HW1 revision

score: 12/20

All 4 Quesgions.

E1 3/5

E2 3/5

P1 4/5

P2 2/5

Math 486

Lesson 1 Homework

Due Tues, May 24 at 11:59 on Gradescope

Instructions

Please refer to the solution guidelines posted on Canvas under Course Essentials.

Exercise 1.

Consider the following strategic situation. Three elected officials, A, B, and C are set to vote on whether to give themselves a pay increase. Each official can vote yes (y) or no (n) and the measure will pass if at least 2 of the 3 officials vote yes (y).

They cast their votes in order:

- A votes first.
- B observes A vote and then casts their vote.
- C observes the votes of A and B and then casts their vote.

- The pay increase has a value v to all three officials.
- Any official who votes yes will pay a (political) cost valued at c (this cost affects them whether or not the measure actually passes).
- Both v and c are positive parameters, with $v > c$.

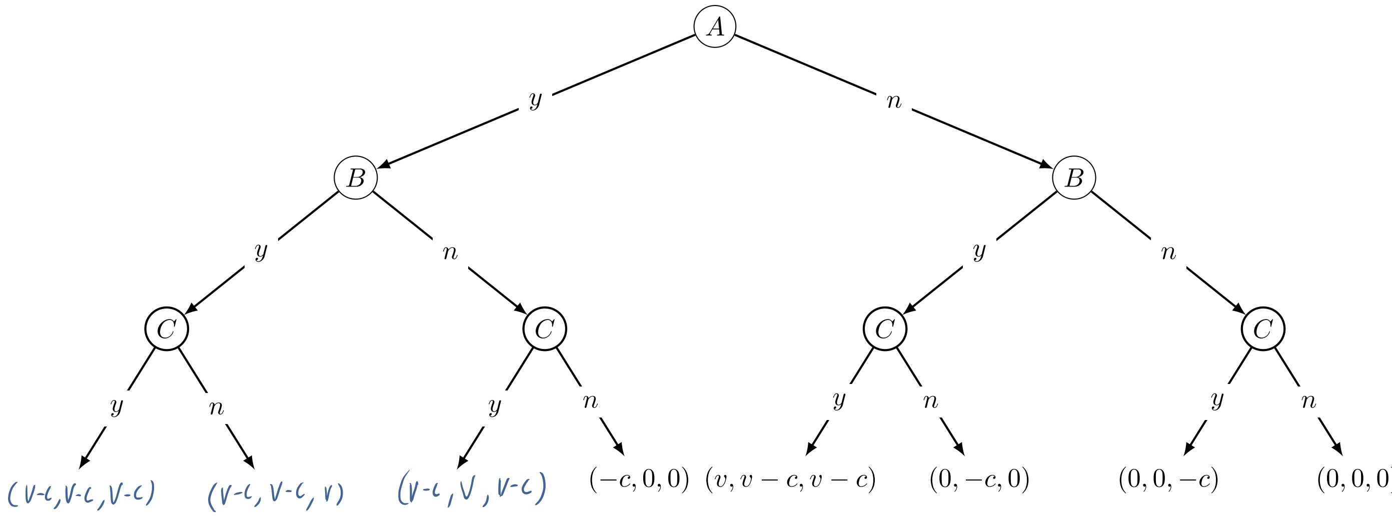
The extensive form game is shown below with payoffs listed in the order (A, B, C).

(a) Determine the payoffs for the three terminal nodes where no payoffs have been given.

(b) Determine the size of each player's strategy set. Please do not write out the strategy sets. Instead compute the size of each strategy set.

(c) Use backward induction to analyze the game and make a prediction about the outcome. Indicate the path through the game tree that will be taken according to your analysis.

