空间解析几何14-1

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直线及其方程 | 直线 方向 $\vec{s}=\{m,n,p\}$, $M_0(x_0,y_0,z_0)$ | 平面 法向 $\vec{n}=\{A,B,C\}$, $M_0(x_0,y_0,z_0)$

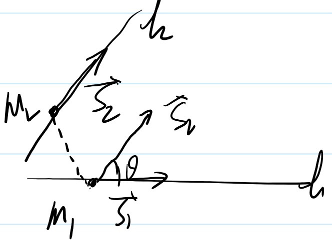
① 标准式（对称式） $\frac{x-x_0}{m}=\frac{y-y_0}{n}=\frac{z-z_0}{p}=t$ $\vec{s}=(m,n,p)$ $M_0(x_0,y_0,z_0)$

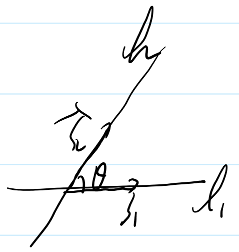
② $\begin{cases} x=x_0+mt \\ y=y_0+nt \\ z=z_0+pt \end{cases}$ $\vec{s}=(m,n,p)$ $M_0(x_0,y_0,z_0)$

③ $\begin{cases} \pi_1: A_1x+B_1y+C_1z+D_1=0 \\ \pi_2: A_2x+B_2y+C_2z+D_2=0 \end{cases}$ $\vec{s}=\vec{n}_1\times\vec{n}_2$ 再选一个点 z_0 代入方程组解出 $M_0(x_0,y_0,z_0)$

④ $\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$ $\vec{s}=\overrightarrow{M_1M_2}$ $M_1(x_1,y_1,z_1)$

⑤ 夹角



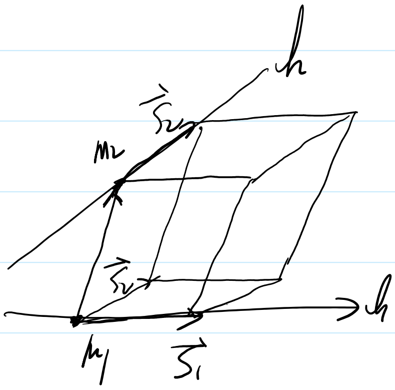


$\cos\theta=\frac{|\vec{s}_1\cdot\vec{s}_2|}{|\vec{s}_1||\vec{s}_2|}$ $\theta\in[0,\frac{\pi}{2}]$

$l_1\perp l_2 \iff \vec{s}_1\perp\vec{s}_2 \iff \vec{s}_1\cdot\vec{s}_2=0$ $m_1m_2+n_1n_2+p_1p_2=0$

$l_1 /\!/ l_2 \iff \vec{s}_1/\!/\vec{s}_2 \iff \vec{s}_1=\lambda\vec{s}_2$ 或 $\vec{s}_1\times\vec{s}_2=0$, $\frac{m_1}{m_2}=\frac{n_1}{n_2}=\frac{p_1}{p_2}$

⑥ 共面、异面 $l_i: \frac{x-x_i}{m_i}=\frac{y-y_i}{n_i}=\frac{z-z_i}{p_i}$ $\vec{s}_i=(m_i,n_i,p_i)$ $M_i(x_i,y_i,z_i)\in l_i$ $i=1,2$

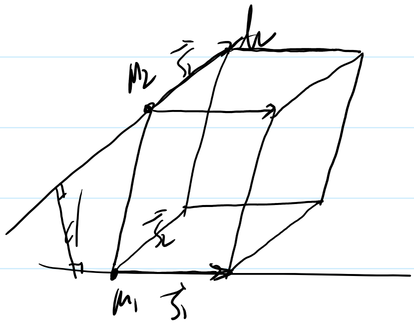


l_1与l_2共面 $\iff [\overrightarrow{M_1M_2}\ \vec{s}_1\ \vec{s}_2]=0$

$\iff \begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ m_1 & n_1 & p_1 \\ m_2 & n_2 & p_2 \end{vmatrix}=0$

