HW13

by shi qiu

Code for

Gaussian elimination method and Gaussian elimination method with partial pivoting

```matlab
format long
%clear all
clc
n=10;
input_a=1;
input_b=4;
input_c=1;
input_b_matrix=zeros(n,1);

info=zeros(n);
info(1,1)=input_b;
info(1,2)=input_c;
info(n,n-1)=input_a;
info(n-1,n)=input_c;
info(n,n)=input_b;
input_b_matrix(1,1)=1;
input_b_matrix(n,1)=1;

%loop
for x=2:n-1
        info(x,x)=input_b;
        info(x,x-1)=input_a;
        info(x,x+1)=input_c;
        input_b_matrix(x,1)=1;
end



%info=[6 3 2; 6 4 3; 20 15 12];
%b=[6; 0; 0];
A=[info input_b_matrix];
for i=1:size(A,1)-1
    %partial pivoting
    [M,P]=(max(abs(A(i:size(A,1),i))));
    C=A(i,:);
    A(i,:)=A(P+i,:);
    A(P+i-1,:)=C;
    for j=i+1:size(A,1)
        key1=A(j,i)./A(i,i);
        A(j,:)=A(j,:)-key1.*A(i,:);
    end
end
x=zeros(1,size(info,2));
for i=size(A,1):-1:1
    hg=sum(A(i,i+1:end-1).*x(i+1:end));
    x(i)=(A(i,end)-hg)./A(i,i);
end

fprintf('Solution is x= %d \n ',x)
```

Handwritten annotations next to the code:

- for n=0, a=c=1, b=4 (next to the setup lines)
- generate matrix (next to the matrix-building lines)
- pivoting: compare each line by loop (next to the partial pivoting lines)
- Gaussian elimination (next to the inner elimination loop)
- ← give value (next to the back-substitution loop)
- ← output (next to the fprintf line)

```matlab
format long
%clear all
clc
n=10;
input_a=1;
input_b=4;
input_c=1;
input_b_matrix=zeros(n,1);

info=zeros(n);
info(1,1)=input_b;
info(1,2)=input_c;
info(n,n-1)=input_a;
info(n-1,n)=input_c;
info(n,n)=input_b;
input_b_matrix(1,1)=1;
input_b_matrix(n,1)=1;

%loop
for x=2:n-1
        info(x,x)=input_b;
        info(x,x-1)=input_a;
        info(x,x+1)=input_c;
        input_b_matrix(x,1)=1;
end



%info=[6 3 2; 6 4 3; 20 15 12];
%b=[6; 0; 0];
A=[info input_b_matrix];
for i=1:size(A,1)
    for j=i+1:size(A,1)
        key1=A(j,i)./A(i,i);
        A(j,:)=A(j,:)-key1.*A(i,:);
    end
end
x=zeros(1,size(info,2));
for i=size(A,1):-1:1
    hg=sum(A(i,i+1:end-1).*x(i+1:end));
    x(i)=(A(i,end)-hg)./A(i,i);
end

fprintf('Solution is x= %d \n ',x)
```

they both have the same output three conditions

for a=c=1, b=4

n=10

```
Solution is x= 2.113246e-01
 Solution is x= 1.547017e-01
 Solution is x= 1.698686e-01
 Solution is x= 1.658241e-01
 Solution is x= 1.668352e-01
 Solution is x= 1.668352e-01
 Solution is x= 1.658241e-01
 Solution is x= 1.698686e-01
 Solution is x= 1.547017e-01
 Solution is x= 2.113246e-01
 >>
```

