1.a $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$

$x_0 = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} \rightarrow X = \begin{bmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ 1 \end{bmatrix} \leftarrow p = 3$

$y = Ax = \begin{bmatrix} \frac{3}{2} \\ \frac{1}{2} \\ 2 \end{bmatrix} \quad \mu_1 = 2 \quad \vec{x_1} = \begin{bmatrix} \frac{3}{4} \\ \frac{1}{4} \\ 1 \end{bmatrix}$

$\leftarrow p = 3$

$y = Ax_1 = \begin{bmatrix} \frac{11}{4} \\ \frac{9}{4} \\ 3 \end{bmatrix} \quad \vec{x_2} = \begin{bmatrix} \frac{11}{12} \\ \frac{3}{4} \\ 1 \end{bmatrix}$

$\mu_2 = 3$

$\leftarrow p = 3$

$y = Ax_2 = \begin{bmatrix} \frac{43}{12} \\ \frac{41}{12} \\ \frac{11}{3} \end{bmatrix} \quad X_3 = \begin{bmatrix} \frac{43}{44} \\ \frac{41}{44} \\ 1 \end{bmatrix}$

$\mu_3 = \frac{11}{3}$

$\leftarrow p = 3$

1.b. $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \quad x_0 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \rightarrow p \approx 3$

$y = Ax_0 = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}, \quad \mu_1 = 0, \quad p = 2, \quad \vec{x_1} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

$y = A\vec{x_1} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \quad \mu_2 = 1, \quad p = 2 \text{ (or 1)}, \quad \vec{x_2} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

$y = A\vec{x_2} = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}, \quad \mu_3 = 2, \quad p = 2 \text{ (or 1)}, \quad \vec{x_3} = \begin{bmatrix} 1 \\ 1 \\ \frac{1}{2} \end{bmatrix}$

2.a $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$ $X_0 = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$

$\|X_0\|_2 = \sqrt{6}$, $\vec{X}_0 = \frac{1}{\sqrt{6}} \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$

$\vec{y} = A\vec{X}_0 = \frac{1}{\sqrt{6}} \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}$, $\mu_1 = (\vec{X}_0, \vec{y}) = \frac{1}{6}(3 - 1 + 8) = \frac{5}{3}$

$\|\vec{y}\| = \frac{1}{\sqrt{6}} \sqrt{26}$, $\vec{X}_1 = \frac{1}{\sqrt{26}} \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}$

$\vec{y} = A\vec{X}_1 = \frac{1}{\sqrt{26}} \begin{bmatrix} 11 \\ 9 \\ 12 \end{bmatrix}$, $\mu_2 = (\vec{X}_1, \vec{y}) = \frac{1}{26}(33 + 9 + 48)$
$$= \frac{45}{13}$$

$\|\vec{y}\|_2 = \frac{1}{\sqrt{26}} \sqrt{121 + 81 + 144} = \frac{1}{\sqrt{26}} \sqrt{346}$

$\vec{X}_2 = \frac{1}{\sqrt{346}} \begin{bmatrix} 11 \\ 9 \\ 12 \end{bmatrix}$,

$\vec{y} = A\vec{X}_2 = \frac{1}{\sqrt{346}} \begin{bmatrix} 43 \\ 41 \\ 44 \end{bmatrix}$, $\mu_3 = \frac{1}{346}(43 \times 11 + 9 \times 41 + 12 \times 42)$
$$= \frac{1370}{346}$$

$\vec{X}_3 = \frac{1}{\sqrt{43^2 + 41^2 + 42^2}} \begin{bmatrix} 43 \\ 41 \\ 42 \end{bmatrix}$

$\underbrace{\hspace{2cm}}_{\sqrt{5466}}$

2.b

$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ $X_0 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ $\to$ $X_0 \frac{1}{\sqrt{2}} \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

$y = AX_0 = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$, $\Rightarrow \mu_1 = 0$, $\vec{X}_1 = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$

$$y = Ax_1 = \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix}, \quad \mu_2 = (\vec{x_1}, \vec{y}) = 1, \quad \vec{x_2} = \frac{1}{\sqrt{2}} \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix}$$

$$y = Ax_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} -2 \\ -2 \\ -1 \end{bmatrix}, \quad \mu_3 = (x_2, \vec{y}) = \frac{1}{2}(2 + 2 + 0) = 2$$

$$\Rightarrow \vec{x_3} = \frac{1}{3} \begin{bmatrix} -2 \\ -2 \\ -1 \end{bmatrix}$$

3. $\quad \lambda_1 \approx 5.2361$