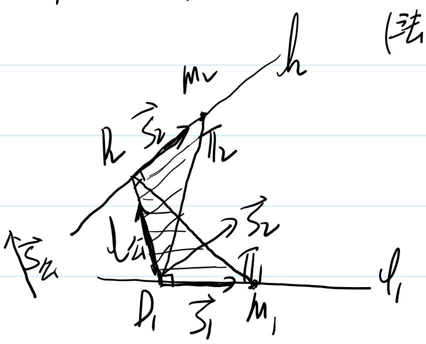
l_1与l_2异面 $\iff [\overrightarrow{M_1M_2}\ \vec{s}_1\ \vec{s}_2]\neq 0$

⑥* 求 l_1 与 l_2 (异面) 公垂线方程，及 l_1 与 l_2 距离



$$d=\frac{V_{平}}{S_{\square}}=\frac{|[\overrightarrow{M_1M_2}\ \vec{s_1}\ \vec{s_2}]|}{|\vec{s_1}\times\vec{s_2}|}$$

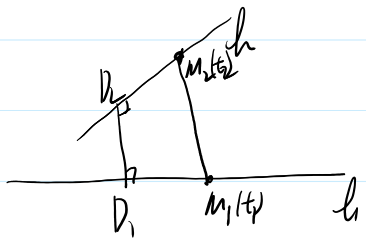
(5)* 异面直线 l_1 与 l_2 公垂线



(法一)

$$l_{公垂}\begin{cases}\pi_1 & M_1\in\pi_1 & \vec{n_1}=\vec{s_1}\times\vec{s_{公}}=\vec{s_1}\times(\vec{s_1}\times\vec{s_2})\\ \pi_2 & M_2\in\pi_2 & \vec{n_2}=\vec{s_2}\times\vec{s_{公}}=\vec{s_2}\times(\vec{s_1}\times\vec{s_2})\end{cases}$$

其中 $\vec{s_{公}}=\vec{s_1}\times\vec{s_2}$



$$l_i:\ \frac{x-x_i}{m_i}=\frac{y-y_i}{n_i}=\frac{z-z_i}{p_i}=t$$

记 $=\ l_i\begin{cases}x=x_i+m_it\\ y=y_i+n_it\\ z=z_i+p_it\end{cases}\quad i=1,2$

$$M_1(t_1)=\begin{cases}x=x_1+m_1t_1\\ y=y_1+n_1t_1\\ z=z_1+p_1t_1\end{cases}(t_1\in\mathbb{R})\qquad M_2(t_2)=\begin{cases}x=x_2+m_2t_2\\ y=y_2+n_2t_2\\ z=z_2+p_2t_2\end{cases}(t_2\in\mathbb{R})$$

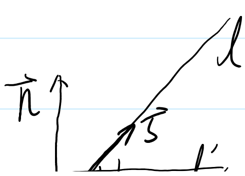
$$\begin{cases}\overrightarrow{M_1M_2}\perp\vec{s_1}\\ \overrightarrow{M_2M_1}\perp\vec{s_2}\end{cases}\iff\begin{cases}\overrightarrow{M_1M_2}\cdot\vec{s_1}=0 & (t_1方程)\\ \overrightarrow{M_1M_2}\cdot\vec{s_2}=0 & (t_2方程)\end{cases}$$

$\Rightarrow t_1$ 与 t_2 值 代回参数式
$\Rightarrow D_1$ 与 D_2 坐标

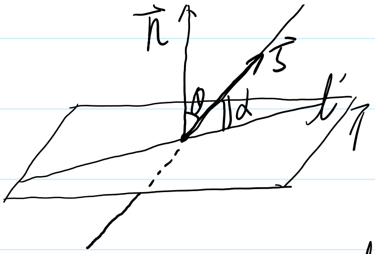
直线与平面夹角

$\pi:\ Ax+By+Cz+D=0\qquad \vec{n}=(A,B,C)$

$l:\ \frac{x-x_0}{m}=\frac{y-y_0}{n}=\frac{z-z_0}{p}\qquad \vec{s}=(m,n,p)\quad M_0(x_0,y_0,z_0)\in l$



