15-16

一，B C　B　A　D　B

二，　π　　$\pi/6$　　$x^2+xz+z^2=2$　　$3\pi/10$　　$x+y+z+1=0$　　$\frac{1}{2}\left(1-\mathrm{e}^{-4}\right)$

三、

1．$\mathrm{d}z=(f_1'+f_2')\mathrm{d}x+(f_1'-f_2')\mathrm{d}y$　　$\frac{\partial^2 z}{\partial x\partial y}=f_{11}''-f_{12}''+f_{21}''-f_{22}''=f_{11}''-f_{22}''$

2．$\begin{cases}\frac{dy}{dx}=\frac{1-x}{2y-1}, & \frac{dz}{dx}=\frac{x-2y}{2y-1}\end{cases}$

3．$\frac{x+1}{1}=\frac{y-2}{-2}=\frac{z-3}{1}$

4．$\iint\limits_D \frac{1}{1+x^2+y^2}\mathrm{d}x\mathrm{d}y=2\int_0^{\frac{\pi}{2}}\mathrm{d}\theta\int_0^1\frac{r}{1+r^2}\mathrm{d}r=\frac{\pi\ln 2}{2}$　$\iint\limits_D \frac{xy}{1+x^2+y^2}\mathrm{d}x\mathrm{d}y=0$，

$$\iint\limits_D \frac{1+xy}{1+x^2+y^2}\mathrm{d}x\mathrm{d}y=\iint\limits_D \frac{1}{1+x^2+y^2}\mathrm{d}x\mathrm{d}y+\iint\limits_D \frac{xy}{1+x^2+y^2}\mathrm{d}x\mathrm{d}y=\frac{\pi\ln 2}{2}$$

四、按要求解答下列各题（共 4 道小题，每小题 8 分，满分 32 分）.

1.　$\mathrm{d}s=\sqrt{r^2+r'^2}\mathrm{d}\theta=4\left|\cos\frac{\theta}{2}\right|\mathrm{d}\theta$　$s=\int_0^{2\pi}4\left|\cos\frac{\theta}{2}\right|\mathrm{d}\theta=\int_0^{\pi}4\cos\frac{\theta}{2}\mathrm{d}\theta-\int_{\pi}^{2\pi}4\cos\frac{\theta}{2}=16$

2.　方向导数为$u_x\cos\alpha+u_y\cos\beta+u_z\cos\gamma=\frac{6}{\sqrt{14}}\times\frac{2}{\sqrt{14}}+\frac{8}{\sqrt{14}}\times\frac{3}{\sqrt{14}}-\sqrt{14}\times\frac{1}{\sqrt{14}}=\frac{6}{7}$

3.　（1）Σ的方程$z=1+x^2+y^2,(1\le z\le 3)$；

（2）$\iiint\limits_{\Omega}\mathrm{e}^z\mathrm{d}x\mathrm{d}y\mathrm{d}z=\int_1^3\mathrm{e}^z\mathrm{d}z\iint\limits_{D_z}\mathrm{d}x\mathrm{d}y=\pi\int_1^3\mathrm{e}^z(\mathrm{z}-1)\mathrm{d}z=e+e^3$

4.　$\int_a^b|x|\mathrm{d}x=\int_a^0-x\mathrm{d}x+\int_0^b x\mathrm{d}x=\frac{1}{2}(a^2+b^2)=\frac{1}{2}\Rightarrow a^2+b^2=1$，　$s=\int_0^{b-a}(bx-x^2-ax)\mathrm{d}x=\frac{1}{6}(b-a)^3$

$F(a,b,\lambda)=\frac{1}{6}(b-a)^3+\lambda(a^2+b^2-1)$，$\begin{cases}F_a=-\frac{1}{2}(b-a)^2+2\lambda a=0,\\ F_b=\frac{1}{2}(b-a)^2+2\lambda b=0,\\ a^2+b^2-1=0\end{cases}$，驻点$(-\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2})$，

$s=\frac{\sqrt{2}}{3}$，$a=0,b=1,s=\frac{1}{6},a=-1,b=0,s=\frac{1}{6}$，面积最大值为$s=\frac{\sqrt{2}}{3}$，最小值为$s=\frac{1}{6}$.

