# X1 - 5 Year 12 - Ext 1 - Trial and HSC Revision - Sheet 5

Name:

**Question 1** {further calculus}

Which of the following is an equation of a curve that intersects at right angles every curve of the family $y = \frac{1}{x} + k$ (where $k$ is a constant)?

(A) $y = x^2$

(B) $y = -x^2$

(C) $y = -\frac{1}{3}x^3$

(D) $y = \frac{1}{3}x^3$

**Question 2** {mathematical induction}

Emma Jane made an error proving that $3^{2n} - 1$ is divisible by 8 (for $n \geq 1$), using mathematical induction. Part of her proof is shown below.

*Step* 2: Assume the result true for $n = k$

$3^{2k} - 1 = 8P$ where $P$ is an integer.

Hence $3^{2k} = 8P + 1$

*Step* 3: To prove the result is true for $n = k + 1$

$RTP: \ 3^{2(k+1)} - 1 = 8Q$ where $Q$ is an integer. Line 1

$LHS = 3^{2k+2} - 1$ Line 2

$= 3^{2k} \times 3^2 - 1$

$= (8P+1) \times 3^2 - 1$ (*using the assumption*) Line 3

$= 72P + 1 - 1$ Line 4

$= 72P$

$= 8(9P)$

$= 8Q$

$= RHS$

In which line did Emma Jane make an error?

(A) Line 1 (B) Line 2 (C) Line 3 (D) Line 4

**Question 3** {trig equations} -

Solve $\sin 2\theta = \cos\theta$ for $0 \le \theta \le \pi$. **2**

Hint: move the $\cos\theta$ to the LHS and factorise, don't just divide by $\cos\theta$

**Question 4** {polynomials}

The cubic polynomial $P(x) = ax^3 + bx^2 - 6x + 8$ has a factor of $(x-1)$ and a remainder of $-24$ when divided by $(x+2)$.

(i) Show that $a = 3$ and $b = -5$. **2**

(ii) Hence without using calculus, sketch *P(x)*, showing all axes intercepts. **3**

Hint: use the Remainder Theorem, it'll be much much easier.

**Question 5** {polynomials}

Find the term independent of $x$ in the expansion of $\left(5x^2 + \frac{1}{x}\right)^{12}$. **3**

Hint: "independent term" means the term that doesn't have an x in it. AKA the constant term.

**Question 6** {polynomials}

Let the cubic polynomial, $P(x) = x^3 - 3x^2 - 4x + 12$ have roots $\alpha, \beta$ and $\gamma$.

(i) Find the value of $\alpha + \beta + \gamma$. **1**

(ii) Find the value of $\dfrac{1}{\alpha} + \dfrac{1}{\beta} + \dfrac{1}{\gamma}$. **2**

(iii) Given that two of its roots have a sum of zero, find the values of $\alpha, \beta$ and $\gamma$. **2**

**Question 7** {vectors}

Alysha hits a golf ball off the ground with velocity $V$ at an angle of projection $\theta$ to the horizontal. The equations of motion are as follows and **do not need to be proven**:

| | |
|---|---|
| $\ddot{x} = 0$ | $\ddot{y} = -g$ |
| $\dot{x} = V\cos\theta$ | $\dot{y} = V\sin\theta - gt$ |
| $x = Vt\cos\theta$ | $y = Vt\sin\theta - \frac{1}{2}gt^2$ |

(i) Show that Alysha's golf ball reaches a maximum height of $\dfrac{V^2\sin^2\theta}{2g}$. **2**

(ii) Kristine hits a second golf ball projected from the same horizontal plane. **4**

It has velocity $V \times \sqrt{\dfrac{5}{2}}$ and is projected at an angle $\dfrac{\theta}{2}$ to the horizontal.

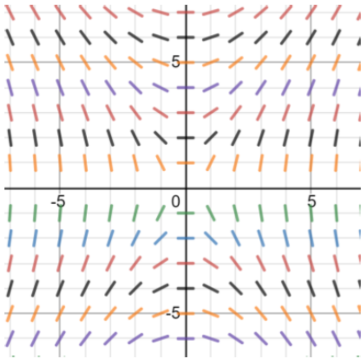
What angles should Alysha and Kristine project their golf balls if they are to reach the same maximum height?

**Question 8** {second derivatives}

Prove that the graph of $y = \log_e x$ is concave down for all $x > 0$. **2**

**Question 9** {differential equations}

The slope field below could represent which of the following differential equations?




A. $\frac{dy}{dx} = \frac{2x}{y}$ B. $\frac{dy}{dx} = \frac{2y}{x}$ C. $\frac{dy}{dx} = \frac{x^2}{y^2}$ D. $\frac{dy}{dx} = \frac{y^2}{x^2}$