HKN ECE 310 Exam 1 Review Session

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Topics

- DTFT
- DFT
- Windowing and Spectral Analysis
- LSIC Systems
- Sampling
- Convolution
- Impulse Response

Discrete Time Fourier Transform

$$X_d(\omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X_d(\omega)e^{j\omega n}d\omega$$

- Important Properties:
  - Periodicity!
  - Linearity
  - Symmetries (Magnitude, angle, real part, imaginary part)
  - Time shift and modulation
  - Product of signals and convolution
  - Parseval's Relation
- Know your geometric series sums!

Discrete Fourier Transform

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{\frac{-j2\pi kn}{N}}$$

$$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] e^{\frac{j2\pi kn}{N}}$$

- What is the relationship between the DTFT and the DFT?

- $\omega_k = \frac{2\pi k}{N}$

Discrete Fourier Transform Properties

- *Circular* shift
- *Circular* modulation
- *Circular* convolution
- Why are these properties circular?
- Parseval’s Relation

DTFT and DFT Examples

- Suppose we have a signal $x[n]$ with DTFT $X_d(\omega)$. Find the DTFT for the following signals in terms of $X_d(\omega)$
  - $y[n] = x[n]\cos(\omega_0 n)$
  - $z[n] = x[n - n_0] + x[n + n_0]$
- Drawing Example! (Should have come to the review session.)
- Suppose we have a signal $x[n] = [1,2,3,4,5,6]$ with DFT $X[k]$. Find the matching signal or DFT that corresponds to the following DFTs in terms of $x[n]$ and $X[k]$
  - $Y[k] = X[k]e^{-j\pi k}$
  - $z[n] = [1,\ -2, 3,\ -4, 5,\ -6]$

Windowing and Spectral Analysis

- Signals cannot go to infinity
  - Therefore, we need to window

- There are many different windows
  - Rectangular (boxcar)
  - Hamming
  - Hanning
  - Triangular
  - Kaiser
- More on advantages/disadvantages later

Windowing and Spectral Analysis

- What happens when we dictate that $x[n] = \cos(\omega_0 n)$ is of finite duration N?
  - Derivation on page 54 of textbook

- Spectral Analysis: Resolving different sinusoidal frequency components in a signal
- The DTFT of a finite sinusoidal signal has main lobe width of $\frac{4\pi}{N}$ where N is the # of samples in the signal
- Resolution can be defined in different ways
  - Full lobe resolution vs. Half lobe resolution

Full-Lobe vs. Half-Lobe Resolution

- Suppose we represent a cosinusoid as $Acos(\Omega T)$
- The lobe centers of two cosinusoids will be located at $\Omega_0 T\ and\ \Omega_1 T$
  - Remember that the half-width of each lobe is $\frac{2\pi}{N}$
- Full-Lobe
  - To prevent crossover: $\Omega_0 T + \frac{2\pi}{N} < \Omega_1 T - \frac{2\pi}{N} \rightarrow \Omega_1 - \Omega_0 > \frac{4\pi}{NT}$
- Half-Lobe
  - $\Omega_0 T < \Omega_1 T - \frac{2\pi}{N} \rightarrow \Omega_1 - \Omega_0 > \frac{2\pi}{NT}$

Zero-Padding

- We can improve the resolution of the DFT simply by adding zeros to the end of the signal
- This doesn’t change the frequency content of the DTFT!
- Instead, it increases the number of samples the DFT takes of the DTFT
- This can be used to improve spectral analysis

Window Comparisons

- Rectangular (boxcar)
  - Maintains width of the main lobe , thus better resolution
  - Poor side lobe attenuation, can lead to resolution errors
- Hamming
  - Doubles the width of the main lobe, thus poorer resolution
  - Greatly reduces side lobes, prevents mistaking side lobes as main lobes of other frequencies
- Kaiser
  - Optimal

LSIC Systems

- Linearity
  - Satisfy Homogeneity and Additivity
  - Can be summarized by Superposition
    - If $x_1[n] \rightarrow y_1[n]$ *and* $x_2[n] \rightarrow y_2[n]$, *then* $ax_1[n] + bx_2[n] \rightarrow ay_1[n] + by_2[n]$
- Shift Invariance
  - If $x[n] \rightarrow y[n]$, *then* $x[n - n_0] \rightarrow y[n - n_0]\ \forall\ n_0$ *and* $x[n]$
- Causality
  - Output cannot depend on future input values

LSIC Examples

- For the following systems, determine whether it is linear, shift-invariant, and causal
- $y[n] = x^2[n]$
- $y[n] = x[|n|]$
- $y[n] = x[n]\cos(\omega_0 n)$

Sampling

- For ideal A/D sampling, the relationship between the CTFT and DTFT is as follows

$$X_d(\omega) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X_a\left(\frac{\omega + 2\pi k}{T}\right)$$

- Why is there a $2\pi k$?
- Why is there a $\frac{1}{T}$ factor?

$$\omega_d = \Omega_\mathrm{a} T$$

- Nyquist Criterion: $\frac{1}{\mathrm{T}} > 2B$

Convolution

- $y[n] = \sum_{m=-\infty}^{\infty} x[m]h[n-m] = \sum_{m=-\infty}^{\infty} h[m]x[n-m]$
- System must be:
  - Linear
  - Shift Invariant
- Popularly done graphically
- Be comfortable doing it algebraically

Impulse Response

- System output to an $x[n] = \delta[n]$ input

- $y[n] = x[n] * h[n]$

- $Y_d(\omega) = X_d(\omega)H_d(\omega)$

Impulse Response and Convolution Examples

- Given $x[n] = [6\,, 12, -3,\ 0,\ 15,\ 3, -9,\ 0]$ and $h[n] = [\frac{1}{3}, \frac{1}{3}, \frac{1}{3}]$, compute the system output.
  - What does this filter do?

- Suppose we have a digital filter $h[n]$ with an unknown impulse response. We do know the system output to the follow two input signals. Determine the impulse response in terms of the two system outputs.
  - $x_1[n] = [2, 4, 2, 4] \rightarrow y_1[n]$
  - $x_2[n] = [0, 2, 1, 2] \rightarrow y_2[n]$