16-17

一　1. $\underline{x^2+y^2=1}$.　2. $\underline{\begin{cases}x+y=1\\z=0\end{cases}}$.　3. $\underline{\mathrm{d}x+\mathrm{d}y}$.　4. $\underline{\quad 0\quad}$.　5. $\underline{2x-4y-z-3=0}$。

二　**1.** D　**2.** C　**3.** B　4. C　**5.**. A

三

1．$A=\int_{-1}^{3}(2x+3-x^2)\mathrm{d}x=\left[x^2+3x-\frac{x^3}{3}\right]_{-1}^{3}=\frac{32}{3}$

2．$2x+z-2+\lambda(y-1)=0$，$\vec{n}\bullet\vec{s}=0\Rightarrow 4-\lambda-2=0,\ \lambda=2$ 所求平面为$2x+2y+z-4=0$.

3．$u_x'=f_1'+2xf_2'$，$u_{xx}''=f_{11}''+4xf_{12}''+4x^2f_{22}''+2f_2'$

4.　$L(x,y,z,\lambda)=x-2y+2z+\lambda\left(x^2+y^2+z^2-1\right)$，　$L_x'=1+2\lambda x=0,\ L_y'=-2+2\lambda y=0,\quad L_z'=2+2\lambda z=0,$　，

$L_\lambda'=x^2+y^2+z^2-1=0$，　$x=\pm\frac{1}{3},\ y=\mp\frac{2}{3},\ z=\pm\frac{2}{3},u=\pm3$

四、.

1.　$\iint\limits_D\left|x^2+y^2-1\right|\mathrm{d}\sigma=\iint\limits_{D_1}\left(1-x^2-y^2\right)\mathrm{d}\sigma+\iint\limits_{D_2}\left(x^2+y^2-1\right)\mathrm{d}\sigma=\int_0^{\frac{\pi}{4}}d\theta\int_0^1\left(1-r^2\right)rdr+\int_0^{\frac{\pi}{4}}d\theta\int_1^{\frac{1}{\cos\theta}}\left(r^2-1\right)rdr=\frac{\pi}{8}-\frac{1}{6}$

2.　$V=\iiint\limits_\Omega dV=8\int_0^{\frac{\pi}{2}}d\theta\int_0^{\frac{\pi}{2}}d\varphi\int_0^1 abcr^2\sin\varphi dr=\frac{4\pi}{3}abc$

3.　$f_x'(0,0)=\lim\limits_{(x,y)\to(0,0)}\frac{f(x,0)-f(0,0)}{x}=0$　　同理$f_y'(0,0)=0$

$$\lim_{(x,y)\to(0,0)}\frac{f(x,y)-f(0,0)-f_x'(0,0)x-f_y'(0,0)y}{\sqrt{x^2+y^2}}=\lim_{(x,y)\to(0,0)}\sqrt{\frac{x^4+y^4}{x^2+y^2}}=0$$

4.　（1）$\iiint\limits_\Omega\frac{dV}{x^2+y^2}=\int_0^{\frac{\pi}{4}}d\theta\int_{\frac{1}{\cos\theta}}^{\frac{2}{\cos\theta}}rdr\int_0^{r\sin\theta}\frac{dz}{r^2}=\frac{1}{2}\ln2$；　（2）$\iiint\limits_\Omega\frac{dV}{x^2+y^2}=\int_0^{\frac{\pi}{4}}d\theta\int_{\operatorname{arccot}\sin\theta}^{\frac{\pi}{2}}d\varphi\int_{\frac{1}{\sin\varphi\cos\theta}}^{\frac{2}{\sin\varphi\cos\theta}}\frac{dr}{\sin\varphi}=\frac{1}{2}\ln2$

17-18

一、B　B　C　D　C　B

二、$\ln 3-\dfrac{1}{2}$　　$\dfrac{1}{2}$　　$-\dfrac{4}{3}$　　$f\left(\pm\sqrt{x^2+y^2},z\right)=0$　　$f_1'+yf_2'$　　$\sqrt{3}$

13. $I=\int_0^1\mathrm{d}x\int_0^{x^2}\dfrac{y\mathrm{e}^y}{1-\sqrt{y}}\mathrm{d}y=\int_0^1\dfrac{y\mathrm{e}^y}{1-\sqrt{y}}\mathrm{d}y\int_{\sqrt{y}}^1\mathrm{d}x \quad =\int_0^1 y\mathrm{d}\mathrm{e}^y=y\mathrm{e}^y\Big|_0^1-\int_0^1\mathrm{e}^y\mathrm{d}y \quad =\mathrm{e}-\mathrm{e}^y\Big|_0^1=1$.

