X1 - 2 Year 12 - Ext 1 - Trial and HSC Revision - Sheet 2

Name:

Question 1 {Polynomials}

a) Show that $(x-4)$ is one of the factors of $x^3-2x^2-11x+12$. HINT: Show that there is no remainder when you divide by $x-4$.

b) Hence or otherwise, express $x^3-2x^2-11x+12$ in fully factored form.

Question 2 {Inverse functions} Find and sketch the inverse of $f(x) = x^2 - 4$ and state the domain and range of the inverse function.

Question 3 {trig} - given $\sin x = \frac{2}{5}$, find the exact value of $\sin 2x$.

Hint: check page 610 of the Year 11 textbook (on Team).

Question 4 {related rates of change}

A stone drops into a pond, creating a circular ripple. The radius of the ripple increases from 0 cm, at a constant rate of 5 cm s^{-1}.

At what rate is the area enclosed within the ripple increasing when the radius is 15 cm?

A. 25π cm^2 s^{-1}

B. 30π cm^2 s^{-1}

C. 150π cm^2 s^{-1}

D. 225π cm^2 s^{-1}

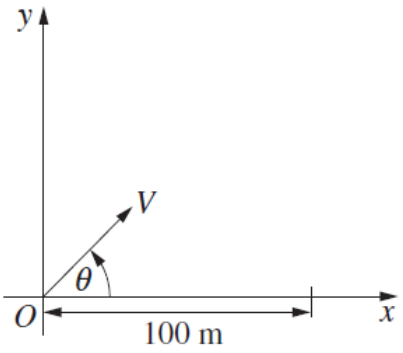
Question 5 {further logs and exponents}

At time t the displacement, x, of a particle satisfies $t = 4 - e^{-2x}$. **3**

Find the acceleration of the particle as a function of x.

Question 6 {vectors}

A golfer hits a golf ball with initial speed V m s^{-1} at an angle θ to the horizontal. The golf ball is hit from one side of a lake and must have a horizontal range of 100 m or more to avoid landing in the lake.



Neglecting the effects of air resistance, the equations describing the motion of the ball are

$$x = Vt\cos\theta$$
$$y = Vt\sin\theta - \frac{1}{2}gt^2,$$

where t is the time in seconds after the ball is hit and g is the acceleration due to gravity in m s^{-2}. Do NOT prove these equations.

(i) Show that the horizontal range of the golf ball is $\dfrac{V^2\sin 2\theta}{g}$ metres. **2**

(ii) Show that if $V^2 < 100g$ then the horizontal range of the ball is less than 100 m. **1**

Question 7 {induction}

Prove by mathematical induction that $8^{2n+1} + 6^{2n-1}$ is divisible by 7, for any integer $n \geq 1$. **3**

Question 8 {binomial distribution}

8 In Havana, Cuba, around 85% of the cars were built before 1960. A sample of 112 cars was taken. Find the probability that in this sample the percentage of cars built before 1960 is:

a less than 80%

b at least 90%

c at least 75%

d between 75% and 80%

Question 9 {differential equations}

2 The number of cattle N on a property is growing over t years according to the equation $\frac{dN}{dt} = 0.18N$.

a Solve this equation, given that the initial number of cattle is 600.

b Find the number of cattle after:

i 5 years **ii** 10 years

c Find how long it will take for the number of cattle to reach 2000.

d Find the rate at which the number of cattle is growing after:

i 5 years **ii** 10 years