MATH 455: HOMEWORK 9

Problem 1 (1 pts). Find the forward and backward error for the following functions, where the root is 1/3 and the approximate root is $x_a = 0.3333$: (1) $f(x) = (3x-1)^3$ (2) $f(x) = (3x-1)^{1/3}$

Problem 2 (1 pts). (a) Find the multiplicity of the root $r = 0$ of $f(x) = 1 - \cos x$. (b) Find the forward and backward errors of the approximate root $x_a = 0.0001$.

Problem 3 (2 pts). (a) Find the multiplicity of the root $r = 0$ of $f(x) = x^2 \sin x^2$. (b) Find the forward and backward errors of the approximate root $x_a = 0.01$.

Problem 4 (2 pts). Let $f(x) = -x^3 - cos(x)$ and $x_0 = -1$. Use two steps of Newton's method to find x_2. Could $x_0 = 0$ be used?

Problem 5 (3 pts). Use Newton's method to solve the equation

$$\frac{1}{2} + \frac{1}{4}x^2 - xsin(x) - \frac{1}{2}cos(2x), \text{ with } x_0 = \frac{\pi}{2}.$$

(1) Do we have quadratic convergence?

(2) Assume r_1 is the root of $f(x)$, show that $f(x)$ has a double root at $r = r_1$.

(3) Do we have linear convergence? If so, please show the convergence rate?

Computer Problem (5 pts). Solve $f_1(x) = x - cos(x) = 0$ and $f_2(x) = x^2 - 2xcos(x) + cos^2(x) = 0$ both with initial guess $x_0 = 0$ by using Newton's mehtod and fill the following table.

P1 : 1) backward $= |f(3x-1)^3|$ $= |(3\cdot 0.3333-1)^3|$ $= (0.0001)^3$ $= 10^{-4^3}$ $= 10^{-12}$

forward $= |r - x_0|$ $= \frac{1}{3} - 0.3333$ $= 0.00003\ldots$ $= 3.\dot{3} \times 10^{-5}$

2) backward $= |f(x_a)|$ $= |(3\cdot 0.3333-1)^{\frac{1}{3}}|$ $= (10^{-4})^{\frac{1}{3}}$

forward $= |r - x_0|$ $= 3.3 \times 10^{-5}$

P 2: $f(0) = 1 - 1 = 0$, $f'(x) = \sin x$, $f'(0) = 0$

$f''(x) = \cos(x)$, $f''(0) = 1$, $m = 2$

backward $= |f(0.0001)|$ $= 1 - \cos(0.0001)$ $= 5\times 10^{-9}$

forward $= |r - x_1|$ $= |0 - 0.0001|$ $= 0.0001$

P3: $f(0) = 0$

$f'(x) = x^2 \cdot \cos(x^2)\cdot 2x + \sin x^2 \cdot 2x$ $= 2x^3\cos(x^2) + 2x\cdot\sin x^2$

$f'(0) = 0 + 0 = 0$

$f''(x) = \cos(x^2)\cdot 6x^2 - \sin(x^2)\cdot 2x\cdot 2x^3 + \sin(x^2)\cdot 2 + \cos(x^2)\cdot 2x\cdot 2x$ $= 2\sin x^2 - 4x^4\sin(x^2) + 10x^2\cos x^2$

$f''(0) = 0 - 0 + 0 = 0$

$f'''(x) = -36x^3\sin(x^2) + 24x\cos(x^2) - 8x^5\cos(x^2)$

$f'''(0) = 0$

$f^4(x) = 16x^6\sin(x^2) - 112x^4\cos(x^2) - 156x\sin(x^2) + 24\cos(x^2)$

$f^4(0) = 0 - 0 - 0 + 24(1)$ $= 24$

$m = 4$

backward $= |f(0.01)|$ $= 0.01^2\sin(0.01^2)$ $= 0.$ $= 9.99999998 \times 10^{-9}$

forward $= |r - x_a|$ $= 0.01$

P 4: $f'(x) = -3x^2 + \sin x$

$f'(-1) = -3 + (-0.84147)$ $= -3.84147$

$f'(0) = 0$

$x_n - \frac{f(x_n)}{f'(x_n)}$ $= -1 - \frac{1 - \cos(1)}{-3(-1)^2 + \sin(-1)}$

$= -0.880333$

$x_2 = -0.88033 - \frac{-0.88033 - \cos(-0.88033)}{\sin(-0.88033) - 3(-0.88033)}$

$= -0.86568$

x_0 can't be used

P5: $f'(x) = \frac{1}{2}x - \sin x - x\cos x + \sin 2x$

$f'(0) = 0$ we do not have quadratic convergence.

2) $\cos 2x = 1 - 2\sin^2 x$

$f(x) = \cancel{\frac{1}{2}} + \frac{1}{4}x^2 - x\sin x - \cancel{\frac{1}{2}} + \sin^2 x$

$= \frac{1}{4}x^2 - x\sin x + \sin^2 x$

$= (\frac{1}{2}x - \sin x)^2$

$f'(x) = 2(\frac{x}{2} - \sin x)(\cos x - \frac{1}{2})$

$r = r_1$ is the double root.

3). Yes

$$\lim_{n\to\infty} \frac{e_{n+1}}{e_n} = P = \frac{m-1}{m} = \frac{1}{2}$$

Stopping	$f_1(x) = 0$		$f_2(x) = 0$	
tolerance	# of iterations	Root	# of iterations	Root
10^{-5}	5	0.739085	8	0.737432
10^{-6}	5	0.739085	10	0.738672
10^{-8}	5	0.739085	13	0.739033
10^{-10}	6	0.739085	17	0.739082