1.

Set $\varphi_0(x) = 1$

$\varphi_1(x) = x + c$

$(\varphi_0, \varphi_1) = 0 \Rightarrow \int_0^{+\infty} e^{-x} x \, dx + c \int_0^{+\infty} e^{-x} dx = 0$

$\int_0^{+\infty} e^{-x} x \, dx = -e^{-x} x \Big|_0^{+\infty} + \int_0^{+\infty} e^{-x} dx = 1$

$\Rightarrow c = -1$

$\varphi_2(x) = x^2 + c_1 \varphi_1 + c_0 \varphi_0$

$(\varphi_2, \varphi_0) = 0 \Rightarrow c_0 = -\dfrac{(x^2, \varphi_0)}{(\varphi_0, \varphi_0)} = -2$

$(\varphi_1, \varphi_0) = 0 \Rightarrow c_1 = -\dfrac{(x^2, \varphi_1)}{(\varphi_1, \varphi_1)} = -4$

$\Rightarrow \varphi_2(x) = x^2 - 4x + 2$

2. $f(x) \approx P_3(x) = c_0\varphi_0 + c_1\varphi_1 + c_2\varphi_2 + c_3\varphi_3$

$\varphi_0 = 1, \ \varphi_1 = x, \ \varphi_2 = \frac{3}{2}x^2 - \frac{1}{2}, \ \varphi_3 = \frac{5}{2}x^3 - \frac{3}{2}x$

$c_0 = \dfrac{(f, \varphi_0)}{(\varphi_0, \varphi_0)} = \dfrac{\int_{-1}^{1} \ln(x+2)\, dx}{\int_{-1}^{1} dx} \approx 0.6479$

$c_1 = \dfrac{(f, \varphi_1)}{(\varphi_1, \varphi_1)} = \dfrac{\int_{-1}^{1} \ln(x+2)\, x \, dx}{\int_{-1}^{1} x^2 \, dx} \approx 0.5281$

$$c_2 = \frac{(f, \phi_2)}{(\phi_2, \phi_2)} \approx -0.0937$$

$$c_3 = \frac{(f, \phi_3)}{(\phi_3, \phi_3)} \approx 0.02$$