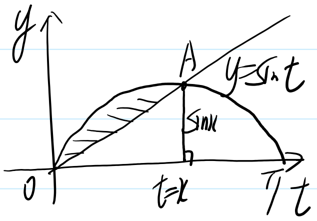
习题 空间解析几何11-1

2022年4月5日 7:19

作业题

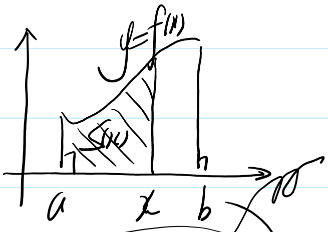
一、2、$A(x,\sin x)$ 为 $y=\sin x\ (0\le x\le\pi)$ 上一点，$S(x)$ 为 OA与 $\sin x$ 围面积



$$S(x)=\int_0^x\left(\sin t-\frac{\sin x}{x}t\right)dt$$

$$=1-\cos x-\frac{\sin x}{x}\frac{1}{2}t^2\Big|_0^x$$

$$=1-\cos x-\frac{1}{2}x\sin x$$



$$S(x)=\int_0^x f(x)dx$$

$$S(x)=\int_0^x f(t)dt$$

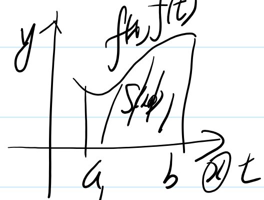
定积分值与积分变量取值无关

$$S(x)=1-\cos x-\frac{1}{2}x\sin x$$

$$=1-\left[1-\frac{1}{2!}x^2+\frac{1}{4!}x^4+\frac{1}{6!}x^6+o(x^6)\right]$$

$$-\frac{1}{2}x\left[x-\frac{1}{3!}x^3+\frac{1}{5!}x^5+o(x^5)\right]$$

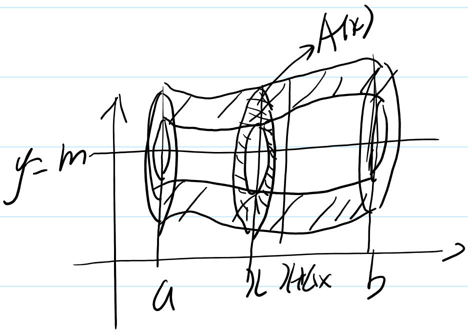
$$=\left(\frac{-1}{24}\quad+\frac{1}{12}\quad\right)x^4+o(x^4)$$



$$S=\int_a^b f(x)dx$$

$$=\int_a^b f(t)dt$$

3、$0<g(x)<f(x)<m$，绕 $y=m$ 旋转



$$V=\int_a^b A(x)dx$$

$$=\int_a^b\left\{\pi[m-g(x)]^2-\pi[m-f(x)]^2\right\}dx$$

$$=\text{— — - - - - —}$$

5、$f(x)=\begin{cases}\dfrac{1}{(x-1)^{\alpha-1}} & 1<x<e\\[2ex] \dfrac{1}{x\ln^{\alpha+1}x} & x\ge e\end{cases}$

$$\int_1^{+\infty}f(x)=\int_1^e\frac{1}{(x-1)^{\alpha-1}}dx+\int_e^{+\infty}\frac{1}{x\ln^{\alpha+1}x}dx$$

$$=\int_1^e\frac{1}{(x-1)^{\alpha-1}}d(x-1)+\int_e^{+\infty}\frac{1}{\ln^{\alpha+1}x}d\ln x$$

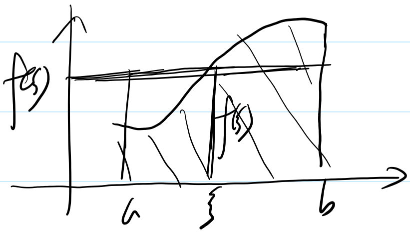
$$\overset{x=x-1}{=}\int_0^{e-1}\frac{1}{x^{\alpha-1}}dx\qquad\int_1^{+\infty}\frac{1}{x^{\alpha+1}}dx$$

