Appendix to "Collateral Constraints and Macroeconomic Asymmetries"

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Appendix A. House Prices and Consumption in U.S. Data Excluding the Great Recession

Figure A.1 shows that the nonlinearity in the scatter plot between house prices and consumption in U.S. data is in post–1975 data also excluding the collapse in house prices which started around 2005.

Appendix B. Equilibrium Conditions of the DSGE Model

We list here the equations describing the equilibrium of the DSGE model.

Let $u_{c,t}$ (and $u_{c',t}$), $u_{h,t}$ (and $u_{h',t}$), $u_{n,t}$ (and $u_{n',t}$) denote the time-t marginal utility of consumption, marginal utility of housing and marginal disutility of labor (inclusive of the shock terms: that is, $u_t = \mathsf{z}_t \left(\Gamma_c \log(c_t - \varepsilon_c c_{t-1}) + \mathsf{j}_t \Gamma_h \log(h_t - \varepsilon_h h_{t-1}) - \frac{1}{1+\eta} n_t^{1+\eta} \right)$, and $u_{c,t}$ is the derivative of u_t with respect to c_t). Let Δ be the first difference operator, and let overbars denote steady states. The set of necessary conditions for an equilibrium is given by:

- Budget constraint for patient households:

$$c_t + q_t \Delta h_t + i_t - \frac{R_{t-1} b_{t-1}}{\pi_t} = \frac{w_t n_t}{x_{w,t}} + r_{k,t} k_{t-1} - b_t + div_t, \tag{B.1}$$

  where lump-sum dividends from ownership of final goods firms and from labor unions are given by $div_t = \frac{x_{p,t}-1}{x_{p,t}} y_t + \frac{x_{w,t}-1}{x_{w,t}} w_t n_t$.

- Capital accumulation equations for patient households:

$$u_{c,t} q_{k,t} \left(1 - \phi \frac{\Delta i_t}{\bar{i}} \right) = u_{ct} - \beta E_t \left(u_{c,t+1} q_{k,t+1} \phi \frac{\Delta i_{t+1}}{\bar{i}} \right), \tag{B.2}$$

$$u_{c,t} \frac{q_{k,t}}{\mathsf{a}_t} = \beta E_t \left(u_{c,t+1} \left(r_{k,t+1} + q_{k,t+1} \frac{1-\delta_k}{\mathsf{a}_{t+1}} \right) \right) \tag{B.3}$$

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$$k_t = \mathsf{a}_t \left(i_t - \frac{\phi}{2} \frac{\Delta i_t^2}{\bar{i}} \right) + (1 - \delta_k) k_{t-1} \tag{B.4}$$

where $q_{k,t}$ is the Lagrange multiplier on the capital accumulation constraint.

- Other optimality conditions for patient households:

$$u_{c,t} = \beta E_t \left(u_{c,t+1} R_t / \pi_{t+1} \right), \tag{B.5}$$

$$\frac{w_t}{x_{w,t}} u_{c,t} = u_{n,t}, \tag{B.6}$$

$$q_t u_{c,t} = u_{h,t} + \beta E_t q_{t+1} u_{c,t+1}. \tag{B.7}$$

- Budget and borrowing constraint and optimization conditions for impatient households:

$$c'_t + q_t \Delta h'_t + \frac{R_{t-1}}{\pi_t} b_{t-1} = \frac{w'_t}{x'_{w,t}} n'_t + b_t + div'_t, \tag{B.8}$$

$$b_t \leq \gamma \frac{b_{t-1}}{\pi_t} + (1 - \gamma) \mathsf{m} q_t h'_t, \tag{B.9}$$

$$(1 - \lambda_t) u_{c',t} = \beta' E_t \left(\frac{R_t - \gamma \lambda_{t+1}}{\pi_{t+1}} u_{c',t+1} \right), \tag{B.10}$$

$$\frac{w'_t}{x'_{w,t}} u_{c',t} = u_{n',t} \ , \tag{B.11}$$

$$q_t u_{c',t} = u_{h',t} + \beta' q_{t+1} u_{c',t+1} + u_{c',t} \lambda_t (1 - \gamma) \mathsf{m} q_t, \tag{B.12}$$

where lump-sum dividends from labor unions are given by $div'_t = \frac{x_{w',t} - 1}{x_{w',t}} w'_t n'_t$, and λ_t is the Lagrange multiplier on the borrowing constraint (normalized by the marginal utility of consumption $u_{c',t}$).

- Firm problem, aggregate production, and Phillips curves:

$$\begin{aligned}
y_t &= n_t^{(1-\sigma)(1-\alpha)} n_t^{\prime\sigma(1-\alpha)} k_{t-1}^{\alpha}, && \text{(B.13)} \\
(1 - \alpha)(1 - \sigma) y_t &= x_{p,t} w_t n_t, && \text{(B.14)} \\
(1 - \alpha) \sigma y_t &= x_{p,t} w'_t n'_t, && \text{(B.15)} \\
\alpha y_t &= x_{p,t} r_{k,t} k_{t-1}, && \text{(B.16)} \\
\log (\pi_t / \overline{\pi}) &= \beta E_t \log (\pi_{t+1} / \overline{\pi}) - \varepsilon_\pi \log (x_{p,t} / \overline{x}_p) + \mathsf{u}_{p,t}, && \text{(B.17)} \\
\log (\omega_t / \overline{\pi}) &= \beta E_t \log (\omega_{t+1} \overline{\pi}) - \varepsilon_w \log (x_{w,t} / \overline{x}_w) + \mathsf{u}_{w,t}, && \text{(B.18)} \\
\log (\omega'_t / \overline{\pi}) &= \beta' E_t \log (\omega'_{t+1} / \overline{\pi}) - \varepsilon'_w \log (x'_{w,t} / \overline{x}'_w) + \mathsf{u}_{w,t}. && \text{(B.19)}
\end{aligned}$$

Above, $\omega_t = \frac{w_t \pi_t}{w_{t-1}}$ and $\omega'_t = \frac{w'_t \pi_t}{w'_{t-1}}$ denote wage inflation for each household type, and $\varepsilon_\pi = \frac{(1-\theta_\pi)(1-\beta\theta_\pi)}{\theta_\pi}$, $\varepsilon_w = \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w}$, $\varepsilon'_w = \frac{(1-\theta_w)(1-\beta'\theta_w)}{\theta_w}$.

