X2 - 4 Year 12 - Ext 2 - Trial and HSC Revision - Sheet 4

Name:

Question 1 {Integration} Evaluate the following intergrals

a) (by substitution) $\int_0^{\frac{\pi}{2}} \frac{\cos x}{(1+\sin x)^2} dx$

b) (by completing the square) $\displaystyle\int \frac{1}{x^2 - 2x + 5}\,dx$

c) (by partial fractions) $$\int \frac{4+7x}{(x+1)^2(2+3x)}dx$$

By re-writing in the form $$\frac{A}{x+a}+\frac{B}{(x+a)^2}+\frac{C}{x+b}$$

Question 2 {proof} -

Show that $1+r+r^2+r^3+...+r^{n-1}=\dfrac{1-r^n}{1-r}$ for $r\neq 1$.

Question 3 {proof}

Use proof by contradiction to prove that if a, b are integers, then $a^2 - 4b - 3 \neq 0$. **3**
You may wish to consider the case when a is even and the case when a is odd.

Question 4 {complex numbers}

(i) Use De Moivre's theorem to show that $(1+i\tan\theta)^5 = \dfrac{\cos 5\theta + i\sin 5\theta}{\cos^5\theta}$. **1**

(ii) Hence find expressions for $\cos 5\theta$ and $\sin 5\theta$ in terms of $\tan\theta$ and $\cos\theta$. **2**

(iii) Show that $\tan 5\theta = \dfrac{5t - 10t^3 + t^5}{1 - 10t^2 + 5t^4}$ where $t = \tan\theta$. **1**

(iv) Use the result of (iii) and an appropriate substitution **3**

to show that $\tan\dfrac{\pi}{5} = \sqrt{5 - 2\sqrt{5}}$.

Question 5 {vectors}

Relative to a fixed origin O, the points A, B and C have position vectors

$\underset{\sim}{a} = \begin{pmatrix} -1 \\ \frac{4}{3} \\ 7 \end{pmatrix}$, $\underset{\sim}{b} = \begin{pmatrix} 4 \\ \frac{4}{3} \\ 2 \end{pmatrix}$ and $\underset{\sim}{c} = \begin{pmatrix} 6 \\ \frac{16}{3} \\ 2 \end{pmatrix}$ respectively.

(i) Find the cosine of $\angle ABC$. **1**

(ii) Hence find the area of the triangle ABC. **1**

(iii) Use a vector method to find the shortest distance between the point A and the line passing through the points B and C. **2**