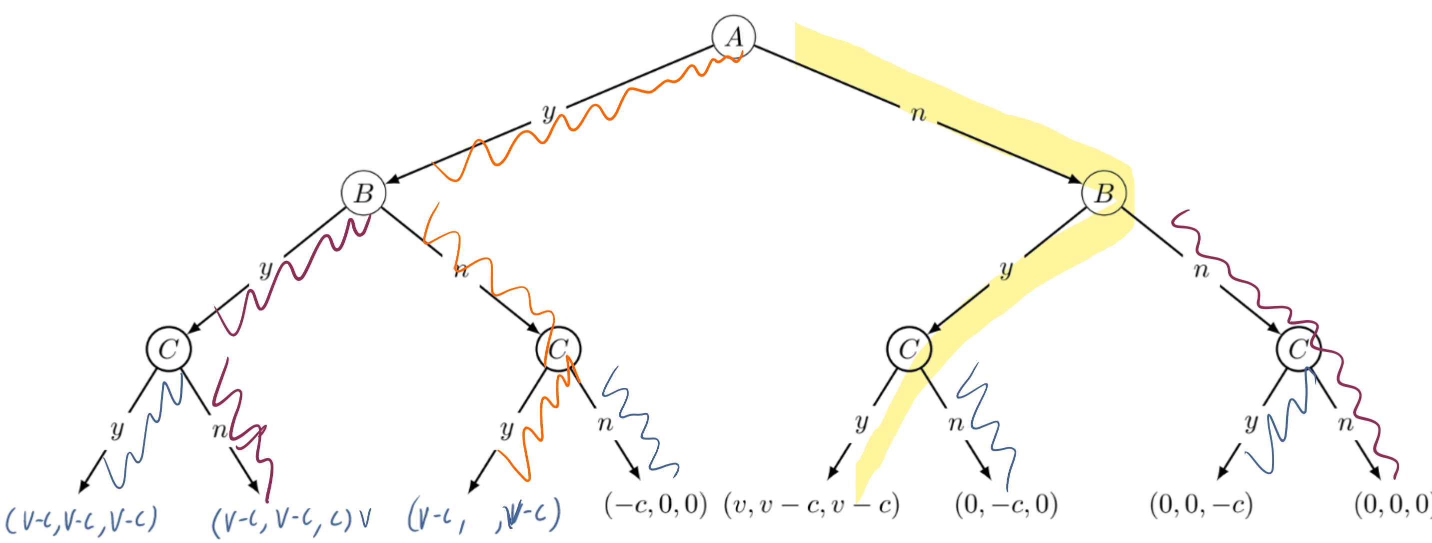
(d) According to your backward induction analysis, what is player C's (rational) strategy?



A : 2

B : 4

C : 16



d)

$S_C = nyyn$

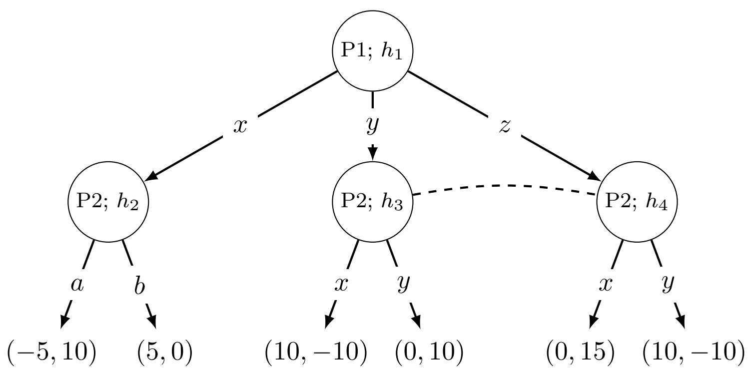
Exercise 2.

Each extensive form game shown below is a two player game with the players $P1$ and $P2$. The nonterminal nodes have been labeled by player and have also been given identifiers for reference: $h_1, h_2, \ldots$

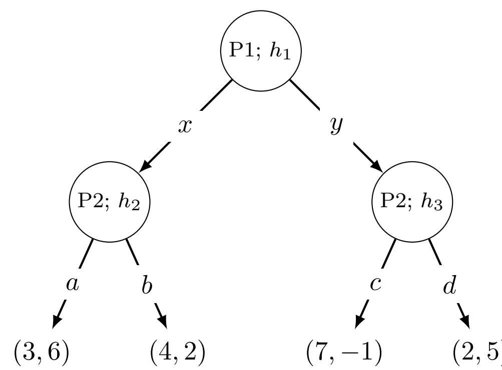
(i) For each game determine the information sets (include any information sets that consist of a single node).

(ii) For each game construct the corresponding normal form game.

