

HOMEWORK 17

Problem 1. Determine constants a, b, c that will produce a quadrature formula

$$\int_{-1}^1 f(x) dx = af(-1) + bf(0) + cf(1)$$

that has degree of precision 3.

$$f(x) = 1, x, x^2$$

$$= \int_{-1}^1 1 dx$$

$$= (x)_{-1}^1 = 1 - (-1) = 2 = a + b + c$$

$$\int_{-1}^1 x dx = 0 = -a + c \quad a = c$$

$$\int_{-1}^1 x^2 dx = \frac{2}{3} = a + c$$

$$2a = 2c = \frac{2}{3}$$

$$a = c = \frac{1}{3}$$

$$a + b + c = 2 \quad a + c = \frac{2}{3}$$

$$b = \frac{4}{3}$$

$$\text{if } a=c=\frac{2}{3}, \quad b=\frac{4}{3} \quad p(x)=x^3$$

$$\text{LHS} = \int_{-1}^1 x^3 dx = \left. \frac{x^4}{4} \right|_{-1}^1 = \frac{x}{4} - \frac{x}{4} = 0$$

$$\text{RHS} = \frac{2}{3} \cdot (-1) + 0 + \frac{2}{3} \cdot 1 = 0$$

$$\text{if } a=c=\frac{2}{5}, \quad b=\frac{4}{3} \quad p(x)=x^4$$

$$\text{LHS} = \left. \frac{x^5}{5} \right|_{-1}^1 = \frac{1}{5} - \frac{-1}{5} = \frac{2}{5}$$

$$\text{RHS} = \frac{1}{5} \cdot 1 + 0 + \frac{1}{5} \cdot 1$$

$$= \frac{2}{5}$$

$$\text{LHS} \neq \text{RHS},$$

$$\text{Dop} = 3$$