Math456/CMPSC456 Homework 5

Due Feb 21 2020

1. (10 points)

Find the first three iterations obtained by the Power method applied to the following matrices.

a. $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$;
Use $\mathbf{x}^{(0)} = (1, -1, 2)^t$.

b. $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$;
Use $\mathbf{x}^{(0)} = (-1, 0, 1)^t$.

2. (10 points)
Find the first three iterations obtained by the Symmetric Power method applied to the following matrices.

a. $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$;
Use $\mathbf{x}^{(0)} = (1, -1, 2)^t$.

b. $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$;
Use $\mathbf{x}^{(0)} = (-1, 0, 1)^t$.

3. (20 points) Computer project: Write computer code and implement (1) the power method; (2) the symmetric power method; Test the methods on the following matrix,

$$\begin{pmatrix} 4 & 1 & 1 & 1 \\ 1 & 3 & -1 & 1 \\ 1 & -1 & 2 & 0 \\ 1 & 1 & 0 & 2 \end{pmatrix}.$$

Use tolerance 10^{-5} and $\vec{x}_0 = (1, 0, 0, 0)^T$. Show the approximate eigenvalues from the iterations.

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \qquad x_0 = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$$

$$x_1 = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}$$

$$x_2 = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 11 \\ 9 \\ 12 \end{bmatrix}$$

$$x_3 = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 11 \\ 9 \\ 12 \end{bmatrix} = \begin{bmatrix} 43 \\ 41 \\ 44 \end{bmatrix}$$

$$x_4 = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 43 \\ 41 \\ 44 \end{bmatrix} = \begin{bmatrix} 171 \\ 164 \\ 172 \end{bmatrix}$$

$$x_5 = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 171 \\ 164 \\ 172 \end{bmatrix} = \begin{bmatrix} 683 \\ 681 \\ 684 \end{bmatrix}$$

$$\lambda = \frac{x_2^T x_3}{x_2^T x_2} = \frac{1370}{346} = 3.9595$$

$$\begin{bmatrix} \frac{43}{44} \\ \frac{41}{44} \\ \frac{44}{44} \end{bmatrix} = \begin{bmatrix} 0.98 \\ 0.93 \\ 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \qquad x_0 = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$$

1^{st}

$$Ax_0 = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix} \leftarrow$$

$$x_1 = \frac{1}{4} \cdot \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 0.75 \\ 0.25 \\ 1 \end{bmatrix}$$

2^{th}

$$Ax_1 = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 0.75 \\ 0.25 \\ 1 \end{bmatrix} = \begin{bmatrix} 2.75 \\ 2.25 \\ 3 \end{bmatrix} \leftarrow$$

$$x_2 = \frac{1}{3} \begin{bmatrix} 2.75 \\ 2.25 \\ 3 \end{bmatrix} = \begin{bmatrix} 0.92 \\ 0.75 \\ 1 \end{bmatrix}$$

3^{th}

$$Ax_2 = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 0.92 \\ 0.75 \\ 1 \end{bmatrix} = \begin{bmatrix} 3.58 \\ 3.42 \\ 3.67 \end{bmatrix} \leftarrow$$

$$x_3 = \frac{1}{3.67} \begin{bmatrix} 3.58 \\ 3.42 \\ 3.67 \end{bmatrix} = \begin{bmatrix} 0.98 \\ 0.93 \\ 1 \end{bmatrix}$$

b $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ $x_0 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

$$x_1 = A \cdot x_0 = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$$

$$x_2 = A \cdot x_1 = \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix}$$

$$x_3 = A \cdot x_2 = \begin{bmatrix} -2 \\ -2 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} \frac{-2}{2} \\ \frac{-2}{2} \\ \frac{-1}{2} \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ -0.5 \end{bmatrix}$$

$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ $x_0 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

1th

$$A x_0 = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$$

$$x_1 = \frac{1}{1} \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$$

2th

$$A x_1 = \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix}$$

$$x_2 = \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix}$$

3th

$$A x_2 = \begin{bmatrix} -2 \\ -2 \\ -1 \end{bmatrix}$$

$$x_3 = \frac{1}{2} \begin{bmatrix} -2 \\ -2 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ -\frac{1}{2} \end{bmatrix}$$

a. $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$ $\quad x_0 = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$

$$y = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}$$

$$\mu_0 = (x_0, y) = 10$$

$$x_1 = \frac{y}{\|y\|_2} = \begin{bmatrix} \frac{3}{\sqrt{26}} \\ \frac{1}{\sqrt{26}} \\ \frac{4}{\sqrt{26}} \end{bmatrix}$$

$$y = Ax_1 = \begin{bmatrix} \frac{11}{\sqrt{26}} \\ \frac{9}{\sqrt{26}} \\ \frac{6\sqrt{2}}{\sqrt{13}} \end{bmatrix} \qquad \mu = \frac{45}{13}$$

$$Y_2 = \frac{Y}{|Y|_2} = \begin{bmatrix} \frac{11\sqrt{173}}{13\sqrt{2}} \\ \frac{9\sqrt{173}}{13\sqrt{2}} \\ \frac{6\sqrt{346}}{13} \end{bmatrix}$$

$$y = A x_2 = \begin{bmatrix} \frac{43\sqrt{173}}{13\sqrt{2}} \\ \frac{41\sqrt{173}}{13\sqrt{2}} \\ \frac{22\sqrt{346}}{13} \end{bmatrix} \qquad \mu = \frac{118528}{169}$$

b) $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \qquad x_0 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

$$y = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \qquad \mu = 0$$

$$x_1 = \frac{y}{\|y_2\|} = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$$

$$y = \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix} \qquad \mu = 1$$

$$x_2 = \frac{y}{\|y_2\|} = \begin{bmatrix} \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \\ 0 \end{bmatrix}$$

$$v = \begin{bmatrix} -\sqrt{2} \\ -\sqrt{2} \\ -\frac{1}{\sqrt{2}} \end{bmatrix} \qquad \mu = 2$$

```
A=[4 1 1 1; 1 3 -1 1; 1 -1 2 0; 1 1 0 2];
x0=[1; 0; 0; 0];

tol=10^-5;
dx=1.0;
step=0;

while dx>tol
    Ax0=A*x0;
    x1=1/max(Ax0);
    x1=x1*Ax0;

    dx=norm(x1-x0);
    x0=x1;
    step=step+1;
end

disp(step)
disp(x0)
```

命令行窗口

```
>> M_20220214456HW5
    29

    1.0000
    0.6180
    0.1180
    0.5000

fx >>
```

```
A=[4 1 1 1; 1 3 -1 1; 1 -1 2 0; 1 1 0 2];
x0=[1; 0; 0; 0];

tol=10^-5;
dx=1.0;
step=0;

%x1=A*x0
%dx=norm(x1)
%x0=x0*x1
while dx>tol
    y=A*x0;
μ   mu=(x0(1)*y(1)+x0(2)*y(2)+x0(3)*y(3)+x0(4)*y(4));
    x1=norm(y);
    dx=norm(x0-y/x1,2);
    x1=y/x1;


    x0=x1;
    step=step+1;
end

disp(step)
disp(x0)
```

窗口

```
M_20220214456HW5_2
 28

 0.7795
 0.4817
 0.0920
 0.3897
```