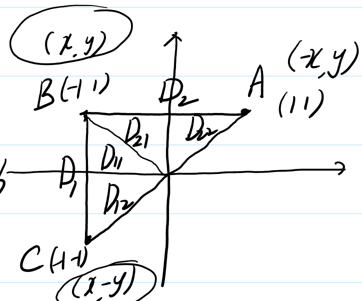
第六次作业

1) $\iint_D (xy+\cos x\sin y)\,d\sigma = \iint_{D_1}(\underbrace{xy}_{奇}+\underbrace{\cos x\sin y}_{奇})\,d\sigma + \iint_{D_2}(xy+\cos x\sin y)\,d\sigma$

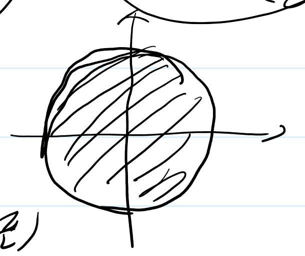


$$= \iint_{D_2}(\underbrace{xy}_{奇}+\underbrace{\cos x\sin y}_{偶})\,d\sigma$$

$D = D_1 \cup D_2$ $\begin{cases} D_1 = D_{11}\cup D_{12} \\ D_2 = D_{21}\cup D_{22}\end{cases}$

$$= \iint_{D_2}\underbrace{\cos x\sin y}_{偶}\,d\sigma = 2\iint_{D_{22}}\cos x\sin y\,d\sigma$$

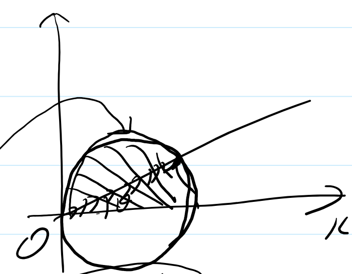
3) $D=\{(x,y)\mid x^2+y^2\le 1\}$ $M=\iint_D (x+y)^3\,d\sigma$ → 关于原点对称 $=0$ $N=\iint_D \cos x^2\sin y^2\,d\sigma > 0$ $P=\iint_D (e^{-(x^2+y^2)}-1)\,d\sigma < 0$



例② $\alpha\in(0,\frac{\pi}{2})$
$(0, 1.5^{+})$

4) $\int_0^{\frac{\pi}{2}} d\theta \int_0^{\cos\theta} f(r\cos\theta, r\sin\theta)\, r\,dr$

$D_\theta \begin{cases} 0\le\theta\le\frac{\pi}{2} \\ 0\le r\le\cos\theta \end{cases}$



$r=\cos\theta$
$r^2=r\cos\theta$
$x^2+y^2=x$

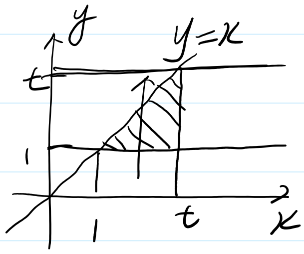
$\begin{cases} x=r\cos\theta \\ y=r\sin\theta \end{cases}$

二、4) $f(x)$连续 $F(t)=\int_1^t dy\int_y^t f(x)\,dx$ 则 $F'(2)=$ ______

$(\int_a^x f(t)\,dt)' = f(x)$ → $D\{1\le y\le t$

$\left(\int_a^t f(x)dx\right)' = f(t)$

$D_y \begin{cases} 1\le y\le t \\ y\le x\le t \end{cases}$



$D_x \begin{cases} 1\le x\le t \\ 1\le y\le x \end{cases}$

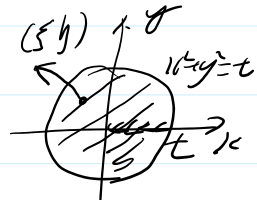
$F(t) = \int_1^t dx \int_1^x f(x)\, dy = \int_1^t (x-1) f(x)\, dx$

$F'(t) = (t-1)f(t)$

5) $\lim\limits_{t\to 0^+} \dfrac{\iint_D f(x,y)d\sigma}{\pi t^2} = \lim\limits_{t\to 0^+} \dfrac{f(\xi,\eta)\cancel{S_D}}{\cancel{\pi t^2}}$

$= \lim\limits_{\substack{\xi\to 0\\ \eta\to 0}} f(\xi,\eta) = f(0,0)$

D



$t\to 0^+ \Rightarrow \begin{cases}\xi\to 0\\ \eta\to 0\end{cases}$

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5 $f(x) = x^2 + x\int_0^{x^2} f(x^2-t)dt + \iint_D f(x,y)d\sigma$ $\quad D\begin{cases} -1\le x\le 1\\ x\le y\le 1\end{cases}$ $\quad f(1)=0$ 求 $\iint_D f d\sigma$