14. $\dfrac{x}{-2}=\dfrac{y-2}{3}=\dfrac{z-4}{1}$.　　$\begin{cases}x=-2t,\\ y=2+3t,\\ z=4+t.\end{cases}$

15. $\mathrm{d}z\big|_{(\mathrm{e},0)}=\dfrac{1}{2\mathrm{e}}\mathrm{d}x-\dfrac{1}{2}\mathrm{d}y$，　$\dfrac{\partial^2 z}{\partial y\partial x}=-\dfrac{\dfrac{\partial z}{\partial x}(1+z)-z\dfrac{\partial z}{\partial x}}{(1+z)^2}=-\dfrac{z}{x(1+z)^3}$，　$\dfrac{\partial^2 z}{\partial y\partial x}\Big|_{(\mathrm{e},0)}=-\dfrac{1}{8\mathrm{e}}$.

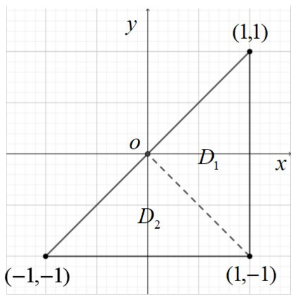
16. $I=\iiint\limits_{\Omega}\sqrt{(r\cos\theta)^2+(r\sin\theta)^2}\,r\mathrm{d}r\mathrm{d}\theta\mathrm{d}z \quad =\int_0^{2\pi}\mathrm{d}\theta\int_0^1 r^2\mathrm{d}r\int_r^1\mathrm{d}z \quad =2\pi\int_0^1 r^2(1-r)\mathrm{d}r=\dfrac{\pi}{6}$.

$I=\int_0^1\mathrm{d}z\iint\limits_{D_z}\sqrt{x^2+y^2}\,\mathrm{d}x\mathrm{d}y=\int_0^1\mathrm{d}z\int_0^{2\pi}\mathrm{d}\theta\int_0^z r^2\mathrm{d}r=2\pi\int_0^1\dfrac{z^3}{3}\mathrm{d}z=\dfrac{\pi}{6}$.

17. $V=\pi\int_{-4}^4\left(5+\sqrt{16-x^2}\right)^2\mathrm{d}x-\pi\int_{-4}^4\left(5-\sqrt{16-x^2}\right)^2\mathrm{d}x=\pi\int_{-4}^4 20\sqrt{16-x^2}\mathrm{d}x=40\pi\int_0^4\sqrt{16-x^2}\mathrm{d}x$

$=40\pi\cdot\dfrac{1}{4}\pi\cdot 4^2=160\pi$.

18. $L=2x-y+1+\lambda(x^2+y^2-5)$，　$\begin{cases}L_x=2+2\lambda x=0,\\ L_y=-1+2\lambda y=0,\\ L_\lambda=x^2+y^2-5=0,\end{cases}$，解得驻点 $(-2,1),(2,-1)$　又 $f(-2,1)=-4, f(2,-1)=6,$ 所以函数 $f(x,y)$ 满足约束条件 $x^2+y^2=5$ 下的最大值为 $f(2,-1)=6,$ 最小值为 $f(-2,1)=-4$.



19. $\int_0^{x^2}f(x^2-t)\mathrm{d}t\overset{x^2-t=u}{=}\int_0^{x^2}f(u)\mathrm{d}u$，故 $f(x)=x^2+x\int_0^{x^2}f(u)\mathrm{d}u+\iint\limits_D f(xy)\mathrm{d}x\mathrm{d}y$.

$f(xy)=(xy)^2+xy\int_0^{(xy)^2}f(u)\mathrm{d}u+\iint\limits_D f(xy)\mathrm{d}x\mathrm{d}y$

令 $\iint\limits_D f(xy)\mathrm{d}x\mathrm{d}y=k$，　$k=\iint\limits_D(xy)^2\mathrm{d}x\mathrm{d}y+\iint\limits_D\left[xy\int_0^{(xy)^2}f(u)\mathrm{d}u\right]\mathrm{d}x\mathrm{d}y+\iint\limits_D k\mathrm{d}x\mathrm{d}y$.

$\iint\limits_D(xy)^2\mathrm{d}x\mathrm{d}y=\int_{-1}^1\mathrm{d}x\int_{-1}^x x^2y^2\mathrm{d}y=\dfrac{2}{9}$.　$k=\dfrac{2}{9}+0+2k, k=-\dfrac{2}{9}$. 于是 $f(x)=x^2+x\int_0^{x^2}f(u)\mathrm{d}u-\dfrac{2}{9}$.

