X2 - 3 Year 12 - Ext 2 - Trial and HSC Revision - Sheet 3

Name:

Question 1 {Complex numbers}

(a) Let $z = \dfrac{2 - 3i}{1 + i}$.

(i) Find $\overline{z}$ in the form $x + iy$. **2**

(ii) Evaluate $|z|$. **1**

Question 2 {Complex numbers}

Consider $w = -\sqrt{3} + i$.

(i) Express w in modulus-argument form. **2**

(ii) Hence or otherwise show that $w^7 + 64w = 0$. **2**

Question 3 {vectors}

Relative to a fixed origin O, the respective position vectors of three points A , B and C are:

$$\begin{pmatrix} 3 \\ 2 \\ 9 \end{pmatrix}, \quad \begin{pmatrix} -5 \\ 11 \\ 6 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 4 \\ 0 \\ -8 \end{pmatrix}.$$

(i) Determine, in component form, the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$. **2**

(ii) Hence find, to the nearest degree, $\angle BAC$. **2**

(iii) Calculate the area of ΔBAC. **2**

Question 4 {integrals}

By completing the square find $\int \frac{1}{\sqrt{6 - x^2 - x}}\, dx$. **2**

Question 5 {proofs}

Prove if $x, y \in \mathbb{Z}$, then $x^2 - 4y \neq 2$. **3**

Question 6 {proofs}

Suppose that a_n $(n \geq 1)$ is a sequence defined by:

$$a_1 = 1, \quad a_2 = 3 \quad \text{and} \quad a_k = a_{k-1} + a_{k-2} \quad \forall\ k \geq 3.$$

Prove that $\forall\ n \geq 1$, we have $a_n \leq \left(\frac{7}{4}\right)^n$. **3**