- Monetary policy:

$$R_t = \max \left(1, R_{t-1}^{r_R} \left(\frac{\pi_t^A}{\overline{\pi}^A} \right)^{(1-r_R) r_\pi} \left(\frac{y_t}{\overline{y}} \right)^{(1-r_R) r_Y} \overline{R}^{1-r_R} \mathsf{e}_{r,t} \right), \tag{B.20}$$

where π_t^A is year-on-year inflation (expressed in quarterly units) and is defined as $\pi_t^A \equiv (P_t/P_{t-4})^{0.25}$.

- Market clearing:

$$h_t + h'_t = 1. \tag{B.21}$$

By Walras' law, the good's market clears, so that $y_t = c_t + c'_t + k_t - (1-\delta_k)\, k_{t-1}$.

Equations B.1 to B.21, together with the laws of motion for the exogenous shocks described in Section 3 of the paper, define a system of 21 equations in the following variables: c, c', h, h', i, k, y, b, n, n', w, w', π, q, R, λ, x_p, x_w, x'_w, r_k, q_k.

We use the methods described in Appendix D and more fully described in Guerrieri and Iacoviello (2015) to solve the model subject to the two occasionally binding constraints given by equations B.9 and B.20.

Appendix C. Estimation Details

Data. Data sources for the estimation are as follows:

1. Consumption

   Model Variable: $\widetilde{C}_t = \log \frac{c_t + c'_t}{\bar{c} + \bar{c}'}$.

   Data: Real Personal Consumption Expenditures, from Bureau of Economic Analysis – BEA – (Haver Analytics code: CH@USECON), log transformed and detrended with one-sided HP filter (smoothing parameter equal to $100,000$).

2. Price Inflation

   Model Variable: $\widetilde{\pi}_t = \log \frac{\pi_t}{\bar{\pi}}$.

   Data: quarterly change in GDP Implicit Price Deflator, from BEA (DGDP@USECON), minus 0.5 percent.

3. Wage Inflation

   Model Variable: $\widetilde{\omega}_t = \log \frac{\sigma\omega_t + (1-\sigma)\omega'_t}{\bar{\pi}}$.

   Data: Real Compensation per Hour in Nonfarm Business Sector (LXNFR@USECON), log transformed, detrended with one-sided HP filter (smoothing parameter equal to $100,000$), first differenced, and expressed in nominal terms adding back price inflation.

4. Investment

   Model Variable: $\widetilde{i}_t = \log \frac{i_t}{\bar{i}}$.

   Data: Real Private Nonresidential Fixed Investment, from BEA (FNH@USECON), log transformed and detrended with one-sided HP filter (smoothing parameter equal to $100,000$).

5. House Prices

   Model Variable: $\widetilde{q}_t = \log \frac{q_t}{\bar{q}}$.

   Data: Corelogic House Price Index (USLPHPIS@USECON) divided by GDP Implicit Price Deflator, log transformed and detrended with one-sided HP filter (smoothing parameter equal to $100,000$).

6. Interest Rate

   Model Variable: $\widetilde{r}_t = R_t - 1$.

   Data: Effective Federal Funds Rate, annualized percent (FEDFUNDS@USECON), divided by 400 to express in quarterly units.

Appendix D. DSGE Model: Solution Method and Accuracy Checks

Solution Method. We use a nonlinear solution method to find the equilibrium allocations of the model in Section 3. The method resolves the problem of computing decision rules that approximate the equilibrium well both when the borrowing constraint binds, and when it does not (similar reasoning applies to the nonnegativity constraint on the interest rate, as described at the end of this section).

The economy features two regimes: a regime when collateral constraints bind; and a regime in which they do not, but are expected to bind in the future.[i] With binding collateral constraints, the linearized system of necessary conditions for an equilibrium can be expressed as

$$\mathcal{A}_1 E_t X_{t+1} + \mathcal{A}_0 X_t + \mathcal{A}_{-1} X_{t-1} + \mathcal{B}\epsilon_t = 0, \tag{D.1}$$

where $\mathcal{A}_1$, $\mathcal{A}_0$, and $\mathcal{A}_{-1}$ are matrices of coefficients conformable with the vector X collecting the model variables in deviation from the steady state for the regime with binding constraints; and ϵ is the vector collecting all shock innovations (and $\mathcal{B}$ is the corresponding conformable matrix). Similarly, when the constraint is not binding, the linearized system can be written as

$$\mathcal{A}_1^* E_t X_{t+1} + \mathcal{A}_0^* X_t + \mathcal{A}_{-1}^* X_{t-1} + \mathcal{B}^*\epsilon_t + \mathcal{C}^* = 0, \tag{D.2}$$

where $\mathcal{C}^*$ is a vector of constants. When the constraint binds, we use standard linear solution methods to express the decision rule for the model as

$$X_t = \mathcal{P} X_{t-1} + \mathcal{Q}\epsilon_t. \tag{D.3}$$

When the collateral constraints do not bind, we use a guess-and-verify approach. We shoot back towards the initial conditions, from the first period when the constraints are guessed to bind again. For example, if the constraints do not bind in t but are expected to bind the next period, the decision rule for period t can be expressed, starting from D.2 and using the result that $E_t X_{t+1} = \mathcal{P} X_t$, as:

$$X_t = -\left(\mathcal{A}_1^*\mathcal{P} + \mathcal{A}_0^*\right)^{-1}\left(\mathcal{A}_{-1}^* X_{t-1} + \mathcal{B}^*\epsilon_t + \mathcal{C}^*\right). \tag{D.4}$$

We proceed in a similar fashion to compute the allocations for the case when collateral constraints are guessed not to bind for multiple periods, or when they are foreseen to be slack starting in periods beyond t. As shown by equation D.4, the model dynamics when constraints are not binding depend both on the current regime (through the matrices $\mathcal{A}_1^*, \mathcal{A}_0^*$ and $\mathcal{A}_{-1}^*$) and on the expectations of future regimes when constraints will bind again (through the matrix $\mathcal{P}$, which is a nonlinear function of the matrices $\mathcal{A}_1$, $\mathcal{A}_0$ and $\mathcal{A}_{-1}$).

[i] If one assumes that the constraints are not expected to bind in the future, the regime with slack borrowing constraints becomes unstable, since borrowers' consumption falls over time and their debt rises over time until it reaches the debt limit, which contradicts the initial assumption.

