IEOR 173 Discussion Notes

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Chapter 1

1.1 Discussion 02

1.1.1 Discrete Random Variables

Discrete random var takes values in a countable set. $\{0, 1, 2, \ldots, n\}$

Probability Mass function (EMF): $P(X = k)$

1.1.2 Common Discrete Dusts

Bernoulli $X \in \{0, 1\}$

Binomial $X \sim binom(n, p)$

Geometric Number of trials until first success.

1.1.3 Expectation and Variance

$$E[X] = \sum k \cdot P(X = x) \tag{1.1}$$

$$Var(X) = E[X^2] - (E[X])^2 \tag{1.2}$$

1.1.4 Key techniques

Indicator Variable: Define $I = 1$ if event A occurs, 0 otherwise

Then, $E[I] = P(A)$

Strategy: Express $X = \sum_i I_i$ and use linearity of expectations.

1.1.5 Binomial Coefficients

$$\binom{n}{k} = \frac{n!}{k!(n-k)!} \tag{1.3}$$

Number of ways to choose k items out of n items.

1.2 Homework Problems

Question 1: Homework 1

Let X denote the number of white balls selected when k ball are chosen at random from an urn containing n white and m black balls.
1.) Compute P(X =i)

$$P(X = i) = \frac{\binom{n}{i} \cdot \binom{m}{k-i}}{\binom{n+m}{k}}$$

2.) Let $X = \sum X_i$

$$E[X] = \sum E[X_i] = \sum_{i=1}^{k} \frac{n}{n+m} = k\frac{n}{n+m}$$

The probability of each ball does not change, because the indicator function just indicates if the ith ball is white. You don't know the whether or not the other balls have been removed or not, thus the probability does not change. However, they are still dependent.
We can also use Y_j as well, where Y is if white ball j is selected.

$$EY_j = P_j = \frac{k}{n+m}$$

$$EX = \sum_{i}^{n+m} EY_i = n \cdot \frac{k}{n+m}$$

3.) Compute Var(X_i) using your expression of X as a function of X_i. Each X_i is a Bern wiht $p = \frac{n}{n+m}$

$$Var(X_i) = pq = \frac{n}{n+m} \cdot \frac{m}{n+m} = \frac{nm}{(n+m)^2}$$

$$Cov(X_i, X_j) = E[X_i, X_j] - E[X_i]E[X_j]$$

Since we are sampling without replacement,

$$P(X_1 = 1, X_2 = 1) = \frac{n}{n+m} \cdot \frac{n-1}{n+m-1}$$

Since if $X_k = 0$, then we only care that they are both equal to one.
Thus,

$$Cov(X_1, X_2) = \frac{n(n-1)}{(n+m)(n+m-1)} - \frac{n^2}{(n+m)^2}$$

TODO: SIMPLIFY
then there is k choose 2 Covariances.