n=50

命令行窗口

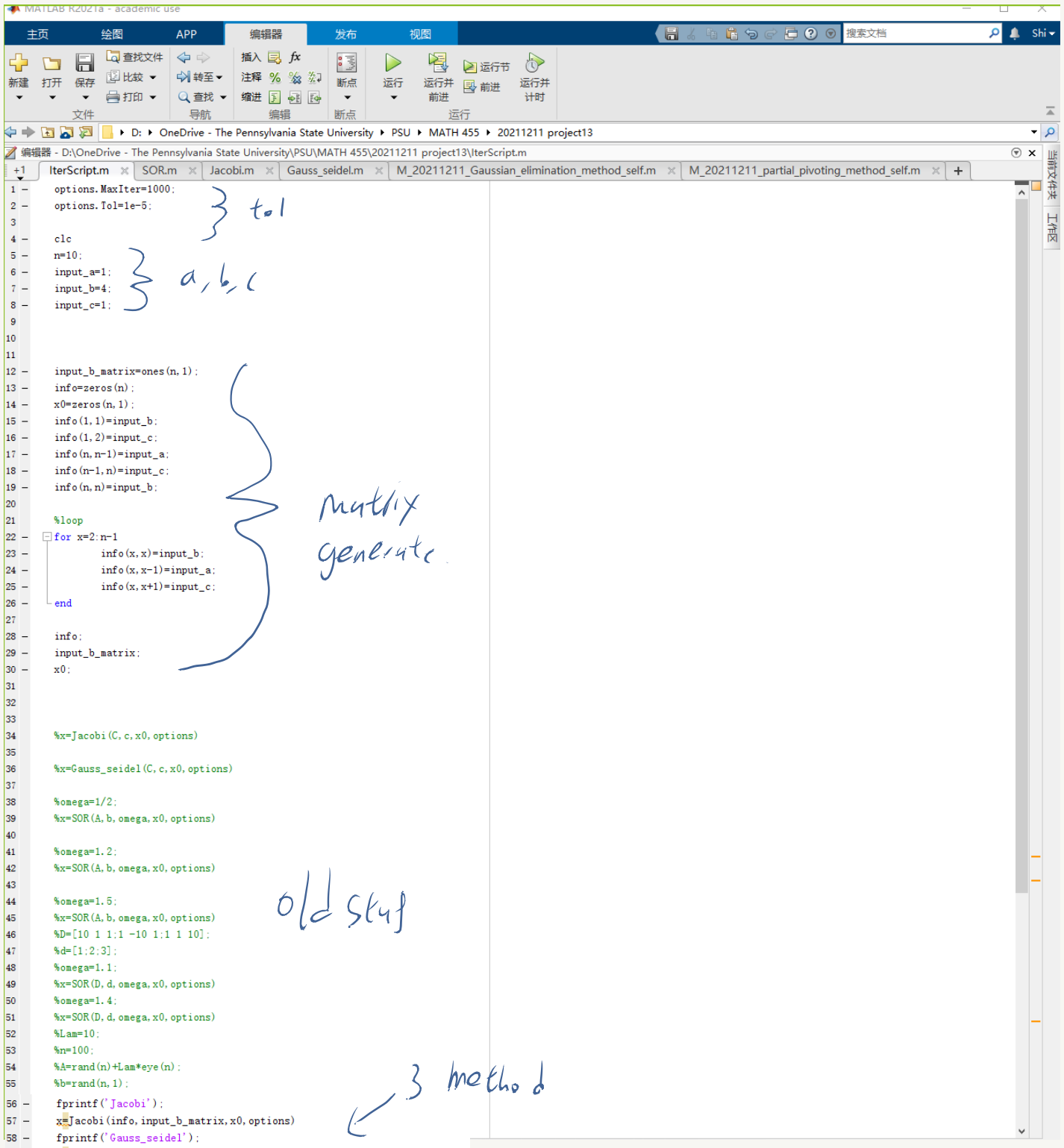
```
Solution is x= 2.113249e-01
 Solution is x= 1.547005e-01
 Solution is x= 1.698730e-01
 Solution is x= 1.658075e-01
 Solution is x= 1.668969e-01
 Solution is x= 1.666050e-01
 Solution is x= 1.666832e-01
 Solution is x= 1.666622e-01
 Solution is x= 1.666679e-01
 Solution is x= 1.666663e-01
 Solution is x= 1.666668e-01
 Solution is x= 1.666666e-01
 Solution is x= 1.666667e-01
 Solution is x= 1.666667e-01
 Solution is x= 1.666667e-01
 Solution is x= 1.666667e-01
 Solution is x= 1.666667e-01
 Solution is x= 1.666667e-01
 Solution is x= 1.666667e-01
 Solution is x= 1.666667e-01
 Solution is x= 1.666667e-01
 Solution is x= 1.666667e-01
 Solution is x= 1.666667e-01
 Solution is x= 1.666667e-01
 Solution is x= 1.666667e-01
 Solution is x= 1.666667e-01
 Solution is x= 1.666667e-01
 Solution is x= 1.666667e-01
 Solution is x= 1.666667e-01
 Solution is x= 1.666667e-01
 Solution is x= 1.666667e-01
 Solution is x= 1.666667e-01
 Solution is x= 1.666667e-01
 Solution is x= 1.666667e-01
 Solution is x= 1.666667e-01
 Solution is x= 1.666667e-01
 Solution is x= 1.666667e-01
 Solution is x= 1.666667e-01
 Solution is x= 1.666666e-01
 Solution is x= 1.666668e-01
 Solution is x= 1.666663e-01
 Solution is x= 1.666679e-01
 Solution is x= 1.666622e-01
 Solution is x= 1.666832e-01
 Solution is x= 1.666050e-01
 Solution is x= 1.668969e-01
 Solution is x= 1.658075e-01
 Solution is x= 1.698730e-01
 Solution is x= 1.547005e-01
 Solution is x= 2.113249e-01
fx >>
```