(a)



(b)



a)

$I_1 = \{h_1\}$

$I_2^1 = \{h_2\}$ $\quad I_2^2 = \{h_3, h_4\}$

$S_1 = \{x, y, z\}$ $\quad S_2 = \{ax, ay, bx, by\}$

$I_1 = \{h_1\}$ $\quad I_2^1 \{h_2\}$ $\quad I_2^2 \{h_3\}$

$S_1 = \{x, y\}$

$S_2 = \{ac, ad, bc, bd\}$

P_1		ax	ay	bx	by
	x	(-5, 10)	(-5, 10)	(5, 0)	(5, 0)
	y	(10, -10)	(0, 10)	(10, -10)	(0, 10)
	z	(0, 15)	(10, -10)	(0, 15)	(10, -10)

P_2

P_1		ac	ad	bc	bd
	x	(3, 6)	(3, 6)	(4, 2)	(4, 2)
	y	(7, -1)	(2, 5)	(7, -1)	(2, 5)

Problem 1.

Consider the following extensive form game with three players, player 1, player 2, and player 3. The payoffs are listed in the usual way, so that (p_1, p_2, p_3) means p_1 is the payoff to player 1, p_2 is the payoff to player 2, and p_3 is the payoff to player 3.

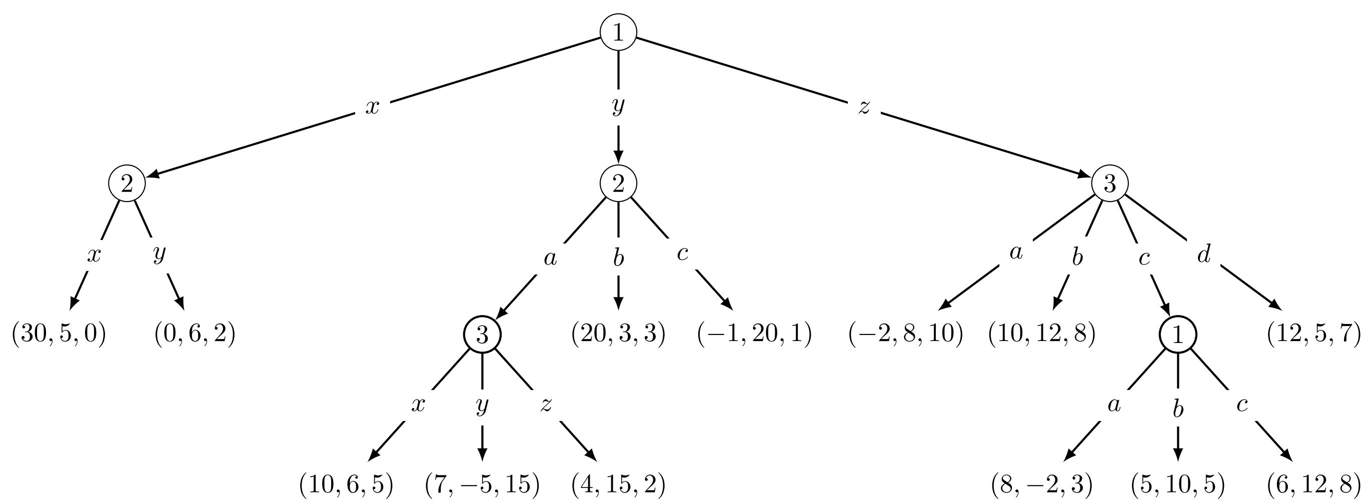
(a) For each player i, determine the size of the strategy set S_i. You do not need to list the strategies in each set.

(b) For player 3 **describe in words** what is meant by the strategy $s_3 = x\ b$.

(c) Use backward induction to analyze the game. Indicate the path of play that will be taken according to your analysis.

(d) Identify at least one outcome that all players would prefer over the outcome reached through the backward induction analysis.

(e) Are there threats or promises that players could make that would lead to this preferred outcome? Assume that players will act on threats or promises made if the path of play reaches the node where the threat or promise applies and show that the threat or promise satisfies the definition given in the lecture.



