

# 2020 HSC Mathematics Extension 1 Marking Guidelines

## Section I

### Multiple-choice Answer Key

Question	Answer
1	A
2	C
3	D
4	B
5	C
6	D
7	A
8	C
9	B
10	A

## Section II

### Question 11 (a) (i)

Criteria	Marks
<ul style="list-style-type: none"> <li>Provides correct solution</li> </ul>	1

**Sample answer:**

$$\begin{aligned}
 P(2) &= 2^3 + 3(2)^2 - 13(2) + 6 \\
 &= 0
 \end{aligned}$$

### Question 11 (a) (ii)

Criteria	Marks
<ul style="list-style-type: none"> <li>Provides correct solution</li> </ul>	2
<ul style="list-style-type: none"> <li>Attempts to divide by <math>x - 2</math>, or equivalent merit</li> </ul>	1

**Sample answer:**

$$\begin{array}{r}
 x^2 + 5x - 3 \\
 x - 2 \overline{) x^3 + 3x^2 - 13x + 6} \\
 \underline{x^3 - 2x^2} \phantom{+ 6} \\
 5x^2 - 13x \phantom{+ 6} \\
 \underline{5x^2 - 10x} \phantom{+ 6} \\
 -3x + 6 \\
 \underline{-3x + 6} \\
 0
 \end{array}$$

$$\therefore P(x) = (x - 2)(x^2 + 5x - 3)$$

### Question 11 (b)

Criteria	Marks
• Provides correct solution	3
• Evaluates the dot product and sets it equal to 0, or equivalent value	2
• Writes a dot product = 0, or equivalent merit	1

**Sample answer:**

Since the vectors are perpendicular,

$$0 = \begin{pmatrix} a \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 2a-3 \\ 2 \end{pmatrix}$$

$$0 = 2a^2 - 3a - 2$$

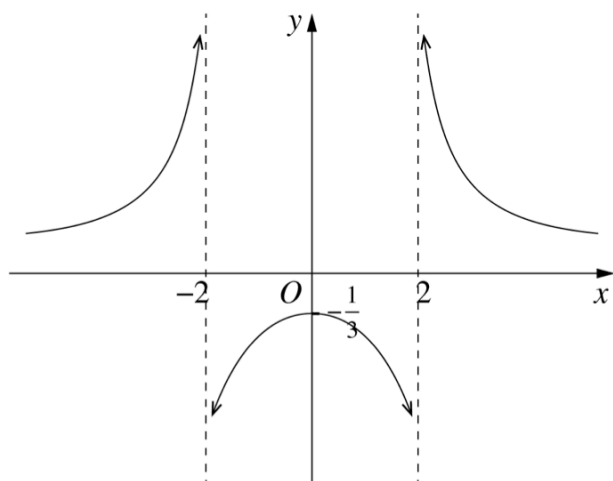
$$0 = (2a+1)(a-2)$$

$$a = -\frac{1}{2} \text{ or } 2$$

### Question 11 (c)

Criteria	Marks
• Provides correct sketch	3
• Provides a sketch with some correct features	2
• Marks asymptotes at $x = -2$ or $2$ , or marks local maximum at $y = -\frac{1}{3}$ , or equivalent merit	1

**Sample answer:**



### Question 11 (d)

Criteria	Marks
• Provides correct solution	4
• Correctly writes $\sqrt{3}\sin x + 3\cos x$ in the form $A\sin(x + \alpha)$ and finds one solution	3
• Finds $A$ and $\alpha$ , or equivalent merit	2
• Finds the value of $A$ , or equivalent merit	1

