X2 - 1

Year 12 - Ext 2 - Trial and HSC Revision - Sheet 1

Name:

Question 1 {Proofs}

Prove by contradiction that $\sqrt{2}$ is irrational.

Question 2 {Complex numbers}

Given $z_1 = 2\left(\cos\dfrac{\pi}{6} + i\sin\dfrac{\pi}{6}\right)$ and $z_2 = 3\left(\cos\dfrac{\pi}{2} + i\sin\dfrac{\pi}{2}\right)$, sketch $z_1 z_2$ on the complex plane.

Question 3 {further induction} Prove by mathematical induction that $3^n + 2^n$ is divisible by 5 for all positive integers n such that n is odd.

Hint: the first step involves proving that it holds for $n = 1$

Question 4 {vectors} Describe / sketch the graph of the vector function $\begin{pmatrix} \cos t \\ \sin t \\ -t \end{pmatrix}$ for $t \geq 0$

Question 5 {integration} Integrate the following

a) $\int \dfrac{1}{\sqrt{3 - 2x - x^2}}\,dx$

b) $\int \dfrac{3x + 1}{(x - 3)(x + 2)}\,dx$