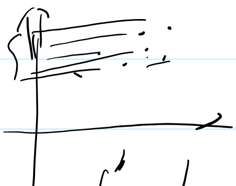
$\overset{x=x-1}{=\!=} \int_0^{e-1} \frac{1}{x^{\alpha-1}}dx$ $\int_1 \frac{1}{x^{\alpha+1}}dx$

$\alpha-1<1$ 收 $\alpha+1>1$ 收

二 1. $f(x)=\frac{1}{1+x^2}$ 在 $[1,\sqrt{3}]$ 平均值 $\frac{\int_1^{\sqrt{3}}\frac{1}{1+x^2}dx}{\sqrt{3}-1}$

定义 $f(\xi)=\frac{\int_a^b f(x)dx}{b-a}$ 为 $y=f(x)$ 在 $[a,b]$ 上平均值





$W=\int_a^b I^2 R\,dt$
$=\int I^2(t) R\,dt$
$=\int_a^b \frac{U^2(t)}{R}dt$

4. $\int_0^{+\infty} x e^{ax^2}dx=1$ 求 $a=$?

$\int_0^{+\infty} e^{-x^2}dx=\frac{\sqrt{\pi}}{2}$

$$1=\int_0^{+\infty} x e^{ax^2}dx=\frac{1}{\sqrt{a}}\int_0^{+\infty} e^{-(\sqrt{a}x)^2}d\sqrt{a}x=\frac{1}{\sqrt{a}}\frac{\sqrt{\pi}}{2}$$

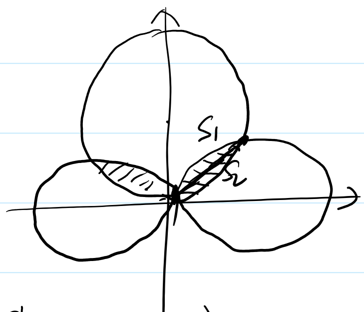
习题 6.4 (A)

1. (4) $r=\sqrt{2}\sin\theta$ $r^2=\cos 2\theta$ 公共面

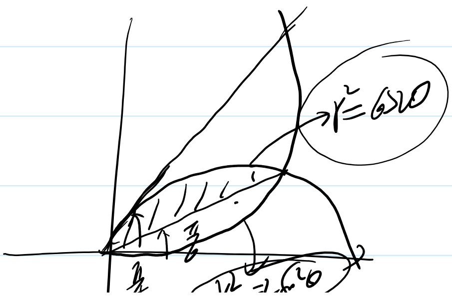
$r^2=\sqrt{2}r\sin\theta$

$x^2+y^2=\sqrt{2}y$

$x^2+(y-\frac{\sqrt{2}}{2})^2=(\frac{\sqrt{2}}{2})^2$



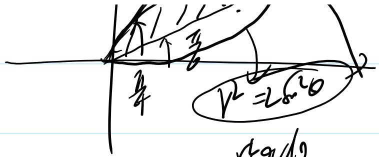
$S=2(S_1+S_2)$



$r^2=(\sqrt{2}\sin\theta)^2=\cos 2\theta$

$2\sin^2\theta=1-2\sin^2\theta$

$\sin^2\theta=\frac{1}{4} \Rightarrow \sin\theta=\pm\frac{1}{2}$



$2\sin\theta = \cos\theta$

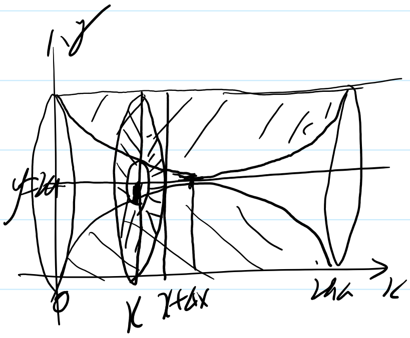
$\sin^2\theta = \frac{1}{4} \Rightarrow \sin\theta = \pm\frac{1}{2}$

$r^2 = \cos 2\theta = 0 \Rightarrow 2\theta = \pm\frac{\pi}{2} \quad \theta = \pm\frac{\pi}{4}$

$r(\theta)d\theta$

$$S_1 = \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \cos 2\theta \, d\theta \qquad S_2 = \frac{1}{2}\int_0^{\frac{\pi}{6}} 2\sin^2\theta \, d\theta$$