**Sample answer:**

$$\begin{aligned}\sqrt{3}\sin x + 3\cos x &= A\sin(x + \alpha) \\ &= A\sin x \cos \alpha + A\cos x \sin \alpha\end{aligned}$$

$$\therefore A\cos \alpha = \sqrt{3} \quad (1)$$

$$A\sin \alpha = 3 \quad (2)$$

$$\text{dividing, } \tan \alpha = \frac{3}{\sqrt{3}} = \sqrt{3} \Rightarrow \alpha = \frac{\pi}{3}.$$

$$\therefore A\cos \frac{\pi}{3} = \sqrt{3} \Rightarrow A = 2\sqrt{3}$$

$$\therefore \sqrt{3}\sin x + 3\cos x = 2\sqrt{3}\sin\left(x + \frac{\pi}{3}\right)$$

$$\text{so } 2\sqrt{3}\sin\left(x + \frac{\pi}{3}\right) = \sqrt{3}$$

$$\sin\left(x + \frac{\pi}{3}\right) = \frac{1}{2}$$

$$x + \frac{\pi}{3} = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6},$$

$$x = \frac{\pi}{2}, \frac{11\pi}{6} \text{ given } 0 \leq x \leq 2\pi.$$

**Question 11 (e)**

Criteria	Marks
• Provides correct solution	2
• Rewrites as $\frac{dx}{dy} = e^{-2y}$ , or equivalent merit	1

**Sample answer:**

$$\frac{dy}{dx} = e^{2y}$$

$$\frac{dx}{dy} = e^{-2y}$$

$$x = \int e^{-2y} dy$$

$$= -\frac{1}{2}e^{-2y} + C$$

## Question 12 (a)

Criteria	Marks
• Provides correct solution	3
• Proves inductive step by assuming true for $k$ (or equivalent) and using that assumption to show true for $k + 1$ , or equivalent merit	2
• Verifies base case, $n = 1$ , or equivalent merit	1

**Sample answer:**

$$(1 \times 2) + (2 \times 5) + (3 \times 8) + \cdots + n(3n - 1) = n^2(n + 1)$$

$$\text{For } n = 1 \quad \text{LHS} = 1(2) = 2 \quad \text{RHS} = 1^2(2) = 2$$

$\therefore$  Statement is true for  $n = 1$

Assume statement true for  $n = k$ , that is,

$$(1 \times 2) + (2 \times 5) + (3 \times 8) + \cdots + k(3k - 1) = k^2(k + 1)$$

Prove true for  $n = k + 1$

That is, we show that

$$(1 \times 2) + (2 \times 5) + \cdots + k(3k - 1) + (k + 1)(3(k + 1) - 1) = (k + 1)^2(k + 2)$$

$$\begin{aligned} \text{LHS} &= k^2(k + 1) + (k + 1)(3k + 2) \\ &= (k + 1)(k^2 + 3k + 2) \\ &= (k + 1)(k + 1)(k + 2) \\ &= (k + 1)^2(k + 2) \\ &= \text{RHS} \end{aligned}$$

$\therefore$  By the principle of mathematical induction the statement is true for  $n \geq 1$ .

### Question 12 (b) (i)

Criteria	Marks
• Provides correct answer	1

**Sample answer:**

$$\begin{aligned}
 E(X) &= np \\
 &= 100 \times \frac{3}{5} \\
 &= 60
 \end{aligned}$$

### Question 12 (b) (ii)

Criteria	Marks
• Provides correct solution	1

**Sample answer:**

$$\begin{aligned}
 \sigma &= \sqrt{np(1-p)} \\
 &= \sqrt{100 \times \frac{3}{5} \times \frac{2}{5}} \\
 &= \sqrt{24} \\
 &\div 5
 \end{aligned}$$

### Question 12 (b) (iii)

Criteria	Marks
• Provides correct solution	1

**Sample answer:**

$$\begin{aligned}
 P(55 \leq X \leq 65) &= P(-1 \leq Z \leq 1) \\
 &\approx 68\%
 \end{aligned}$$

### Question 12 (c)

Criteria	Marks
• Provides correct explanation	2
• Evaluates $\binom{8}{3}$ , or obtains the expression $\frac{400}{\binom{8}{3}}$ , or equivalent merit	1

**Sample answer:**

There are  $\binom{8}{3} = 56$  possible choices of 3 topics and  $\frac{400}{56} = 7.14$ .