见到 积分上限 即求导

$\left(\int_a^x f(t)dt\right)' = f(x)$

$\left(\int_{v(x)}^{u(x)} f(t)dt\right)' = f[u(x)]u'(x) - f[v(x)]v'(x)$

变型 ① $\int_0^x (x-t)f(t)dt = x\int_0^x f(t)dt - \int_0^x t f(t)dt$

② $\int_0^x x f(\underset{换元}{\underline{x-t}})dt$

$\xrightarrow[-dt=du]{x-t=u} \int_x^0 x f(u)(-du)$

$= \int_0^x x f(u)\,du$

$\int_0^{x^2} f(x^2-t)dt \xrightarrow[-dt=du]{x^2-t=u} \int_{x^2}^0 f(u)(-du)$

$= \int_0^{x^2} f(u)\,du$

$f(x) = x^2 + x\int_0^{x^2} f(u)du + \iint_D f(x,y)d\sigma = A$

$\iint_{D_{xy}} f(x,y)d\sigma = A$

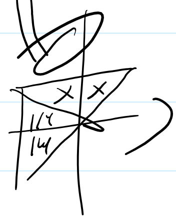
$f(x,y) = x^2y^2 + xy\int_0^{x^2y^2} f(u)du + \iint_D f(x,y)d\sigma = A$

$\iint_{D_{uv}} f(u,v)d\sigma = A$

$f(x,y) = x^2y^2 + xy\int_0^{x^2y^2} f(u)du + \iint_D f(x,y)d\sigma$ $=A$ $\iint_{D_{uv}} f(u,v)d\sigma = A$

两侧取 $\iint_D$

$$A=\iint_D f(x,y)\,d\sigma = \underbrace{\iint_D x^2y^2\,d\sigma}_{可求} + \iint_D \left[xy \int_0^{x^2y^2} f(u)\,du\right] d\sigma + \underbrace{\iint_D A\,d\sigma}_{=AS_D\ 可求}$$



第十次

1) $\Omega_1 \begin{cases} x^2+y^2+z^2\le R^2 \\ z\ge 0\end{cases}$ $\Omega_2 \begin{cases} x^2+y^2+z^2\le R^2 \\ x\ge 0,\ y\ge 0,\ z\ge 0\end{cases}$

A $\iiint_{\Omega_1} x\,dV = 4\iiint_{\Omega_2} z\,dV$ $=0$ >0

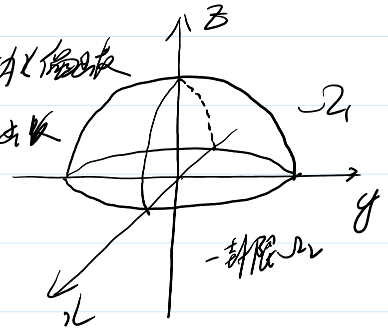
B $\iiint_{\Omega_1} y\,dV = 4\iiint_{\Omega_2} z\,dV$ $=0$ >0

C $\iiint_{\Omega_1} z\,dV = 4\iiint_{\Omega_2} z\,dV$

D $\iiint_{\Omega_1} xyz\,dV = 4\iiint_{\Omega_2} xyz\,dV$ $=0$ >0

$f(x,y,z)=z$ 对x偶函数

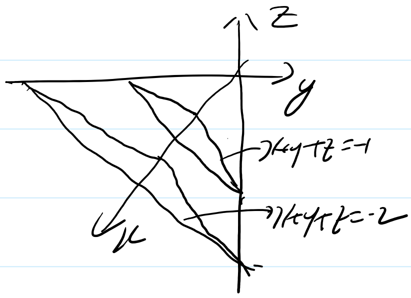
也对y偶函数



2) Ω 由 $x+y+z+1=0$ 与 $x+y+z+2=0$ 及 $x=0$ $y=0$ $z=0$ 围成

$$I_1 = \iiint_\Omega [\underbrace{\ln(x+y+z+3)}_{小数<1}]^2\,dV$$

$$I_2 = \iiint_\Omega \underbrace{(x+y+z)^2}_{\in(1,4)}\,dV$$



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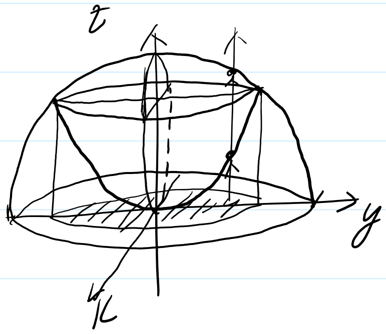
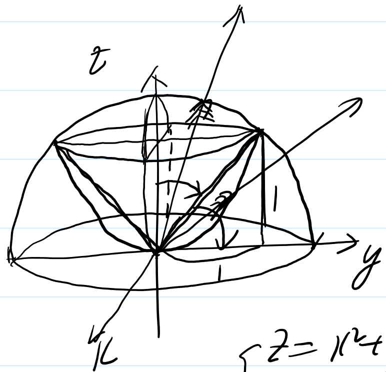
二