It is straightforward to generalize the solution method described above for multiple occasionally binding constraints.[ii] The extension is needed to account for the zero lower bound (ZLB) on policy interest rates as well as the possibility of slack collateral constraints. In that case, there are four possible regimes: 1) collateral constraints bind and policy interest rates are above zero, 2) collateral constraints bind and policy interest rates are at zero, 3) collateral constraints do no bind and policy interest rates are above zero, 4) collateral constraints do not bind and policy interest rates are at zero. Apart from the proliferation of cases, the main ideas outlined above still apply.

Local linearity of the Policy Functions. The solution of the model can be described by a policy function of the form:

$$X_t = \mathbf{P}(X_{t-1}, \epsilon_t)X_{t-1} + \mathbf{D}(X_{t-1}, \epsilon_t) + \mathbf{Q}(X_{t-1}, \epsilon_t)\epsilon_t. \tag{D.5}$$

The vector X_t collects all the variables in the model, except the innovations to the shock processes, which are separated in the vector ϵ_t. The matrix of reduced-form coefficients $\mathbf{P}$ is state-dependent, as are the vector $\mathbf{D}$ and the matrix $\mathbf{Q}$. These matrices and vector are functions of the lagged state vector and of the current innovations. However, while the current innovations can trigger a change in the reduced-form coefficients, X_t is still locally linear in ϵ_t. To illustrate this point, Figure A.2 shows how the policy function for impatient agents' consumption c'_t – one of the elements of X_t – depends on the realization of the housing preference shock $u_{j,t}$ – one of the elements of ϵ_t – when all the other elements of X_{t-1} are held at their steady–state value. The top panel shows the consumption function. This function is piecewise linear, with each of the rays corresponding to a given number of periods in which the borrowing constraint is expected to be slack. The bottom panel shows the derivative of the consumption function with respect to $u_{j,t}$. As the consumption function is piecewise linear, the derivative is not defined at the threshold values of the shock $u_{j,t}$ that change the expected duration of the regime. However, each of these threshold points for different shocks is a set of measure zero.[iii]

Realizations of the shock $u_{j,t}$ above a threshold will imply that the borrowing constraint is temporarily slack. When the constraint is slack, the constraint will be expected to be slack for a number of periods which increases with the size of the shock. Accordingly, consumption will respond proportionally less, and the $\mathbf{Q}_{c',u_j}$ element of the matrix $\mathbf{Q}$ that defines the impact sensitivity of c' to u_j will be smaller.

Accuracy of Numerical Solution. We assess the accuracy of the solution method for the DSGE model by computing the errors of the model's intertemporal equations. The errors arise both because of the linearization of the original nonlinear model, and because the method abstracts from precautionary motives due to possibility of future regime switches. We focus here on the intertemporal errors for the consumption and housing demand equations of patient and impatient agents, equations B.5, B.7, B.10 and B.12, and we compute the errors using standard monomial integration for the expectation terms and simulating the model under the estimated filtered shocks (see Judd, Maliar, and Maliar 2011 for a description of the monomial method). Across these equations, the mean absolute errors – expressed in consumption units – are about 4×10^{-4}, that is, \$4 for every \$10,000 spent, a level that can be deemed negligible.

[ii] For an array of models, Guerrieri and Iacoviello (2015) compare the performance of the piecewise perturbation solution described above against a dynamic programming solution obtained by discretizing the state space over a fine grid. Their results show that this solution method efficiently and quickly computes accurate policy functions.

[iii] It is straightforward to prove that the points where the derivative of the decision rule is not defined are of measure zero given a choice of process for the stochastic innovations. By construction, there are only countably many of these points. If there were uncountably many, a shock could lead to a permanent switch in regimes, which is ruled out by the solution method.

As standard practice (for instance, see Bocola 2016), Figure A.4 plots the histogram of the logarithm with base 10 of the absolute value of these errors. On average, residual log errors are -2.9 and -2.92 for the consumption and the housing Euler equations of the borrower, and -4.1 and -3.6 for the consumption and housing Euler equations of the saver, indicating a miss of about \$1 per \$1000 of consumption for the borrower's Euler equations, and of about \$1 per \$10,000 of consumption for the saver's Euler equations.

The small intertemporal errors indicate that precautionary motives, while potentially important, are, on average, quantitatively small. Nonetheless, one may suspect that, in the housing boom that preceded the crisis, agents would have wanted to engage in precautionary saving to insure against bad shocks, and that, during the crisis, uncertainty about the path and duration of the zero lower bound on interest rates would have affected macroeconomic outcomes through precautionary behavior. To assess this possibility, we modify our solution algorithm to account for the possibility of future shocks. At each point for which the solution is sought, this alternative algorithm augments the state space with sequences of anticipated shocks and corrects current decisions by gauging the difference between the augmented expectations and the original expectations.

To understand the modifications of our solution algorithm that allow for precautionary behavior, consider, as an example, a forward-looking equation of the form:

$$q_t = \max\left(0, \beta E_t q_{t+1} + \varepsilon_t\right), \quad \varepsilon_t \sim \text{NIID}\left(0, \sigma^2\right). \tag{D.6}$$

The perfect foresight solution assumes that the variance of ε_{t+j} is zero for $j > 0$. Under this assumption, $E_t^{PF} q_{t+1} = 0$, where E^{PF} denotes the expectation operator under perfect foresight. Accordingly, the solution under perfect foresight is $q_t = \varepsilon_t$ if $\varepsilon_t \geq 0$, and $q_t = 0$ if $\varepsilon_t < 0$ or, more succinctly:

$$q_t^{PF} = \max\left(0, \varepsilon_t\right). \tag{D.7}$$

We modify the solution by extending the expectation operator E_t as follows. We augment the time-t state space with two anticipated shocks to ε_{t+1} of equal size, opposite sign and equal probability. When integrating the expectations of ε_{t+1}, this approach is equivalent to considering two integration nodes and weights, following the lead of Judd, Maliar, and Maliar (2011). Under this scheme, the two integration nodes are σ are $-\sigma$, each with weight $1/2$. Accordingly, the expectation of q_{t+1} can be defined as follows:

$$E_t^{RE1} q_{t+1} = (1/2)\max\left(0, \sigma\right) + (1/2)\max\left(0, -\sigma\right) = (1/2)\,\sigma, \tag{D.8}$$

where E_t^{RE1} denotes the expectation taken assuming knowledge that additional shocks will occur in period $t+1$. The solution for q_t becomes:

$$q_t^{RE1} = \max\left(0, \beta\sigma/2 + \varepsilon_t\right). \tag{D.9}$$

We can proceed in similar fashion to add $n-$period ahead anticipated shocks (to $\varepsilon_{t+1}, \varepsilon_{t+2}, \ldots\, \varepsilon_{t+n}$), and choose optimally the number of anticipated shocks that yield the largest reduction in intertemporal errors. We have found that $4-$period ahead anticipated shocks yield the largest decline in the errors to the intertemporal equations in the proximity of regime switches.[iv]