6 $\begin{cases} x = a(t-\sin t) \\ y = a(1-\cos t) \end{cases}$



$$V = \quad A(x) = \pi\int_0^{2\pi a}(2a)^2 dx - \pi\int_0^{2\pi a}(2a-y)^2 dx$$

$$= \pi\int_0^{2\pi a}\left[(2a)^2 - (2a-y)^2\right]dx = \text{——}$$

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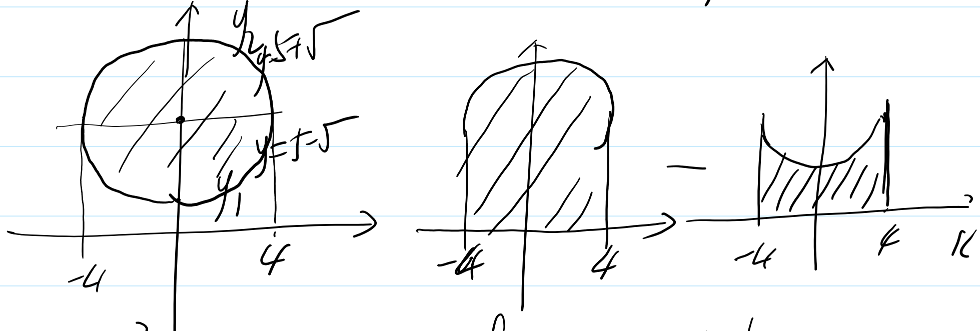

$x^2 + (y-5)^2 = 16$

$y = 5 \pm \sqrt{16 - x^2}$

$$V = \int \pi(y_2^2 - y_1^2)\,dx$$

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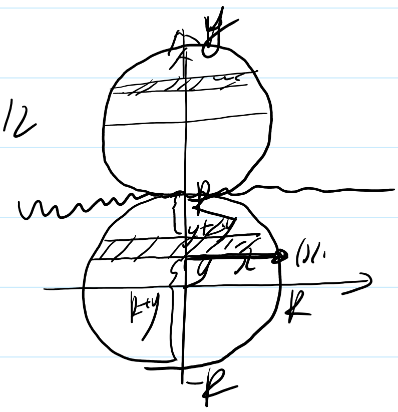


对$[-R, R]$上求W

1) $[y, y+dy]$上$\Delta W \approx dW = \rho\cdot\pi x^2 dy\, g \cdot (2R-(R-y))$

2) $W = \int dW = \int_{-R}^{R} \rho\pi x^2 g(R+y)dy = \text{——}$

$x^2 = R^2 - y^2$

有问题的看这个

$$\int_0^{+\infty} \frac{\ln(1+x)}{(1+x)^2} d(x+1) = \int_0^{+\infty} \ln(1+x)\left(-d\frac{1}{1+x}\right)$$

$= \frac{1}{(1+x)^2}dx$

分部积分即可

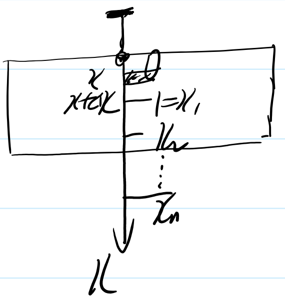
$\int_0 \frac{\quad}{(1+x)^2} d(x+1) = \int_0 \ln(1+x)\left(-d\frac{1}{1+x}\right)$

你觉得记错了 ~~在这~~ $\int_0^{+\infty} e^{-x^2}dx = \frac{\sqrt{\pi}}{2}$, $\int_0^{+\infty} e^{-ax^2}dx = \frac{1}{\sqrt{a}}\int_0^{+\infty} e^{-(\sqrt{a}x)^2} d\sqrt{a}x$

$\frac{1}{\sqrt{a}}\int_0^{+\infty} e^{-u^2}du = 1$

即 $\frac{1}{\sqrt{a}}\frac{\sqrt{\pi}}{2} = 1$

14.