n=250

```
Solution is x= 2.113249e-01
 Solution is x= 1.547005e-01
 Solution is x= 1.698730e-01
 Solution is x= 1.658075e-01
 Solution is x= 1.668969e-01
 Solution is x= 1.666050e-01
 Solution is x= 1.666832e-01
 Solution is x= 1.666622e-01
 Solution is x= 1.666679e-01
 Solution is x= 1.666663e-01
 Solution is x= 1.666668e-01
 Solution is x= 1.666666e-01
 Solution is x= 1.666667e-01
 Solution is x= 1.666667e-01
 Solution is x= 1.666667e-01
 Solution is x= 1.666667e-01
 Solution is x= 1.666667e-01
 Solution is x= 1.666667e-01
 Solution is x= 1.666667e-01
 Solution is x= 1.666667e-01
 Solution is x= 1.666667e-01
 Solution is x= 1.666667e-01
 Solution is x= 1.666667e-01
 Solution is x= 1.666667e-01
 Solution is x= 1.666667e-01
 Solution is x= 1.666667e-01
 Solution is x= 1.666667e-01
 Solution is x= 1.666667e-01
 Solution is x= 1.666667e-01
```

x= 1.666667e-01

```
 Solution is x= 1.666667e-01
 Solution is x= 1.666667e-01
 Solution is x= 1.666667e-01
 Solution is x= 1.666667e-01
 Solution is x= 1.666666e-01
 Solution is x= 1.666668e-01
 Solution is x= 1.666663e-01
 Solution is x= 1.666679e-01
 Solution is x= 1.666622e-01
 Solution is x= 1.666832e-01
 Solution is x= 1.666050e-01
 Solution is x= 1.668969e-01
 Solution is x= 1.658075e-01
 Solution is x= 1.698730e-01
 Solution is x= 1.547005e-01
 Solution is x= 2.113249e-01
fx >>
```



output

```
Jacobi-th: Residual is 1.4577
2-th: Residual is 0.69034
3-th: Residual is 0.32924
4-th: Residual is 0.15751
5-th: Residual is 0.075467
6-th: Residual is 0.036182
7-th: Residual is 0.017353
8-th: Residual is 0.0083237
9-th: Residual is 0.0039929
10-th: Residual is 0.0019155
11-th: Residual is 0.00091893
12-th: Residual is 0.00044084
13-th: Residual is 0.00021149
14-th: Residual is 0.00010146
15-th: Residual is 4.8675e-05
16-th: Residual is 2.3352e-05
17-th: Residual is 1.1203e-05

x =

   0.211324792937376
   0.154702164116316
   0.169869155040942
   0.165824805735610
   0.166835982876364
   0.166835982876364
   0.165824805735610
   0.169869155040942
   0.154702164116316
   0.211324792937376
```

```
SOR
w =

     1

1-th: Residual is 0.5968
2-th: Residual is 0.11327
3-th: Residual is 0.021547
4-th: Residual is 0.0042455
5-th: Residual is 0.00089928
6-th: Residual is 0.00021759
7-th: Residual is 6.0006e-05
8-th: Residual is 1.8044e-05
9-th: Residual is 5.636e-06
```

```
Gauss_seidel1-th: Residual is 0.5968
2-th: Residual is 0.11327
3-th: Residual is 0.021547
4-th: Residual is 0.0042455
5-th: Residual is 0.00089928
6-th: Residual is 0.00021759
7-th: Residual is 6.0006e-05
8-th: Residual is 1.8044e-05
9-th: Residual is 5.636e-06

x =

   0.211324965464883
   0.154700622340897
   0.169870014044136
   0.165822729417414
   0.166836140762825
   0.166834622399506
   0.165824349483486
   0.169868431488059
   0.154701762413253
   0.211324559396687
```

by comparison, G-S and SOR both only have to only do 9 iterations.

For $n=50$ and $n=250$, omega $=1$,

both G-S and SOR still

have the same 9 iterations.