$\theta+\alpha=\frac{\pi}{2}\qquad \alpha\in[0,\frac{\pi}{2}]$



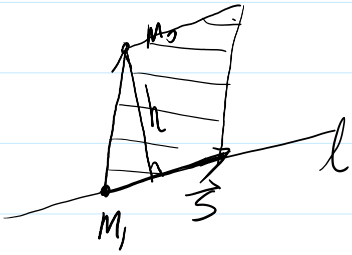
$\theta+\alpha=\frac{\pi}{2}$ $\alpha\in[0,\frac{\pi}{2}]$

$\sin\alpha=\cos\theta=\frac{|\vec n\cdot\vec s|}{|\vec n||\vec s|}=\frac{|Am+Bn+Cp|}{\sqrt{A^2+B^2+C^2}\sqrt{m^2+n^2+p^2}}$

$l\parallel\pi \iff \vec n\perp\vec s \iff \vec n\cdot\vec s=0 \iff Am+Bn+Cp=0$

$l\perp\pi \iff \vec n\parallel\vec s \iff \vec n\times\vec s=0 \iff \vec n=\lambda\vec s \iff \frac{A}{m}=\frac{B}{n}=\frac{C}{p}$

点到直线距离

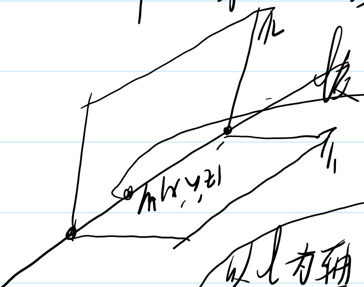


$M_0(x_0,y_0,z_0)$ 为直线 $l: \frac{x-x_1}{m}=\frac{y-y_1}{n}=\frac{z-z_1}{p}$ 外一点

$M_1(x_1,y_1,z_1)\in l$, $\vec s=(m,n,p)\parallel l$

$$h=\frac{|\vec s\times\overrightarrow{M_1M_0}|}{|\vec s|}$$

※平面束 设 $\pi_1: A_1x+B_1y+C_1z+D_1=0$, $\pi_2: A_2x+B_2y+C_2z+D_2=0$ 不平行



交 $l: \begin{cases}A_1x+B_1y+C_1z+D_1=0\\A_2x+B_2y+C_2z+D_2=0\end{cases}$

以 l 为轴平面束方程 $A_1x+B_1y+C_1z+D_1+\lambda(A_2x+B_2y+C_2z+D_2)=0$, $(\lambda\in\mathbb{R})$

即 $(A_1+\lambda A_2)x+(B_1+\lambda B_2)y+(C_1+\lambda C_2)z+(D_1+\lambda D_2)=0$ 称为

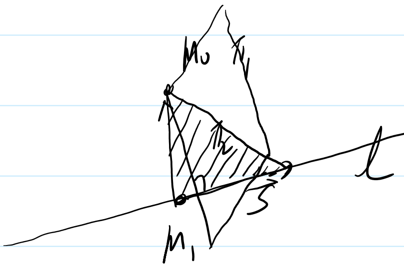
此平面束包含了过 l 的无穷多个平面，(但不包含 π_2 $A_2x+B_2y+C_2z+D_2=0$)

或 $\mu(A_1x+B_1y+C_1z+D_1)+\lambda(A_2x+B_2y+C_2z+D_2)=0$ 包含了所有过 l 的平面 $(\lambda,\mu\in\mathbb{R})$

