5.4-4. Generalize Exercise 5.4-3 by showing that the sum of n independent Poisson random variables with respective means $\mu_1, \mu_2, \ldots, \mu_n$ is Poisson with mean

$$\mu_1 + \mu_2 + \cdots + \mu_n.$$

$$X_1(s) = e^{-\mu_1(1-s)}$$

$$\Pi_{X_3}(s) = e^{-\mu(1-s)}$$

$$\Pi_{X_1+X_2+\cdots X_n}(s) = \Pi_{X_1}(s)\,\Pi_{X_2}(s)\ \cdots\ \Pi_{X_n}(s)$$

$$= e^{-(\mu_1+\mu_2+\cdots\mu_n)(1-s)}$$

So sum of n independent Poisson random variables with respective means $\mu_1 \cdots \mu_n$ is Poisson with mean $\mu_1 + \mu_2 + \cdots \mu_n$

5.4-5. Let $Z_1, Z_2, \ldots, Z_7$ be a random sample from the standard normal distribution $N(0, 1)$. Let $W = Z_1^2 + Z_2^2 + \cdots + Z_7^2$. Find $P(1.69 < W < 14.07)$.

$$P(1.69 < W < 14.07) = P(W \geq 1.69) - P(W \geq 14.07)$$

$$= 0.975 - 0.05$$

$$= 0.925$$

5.4-14. The number of accidents in a period of one week follows a Poisson distribution with mean 2. The numbers of accidents from week to week are independent. What is the probability of exactly seven accidents in a given three weeks? HINT: See Exercise 5.4-4.

$3 \cdot 2 = 6$

$$p(x) = \frac{e^{-6} 6^{x}}{x!}$$

$x = 7$

$$p(x) = \frac{e^{-6} 6^{7}}{7!} = 0.1377$$

5.5-7. Suppose that the distribution of the weight of a prepackaged "1-pound bag" of carrots is $N(1.18, 0.07^2)$ and the distribution of the weight of a prepackaged "3-pound bag" of carrots is $N(3.22, 0.09^2)$. Selecting bags at random, find the probability that the sum of three 1-pound bags exceeds the weight of one 3-pound bag. HINT: First determine the distribution of Y, the sum of the three, and then compute $P(Y > W)$, where W is the weight of the 3-pound bag.

$P(Y > W)$

$Y = 3 \cdot$ 1pound bag

$= 3 \cdot X$

$E(Y) = E(X) + E(X) + E(X)$

$= 3.54$

$Var(Y) = Var(X) + Var(X) + Var(X) = 0.0147$

$E(Y) - E(W) = 3.54 - 3.22 = 0.32$

$Var(Y) - Var(W) = 0.0147 - 0.09^2 = 0.0228$

$P(Y > W) = P(Y - W > 0) = P\left(Z > \frac{0 - 0.32}{\sqrt{0.0228}}\right)$

$= P(Z > -2.12)$

$= 0.9830$

5.6-2. Let $Y = X_1 + X_2 + \cdots + X_{15}$ be the sum of a random sample of size 15 from the distribution whose pdf is $f(x) = (3/2)x^2$, $-1 < x < 1$. Using the pdf of Y, we find that $P(-0.3 \le Y \le 1.5) = 0.22788$. Use the central limit theorem to approximate this probability.

$f(x) = \frac{3}{2}x^2 \quad -1 < x < 1$

$P(-0.3 \le Y \le 1.5) = 0.22788$

size 15

$E(x) = \int_{-1}^{1} \frac{3}{2}x^3 \, dx$

$= \frac{3}{2} \frac{x^4}{4} \Big|_{-1}^{1}$

$= 0$

$E(x^2) = \int_{-1}^{1} \frac{3}{2}x^4 \, dx$

$= \frac{3}{2} \frac{x^5}{5} \Big|_{-1}^{1}$

$= \frac{3}{2} \cdot \frac{2}{5}$

$= \frac{6}{10} = \frac{3}{5}$

$Var(x) = E(x^2) - (E(x))^2$

$= \frac{3}{5}$

$$\sigma_{\bar{X}} = \frac{\sqrt{\frac{3}{5}}}{\sqrt{15}} = 0.2$$

$$P(-0.3 \le Y \le 1.5) = P(Y > -0.3) - P(Y > 1.5)$$

$$= P(Y \le 1.5) - P(Y < -0.3)$$

$$= P\left(\frac{Y}{n} < \frac{1.5}{15}\right) - P\left(\frac{Y}{n} < \frac{-0.3}{15}\right)$$

$$= P(\bar{X} < 0.1) - P(\bar{X} < -0.02)$$

$$= P\left(Z < \frac{0.1-0}{0.2}\right) - P\left(\le \frac{-0.02-0}{0.2}\right)$$

$$= P(Z \le 0.5) - P(Z \le -0.1)$$

$$= 0.6914 - 0.4601$$

$$= 0.2313$$

5.6-7. Let X equal the maximal oxygen intake of a human on a treadmill, where the measurements are in milliliters of oxygen per minute per kilogram of weight. Assume that, for a particular population, the mean of X is $\mu = 54.030$ and the standard deviation is $\sigma = 5.8$. Let $\overline{X}$ be the sample mean of a random sample of size $n = 47$. Find $P(52.761 \le \overline{X} \le 54.453)$, approximately.

Mean $= \mu = 54.030$

Var $= \sigma = 5.8$

Sample size $= 47$

$P(52.761 \le \bar{x} \le 54.453)$

$= P(\bar{x} \le 54.453) - P(\bar{x} \le 52.761)$

$= P\left(Z < \dfrac{54.453 - 54.030}{\left(\frac{5.8}{\sqrt{47}}\right)}\right) - P\left(Z \le \dfrac{52.761 - 54.030}{\left(\frac{5.8}{\sqrt{47}}\right)}\right)$

$= P(Z \le 0.5) - P(Z \le -1.50)$

$= 0.6915 - 0.0668$

$= 0.6247$