For $a=c=-1$ $b=2$

Gaussian elimination method and Gaussian elimination method with partial pivoting

```
Solution is x= 5.000000e+00
 Solution is x= 9.000000e+00
 Solution is x= 1.200000e+01
 Solution is x= 14
 Solution is x= 15
 Solution is x= 1.500000e+01
 Solution is x= 14
 Solution is x= 12
 Solution is x= 9
 Solution is x= 5
```

$n=10$

```
Solution is x= 2.500000e+01
 Solution is x= 4.900000e+01
 Solution is x= 7.200000e+01
 Solution is x= 9.400000e+01
 Solution is x= 1.150000e+02
 Solution is x= 1.350000e+02
 Solution is x= 1.540000e+02
 Solution is x= 1.720000e+02
 Solution is x= 1.890000e+02
 Solution is x= 2.050000e+02
 Solution is x= 2.200000e+02
 Solution is x= 2.340000e+02
 Solution is x= 2.470000e+02
 Solution is x= 2.590000e+02
 Solution is x= 2.700000e+02
 Solution is x= 2.800000e+02
 Solution is x= 2.890000e+02
 Solution is x= 2.970000e+02
 Solution is x= 3.040000e+02
 Solution is x= 3.100000e+02
 Solution is x= 3.150000e+02
 Solution is x= 3.190000e+02
 Solution is x= 3.220000e+02
 Solution is x= 3.240000e+02
 Solution is x= 3.250000e+02
 Solution is x= 3.250000e+02
 Solution is x= 3.240000e+02
 Solution is x= 3.220000e+02
 Solution is x= 3.190000e+02
 Solution is x= 3.150000e+02
 Solution is x= 3.100000e+02
 Solution is x= 3.040000e+02
 Solution is x= 2.970000e+02
 Solution is x= 2.890000e+02
 Solution is x= 2.800000e+02
 Solution is x= 2.700000e+02
 Solution is x= 2.590000e+02
 Solution is x= 2.470000e+02
 Solution is x= 2.340000e+02
 Solution is x= 2.200000e+02
 Solution is x= 2.050000e+02
 Solution is x= 1.890000e+02
 Solution is x= 1.720000e+02
 Solution is x= 1.540000e+02
 Solution is x= 1.350000e+02
 Solution is x= 1.150000e+02
 Solution is x= 9.400000e+01
 Solution is x= 7.200000e+01
 Solution is x= 4.900000e+01
 Solution is x= 2.500000e+01
```

$n=50$

```
Solution is x= 1.250000e+02
 Solution is x= 2.490000e+02
 Solution is x= 3.720000e+02
 Solution is x= 4.940000e+02
 Solution is x= 6.150000e+02
 Solution is x= 7.350000e+02
 Solution is x= 8.540000e+02
 Solution is x= 9.720000e+02
 Solution is x= 1.089000e+03
 Solution is x= 1.205000e+03
 Solution is x= 1.320000e+03
 Solution is x= 1.434000e+03
 Solution is x= 1.547000e+03
 Solution is x= 1.659000e+03
 Solution is x= 1.770000e+03
 Solution is x= 1.880000e+03
 Solution is x= 1.989000e+03
 Solution is x= 2.097000e+03
 Solution is x= 2.204000e+03
 Solution is x= 2.310000e+03
 Solution is x= 2.415000e+03
```

```
Solution is x= 1.880000e+03
Solution is x= 1.770000e+03
Solution is x= 1.659000e+03
Solution is x= 1.547000e+03
Solution is x= 1.434000e+03
Solution is x= 1.320000e+03
Solution is x= 1.205000e+03
Solution is x= 1.089000e+03
Solution is x= 9.720000e+02
Solution is x= 8.540000e+02
Solution is x= 7.350000e+02
Solution is x= 6.150000e+02
Solution is x= 4.940000e+02
Solution is x= 3.720000e+02
Solution is x= 2.490000e+02
Solution is x= 1.250000e+02
```

$n=250$

1.59

1.89

1.98

n = 10

Jacobi = 289 th

Gauss seidel = 154 th

SOR = 28 th

n = 50

Jacobi = 6681 th

Gauss seidel = 3524 th

SOR = 145 th

n = 250

Jacobi = 172060 th

Gauss seidel = 90455 th

SOR = 841 th

$a = b = c = 1$

```
Solution is x= NaN
 Solution is x= NaN
 Solution is x= NaN
 Solution is x= NaN
 Solution is x= NaN
 Solution is x= NaN
 Solution is x= NaN
 Solution is x= NaN
 Solution is x= NaN
fx >>
```

$n = 10$ and same with $n = 50$ & $n = 250$

$X = NaN$

for

$n = 10, 50$ & 250

both Jacobi, Gauss seidel and SOR can not figure out