As there are 56 possible combinations, we can have at most 392 students without exceeding 7 students per combination. But we have 400 students, so at least one combination has 8 or more students.

### Question 12 (d)

Criteria	Marks
• Provides correct solution	3
• Finds correct primitive, or equivalent merit	2
• Uses product to sum result, or equivalent merit	1

**Sample answer:**

$$\begin{aligned}
 \int_0^{\frac{\pi}{2}} \cos 5x \sin 3x \, dx &= \int_0^{\frac{\pi}{2}} \frac{1}{2} (\sin 8x - \sin 2x) \, dx \\
 &= \left[ \frac{1}{2} \left( -\frac{\cos 8x}{8} + \frac{\cos 2x}{2} \right) \right]_0^{\frac{\pi}{2}} \\
 &= \frac{1}{2} \left( -\frac{1}{8} - \frac{1}{2} \right) - \frac{1}{2} \left( -\frac{1}{8} + \frac{1}{2} \right) \\
 &= -\frac{1}{2}
 \end{aligned}$$



### Question 12 (e)

Criteria	Marks
• Provides correct solution	3
• Obtains $x^2 + y^2 = \text{constant}$ , or equivalent merit	2
• Separates the variable, or equivalent merit	1

**Sample answer:**

$$\frac{dy}{dx} = \frac{-x}{y}$$

Separating variables,  $\int y \, dy = \int -x \, dx$

$$\therefore \frac{y^2}{2} = \frac{-x^2}{2} + c$$

$$x^2 + y^2 = d, \text{ where } d = 2c$$

The curve passes through (1, 0)

$$\text{So } d = 1^2 + 0^2 = 1$$

Hence the equation of  $D$  is

$$x^2 + y^2 = 1 \quad (\text{unit circle})$$

### Question 13 (a) (i)

Criteria	Marks
• Provides correct derivative	1

**Sample answer:**

$$\frac{d}{d\theta}(\sin^3 \theta) = 3\sin^2 \theta \cos \theta$$

### Question 13 (a) (ii)

Criteria	Marks
• Provides correct solution	4
• Obtains correctly simplified integrand in terms of $\sin\theta$ and $\cos\theta$ , or equivalent merit	3
• Correctly substitutes and attempts to simplify (ignoring limits), or equivalent merit	2
• Uses given substitution, or equivalent merit	1

**Sample answer:**

Let  $x = \tan\theta$

$$\frac{dx}{d\theta} = \sec^2\theta$$

At  $x = 0$ ,  $\theta = 0$

at  $x = 1$ ,  $\theta = \frac{\pi}{4}$

$$\begin{aligned}
 \therefore \int_0^1 \frac{x^2}{(1+x^2)^{\frac{5}{2}}} dx &= \int_0^{\frac{\pi}{4}} \frac{\tan^2\theta}{(1+\tan^2\theta)^{\frac{5}{2}}} \sec^2\theta d\theta \\
 &= \int_0^{\frac{\pi}{4}} \frac{\tan^2\theta}{\sec^3\theta} d\theta \\
 &= \int_0^{\frac{\pi}{4}} \frac{\sin^2\theta}{\cos^2\theta} \cos^3\theta d\theta \\
 &= \int_0^{\frac{\pi}{4}} \sin^2\theta \cos\theta d\theta \\
 &= \left[ \frac{1}{3} \sin^3\theta \right]_0^{\frac{\pi}{4}} \quad \text{by part (i)} \\
 &= \frac{1}{3} \times \left( \frac{\sqrt{2}}{2} \right)^3 \\
 &= \frac{2\sqrt{2}}{3 \times 2^3} \\
 &= \frac{\sqrt{2}}{12}
 \end{aligned}$$

### Question 13 (b)

Criteria	Marks
• Provides correct solution	4
• Obtains a correct expression for the volume and uses a double-angle formula, or equivalent merit	3
• Finds point of intersection and writes volume as a difference of volumes OR • Finds point of intersection and finds volume generated by $y = \sin x$ , or equivalent merit	2
• Finds point of intersection, or writes volume as a difference of two volumes, or equivalent merit	1