第一次击打 击入1cm 阻力与钉入木板~~深度成正比~~

$F = kx$

$W_1 = \int_0^1 kx\,dx = \frac{1}{2}kx^2\Big|_0^1 = \frac{1}{2}k$

$W_2 = W_1 = \frac{1}{2}k = \int_1^{x_2} kx\,dx = \frac{1}{2}kx^2\Big|_1^{x_2}$

$\frac{1}{2}k = \frac{1}{2}kx_2^2 - \frac{1}{2}k$

$W_1 + W_2 + \cdots + W_n = \int_0^{x_n} kx\,dx$

题6.5 (A) 3. (2) $\int_1^{+\infty} \frac{\arctan x}{x}dx$

$\int_0^{+\infty} \frac{\arctan x}{x}dx$

$\lim_{x\to 0^+} \frac{\arctan x}{x} = 1$ x=0不是瑕点

$\lim_{x\to+\infty} x^{\mu}\cdot\frac{\arctan x}{x} \overset{\mu=1}{=\!=} \frac{\pi}{2}$

$\int_1^{+\infty} \frac{1}{x^{\mu}}dx$, $\mu=1$ 时发散 $\Rightarrow$ ~~原式发散~~

(3) $\int_1^{+\infty} \frac{\sin x}{\sqrt{x^3}}dx$ $\quad \left|\frac{\sin x}{\sqrt{x^3}}\right| \le \frac{1}{\sqrt{x^3}}$

(4) $\int_0^{+\infty} \frac{x^m}{1+x^n}$ $(n\ge 0,\ m\ge 0)$ $\quad \lim_{x\to+\infty} x^{\mu}\cdot\frac{x^m}{1+x^n} = \lim_{x\to+\infty}\frac{x^{\mu+m}}{1+x^n} \overset{\mu+m=n}{=\!=} 1$

$\int_0^{+\infty} \frac{1}{x^{\mu}}dx$ $\quad \mu = n-m > 1$ ~~收敛~~

4 $\int_{-\infty}^{+\infty} \frac{x}{1+x^2}dx = \lim_{a\to+\infty} \int_{-a}^{a} \frac{x}{1+x^2}dx = 0$ ×

$= \int_{-\infty}^{c} \frac{x}{1+x^2}dx + \int_{c}^{+\infty} \frac{x}{1+x^2}dx$

$\lim_{x\to+\infty} x \cdot \frac{x}{1+x^2} = 1 \quad \lambda = 1$ 发

5 $\int_1^{+\infty} \frac{1}{x(1+x)}dx = \lim_{b\to+\infty} \ln x \Big|_1^b - \lim_{b\to+\infty} \ln(1+x)\Big|_1^b$ ×

$= \lim_{b\to+\infty} \int_1^b \frac{1}{x(1+x)}dx = \lim_{b\to+\infty} f(b) = ?$

B 5(1) $\int_0^{\frac{\pi}{2}} \frac{dx}{\sin^p x \cos^q x} \quad (p,q>0) = \int_0^{\frac{\pi}{4}} + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}}$

法一

$-p = 2a-1 \qquad 1-p = 2a > 0$

$-q = 2b-1 \qquad 1-q = 2b > 0$

$\Rightarrow \begin{cases} p<1 \\ q<1 \end{cases}$

法二. $\lim_{x\to 0^+} x^{\lambda} \frac{1}{\sin^p x \cos^q x} \xlongequal{\lambda = p} 1 \qquad \lambda = p < 1$ 收

$\lim_{x\to \frac{\pi}{2}^-} (\frac{\pi}{2}-x)^{\lambda} \cdot \frac{1}{\sin^p x \cos^q x} = \lim_{x\to\frac{\pi}{2}^-} (\frac{\pi}{2}-x)^{\lambda} \frac{1}{\sin^q(\frac{\pi}{2}-x)} \xlongequal{\lambda = q} 1$

$\lambda = q < 1$ 收

$B(p,q) = \int_0^1 x^{p-1}(1-x)^{q-1}dx$

$\xlongequal{x=\sin^2 t} 2\int_0^{\frac{\pi}{2}} \sin^{2p-1} t \cos^{2q-1} t \, dt$

$p>0 \quad q>0$

$B(a,b) = 2\int_0^{\frac{\pi}{2}} \sin^{2a-1} t \cos^{2b-1} t \, dt$

$a>0 \quad b>0$