a) $s_1 : 9$

$s_2 : 6$

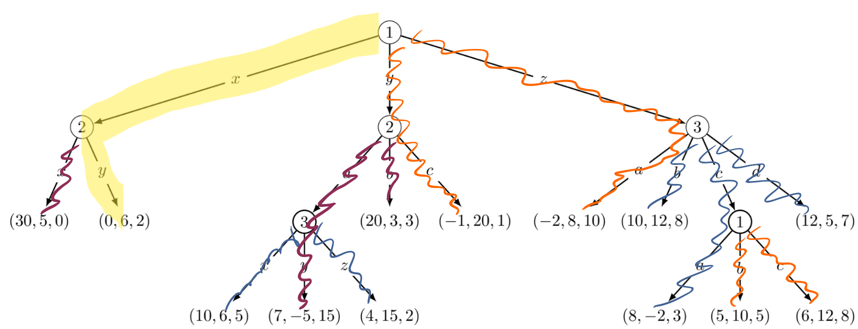
$s_3 : 12$

b) $s_3 = xb$

means if Player 1 choose y and player 2 choose a, player 3 will choose x resulting (10, 6, 5)

and if player 1 choose z, player 3 will choose b, resulting (10, 12, 8)

c)



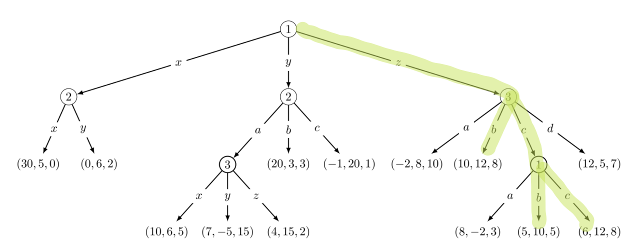
player 1 : x

player 2 : y

d) player 1 : z

player 3 : b

result : (10, 12, 8)



better than (0, 6, 2)

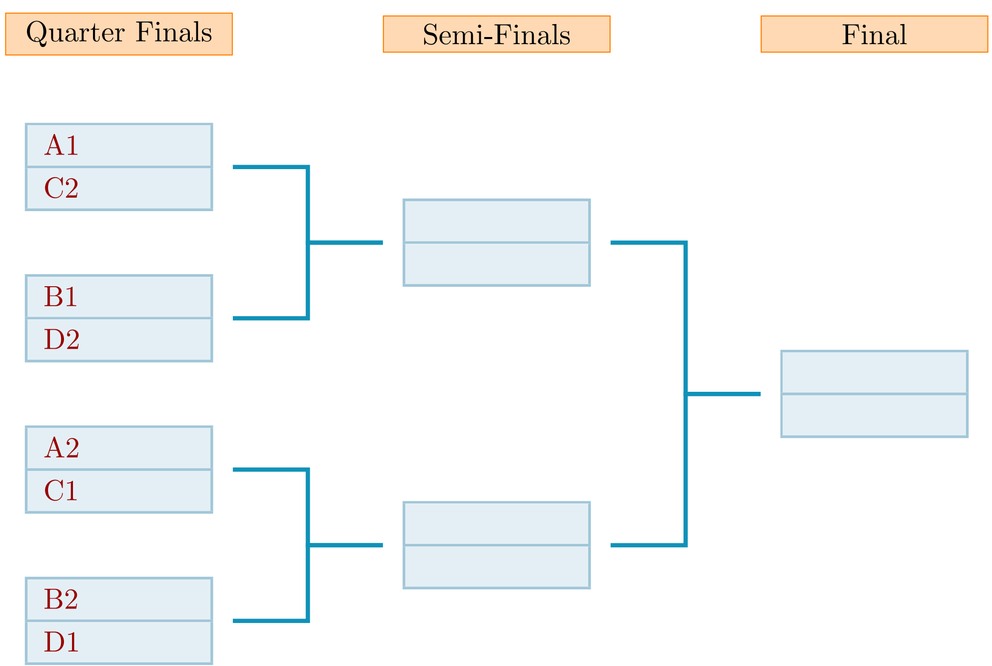
e) Yes, if player 3 promises player 1 to choose b if player 1 chose z, it will result in (10, 12, 8)

it is a promise because $\pi_3' < \pi$ and $\pi_1' > \pi_1$

Problem 2:

Suppose you are a member of an Olympic doubles badminton team. The setting we are considering is the 2012 Summer Olympics in London and your team is one of 16 teams competing.

