X2 - 2 Year 12 - Ext 2 - Trial and HSC Revision - Sheet 2

Name:

Question 1 {Proofs}

Consider the proposition:

'If $2^n - 1$ is not prime, then n is not prime'.

Given that each of the following statements is true, which statement disproves the proposition?

A. $2^5 - 1$ is prime

B. $2^6 - 1$ is divisible by 9

C. $2^7 - 1$ is prime

D. $2^{11} - 1$ is divisible by 23

Question 2 {Complex numbers}

Consider the complex numbers $w = -1 + 4i$ and $z = 2 - i$.

(i) Evaluate $|w|$. **1**

(ii) Evaluate $w\overline{z}$. **2**

Question 3 {vectors}

Consider the two lines in three dimensions given by **3**

$$\underset{\sim}{r} = \begin{pmatrix} 3 \\ -1 \\ 7 \end{pmatrix} + \lambda_1 \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \quad \text{and} \quad \underset{\sim}{r} = \begin{pmatrix} 3 \\ -6 \\ 2 \end{pmatrix} + \lambda_2 \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix}.$$

By equating components, find the point of intersection of the two lines.

Question 4 {induction}

Prove by mathematical induction that, for $n \geq 2$, **4**

$$\frac{1}{2^2} + \frac{1}{3^2} + \cdots + \frac{1}{n^2} < \frac{n-1}{n}.$$

Question 5 {integration}

Use integration by parts to evaluate $\int_1^e x \ln x \, dx$. **3**