$\mu\pi_1+\lambda\pi_2=0$

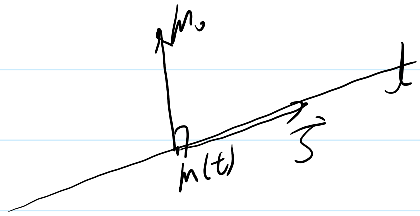
例 求过点 $M_0(2,1,3)$ 且与 l $\frac{x+1}{3}=\frac{y-1}{2}=\frac{z}{-1}$ 垂直相交的直线方程

M_0 π 法一 $M_1(-1,1,0)$ $\vec s=(3,2,-1)$



法一: $M_1(-1,1,0)$ $\vec{s}=(3,2,-1)$

$$\begin{cases}\pi_1(M_0,\ \vec{n_1}=\vec{s}) & 3(x-2)+2(y-1)-1(z-3)=0 \\ \pi_2(M_0,\ \vec{n_2}=\vec{s}\times\overrightarrow{M_1M_0}) & \vec{n_2}=\vec{s}\times\overrightarrow{M_1M_0}=\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 2 & -1 \\ 3 & 0 & 3\end{vmatrix}\end{cases}$$ 点法式



$$M(t)\begin{cases}x=-1+3t \\ y=1+2t \\ z=0+(-1)t=-t\end{cases} \qquad M(t)(-1+3t,\ 1+2t,\ -t)$$

$\overrightarrow{M(t)M_0}\perp\vec{s}$

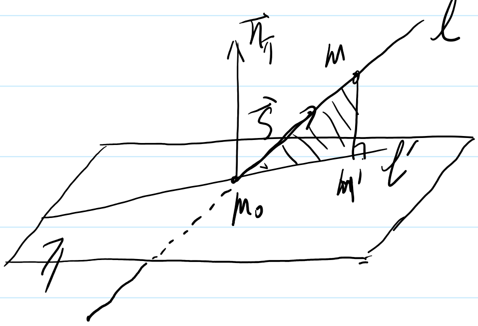
$\overrightarrow{M(t)M_0}\cdot\vec{s}=0$ ∴ $(-1+3t,\ 1+2t,\ -t)\cdot(3,2,-1)=0$

$t=t_0$

二点式即可

四 求直线 $L:\begin{cases}x+y-z-1=0 & \pi_1 \\ x-y+z+1=0 & \pi_2\end{cases}$ 在 π $x+y+z=0$ 上的投影方程

$\vec{s}=\vec{n_1}\times\vec{n_2}$



法一 $M_0\begin{cases}x+y-z-1=0 \\ x-y+z+1=0 \\ x+y+z=0\end{cases}$ $\vec{n}_{法}=\vec{s}\times\vec{n}_\pi=(\vec{n_1}\times\vec{n_2})\times\vec{n}_\pi$

点法式即可求 $\pi_\triangle$

$l'\begin{cases}\pi \\ \pi_\triangle\end{cases}$

法二 求 $\pi_\triangle$

以 l 为轴的平面束 π_λ, $\pi_1+\lambda\pi_2=0$, $(x+y-z-1)+\lambda(x-y+z+1)=0$

即 π_λ: $(1+\lambda)x+(1-\lambda)y+(\lambda-1)z+\lambda-1=0$ △

$\vec{n_\lambda}=(1+\lambda,\ 1-\lambda,\ \lambda-1)$

$\vec{n_\pi}\cdot\vec{n_\lambda}=0$, 即 $(1,1,1)(1+\lambda,\ 1-\lambda,\ \lambda-1)$

$=1+\lambda+1-\lambda+\lambda-1=1+\lambda=0\Rightarrow\lambda=-1$

将 $\lambda=-1$ 代入 $\pi_\lambda\Rightarrow\pi_\triangle$ $y-z-1=0$ $\Rightarrow l'\begin{cases}y-z-1=0 \\ x+y+z=0\end{cases}$