令 $x=1$，得 $\int_0^1 f(x)\mathrm{d}x=-\dfrac{7}{9}$.

18－19　A2

一、$D\ \ A\ \ D\ \ B\ \ C\ \ B$

二、$\mathrm{e}^{xy}\sin\mathrm{e}^{xy}\left(y\mathrm{d}x+x\mathrm{d}y\right)$; $\frac{1}{4}\ln 17$; $-\frac{1}{2}$; $\frac{3\pi}{10}$; 3; $\begin{cases}x^2+2z^2=4\\ y=0\end{cases}$

三、

$8x-9y-22z-59=0$

$z_x{}'=3x^2f+x^3yf_1'-xyf_2'$, $z_{xy}{}''=4x^3f_1'+2xf_2'+x^4yf_{11}''-yf_{22}''$

$5\pi+\frac{32}{5}$

$\mathrm{grad}u\left(P_0\right)=\left(1,-3,-3\right)$, $\frac{\partial u}{\partial\vec{l}}\left(P_0\right)=-\frac{1}{3}$

$z_{\max}=25$, $z_{\min}=0$

$\left(x-1\right)+4\left(y-2\right)+6\left(z-2\right)=0$ 或 $\left(x+1\right)+4\left(y+2\right)+6\left(z+2\right)=0$

$\frac{4}{5}\pi abc$

19-20

C　A　A　D　C　C

4,　$x+3y+z-3=0$,　$\sqrt{3}$,　1,　$(t-1)f(t)$,　$\frac{2\pi}{3}$

13.求 $y=2x-x^2$ 与 $y=0$ 所围的封闭区域绕 x 轴旋转一周生成旋转体的体积.

$$V=\pi\int_0^2\left(2x-x^2\right)^2dx=\frac{16\pi}{15}$$

14.求过直线 $L_1:\frac{x-1}{1}=\frac{y-1}{0}=\frac{z-1}{-2}$ 且平行于直线 $L_2:\frac{x+2}{2}=\frac{y-1}{-1}=\frac{z}{-2}$ 的平面方程.

$$\vec{n}=\vec{s}_1\times\vec{s}_2=-(2,2,1),\quad 2(x-1)+2(y-1)+(z-1)=0,\quad 2x+2y+z-5=0$$

15.求椭球面 $2x^2+3y^2+z^2=9$ 上点 $M(1,1,2)$ 处的切平面方程与法线方程.

$$\vec{n}=\{2x,3y,z\}_M=(2,3,2)\quad 2(x-1)+3(y-1)+2(z-2)=0\quad \frac{x-1}{2}=\frac{y-1}{3}=\frac{z-2}{2}.$$

16.设 $u=f(x,xy,xyz)$，f 具有二阶连续偏导数，求 $\frac{\partial u}{\partial y},\frac{\partial^2 u}{\partial y^2}$.

$$\frac{\partial u}{\partial y}=xf_2'+xzf_3'\qquad \frac{\partial^2 u}{\partial y^2}=x^2\left(f_{22}''+2zf_{23}''+z^2f_{33}''\right)$$

17.求函数 $z=x^2+y^2-2x-y$ 在 $D=\{(x,y)|2x+y\le 4,x\ge 0,y\ge 0\}$ 上的最值.

$$\begin{cases}z_x'=2x-2=0\\ z_y'=2y-1=0\end{cases},\ z_{\min}\left(1,\frac{1}{2}\right)=-\frac{5}{4}$$

$$y=0,\ z=x^2-2x=0\ \ (0\le x\le 2),\ z(1,0)=-1$$

$$x=0,\ z=y^2-y=0\ \ (0\le y\le 4),\ z\left(0,\frac{1}{2}\right)=-\frac{1}{4}$$

$$2x+y=4,\ z=5x^2-16x+12=0\ \ (0\le x\le 2),\ z\left(\frac{8}{5},\frac{4}{5}\right)=-\frac{4}{5}$$

$$z(0,0)=0,\ z(2,0)=0,\ z_{\max}(0,4)=12$$

18．求在上半球体 $x^2+y^2+z^2\le 1\ (z\ge 0)$ 除去柱体 $x^2+y^2\le x$ 的空间立体的体积.

