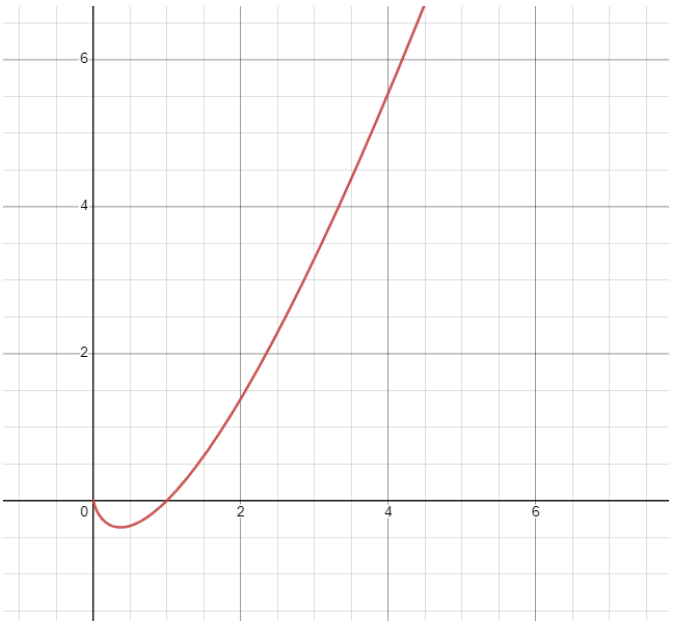
Year 12 - Ext 1 - Trial and HSC Revision - Sheet 1

Name:

Question 1 {Perms and Combs} - a group of 20 people are seated around a table. If a family of 5 have to sit next to each other, how many ways are there for this group to be seated?

Question 2 {Inequalities} solve $x^2 - 4x - 12 \geq 0$

Question 3 {inverse functions} - given the graph of $f(x)$ below, sketch the graph of $f^{-1}(x)$



Question 4 {related rates of change}

A cone-shaped candle whose height is 3 times its radius is melting at the constant rate of $1.4\ \text{cm}^3\ \text{s}^{-1}$. If the proportion of radius to height is preserved, find the rate at which the radius will be decreasing when it is 3.7 cm.

Question 5 {further logs and exponents}

2 a Show that $N = 45 + Ae^{0.14t}$ is a solution of $\frac{dN}{dt} = 0.14(N - 45)$.

b Given $N = 82$ when $t = 2$, find A to 2 decimal places.

c What is N when $t = 5$?

d Find t when $N = 120$.

e Sketch the graph of this function for values of t from 0 to 25.

Question 6 {vectors} for what value of a is the vector $\begin{pmatrix} a \\ 4 \end{pmatrix}$ perpendicular to $\begin{pmatrix} -3 \\ -2 \end{pmatrix}$?

Question 7 {differentiation of inverse trig}

3 What is the derivative of $\tan^{-1}\frac{x}{2}$?

A. $\frac{1}{2(4+x^2)}$

B. $\frac{1}{4+x^2}$

C. $\frac{2}{4+x^2}$

D. $\frac{4}{4+x^2}$

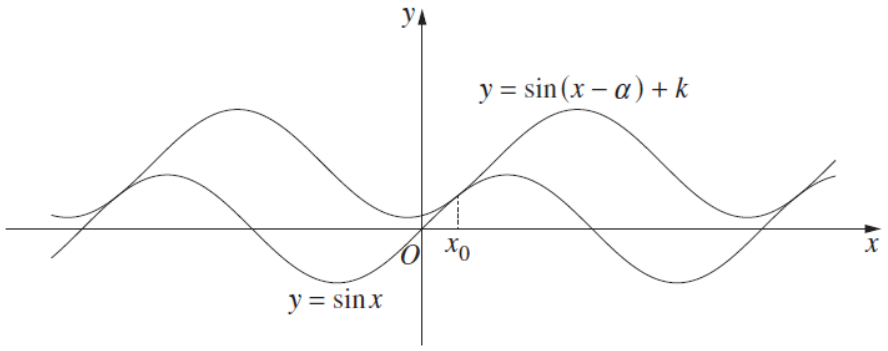
Question 8 {mathematical induction}

(a) Prove by mathematical induction that, for all integers $n \geq 1$, **3**

$$1(1!) + 2(2!) + 3(3!) + \cdots + n(n!) = (n + 1)! - 1.$$

Question 9 {trig equations}

The diagram shows the two curves $y = \sin x$ and $y = \sin(x - \alpha) + k$, where $0 < \alpha < \pi$ and $k > 0$. The two curves have a common tangent at x_0 where $0 < x_0 < \frac{\pi}{2}$.



(i) Explain why $\cos x_0 = \cos(x_0 - \alpha)$. **1**

(ii) Show that $\sin x_0 = -\sin(x_0 - \alpha)$. **2**

(iii) Hence, or otherwise, find k in terms of α. **2**