As this modification of the solution algorithm comes at a very large cost in terms of speed, we make comparisons holding the filtered shocks and the estimated model parameters unchanged.[v] The modified algorithm reduces the errors of the intertemporal equations, particularly in periods when the constraints are close to switching. For instance, when the collateral constraint is slack but expected to bind in the future, or vice versa – an occurrence which happens, according to our estimates, with some frequency

[iv] The optimal number of anticipated shocks is, in general, depends on the stochastic structure of model and its calibration. For the simple example in the text, the optimal number of anticipated shocks is 3 when $\beta = 0.99$ and $\sigma = 0.05$.

[v] Solving the model for a particular sequence of shocks takes a fraction of a second using the algorithm that we employ for Bayesian estimation. The modified algorithm that corrects for uncertainty about future shocks takes several minutes to solve. While useful for assessing the accuracy of the solution, it is unusable for filtering and estimation.

between 1998 and 2006, the consumption Euler errors (expressed in base 10 logs) for the borrower and saver fall from −3.0 and −4.0 to −3.1 and −4.3, respectively. Despite the smaller intertemporal errors, the modified solution method implies only negligible differences in the model's business cycle properties: as shown by Figure A.5, the discrepancies between the model's simulated series are barely visible, except perhaps in periods close to regime switches, when precautionary considerations ought to be heightened.

The modified solution method predicts a slightly smaller increase in total consumption (0.05 percentage point lower) at the peak of the housing boom, and a slightly larger decline in consumption at the trough of the housing price collapse (half a percentage point larger). The smaller increase in consumption during the boom owes to the presence of precautionary saving, as borrowers take on less debt in anticipation of future negative shocks. The larger decrease in consumption during the bust owes to the uncertainty about the zero lower bound. As elucidated, for instance, by Basu and Bundick (2017), at the ZLB, uncertainty about future shocks causes an additional decline in output over and above the decline caused by the shocks that led to the ZLB in the first place. Aside from these two episodes, the differences between the solution methods are barely visible. Accordingly, we conclude that the shocks and frictions in our model do not imply large precautionary motives.

As a further check, we confirmed that our solution method works well in comparison to standard – yet slower – global solution methods when global methods can be easily deployed, as is the case for the partial equilibrium model of Section 2. Starting from the non-stochastic steady state of the model, Figure A.6 plots the responses of consumption, leverage and debt to positive and negative house price shocks using both a standard global method (value function iteration) and the algorithm used to solve the DSGE model for estimation purposes. As the figure shows, the two algorithms deliver very similar dynamics for the variables of interest.

Appendix E. Specification Checks and Sensitivity Analysis

First, we check that our findings are insensitive to the assumption that the initial vector of endogenous variables, X_0, is equal to its steady-state value. Second, we show that our algorithm can accurately recover the "true" structural shocks when the structural parameters are known. Third, we show that when our estimation strategy is applied to data generated from the posterior mode of the model, the estimated parameters are close to their true values. As for sensitivity analysis, we consider an alternative detrending strategy, different shock structure and variable capacity utilization.

Initialization Scheme. Our estimation procedure makes use of the assumption that all variables are known and equal to their nonstochastic steady state in the first period. The first 20 observations are used to train the filter. As a robustness exercise, we have estimated our model under different assumptions about the values of the initial state vector X_0.[vi] We confirmed the initial conditions were essentially irrelevant by period 20 and that our estimated parameters were minimally affected by the initial condition. Table A.1 compares the benchmark results with the estimation results assuming a different known initial condition (see Column 4), randomly sampled from the distribution of the model state variables based on the model's estimated mode.

Filtering. Our estimation procedure relies on using a nonlinear equation solver in order to filter, in each period t, the sequence of shocks ϵ_t that reproduces the observations in the vector Y_t. It is possible

[vi] By treating the initial distribution of X_0 as known, we eliminate the conditionality of the likelihood function for the observed data Y^T on both X_0 and Y_0. Without this assumption, one needs to integrate the likelihood for Y^T over the distribution for X_0 implied by the specification of the model and the observed data, and simulation-based methods (such as the particle filter or the unscented Kalman filter) become necessary.

that small numerical errors in retrieving ϵ_t at each point in time may propagate over time and lead to inaccuracies in computing the filtered shocks. To explore the practical relevance of this possibility, we generate an artificially long sample of observables from our model. Drawing from the posterior mode, we generate a time series of artificial observations of length $T = 500$. We then use our procedure to filter these shocks back and compare the filtered shocks to the "true" ones used to generate our artificial data set. The correlation between the "true" shocks and the filtered ones is, for all shocks, extremely high, ranging from 0.998 for the monetary shock to 0.999992 for the wage shock.

Identifiability. Following Schmitt-Grohé and Uribe (2012), with estimated parameters set to their posterior mode, we generate a sample of 120 observations (comparable in size to our actual dataset) and then estimate the model parameters using the same methods and procedures applied to the observed data, both with our Bayesian approach and with uninformative priors – maximum likelihood estimation (MLE). At no point does our estimation procedure make use of knowledge of the true parameter values. In this case, too, our estimation strategy comes close to uncovering the true shocks and the true values of the parameters in question. For instance, the estimated wage share of impatient households at the mode is 0.52 in the Bayesian approach, 0.38 in the MLE case. All the other estimated coefficients are reported in columns 5 and 6 of Table A.1.