$$V=\frac{2\pi}{3}-2\int_0^{\frac{\pi}{2}}d\theta\int_0^{\cos\theta}\sqrt{1-r^2}\,rdr=\frac{3\pi+4}{9}\quad 或\quad D=\left\{(x,y)\middle|x^2+y^2\le 1,\ x^2+y^2\ge x\right\}$$

$$V=\iint_D\sqrt{1-x^2-y^2}\,dxdy=2\left[\int_0^{\frac{\pi}{2}}d\theta\int_{\cos\theta}^1\sqrt{1-r^2}\,rdr+\int_{\frac{\pi}{2}}^{\pi}d\theta\int_0^1\sqrt{1-r^2}\,rdr+\right]=\frac{3\pi+4}{9}$$

19.已知 $\Omega=\left\{(x,y,z)\middle|\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^4}{c^4}\le 1\right\}$，计算 $I=\iiint_\Omega\left(\frac{x}{a}+\frac{y}{b}+\frac{z^2}{c^2}\right)^2dV$.

$$I=\iiint_\Omega\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^4}{c^4}\right)dV=8\int_0^{\frac{\pi}{2}}d\theta\int_0^1dr\int_0^{c\left(1-r^2\right)^{\frac{1}{4}}}abr\left(r^2+\frac{z^4}{c^4}\right)dz=\frac{8\pi}{9}abc\quad 或$$

$$I=\iiint_\Omega\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^4}{c^4}\right)dV=8\int_0^c dz\int_0^{\frac{\pi}{2}}dr\int_0^1 ab\left(1-\frac{z^4}{c^4}\right)\left[r^2\left(1-\frac{z^4}{c^4}\right)+\frac{z^4}{c^4}\right]rdr=\frac{8\pi}{9}abc$$

20-21

一、**1.**（A） **2.**（C）. **3.**（B）. **4.**（D）. **5.**（C）. **6.**（A）.

二、**7.** $\frac{3}{10}\pi$. **8.** $\underline{1}$. **9.** $\underline{x^2+y^2+z=1}$. **10.** $\underline{2/5}$，**11.** $\frac{1}{y}\left(3\sin y^3-2\sin y^2\right)$. **12.** $\underline{\quad 0\quad}$.

三、

13. $S=\int_{-1}^{2}\left(2x-x^2-x+2\right)dx=\frac{9}{2}$

14. $x-y+z=0$

15. $\frac{\partial u}{\partial x}=yf_1'+2xf_2' \quad \frac{\partial^2 u}{\partial x\partial y}=xyf_{11}''+2\left(x^2+y^2\right)f_{12}''+4xyf_{22}''+f_1'$

16. $(x,y,z)=\left(\pm\frac{1}{3},\pm\frac{2}{3},\pm\frac{2}{3}\right)$

$f_{\max}=3, f_{\min}=-3$

17. $I=\int_0^1\frac{\sin y}{y}\mathrm{d}y\int_{y^2}^{y}\mathrm{d}x=\int_0^1(1-y)\sin y\mathrm{d}y=1-\sin 1$

18 .

$$I=\iiint_{\Omega_1}+\iiint_{\Omega_2}=\int_0^{\frac{\pi}{2}}d\theta\int_0^{\frac{\pi}{4}}d\varphi\int_1^{\frac{1}{\cos\varphi}}r^6\sin\varphi dr+\int_0^{\frac{\pi}{2}}d\theta\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}d\varphi\int_1^{\frac{1}{\sin\varphi}}r^6\sin\varphi dr$$

$$=\frac{\pi}{84}++\frac{\sqrt{2}\pi}{28}+\frac{2\pi}{15}-\frac{\sqrt{2}\pi}{28}=\frac{61\pi}{420}$$

19.

$$I=\iiint_{\Omega_1}+\iiint_{\Omega_2}=\int_{-\frac{\pi}{4}}^{\frac{3\pi}{4}}d\theta\int_0^{\sin\left(\theta+\frac{\pi}{4}\right)}rdr\int_0^r\left(\sqrt{2}\frac{\cos\theta+\sin\theta}{r}-2\right)zdz$$

$$+\int_{-\frac{\pi}{4}}^{\frac{3\pi}{4}}d\theta\int_{\sin\left(\theta+\frac{\pi}{4}\right)}^{1}rdr\int_0^r\left(2-\sqrt{2}\frac{\cos\theta+\sin\theta}{r}\right)zdz$$

$$=\frac{\pi}{32}+\left(\frac{\pi}{4}-\frac{2}{3}+\frac{\pi}{32}\right)=\frac{5\pi}{16}-\frac{2}{3}$$