HKN ECE 310 Quiz 6 Review Session

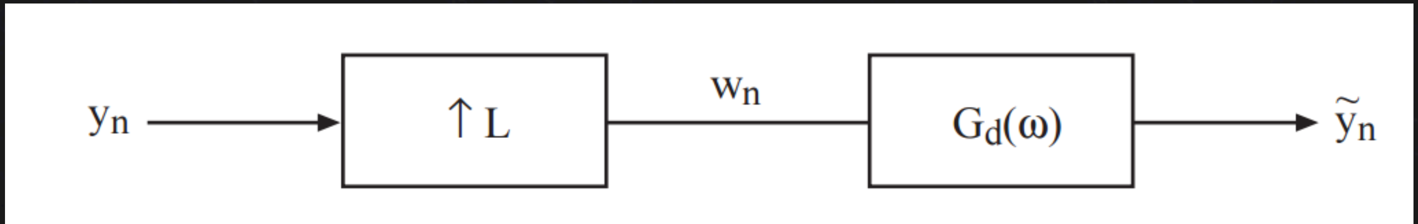
Corey Snyder

Topics

- Digital Rate Conversion: Upsampling, Downsampling
- D/A: Review of Ideal and ZOH, Upsampled D/A
- FFT: Decimation in Frequency and Time
- Circular Convolution, Linear Convolution using FFT

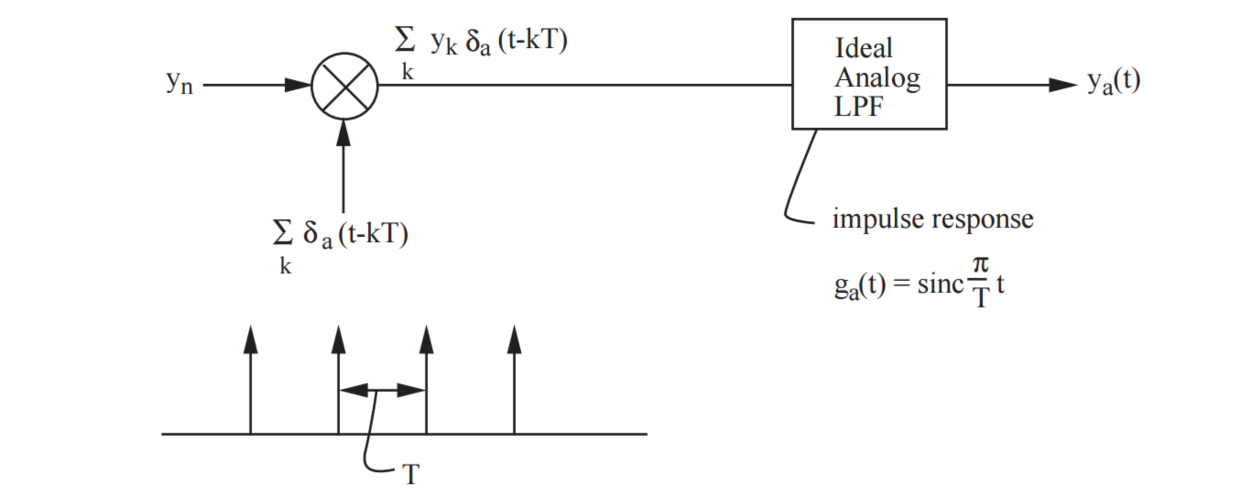
Upsampling

- If we upsample by L, we will interpolate L – 1 zeros between each sample
  - $y[n]=x[n/L]$ if n mod L = 0
  - 0 else
- What happens in the frequency domain?
  - Think about what happens when we oversample and signal, i.e. above Nyquist?
- What does the frequency response look like after upsampling?
  - Shrink x-axis by factor of L
- What should $G_D(\omega)$ be in order to obtain a desirable frequency response?
  - Remove extra copies and correct amplitude for conservation of energy