Detrending Method. We use a one-sided HP filter to construct the data analogues to our model variables prior to estimation. As an alternative, we have incorporated linear deterministic trends in the model and estimated the parameters governing the trends jointly with the other parameters. Specifically, we have assumed three separate deterministic trends for neutral technology, investment goods technology, and housing supply technology. Given our assumptions about preferences and technology, these three separate trends yield a balanced growth path in which real consumption (together with real wages), real investment, and real house prices grow at different rates (even if the nominal shares of consumption, investment and housing expenditures remain constant). The model with deterministic trends implies slightly more persistent and more volatile shocks, presumably in order to account for the larger and more persistent deviations of the observations around their constant trends. The additional estimation results are reported in Column 7 of Table A.1.

Allowing for Neutral Technology Shocks and Variable Capacity Utilization. Our benchmark specification includes six shocks (investment-specific shocks, wage markup, price markup, monetary policy, intertemporal preferences, and preferences for housing). As a robustness exercise, we also considered an additional shock, a shock to the level of neutral technology (denoted by a^N) paired with variable capacity utilization (denoted by z_t). To introduce shocks to neutral technology, Equation (B.13) is modified as follows:

$$y_t = \mathsf{a}_t^N n_t^{(1-\sigma)(1-\alpha)} n_t'^{\sigma(1-\alpha)} (z_t k_{t-1})^{\alpha}. \tag{E.1}$$

where the process for a^N is

$$\log(\mathsf{a}_t^N) = \rho_A \log(\mathsf{a}_{t-1}^N) + u_{a,t}. \tag{E.2}$$

The parameter ρ_A is the AR(1) coefficient and $u_{a,t}$ is an innovation which is normally, independently and identically distributed. With variable utilization, the capital accumulation equation becomes

$$k_t = \mathsf{a}_t \left(i_t - \phi \frac{(i_t - i_{t-1})^2}{\bar{i}} \right) + (1 - \delta_{k,t}) k_{t-1}, \tag{E.3}$$

and, in turn, the depreciation rate $\delta_{k,t}$ becomes time-varying and evolves according to

$$\delta_{k,t} = \delta_k + b_K \zeta_K z_t^2/2 + b_K (1 - \zeta_K) z_t + b_K (\zeta_K/2 - 1), \tag{E.4}$$

where the parameter $\zeta_K > 0$, between 0 and 1, measures the curvature of the capital-utilization function, and where $b_K = 1/\beta - (1 - \delta_K)$ is a normalization that guarantees that steady-state utilization is unity. We assume a decentralization (as in Iacoviello and Neri 2010) where patient households rent capital services (given by $z_t k_{t-1}$) to wholesale firms and choose the capital utilization rate subject to the additional constraint given by equation (E.4).

We also introduce an additional observation equation:

$$TFP_t = (\hat{\mathtt{a}}_t^N) + Sh_i(\hat{\mathtt{a}}_t); \tag{E.5}$$

where the observed measure of total factor productivity, TFP_t, is the (detrended) utilization-adjusted total-factor productivity of Fernald 2012, where Sh_i is the investment share in total absorption, and where the hat symbol denotes a variable expressed in deviation from its steady state. The observation equation takes into account that there are two sources of changes for total factor productivity in our model, the neutral technology $\mathtt{a}^N$ and the investment-specific technology $\mathtt{a}$.

The model with the neutral technology shocks and variable capacity implies a higher fraction of impatient households and minor changes in the rest of the estimated parameters. As a result, the housing collapse plays a larger role than in our benchmark model in accounting for the consumption decline in the Great Recession. The additional estimation results are reported in Column 8 of Table A.1.

As an alternative to the level shocks for neutral technology, we also consider growth rate shocks, allowing for a small error correction component. In this case,

$$\log(\mathtt{a}_t^N) - \log(\mathtt{a}_{t-1}^N) = \rho_A \left(\log(\mathtt{a}_{t-1}^N) - \log(\mathtt{a}_{t-2}^N)\right) - \rho_{ECM} \log(\mathtt{a}_{t-1}^N) + u_{a,t}; \tag{E.6}$$

where $\rho_{ECM} = 0.0025$, a small number that ensures long-run convergence of neutral technology towards its non stochastic steady state at the rate of one percent a year. The mode of the estimated parameters under this configuration is reported in Column 9 of Table A.1

Figure A.7 provides a comparison of the contribution of technology shocks for the evolution of key variables across specifications. In each case, the solid lines show the observed variables (detrended). The shaded areas show the evolution of those same variables when only the estimated technology shocks are turned on – both investment-specific and neutral, if present. (Recall that, by construction, the model matches the observations when all shocks are turned on.) The top row of the figure shows the results for our baseline setup, that considers only investment-specific technology shocks. The middle and bottom rows show sensitivity analysis for the inclusion of neutral technology shocks, in levels for the middle row, and in growth rates for the bottom row.

The figure shows some common patterns: 1) In all cases, the technology shocks account for the bulk of the observed investment movements; 2) The addition of the neutral shocks has an impact on consumption, but the timing is generally off, so that the contributions of the technology shocks need to be offset by other shocks; 3) in all cases, the technology shocks only make a modest contribution to the evolution of house prices, though this contribution is marginally beefier when including shocks to the growth rate of technology.

Appendix F. State-Level Evidence on Mortgage Originations

Because the effects of low and high house prices on consumption work in our model through tightening or relaxing borrowing constraints, it is important to check whether measures of credit also depend asymmetrically on house prices. Table A.2 shows how mortgage originations at the state level respond to changes in house prices. We choose mortgage originations because they are available for a long time period, and because they are a better measure of the flow of new credit to households than the stock of existing debt. In all of the specifications in Table A.2, mortgage originations depend asymmetrically on house prices, too.

