begin{bmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ -\frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$

$A = QR$

$R^T Q^T Q R \vec{x} = R^T Q^T \vec{b}$

$R^T R \vec{x} = R^T Q^T \vec{b}$

$Q = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$ $R = \begin{bmatrix} 2 & 10 \\ 0 & 8 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 \\ 1 & 9 \\ 1 & 9 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{bmatrix} \cdot \begin{bmatrix} 2 & 10 \\ 0 & 8 \end{bmatrix}$

A QR

3. (20 points)

Find the least squares polynomials of degrees 1, 2, and 3 for the data in the following table. Compute the error E in each case. Graph the data and the polynomials.

x_i	1.0	1.1	1.3	1.5	1.9	2.1
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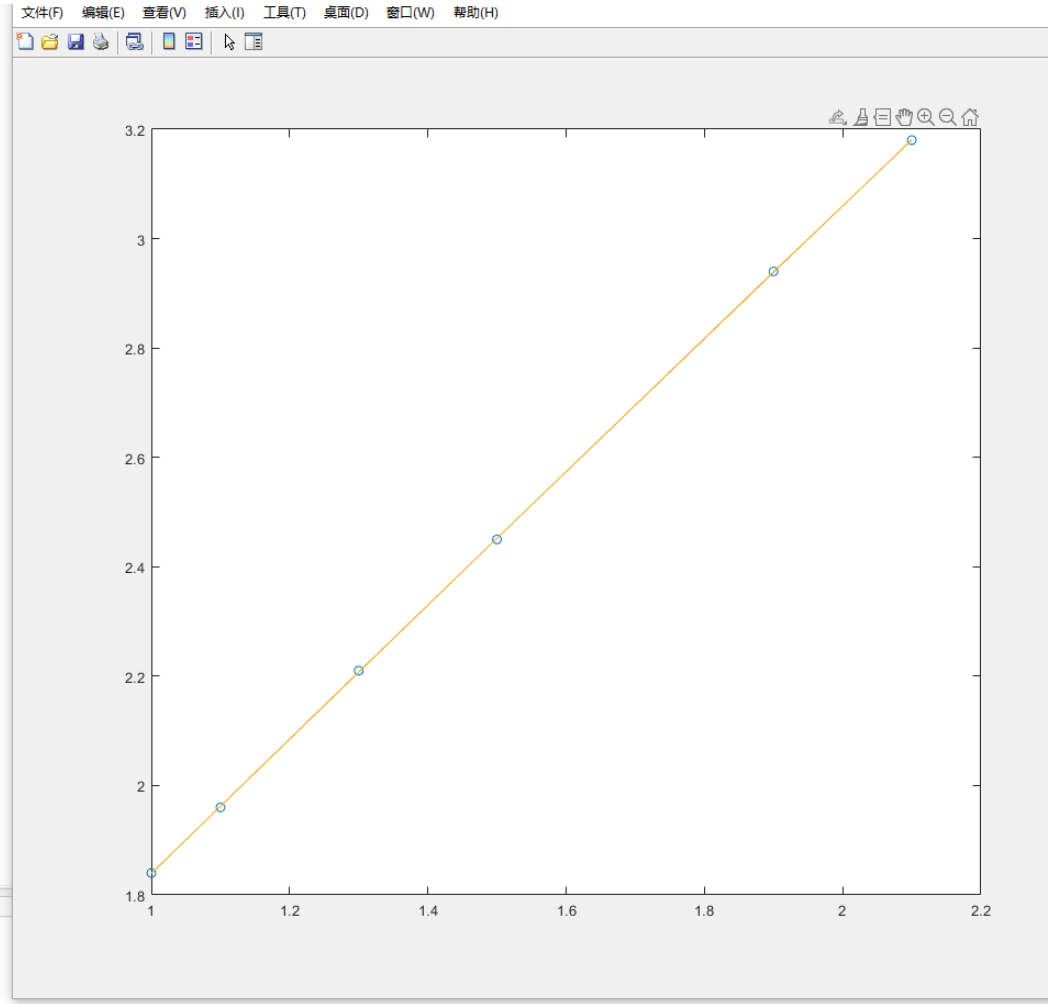
The error here refers to the square error.

```
x=[1.0; 1.1; 1.3; 1.5; 1.9; 2.1];
y=[1.84; 1.96; 2.21; 2.45; 2.94; 3.18];
tol=10^-6;

%plot(x,y,'o')

%L = lsqr(x,y,tol)
t2 = 0:0.1:3.5;

p0 = polyfit(x,y,1);
y1 = polyval(p0,x);
Epp1=y1-y;
Ep1=sum(Epp1.^2);
display(Ep1);



p1 = polyfit(x,y,2);


y2 = polyval(p1,x);
figure

Epp2=y2-y;
Ep2=sum(Epp2.^2);
display(Ep2);

%plot(x,y,'o',t2,y2)

p2 = polyfit(x,y,3);

y3 = polyval(p2,x);
figure

Epp3=y3-y;
Ep3=sum(Epp3.^2);
display(Ep3);

plot(x,y,'o',x,y2,x,y3);
```




```
命令行窗口
Ep1 =

   2.7194e-05


Ep2 =

   1.8015e-05


Ep3 =

   1.7407e-05

>>
```

error