3) Ω $z=\sqrt{2-x^2-y^2}$ 与 $z=x^2+y^2$ 围成　$\iiint_\Omega f(x,y,z)dv$ 用柱、球.



$$\begin{cases} z=\sqrt{2-x^2-y^2} \\ z=x^2+y^2 \end{cases} \Rightarrow z=1=x^2+y^2$$

柱 $\Omega_{\theta r}\begin{cases} 0\le\theta\le 2\pi \\ 0\le r\le 1 \\ r^2\le z\le\sqrt{2-r^2} \end{cases}$



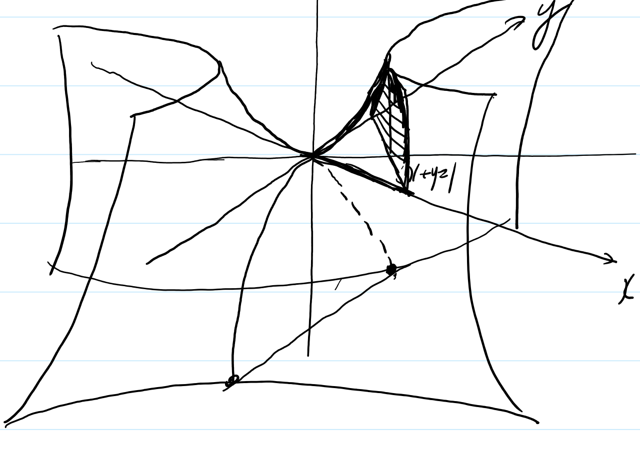
球 $\Omega_{1\theta\varphi}\begin{cases} 0\le\theta\le 2\pi \\ 0\le\varphi\le\frac{\pi}{4} \\ 0\le r\le\sqrt{2} \end{cases}$　$\Omega_{2\theta\varphi}\begin{cases} 0\le\theta\le 2\pi \\ \frac{\pi}{4}\le\varphi\le\frac{\pi}{2} \\ 0\le r\le\frac{\cos\varphi}{\sin^2\varphi} \end{cases}$

$$\begin{cases} z=x^2+y^2 \\ z=\sqrt{2-x^2-y^2} \end{cases} \Rightarrow x^2+y^2=1=z$$

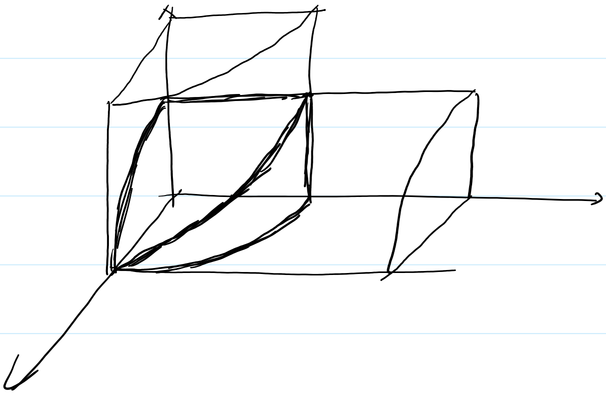
$$z=x^2+y^2 \quad r\cos\varphi=r^2\sin^2\varphi\cos^2\theta+r^2\sin^2\varphi\sin^2\theta=r^2\sin^2\varphi$$

$$\Rightarrow r=\frac{\cos\varphi}{\sin^2\varphi}$$

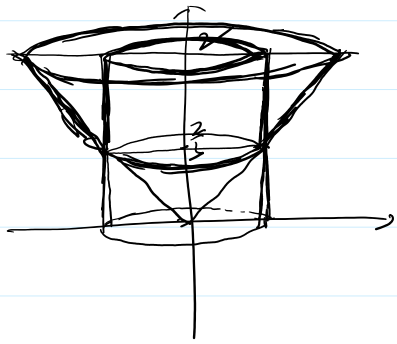
习题 3.3　A 1 (2)　Ω $z=xy$, $z=0$, $x+y=1$ 围



A 5 (3) $x^2+y^2=a^2$ $x^2+z^2=a^2$ $(a>0)$



B 1 (2) $z=\frac{2}{\sqrt{5}}\sqrt{x^2+y^2}$ $x^2+y^2=\frac{4}{5}$ $z=2$



3 $\iiint\limits_{\Omega} z\sin(x+y+z)\,dV$ $x\geq0,\ y\geq0,\ z\geq0,\ x+y+z\leq\frac{\pi}{2}$