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Table A.1: Estimation Results: Robustness Analysis

				Specification Checks			Sensitivity		
	1.	2.	3.	4.	5.	6.	7.	8.	9.
	Bench.	No Constr.	Always Binding	Different Init.Cond.	Bayes Artif.Data	MLE Artif.Data	Lin. Trend	N. Tech. Level	N. Tech. Growth
β'	0.9922		0.9860	0.9915	0.9868	0.9898	0.9844	0.9910	0.9874
ε_c	0.6842	0.8211	0.7005	0.7002	0.6568	0.6755	0.7178	0.7534	0.7540
ε_h	0.8799	0.7717	0.8622	0.8973	0.9057	0.8873	0.9047	0.9095	0.9045
ϕ	4.1209	5.1604	4.5890	6.3518	3.8579	5.4071	4.9318	1.4724	2.1126
σ	0.5013		0.4421	0.5383	0.3760	0.5210	0.4343	0.5729	0.3395
r_π	1.7196	1.8042	1.8773	1.9417	1.8660	1.9267	1.8554	1.6403	1.0058
r_R	0.5509	0.5412	0.5616	0.6133	0.5771	0.5017	0.6059	0.5429	0.5767
r_Y	0.0944	0.0905	0.0955	0.0946	0.1195	0.1114	0.0630	0.1112	0.1043
θ_π	0.9182	0.9035	0.9075	0.9184	0.8430	0.8975	0.9467	0.9271	0.9491
θ_w	0.9163	0.8857	0.9125	0.9120	0.8717	0.9209	0.9274	0.9290	0.9655
γ	0.6945		0.6181	0.7189	0.6610	0.7188	0.6425	0.7858	0.6901
ρ_J	0.9835	0.9863	0.9806	0.9829	0.9332	0.9798	0.9867	0.9857	0.9865
ρ_K	0.7859	0.7888	0.7834	0.7499	0.7969	0.8403	0.8142	0.8232	0.8136
ρ_R	0.6232	0.6537	0.6296	0.6210	0.5430	0.6559	0.7442	0.6013	0.5929
ρ_Z	0.7556	0.6639	0.7419	0.7695	0.7084	0.7404	0.8251	0.6996	0.7633
σ_J	0.0737	0.0745	0.0820	0.0745	0.1612	0.0822	0.0710	0.0647	0.0663
σ_K	0.0360	0.0409	0.0395	0.0535	0.0366	0.0434	0.0377	0.0192	0.0206
σ_P	0.0030	0.0030	0.0031	0.0029	0.0034	0.0031	0.0031	0.0030	0.0030
σ_R	0.0013	0.0013	0.0013	0.0013	0.0016	0.0013	0.0014	0.0013	0.0013
σ_W	0.0100	0.0103	0.0100	0.0103	0.0104	0.0098	0.0097	0.0099	0.0096
σ_Z	0.0163	0.0297	0.0172	0.0177	0.0154	0.0166	0.0197	0.0208	0.0214
τ_C							0.0073		
τ_K							0.0107		
τ_q							0.0043		
ζ_K								0.5234	0.5474
ρ_A								0.8427	0.7452
σ_A								0.0078	0.0067

Note: Each column reports the mode of the estimated parameters for various specifications. Column (1) reports estimates for the benchmark model. Column (2) refers to model without collateral-constrained households. Column (3) refers to the model where the collateral constraint if always binding. Column (4) is the estimated mode using a different initial condition. Columns (5) and (6) report Bayesian and maximum likelihood estimates (MLE) using artificial data generated by the model with parameters set at the values in (1). Column (7) report the model with linear deterministic trends, where τ_C, τ_K, τ_q are the implied growth rates for real consumption, real investment, real house prices. Column (8) refers to the model with shocks to the level of neutral technology and variable utilization, where ρ_A and σ_A are the AR(1) coefficient and standard deviation of those shocks. The parameter ζ_K is the curvature (between 0 and 1) of the utilization cost function, where 0 (1) indicates that utilization can be changed at a small (large) cost. Column (9) reports estimates for the model with shocks to the growth rate of neutral technology, where ρ_A and σ_A are the AR(1) coefficient and standard deviation of those shocks.

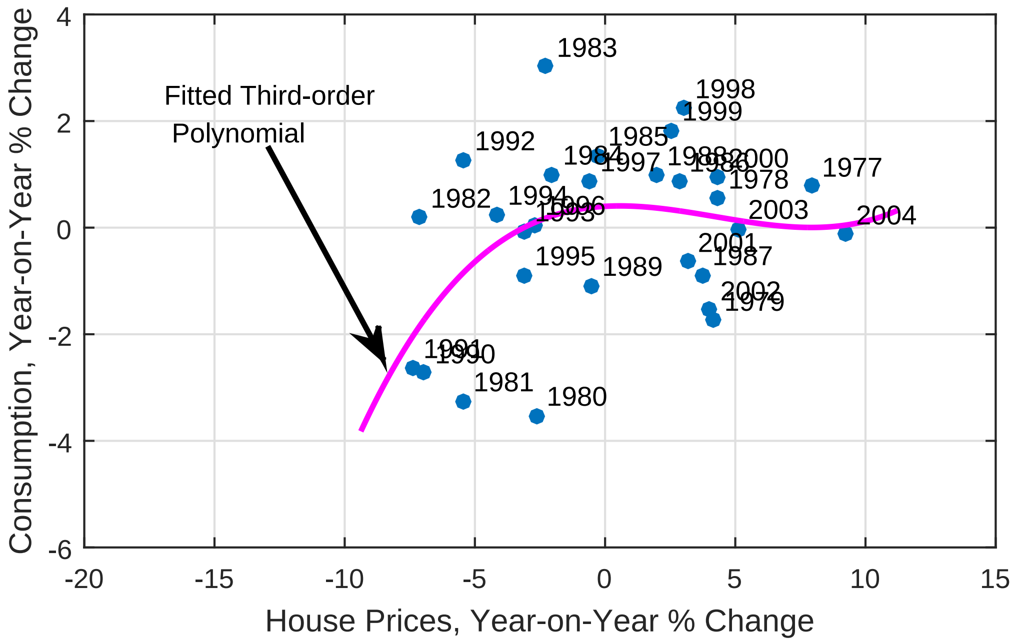
Table A.2: State-Level Regressions: Mortgage Originations and House Prices

	% Change in Mortgage Originations ($\Delta mori_t$)				
Δhp_{t-1}	**1.10*** **				
	(0.18)				
Δhp_high_{t-1}		**-0.41***	**1.08*** **	**1.46*** **	**1.54*** **
		(0.24)	(0.16)	(0.21)	(0.33)
Δhp_low_{t-1}		**3.13*** **	**1.85*** **	**2.53*** **	**2.67** **
		(0.59)	(0.68)	(0.90)	(1.11)
$\Delta mori_{t-1}$				**-0.20*** **	**-0.20*** **
				(0.02)	(0.02)
$\Delta income_{t-1}$					**-0.63**
					(1.04)
pval difference		**0.000**	**0.211**	**0.160**	**0.181**
Time effects	**no**	**no**	**yes**	**yes**	**yes**
Observations	1020	1020	1020	969	969
States	51	51	51	51	51
R-squared	0.01	0.03	0.58	0.53	0.53

Note: The table report state-level regressions using annual observations from 1992 to 2011 for 50 States and the District of Columbia. Robust standard errors are shown in parentheses. The symbols ***,**,* denote estimates statistically different from zero at the 1, 5 and 10% confidence level. In the table, pval is the p-value of the test for differences in the coefficients for high and low house prices.