**Sample answer:**

The curves intersect when

$$\cos 2x = \sin x$$

$$1 - 2\sin^2 x = \sin x$$

$$2\sin^2 x + \sin x - 1 = 0$$

$$(2\sin x - 1)(\sin x + 1) = 0$$

$$\therefore \sin x = \frac{1}{2}, (\sin x \neq -1).$$

$$\therefore x = \frac{\pi}{6}$$

The volume is given by

$$\begin{aligned}
 V &= \pi \int_0^{\frac{\pi}{6}} \cos^2 2x \, dx - \pi \int_0^{\frac{\pi}{6}} \sin^2 x \, dx \\
 &= \pi \left( \int_0^{\frac{\pi}{6}} \frac{1 + \cos 4x}{2} \, dx - \int_0^{\frac{\pi}{6}} \frac{1 - \cos 2x}{2} \, dx \right) \\
 &= \pi \left[ \frac{\sin 4x}{8} + \frac{x}{2} - \frac{x}{2} + \frac{\sin 2x}{4} \right]_0^{\frac{\pi}{6}} \\
 &= \pi \left( \frac{\sin \frac{2\pi}{3}}{8} + \frac{\sin \frac{\pi}{3}}{4} \right) \\
 &= \frac{\pi}{8} \left( \frac{\sqrt{3}}{2} + \sqrt{3} \right) \\
 &= \frac{3\pi\sqrt{3}}{16}
 \end{aligned}$$

**Question 13 (c) (i)**

Criteria	Marks
• Provides correct solution	4
• Obtains one correct derivative and makes some progress towards obtaining the other, or equivalent merit	3
• Obtains one correct derivative or attempts both, or equivalent merit	2
• Attempts one derivative, or equivalent merit	1

**Sample answer:**

$$\begin{aligned}
 f'(x) &= \sec^2(\cos^{-1} x) \cdot \frac{-1}{\sqrt{1-x^2}} \\
 &= \frac{1}{\cos^2(\cos^{-1} x)} \cdot \frac{-1}{\sqrt{1-x^2}} \\
 &= \frac{-1}{x^2 \sqrt{1-x^2}} \\
 g'(x) &= \frac{-\frac{x}{\sqrt{1-x^2}} \times x - \sqrt{1-x^2} \times 1}{x^2} \\
 &= \frac{-x^2 - (1-x^2)}{x^2 \sqrt{1-x^2}} \\
 &= \frac{-1}{x^2 \sqrt{1-x^2}}
 \end{aligned}$$

$$\therefore f'(x) = g'(x)$$

### Question 13 (c) (ii)

Criteria	Marks
• Provides correct solution for both parts of the domain	3
• Provides a correct solution for one part of the domain, or equivalent merit	2
• Observes that $f(x) - g(x)$ is a constant, or equivalent merit	1

**Sample answer:**

$$f'(x) = \frac{-1}{x^2 \sqrt{1-x^2}} = g'(x)$$

And so  $f'(x) - g'(x) = 0$

$\Rightarrow f(x) - g(x)$  is a constant

For  $x < 0$ , say  $x = -1$

$$\begin{aligned} f(-1) - g(-1) &= \tan(\cos^{-1}(-1)) - \frac{\sqrt{1-(-1)^2}}{(-1)} \\ &= \tan(\pi) + 0 \\ &= 0 \end{aligned}$$

So for  $x < 0$ ,  $f(x) = g(x)$ .

For  $x > 0$ , say  $x = 1$ ,

$$\begin{aligned} f(1) - g(1) &= \tan(\cos^{-1}(1)) - \frac{\sqrt{1-(1)^2}}{(1)} \\ &= \tan(0) + 0 \\ &= 0 \end{aligned}$$

So for  $x > 0$ ,  $f(x) = g(x)$

So for all  $x$  in the domain

$$f(x) = g(x)$$

### Question 14 (a) (i)

Criteria	Marks
• Provides correct solution	2
• Identifies coefficient of $x^n$ on the left hand side, or equivalent merit	1

**Sample answer:**

The left hand side is  $\binom{2n}{n}$  which is the coefficient of  $x^n$  in the expansion of  $(1+x)^{2n}$ .

