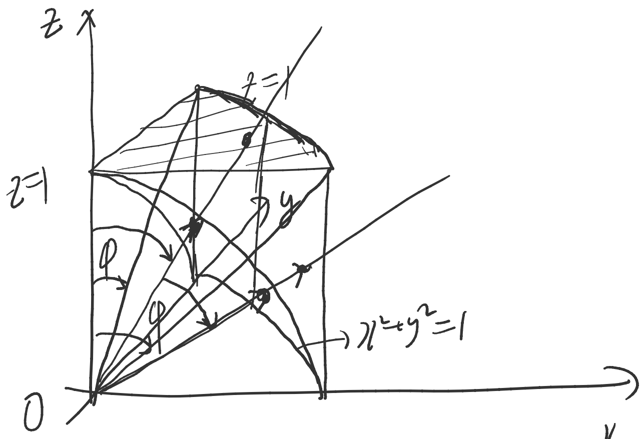
2022年8月19日 18:28



$x^2+y^2 \le 1$

$1 \ge z \ge \sqrt{1-x^2-y^2}$

$x \ge 0$

$y \ge 0$

$$\int_0^{\frac{\pi}{2}} d\theta \int_0^{\frac{\pi}{4}} d\varphi \int_1^{\frac{1}{\cos\varphi}} r^4 r^2 \sin\varphi \, dr +$$

$$\int_0^{\frac{\pi}{2}} d\theta \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} d\varphi \int_1^{\frac{1}{\sin\varphi}} r^4 r^2 \sin\varphi \, dr$$

$$\begin{cases} x = r\sin\varphi\cos\theta \\ y = r\sin\varphi\sin\theta \\ z = r\cos\varphi \end{cases}$$

$$\begin{cases} z=1 \\ x^2+y^2=1 \end{cases} \begin{cases} r\cos\varphi=1 \\ r^2\sin^2\varphi=1 \\ r\sin\varphi=1 \end{cases} \Rightarrow \tan\varphi=1$$

$\varphi = \arctan 1 = \frac{\pi}{4}$

$z=1 \quad r\cos\varphi=1 \Rightarrow r=\frac{1}{\cos\varphi}$

$x^2+y^2=1$

$r^2\sin^2\varphi=1$

$r\sin\varphi=1 \Rightarrow r=\frac{1}{\sin\varphi}$

$\sin^{-2}\varphi \, d\varphi = -d\cot\varphi$

$(1+\cot^2\varphi)^2 = \left(\frac{1}{\sin^2\varphi}\right)^2 \cos^4\varphi$

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1}{\sin^6\varphi} d\varphi$$

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (1+\cot^2\varphi)^2 \cdot -d\cot\varphi$$

$$= \int_0^1 (1+u^2)^2 du$$

$$\varphi(y) = \int_0^y \frac{\ln(1+yx)}{x} dx \quad 求 \varphi'(y)$$

法一： $F(y) = \int_{a(y)}^{b(y)} f(x,y)\,dx$

$$F'(y) = \int_{a(y)}^{b(y)} f'_y(x,y)\,dx + f(b(y),y)\cdot b'(y) - f(a(y),y)\cdot a'(y)$$

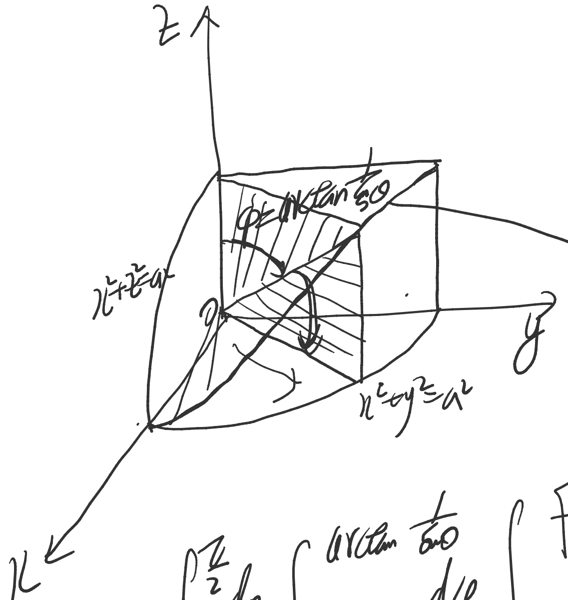
法二 用 $\left[\int_0^{x} f(t)\,dt\right]' = f(x)$