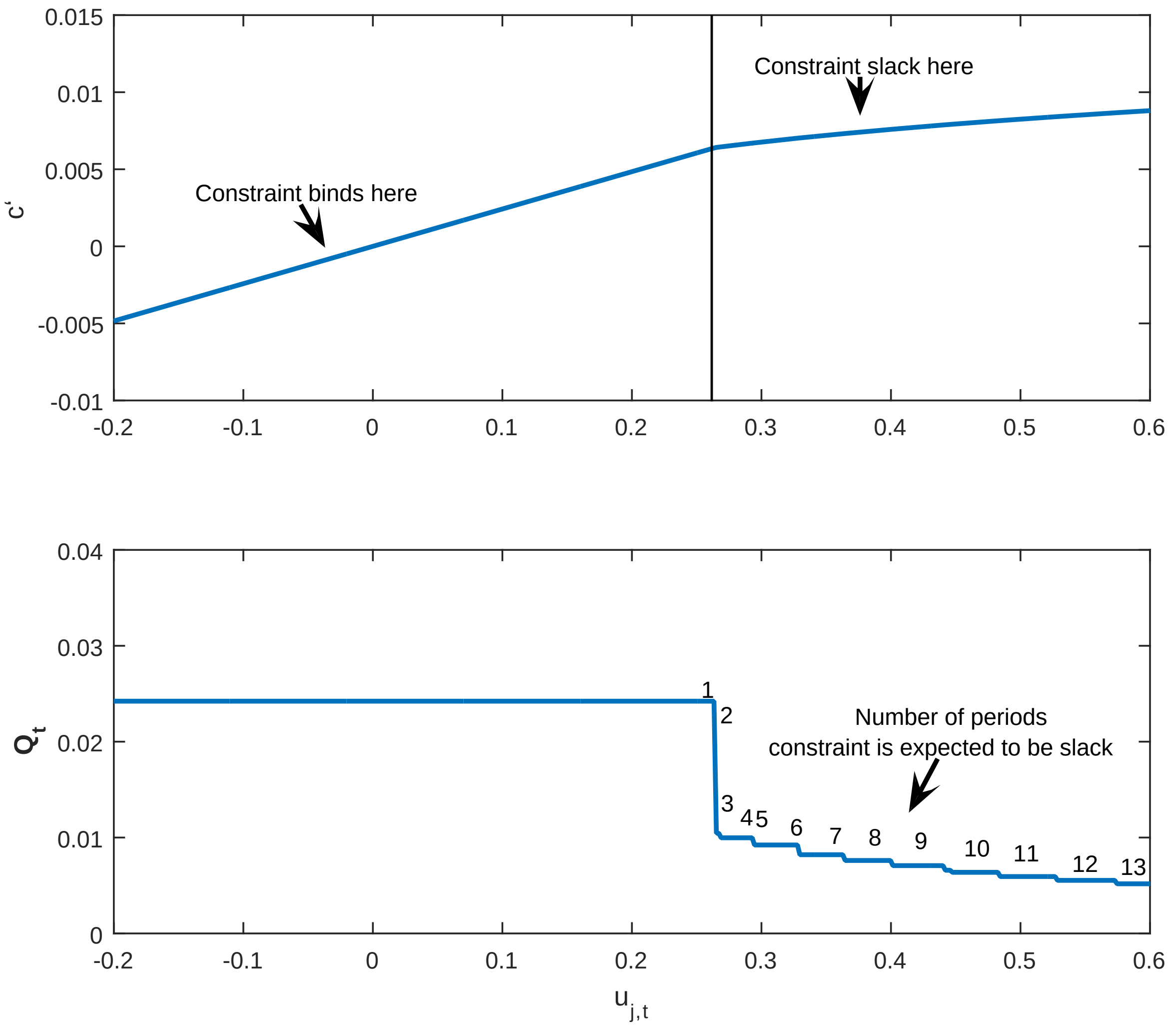
Data Sources and Definitions: $\Delta mori$ is the percent change in "Mortgage originations and purchases: Value" from the U.S. Federal Financial Institutions Examination Council: Home Mortgage Disclosure Act. See Table 2 for other variable definitions.

Figure A.1: House Prices and Consumption in U.S. National Data, excluding the post-2005 period



