

Data140 Disc Notes

Ryan Lin

Contents

Chapter 1	Page 2
1.1 Discussion 02	2
1.2 Sup 02: Waiting Times, Bionmial/Hypergeomotric/Multinomial	2

Chapter 1

1.1 Discussion 02

Example 1.1.1 (Ch 5 Ex 12)

Small org has $n=12$ workers. Assume each worker's birthday is uniformly distributed. Find the chance there is at least one month in which none of the workers has their birthday.

Let M_i be if month i has no birthdays.

$$\begin{aligned} P(\cup M_i) &= \sum_i p(M_i) - \sum_i \sum_{j=i}^N P(M_i, M_j) + \dots \\ &= \sum_i P(M_i) - \sum_k P(M_1 M_k) + \dots \sum_k (-1)^{k+1} 12C(12-k) \\ &= 12P(M_1)^n - 12C2P(M_1, M_2)^n + \dots \\ &= \frac{11^n}{12} - \frac{10^n}{12} + \dots \end{aligned}$$

Example 1.1.2 (Ch5 Ex 9)

Consider 5 card hand dealt from a standard card deck. find probability of being dealt.

a.) four of a kind

$$P(\text{four of a kind}) = \frac{\# \text{outcomes}}{\text{total outcomes}} = \frac{13 \cdot 12 \cdot 4C4 \cdot 4C1}{12C5}$$

Make sure to check for distinction. It does matter what

b.) one pair

$$P(\text{one pair}) = \frac{13 \cdot 12C3 \cdot 4C2 \cdot 4(4C1)}{52C5}$$

In this example, bcd are not distinct. We do not care about order for this. Therefor the $12C3$.

1.2 Sup 02: Waiting Times, Binomial/Hypergeometric/Multinomial

Example 1.2.1 (Chp 6 Ex 2)

If you bet on "red" at roulette, chance of winning is $18/38$. They are independent. Suppose you keep betting on red and stop when you have won 6 bets

There are independent bets, pointing us to a binomial distribution.

a.) chance you place exactly 10 bets?

$$P(\text{exactly 10 bets}) = \binom{9}{5} \frac{18^5}{38} \frac{20^4}{38} \cdot \frac{18}{38}$$

This is $P(\text{exactly 10 bets}) = P(X = 5, 6\text{th red on 10th bet}) = P(5 \text{ reds on 1-9}) \cdot P(\text{Red on 10th})$

b.) What is the chance that you place more than 10 bets?

Let Y be number of reds in 10 bets. $Y \sim \text{Binom}(10, \frac{18}{38})$

$$P(Y \leq 5) = \sum_{k=0}^{5} P(Y = k) = \sum_{k=0}^{10} \binom{10}{k} \frac{18^k}{38} \frac{28^{10-k}}{38}$$

Example 1.2.2 (Ch 6 Ex 10)

Test for disease predicts correct with chance .99. Suppose the test is run on 300 patients, independently

a.) Chance for at least 295 patients the result is correct? Find numeral value.

Let X be number of correct results. $X \sim \text{binom}(300, .99)$

$$P(X \geq 295) = \sum_{k=295}^{300} P(X = k) = \sum_{k=295}^{300} \binom{300}{k} (.99^k) (.01)^{300-k}$$

b.) Justify a Poisson approx for the chance in (a) and find the value of the approx.

In general, if you have a $\text{Binomial}(n,p) \approx \text{Poisson}(np)$, when n is large and p is small.

Let Y be the number of patients with incorrect test results (failures)

$$Y \sim \text{binom}(300, 0.01)$$

Now we have a big N and small p , therefore $Y \approx \text{Poisson}(3)$. This is useful because we know $X + Y = 300$, therefore if we want to find $P(x \geq 295) = P(300 - Y \geq 295) = P(Y \leq 5)$

$$P(Y \leq 5) = \sum_{k=0}^{5} e^{-3} \frac{3^k}{k!} \approx .91608$$

Example 1.2.3 (Ch 6 Ex 4)

class consists of 40 freshmen, 60 sophomores, 30 juniors, and 20 seniors. SRS of 10 students

a.) distribution of number of sophomores

Model: without replacement.

Let S be the number of sophomores in the sample.

$$S \sim \text{Hypergeometric}(150, 60, 10)$$

Hyper geometric has 3 arguments, # in population, # of Good, # sample size.

b.) joint dist of number of freshmen and sophomores

Let F be # Freshmen in the sample

$$P(F = f, S = s) = \frac{\binom{40}{f} \binom{60}{s} \binom{50}{10-(f+s)}}{\binom{150}{10}}, \quad 0 \leq f + s \leq 10$$

We know that $0 \leq f + s \leq 10$.

c.) conditional dist of the number of freshmen in the sample given that there are 4 sophomores.

What we do here is the division rule combining parts a and b.

Example 1.2.4 (Ch 6 Ex 11)

Seven dice rolled. Find prob of

a.) exactly 2 6s

Let X be the number of 6s rolled. $X \sim \text{binom}(7, \frac{1}{6})$

$$P(X = 2) = \binom{7}{2} \frac{1^2}{6} \frac{5^5}{6}$$

b.) two fours, two fives, and 3 sixes

$$P(4, 4, 5, 5, 6, 6, 6) = \frac{1^2}{6} \cdot \frac{1^2}{6} \cdot \frac{1^3}{6}$$

Multiply this by number of ways we can get this combination of numbers. Therefore, multiply the original sequence by $\binom{7}{2} \binom{5}{2} \binom{3}{3}$

- c.) three of one face and four of another
- d.) each face appears
- e.) three each of two different faces