This means that  $\binom{n}{0}\binom{n}{0} + \binom{n}{1}\binom{n}{1} + \dots + \binom{n}{n}\binom{n}{n}$  should be the coefficient of  $x^n$  in the expansion of  $(1+x)^n(1+x)^n$ .

The first term,  $\binom{n}{0}$  is the constant term in the expansion of  $(1+x)^n$  and so should be multiplied by an  $x^n$  term. But  $\binom{n}{0} = \binom{n}{n-0} = \binom{n}{n}$  is equal to the coefficient of  $x^n$  in the expansion  $(1+x)^n$ .

Thus  $\binom{n}{0}^2$  is the coefficient of the  $x^n$  term that comes from the constant in the expansion of  $(1+x)^n$  times the coefficient of the  $x^n$  term in the expansion of  $(1+x)^n$ .

Similarly  $\binom{n}{1}^2$  is the coefficient of the  $x^n$  term that comes from the coefficient of  $x$  term in the expansion of  $(1+x)^n$  times the coefficient of the  $x^{n-1}$  term in the expansion of  $(1+x)^n$ .

Therefore, the right hand side is the coefficient of the  $x^n$  term in the expansion of  $(1+x)^n(1+x)^n$ .

OR

Question 14 (a) (i) (continued)

The coefficient of  $x^n$  in expansion of  $(1+x)^{2n}$  is  $\binom{2n}{n}$ .

The coefficient of  $x^n$  in  $(1+x)^n(1+x)^n$  is

$$= \binom{n}{0}\binom{n}{n} + \binom{n}{1}\binom{n}{n-1} + \cdots + \binom{n}{k}\binom{n}{n-k} + \cdots + \binom{n}{n}\binom{n}{0}$$

$$\text{But } \binom{n}{n-k} = \binom{n}{k}$$

So the coefficient is  $\binom{n}{0}\binom{n}{0} + \binom{n}{1}\binom{n}{1} + \cdots + \binom{n}{n}\binom{n}{n}$

$$\text{Hence } \binom{2n}{n} = \binom{n}{0}^2 + \binom{n}{1}^2 + \cdots + \binom{n}{n}^2$$

### Question 14 (a) (ii)

Criteria	Marks
• Provides correct solution	2
• Explains why one of the terms is $\binom{n}{k} \times \binom{n}{k}$ , or equivalent merit	1

**Sample answer:**

We can choose:

0 men and 0 women in  $\binom{n}{0}\binom{n}{0} = \binom{n}{0}^2$  ways

or 1 man and 1 woman in  $\binom{n}{1}^2$  ways

⋮

or  $n$  men and  $n$  women in  $\binom{n}{n}^2$  ways

Hence the total number of ways is

$$\binom{n}{0}^2 + \cdots + \binom{n}{n}^2 = \binom{2n}{n} \quad \text{from part (i)}$$



### Question 14 (a) (iii)

Criteria	Marks
• Provides correct solution	2
• Explains why one of the terms is $k^2 \binom{n}{k}^2$ , or equivalent merit	1

**Sample answer:**

We can choose 1 woman and 1 leader in  $\binom{n}{1} \times 1$  ways and similarly for the men. This gives

$$\binom{n}{1} \times 1 \times \binom{n}{1} \times 1 = \binom{n}{1}^2 \times 1^2 \text{ for this case.}$$

We can choose 2 women and 1 leader in  $\binom{n}{2} \binom{2}{1} = \binom{n}{2} \times 2$  ways and similarly for the men.

This gives  $\binom{n}{2} \times 2 \times \binom{n}{2} \times 2 = \binom{n}{2}^2 \times 2^2$  for this case.

