IEOR 173 Problem Set 2

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Chapter 1

1.1 Textbook Problems

Question 1: 23

Coin having prob p is successively flipped until the rth head appears. Argue that X, the number of flips required will be n, $n \geq r$, with prob

$$P\{X = n\} = \binom{n-1}{r-1} p^r (1-p)^{n-r}, n \geq r$$

Since X is the number of flips required to get r flips, then that means that in the first $n-1$ trials, there has to be $r-1$ flips. Thus, since each flip is independent, X $\sim$ Binom$(n-1, p)$

$$P(X = n) = \binom{n-1}{r-1} p^{r-1} (1-p)^{n-r} \cdot p = \binom{n-1}{r-1} p^r (1-p)^{n-r}$$

Question 2: 26

Suppose that two teams are playing series of games, independent. Team A wins with prob p and Team B wins with prob $(1-p)$. The winner is the first team to win i games.
Find the expected number of games played when
i.) i=2
Let X be the number of games until one team wins.
Let A_i be if team A wins game i
Let B_i be if team B wins game i
Let $q = q - p$

There are 6 ways the game can end.

- A_1A_2
- B_1B_2
- $A_1B_2A_3$
- $A_1B_2B_3$
- $B_1A_2B_3$
- $B_1A_2A_3$

$$E[X] = \sum (n \cdot P(X = n)$$

$$E[X] = 2 \cdot P(X = 2) + 3 \cdot P(X = 3)$$

$$P(X = 2) = p \cdot p + q \cdot q = p^2 + q^2$$

$$P(X=3) = p\cdot q\cdot p + p\cdot q\cdot q + q\cdot p\cdot q + q\cdot p\cdot p = p^2q + pq^2 + q^2p + qp^2 = 2(p^2q + pq^2) = 2pq(q+p) = 2pq$$

$$E[X] = 2p^2 + 2q^2 + 6pq = 2(2p^2 - 2p + 1) + 6(p - p^2) = 2 + 2p - 2p^2$$

ii.) i=3 when $i = 3$, the game can end in 3, 4, 5 matches.

$$P(X=3) = p^3 + q^3$$

$$P(X=4) = P(\text{A wins 2/3 games})p + P(\text{B wins 2/3 games}q) = \binom{3}{2}p^2qcdotp + \binom{3}{2}q^2p\cdot q$$

Question 3: 28

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Question 6: 51

1.2 Fun Problems

Question 7: 1