$$\text{则} = \text{用}\left[\int_a^{\varphi(x)} f(t)\,dt\right]' = f(x)$$

设 $xy = u \qquad y\,dx = du$

$$\varphi(y) = \int_0^{y^2} \frac{\ln(1+u)}{\cancel{y}u}\,\cancel{y}\,du = \int_0^{y^2} \ln(1+u)\,du$$

$$\varphi'(y) = \ln(1+y^2)\cdot 2y$$



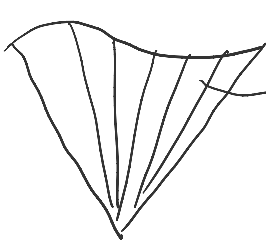
$0 \le \theta \le \frac{\pi}{2}$

$0 \le \varphi \le \frac{\pi}{2}$

$$\begin{cases} x^2+z^2=a^2 \\ x^2+y^2=a^2 \end{cases} \qquad \begin{cases} r^2\sin^2\varphi\cos^2\theta + r^2\cos^2\varphi = a^2 \\ r^2\sin^2\varphi = a^2 \end{cases}$$

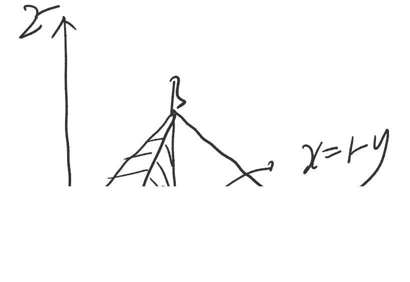
$$\int_0^{\frac{\pi}{2}} d\theta \int_0^{\arctan\frac{1}{\sin\theta}} d\varphi \int_0^{\sqrt{\frac{a^2}{\sin^2\varphi\cos^2\theta+\cos^2\varphi}}} r^2\sin\varphi\,dr \qquad \tan\varphi = \frac{1}{\sin\theta}$$

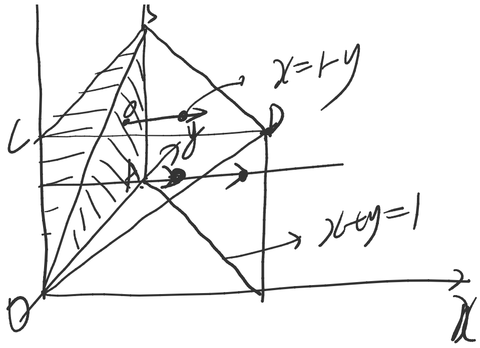
$$\int_0^{\frac{\pi}{2}} d\theta \int_{\arctan\frac{1}{\sin\theta}}^{\frac{\pi}{2}} d\varphi \int_0^{\frac{a}{\sin\varphi}} r^2\sin\varphi\,dr \qquad \int_\alpha^\beta d\theta \int_{\varphi_1(\theta)}^{\varphi_2(\theta)} d\varphi \int_{r_1(\theta,\varphi)}^{r_2(\theta,\varphi)} r^2\sin\varphi\,dr$$

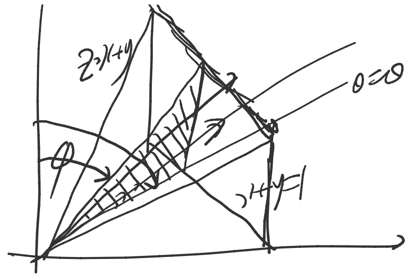


$\varphi = \varphi(\theta)$

一般锥面







$$\begin{cases} z=x+y \\ x+y=1 \end{cases} \quad z=1 \quad r\cos\varphi=1$$

$$r\cos\varphi = r\sin\varphi\cos\theta + r\sin\varphi\sin\theta$$

$$r\sin\varphi\cos\theta + r\sin\varphi\sin\theta = 1$$

$$\Rightarrow \varphi=\varphi(\theta)$$

$z=x+y \Rightarrow x = z-y$

$x+y=1 \Rightarrow x=1-y$

$$\int_0^{\frac{\pi}{2}} d\theta \int_{\varphi(\theta)}^{\frac{\pi}{2}} d\varphi \int_0^{\frac{1}{\sin\varphi\cos\theta+\sin\varphi\sin\theta}} f \; r^2\sin\varphi \, dr$$