⋮

We can choose  $n$  women and 1 leader in  $\binom{n}{n} \binom{n}{1} = \binom{n}{n} \times n$  ways and similarly for the men.

This gives  $\binom{n}{n} \times n \times \binom{n}{n} \times n = \binom{n}{n}^2 \times n^2$  for this case.

And so the total is  $1^2 \binom{n}{1}^2 + 2^2 \binom{n}{2}^2 + \cdots + n^2 \binom{n}{n}^2$ .

### Question 14 (a) (iv)

Criteria	Marks
• Provides correct solution	2
• Recognises that choosing the leaders first reduces the problem to using part (ii) with $n - 1$ women and $n - 1$ men, or equivalent merit	1

**Sample answer:**

There are  $n$  ways to choose a woman leader, leaving  $(n - 1)$  women from which to choose.

There are  $n$  ways to choose a man leader, leaving  $(n - 1)$  men from which to choose.

By part (ii) there are  $\binom{2(n-1)}{(n-1)}$  ways to choose the  $(n - 1)$  women and  $(n - 1)$  men.

$$\text{Hence } 1^2 \binom{n}{1}^2 + 2^2 \binom{n}{2}^2 + \dots + n^2 \binom{n}{n}^2 = n^2 \binom{2n-2}{n-1}.$$

### Question 14 (b) (i)

Criteria	Marks
• Provides correct solution	2
• Expands $\sin(2\theta + \theta)$ , or equivalent merit	1

**Sample answer:**

$$\begin{aligned} \sin 3\theta &= \sin(2\theta + \theta) \\ &= \sin 2\theta \cos \theta + \cos 2\theta \sin \theta \\ &= 2\sin \theta \cos^2 \theta + (1 - 2\sin^2 \theta) \sin \theta \\ &= 2\sin \theta (1 - \sin^2 \theta) + \sin \theta - 2\sin^3 \theta \\ &= 3\sin \theta - 4\sin^3 \theta \end{aligned}$$

$$\therefore 4\sin^3 \theta - 3\sin \theta + \sin(3\theta) = 0$$

$$\therefore \sin^3 \theta - \frac{3}{4}\sin \theta + \frac{\sin(3\theta)}{4} = 0$$

**Question 14 (b) (ii)**

Criteria	Marks
• Provides correct solution	2
• Obtains $\sin^3 \theta - \frac{3}{4}\sin \theta + \frac{8}{64} = 0$ , or equivalent merit	1

**Sample answer:**

$$x^3 - 12x + 8 = 0$$

$$x = 4\sin\theta, \text{ so}$$

$$64\sin^3\theta - 48\sin\theta + 8 = 0$$

$$\therefore \sin^3\theta - \frac{3}{4}\sin\theta + \frac{8}{64} = 0$$

comparing with (i),

$$\frac{8}{64} = \frac{\sin 3\theta}{4} \Rightarrow \sin 3\theta = \frac{32}{64} = \frac{1}{2}.$$

### Question 14 (b) (iii)

Criteria	Marks
• Provides correct proof	3
• Obtains all the roots of the cubic, or equivalent merit	2
• Obtains $4\sin\frac{\pi}{18}$ as a solution of the cubic in part (b) (ii), or equivalent merit	1

**Sample answer:**

$$\text{From } \sin 3\theta = \frac{1}{2}$$

$$\text{we have } 3\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{6} + 2\pi, \frac{5\pi}{6} + 2\pi, \frac{\pi}{6} + 4\pi, \dots$$

$$\theta = \frac{\pi}{18}, \frac{5\pi}{18}, \frac{13\pi}{18}, \frac{17\pi}{18}, \frac{25\pi}{18}, \dots$$

So  $4\sin\frac{\pi}{18}$ ,  $4\sin\frac{5\pi}{18}$ ,  $4\sin\frac{13\pi}{18}$  etc are roots of the cubic, but they are not all distinct.