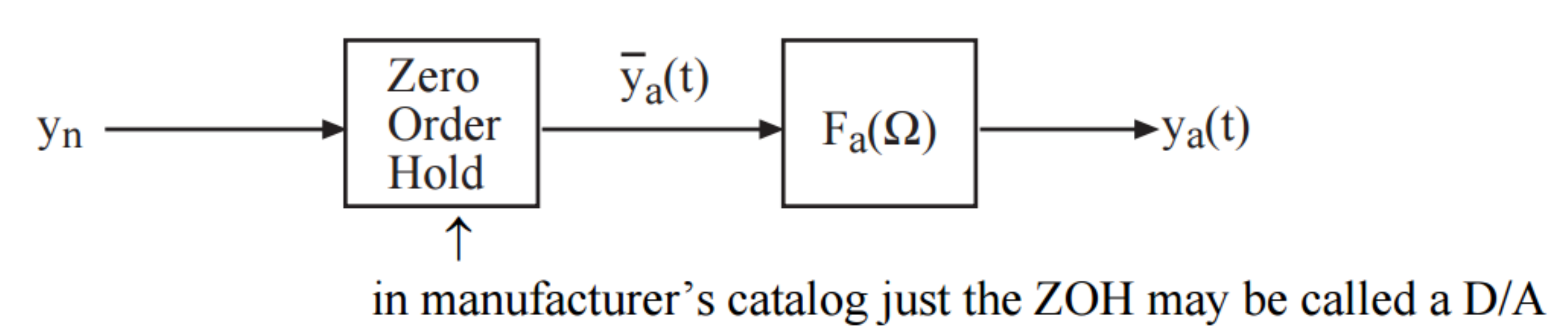
Review: Ideal D/A

- Want $Y_a(\Omega)=Y_d(\Omega T)$, but we want to only take one copy of the DTFT
  - Thus, we should low-pass filter from $\Omega=\pm\pi/T$ (domain of the central copy of the DTFT)
- Inverse CTFT of $T\,rect(\Omega/\tfrac{2\pi}{T})=sinc(\tfrac{\pi}{T}t)$
- Remember that multiplication in the frequency domain is convolution in the time domain
  - Thus, $y_a(t)=\sum_{n=-\infty}^{\infty} y_n\, sinc[\tfrac{\pi}{T}(t-nT)]$, where y_n is obtained by multiplying y[n] by an impulse train



Review: Zero-Order Hold

- Ideal D/A is not practical because generating delta impulses is not achievable
- Zero-Order Hold (ZOH) gives us a suitable approximation to the Ideal D/A
- The ZOH multiplies each sample by a rectangular pulse of width T (our sampling rate)
  - Thus, $y_a(t) = \sum_n y_n \, p_a(t - nT)$ where p_a is the rectangular pulse provided by the ZOH
- $F_a(\Omega)$ is an analog filter that corrects the distortion presented by the ZOH



Upsampled D/A

- Upsampling prior to D/A conversion can make recovery simpler
  - i.e. Compensator $F_a(\Omega)$ can be simpler to implement
- Upsampling effectively increases our sampling frequency, thus our ZOH pulse can be narrower and give us a better staircase approximation
- This 'smoother staircase' will be easier to rectify with the compensator
  - i.e. the transition bandwidth will be larger
- In the frequency domain, we see the frequency axis compress by L; however, the analog frequencies upon recovery do not change!

Downsampling

- If we downsample by D, we keep every D^{th} sample (decimate the rest)
  - $y[n]=x[Dn]$
- What happens in the frequency domain?
  - Frequency response stretches by a factor of D
  - Amplitude reduces by a factor of D (think conservation of energy)
- Anti-aliasing filter prevents downsampling from aliasing our signal
  - $A(\omega)=LPF\ with\ \omega_c = \pi/D$

Fast Fourier Transform

- Computational efficient implementation of the DFT
  - Ordinary DFT requires N^2 multiplies and N(N – 1) adds
  - FFT requires only $O(N \log_2 N)$ computations
- Two main forms of FFT
  - Decimation in Time
  - Decimation in Frequency
- We will consider the Radix 2 FFT, but other Radices may be used
- Use Butterfly Structures to represent multiplies and adds

Decimation in Time

- Divide sequence into two groups
  - $y[n]=x[2n]$ and $z[n]=x[2n+1]$
  - Before we continue, remember that $W_N = e^{-j2\pi/N}$
- We can form the first half of the DFT from these two sequences
  - $X_k = \sum_n x_{2n} W_N^{2kn} + x_{2n+1} W_N^{k(2n+1)} = \sum_n y_n W_{N/2}^{kn} + W_N^k \sum_n z_n W_{N/2}^{kn}$ $for\ 0 \le k \le N/2 - 1$
- And the second half…
  - $X_k = \sum_n y_n W_{N/2}^{kn} - W_N^k \sum_n z_n W_{N/2}^{kn}$ $for\ N/2 \le k \le N$
- Continually halve the sequences until you reach size 2

