X1 - 4 Year 12 - Ext 1 - Trial and HSC Revision - Sheet 4

Name:

Question 1 {Polynomials}

Given that $\alpha, \beta and \gamma$ are roots of $2x^3 + 5x^2 - x - 3$, find the value of $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$

Hint 1: When adding fractions you need a common denominator
Hint 2: Year 11 textbook, page305

Question 2 {inequalities} Solve $\dfrac{3}{x-2} \geq 4$

Hint: multiply both sides by the square of the denominator (Yr 11 Textbook, p 78)

Question 3 {trig} - Show that $1 + \tan^2 \theta = \sec^2 \theta$

Question 4 {related rates of change}

A spherical meteor enters the Earth's atmosphere and burns up (loses volume) at a rate that is proportional to its surface area. Assuming the meteor stays spherical, show that the rate of change of the radius is a constant.

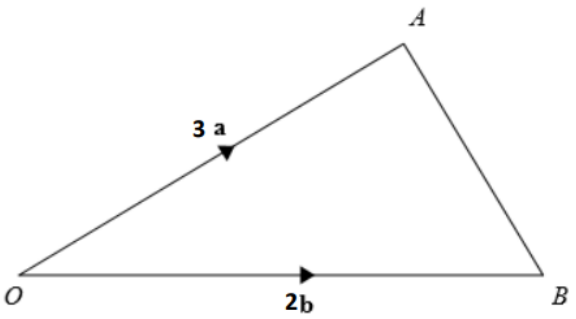
Hint: mathematically, you start with two ideas

1. $\frac{dV}{dt} = \frac{dV}{dr} \cdot \frac{dr}{dt}$ (standard chain rule)
2. $\frac{dV}{dt} = kS$ (the information from the question)

Question 5 {mathematical induction}

Prove that $n^3 + 2n$ is divisible by 3 for all integers n.

Question 6 {vectors}



OAB is a triangle
$\overrightarrow{OA}$ = **3a**
$\overrightarrow{OB}$ = **2b**

P is a point on *AB* so that *AP : PB* is 1 : 3

Given that $\overrightarrow{OP} = k$ (9**a** + 2**b**)

Find the value of k

Question 7 {Binomial distribution}

In externally marked exam papers, an average of 7.5% of students miss doing the questions on the back page. A random sample of 100 students' exam papers were checked for this student error.

a How many students in the sample would be expected to make this error?

b If the sample proportion is approximately normally distributed, find its mean and standard deviation.

c Find the z-score for each percentage of students making this error:

i 4% **ii** 5% **iii** 8% **iv** 10%

d Find the probability that the percentage of students making this error is:

i less than 5% **ii** less than 10% **iii** more than 8%

iv more than 4% **v** between 4% and 10%

Hint: you will need the table on pages 633 and 634 of the textbook.

Question 8 {further calculus}

Find the derivative of the inverse function of $f(x) = x^2 e^x$

Question 9 {differential equations}

An element of mass M is decaying over t years according to the formula

$$\frac{dM}{dt} = -0.045M.$$

The initial mass is 100 g.

a Solve the differential equation to find the equation for the mass of the element.

b Find the mass after 20 years.

c What is the rate at which the mass is decaying after 20 years?

d Find the half-life of the element (the time it takes to halve its mass).