The angles  $\frac{\pi}{18}$  and  $\frac{5\pi}{18}$  are distinct angles in the first quadrant and so have different positive

sine values. The angle  $\frac{25\pi}{18}$  is in the third quadrant and so has a negative sine value. Hence

we can take  $\alpha = 4\sin\frac{\pi}{18}$ ,  $\beta = 4\sin\frac{5\pi}{18}$ ,  $\gamma = 4\sin\frac{25\pi}{18}$  as 3 distinct roots,

$$\text{Now } \alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \alpha\gamma + \beta\gamma)$$

$$= 0^2 + 2 \times 12$$

$$= 24$$

$$\therefore 16\left(\sin^2\frac{\pi}{18} + \sin^2\frac{5\pi}{18} + \sin^2\frac{25\pi}{18}\right) = 24$$

$$\therefore \sin^2\frac{\pi}{18} + \sin^2\frac{5\pi}{18} + \sin^2\frac{25\pi}{18} = \frac{24}{16} = \frac{3}{2}.$$

# 2020 HSC Mathematics Extension 1 Mapping Grid

## Section I

Question	Marks	Content	Syllabus outcomes
1	1	ME-F1 Further Work with Functions	ME11–2
2	1	ME-F1 Further Work with Functions	ME11–1
3	1	ME-C2 Further Calculus Skills	ME12–1
4	1	ME-V1 Introduction to Vectors	ME12–2
5	1	ME-F2 Polynomials	ME11–1
6	1	ME-V1 Introduction to Vectors	ME12–2
7	1	ME-C3 Applications of Calculus	ME12–4
8	1	ME-A1 Working with Combinatorics	ME11–5
9	1	ME-V1 Introduction to Vectors	ME12–2
10	1	ME-C1 Rates of Change	ME11–4

## Section II

Question	Marks	Content	Syllabus outcomes
11 (a) (i)	1	ME-F2 Polynomials	ME11–2
11 (a) (ii)	2	ME-F2 Polynomials	ME11–2
11 (b)	3	ME-V1 Introduction to Vectors	ME12–2
11 (c)	3	ME-F1 Further Work with Functions	ME11–2
11 (d)	4	ME-T3 Trigonometric Equations	ME12–3
11 (e)	2	ME-C3 Applications of Calculus	ME12–4
12 (a)	3	ME-P1 Proof by Mathematical Induction	ME12–1
12 (b) (i)	1	ME-S1 The Binomial Distribution	ME12–5
12 (b) (ii)	1	ME-S1 The Binomial Distribution	ME12–5
12 (b) (iii)	1	ME-S1 The Binomial Distribution	ME12–5
12 (c)	2	ME-A1 Working with Combinatorics	ME11–5, ME12–7
12 (d)	3	ME-T2 Further Trigonometric Identities ME-C2 Further Calculus Skills	ME12–1
12 (e)	3	ME-C3 Applications of Calculus	ME12–4
13 (a) (i)	1	ME-C2 Further Calculus Skills	ME12–1

Question	Marks	Content	Syllabus outcomes
13 (a) (ii)	4	ME-C2 Further Calculus Skills	ME12–1
13 (b)	4	ME-C3 Applications of Calculus	ME12–4
13 (c) (i)	4	ME-C2 Further Calculus Skills	ME12–4
13 (c) (ii)	3	ME-C2 Further Calculus Skills	ME12–4, ME 12 -7
14 (a) (i)	2	ME-A1 Working with Combinatorics	ME11–5
14 (a) (ii)	2	ME-A1 Working with Combinatorics	ME11–5
14 (a) (iii)	2	ME-A1 Working with Combinatorics	ME11–5
14 (a) (iv)	2	ME-A1 Working with Combinatorics	ME11–5
14 (b) (i)	2	ME-T3 Trigonometric Equations	ME12–3
14 (b) (ii)	2	ME-T3 Trigonometric Equations	ME12–3
14 (b) (iii)	3	ME-F2 Polynomials ME-T3 Trigonometric Equations	ME12–3