- The teams are divided into 4 groups, A, B, C, and D, with four teams assigned to each group. Your team (YT) is in group A.
- In the preliminary round of competition, the four teams in each group compete in a round robin tournament (each team plays a match against the other 3 teams in their group). The two teams with the best records from each group move to an *elimination tournament.*
- In the elimination tournament, the eight teams from the round robin tournament are matched for a quarterfinal. The four winners of the quarter-final matches advance to a semifinal. The two winners of the semifinal matches advance to a final match. The winner of the final match receives the gold medal; the loser receives the silver medal. The losing teams from the semifinal play each other to determine the bronze medal winner.
- Teams are matched in the elimination round as shown in the chart below, where "A1" and "A2" are the teams from group A with the best and second best record, respectively, in the group A round robin tournament. The other slots are determined the same way.



- Suppose that your team (YT) is about to play the final round robin match of the tournament against another group A team (AA). Both teams are already guaranteed a slot in the quarter finals. The winner of this game will take the A1 slot and the loser will take the A2 slot. All other slots in the elimination round have been determined. However, there was a surprise upset in group D so that a highly ranked team has ended up in slot D2, and a low ranked team has taken the D1 slot.

- Your coaches estimate:
  - The winner of this round robin match has a 20% chance of advancing to the finals (winning at least a silver medal).
  - The loser of this round robin match will be matched against lower ranked teams in the elimination tournament and has a 60% of advancing to the finals (winning at least a silver medal).
- If Your Team (YT) and the other team (AA) both play well (W), you each have a 50% of winning this match. If one team chooses to play badly (B) and the other team plays well (W), then the team that plays badly will surely lose. If both teams play badly, the judges will surely notice and both teams will be disqualified.

(a) Create both an extensive form and a normal form game where the players are your team (YT) and the other team (AA). Each has the option to play well (W) or badly (B). The teams choose simultaneously (prior to the beginning of the match). The payoffs are the probabilities of winning at least a silver medal. One of the outcomes requires computing an **expected payoff**. An expected payoff is the expected value of the payoff. A brief overview of expected value for random variables with finitely many outcomes can be found on Wikipedia here.

(b) Explain briefly what you think you might do if you were in this situation. There is no correct or incorrect answer here.

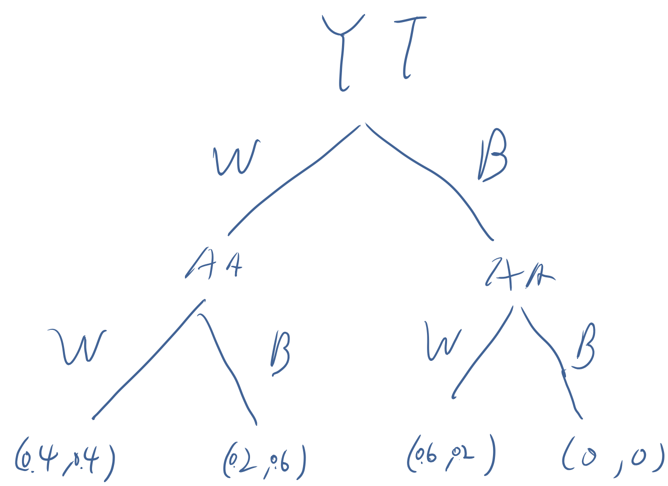
(c) Read the Aug 2012 Huffpost article Badminton and the Science of Rule Making and briefly summarize what happened in the 2012 Olympic Women's Doubles badminton. You can also view the main match here. Keep in mind as you watch that the players are among the best in the world.

a) 20% chance of advancing to final values 2

60% 〃 values 6

disqualified values -1

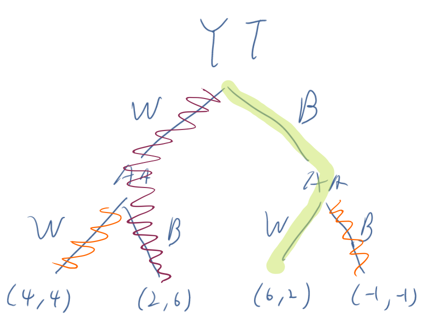
$$\frac{(0.2,0.6)}{2} + \frac{(0.2,0.6)}{2} = (0.4,0.4)$$



AA (,2)

YT (1,)	W	B
W	0.4, 0.4	0.2, 0.6
B	0.6, 0.2	0, 0

b)



according to back ward, best solution is to play bad

but if I can decide, I will juse enjoy the game and play well.

c) both team are attempting to lose the game in order to aviod matching up with another stronger team in Quator finals.