Decimation in Frequency

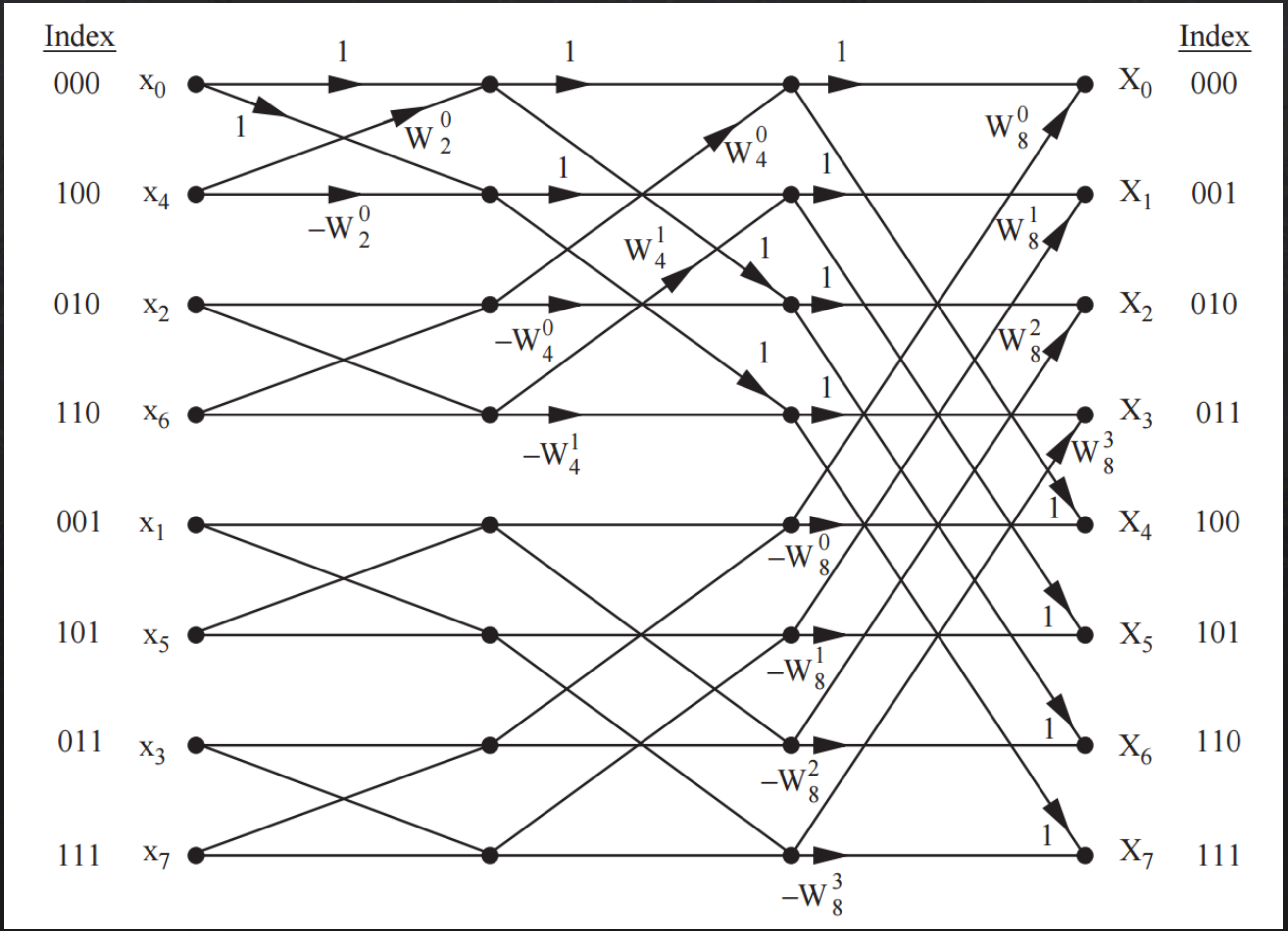
- Reverse of Decimation of Time
- Compute points over entire DFT in even and odd groups
  - DIT divides even and odd groups *then* computes points over smaller groups
- Even k's: $X_{2k} = \sum_{n=0}^{N/2-1} (x_n + x_{n+N/2}) W_{N/2}^{nk}$
- Odd k's: $X_{2k+1} = \sum_{n=0}^{N/2-1} (x_n - x_{n+N/2}) W_N^n W_{N/2}^{nk}$

