160 Lecture 2

26 Jan 2026

1 Unit Variation over an Interval

How do we minimize a function over an Interval (a, b]?

Terminology:

$$x \in \mathbb{R}$$
$$F : \mathbb{R} \to \mathbb{R}$$

X in the real numbers, and F is a function mapping from Real numbers to Real numbers

x is the variable or parameter or decision.

In a convex form, we write it as

$$\begin{aligned} \min \quad & f(x) \\ s.t. \quad & x\ in(a, b]. \end{aligned}$$

We can assume $a, b = \pm\infty$ as well.

Minima and Maxima

A point is a global min if $f(x*) \leq f(x) \forall x \in (a, b]$. similarily, global max just flips the sign to $\geq$

However, this is often hard to find. Local minimums are easier to find. $f(x*) \leq f(x) \forall I\ \{x\}$, or in other words $\exists f(x*) \leq f(x) \forall x \in X \cap \{x \pm \Delta\}$. Intuitively, a large Δ is not going to work here, because it might encompass more local mins. However, it still works because we say there exists.

A strict local min is where $F(x^*) < f(x) \forall x \in \{x - \delta, x + \delta\} \backslash \{x*\}$. Basically, it is a local min that is not repeated, such as the function $f(x) = 3$. On the interval from [4,5], there is a local min of 3, however, it is not strict because there are infinitely many local mins in the neighborhood.

These naturally let us ask the following two questions about optimization.

- How to characterize those points?
- How to find the numerical values of those points?

To answer these questions we use Taylor Series Approximations or Expansions.

Taylor Series

1.) Dominance: The idea that in a taylor series, if Δ is small, then the upper term polynomial term dominates the rest. How small is small? EH Who cares? The basic idea is ignore high order terms

Mean Value Theory:

1. First order approximation (Linear) : $F(x+\Delta) = f(x) \sim f(x) + f'(x)\Delta$. There $\exists z$ from x to $x+\Delta$ $f(x+\Delta) = f(x) + f'(z)\Delta$
2. Second order approximation (quadratic) $f(x+\Delta) \sim f'(x)\Delta + \frac{f''(x+\Delta)}{2}$. Then follows first order approx steps

EX: Imagine $f(x) = e^x$. find the Linear approx of this function.

So we need to find the approximation around the nominal point. here we will choose 0.

$$f(0+\Delta) = f(0) + f'(z)\Delta$$

$$f(0+\Delta) = 0 + e^z\Delta$$

If $f(x^*)$ is a local min of a function, then $f'(x^*) = 0$. Intuition is picking a neighborhood, zoom in and it slowly becomes flat. Later we will define tangent plan and others.

Proof: If $F(x*) \neq 0$, then $x*$ is not a min.

2.) Truncation.