

IEOR 164: Assignment 1

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Question 1: minimize following functions

a.) Min $\|x\|_1$ s.t $\|Ax - b\|_\infty \leq 1$

$$\begin{aligned} \min : & \sum_i t_i \\ \text{s.t} : & t_i \geq x_i \\ & t_i \geq -x_i \\ & 1 \geq a_i^T x - b_i \\ & 1 \geq -a_i^T x + b_i \\ & t_i \geq 0 \end{aligned}$$

b.) Min $\|Ax - b\|_1 + \|x\|_\infty$

$$\begin{aligned} \min : & \sum t_i + u \\ \text{s.t} : & t_i \geq (a_i^T x - b)_i \\ & t_i \geq -(a_i^T x - b)_i \\ & u \geq x_i \\ & u \geq -x_i \\ & t_i \geq 0, u \geq 0 \end{aligned}$$

Question 2: Hospital Shift Schedule

A hospital wants to make a weekly night shift schedule for its nurses. The demand for the night shift on different days is given. Every nurse works 5 days in a row.

a.) Formulate a linear program to minimize total number of nurses used while meeting all demand.

- Let x_i be the number of nurses starting on day i for $i \in \{1, 2, \dots, 7\}$
- Let d_i be the number of nurses needed on day i for $i \in \{1, 2, \dots, 7\}$

$$\begin{aligned} \min : & \sum_{i=1}^7 x_i \\ \text{s.t} : & \begin{array}{llll} x_1 + & & x_4 + x_5 + & x_6 + x_7 \geq d_1 \\ x_1 + & x_2 + & x_5 + & x_6 + x_7 \geq d_2 \\ x_1 + & x_2 + x_3 + & & x_6 + x_7 \geq d_3 \\ x_1 + & x_2 + x_3 + & x_4 + & x_7 \geq d_4 \\ x_1 + & x_2 + x_3 + & x_4 + x_5 & \geq d_5 \\ & x_2 + x_3 + & x_4 + x_5 + & x_6 \geq d_6 \\ & x_3 + & x_4 + x_5 + & x_6 + x_7 \geq d_7 \end{array} \\ & x_i \geq 0 \forall i \end{aligned}$$

b.) The optimal objective value is 110, with 60 nurses on day 1, 10 on day 3, 10 on day 4, 20 on day 5, and 10 on day 6.