Butterfly Structure Exercise

- Draw the butterfly structure of a length 8 Decimation in Time FFT for x_n

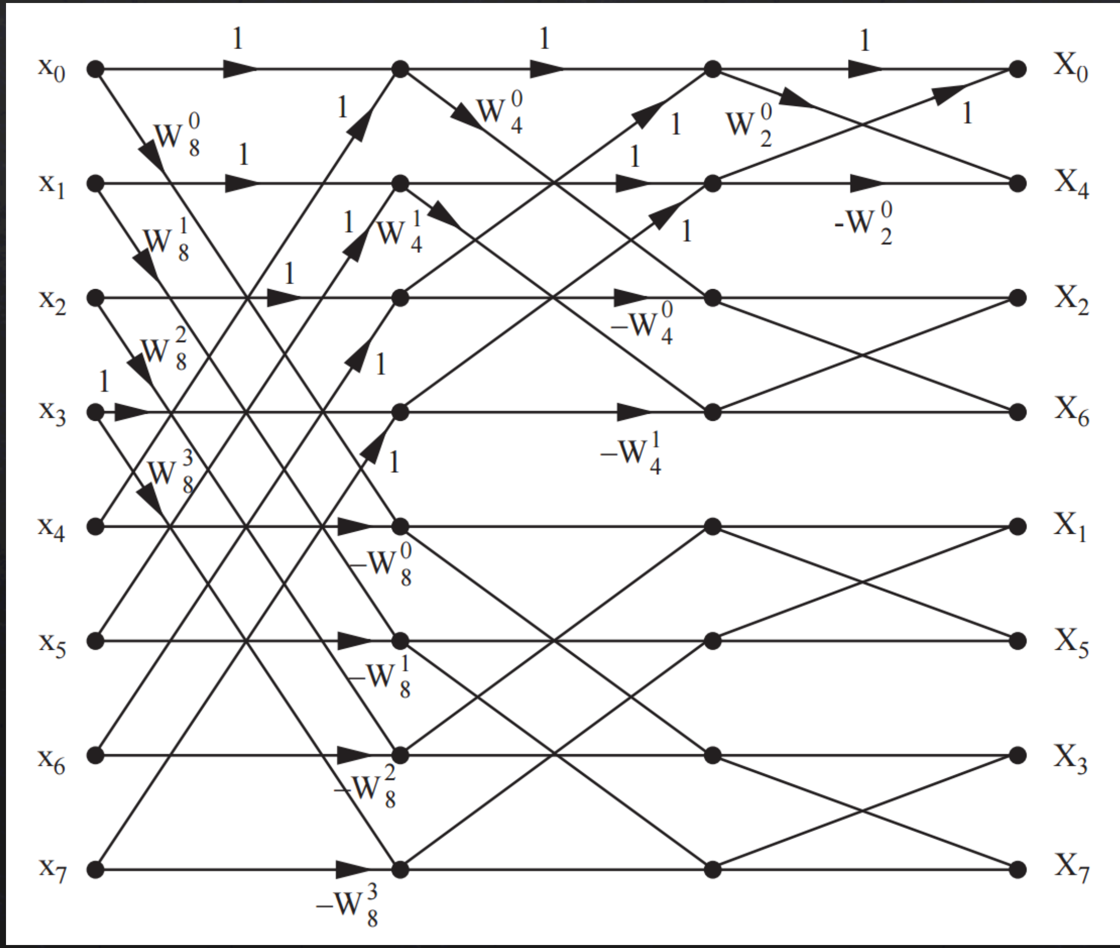
Butterfly Structure Exercise

- Draw the butterfly structure of a length 8 Decimation in Time FFT for x_n



Butterfly Structure Exercise

- Decimation in Frequency is reverse of Decimation in Time



Fast Linear Convolution

- In order to obtain system response, we can multiply DFTs and take inverse DFT
  - Be careful, this is not the same as convolution in time, but rather cyclic convolution in time
- Therefore, in order to perform linear convolution from DFTs, we must first zero pad signals in order to make wrap-around terms go to zero
- If x_n has length N and h_n has length M, then the resulting convolution is of length N+M – 1
  - We must pad x_n with M – 1 zeros
  - h_n with N – 1 zeros
- This will allow multiplication of FFTs to produce linear convolution result from cyclic convolution
- Note: we also typically do extra padding so that the signals are of power of 2 length to optimize the FFT

References

- Chapters 13 and 14 in Singer's Textbook
- Overlap Add and Overlap Save Stuff @ Chapter 14 Page 16+