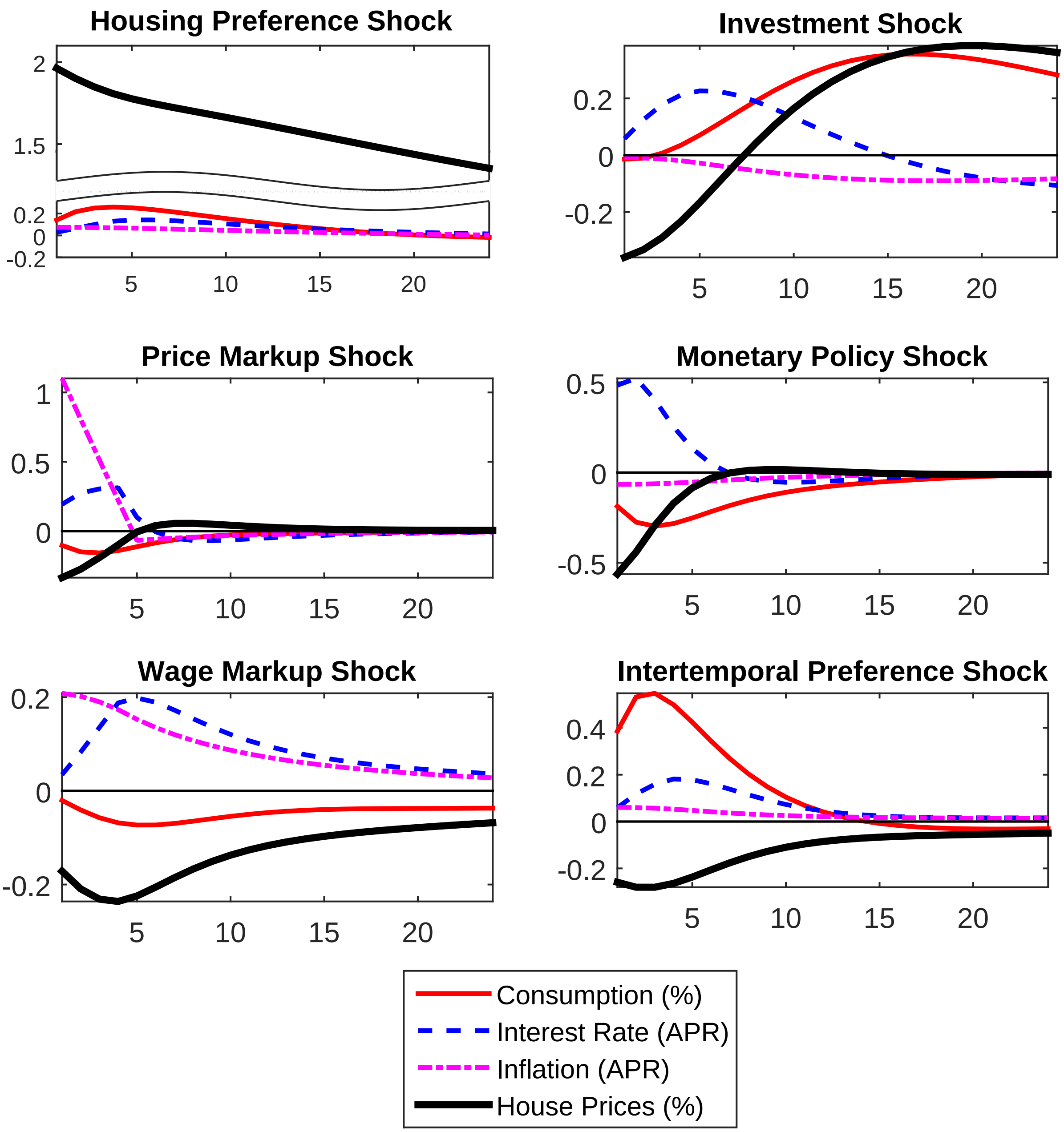
Consumption growth and house price growth are expressed in deviation from their sample mean. The data sample is from 1976Q1 to 2004Q4.

Figure A.2: Local Linearity of the Policy Functions



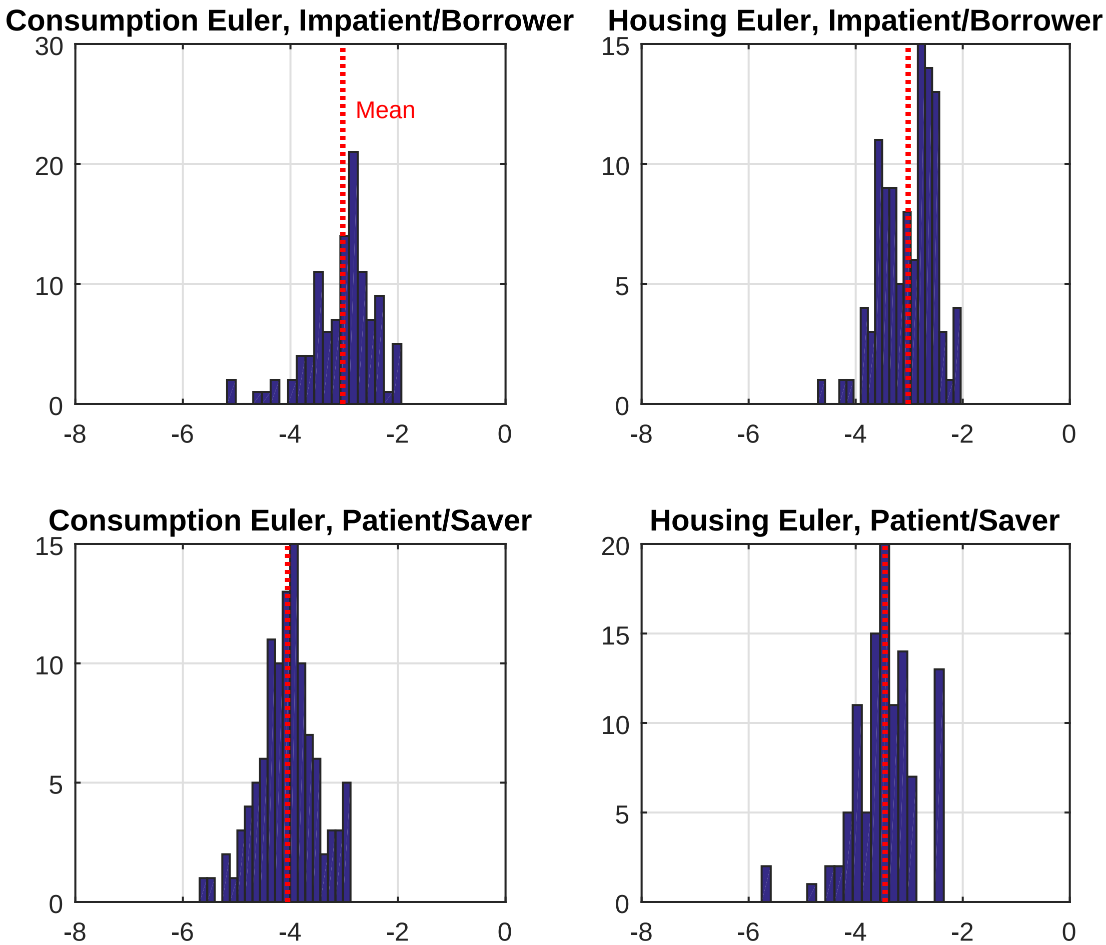
Note: The top panel plots consumption of the impatient agent (in deviation from the steady state) as a function of various realizations of the housing preference shock. The bottom panel plots the slope of the consumption function. The consumption function has a kink when the borrowing constraint becomes binding, and becomes flatter the larger the realization of the housing preference shock.

Figure A.3: Impulse Responses to All Shocks for the Estimated Model



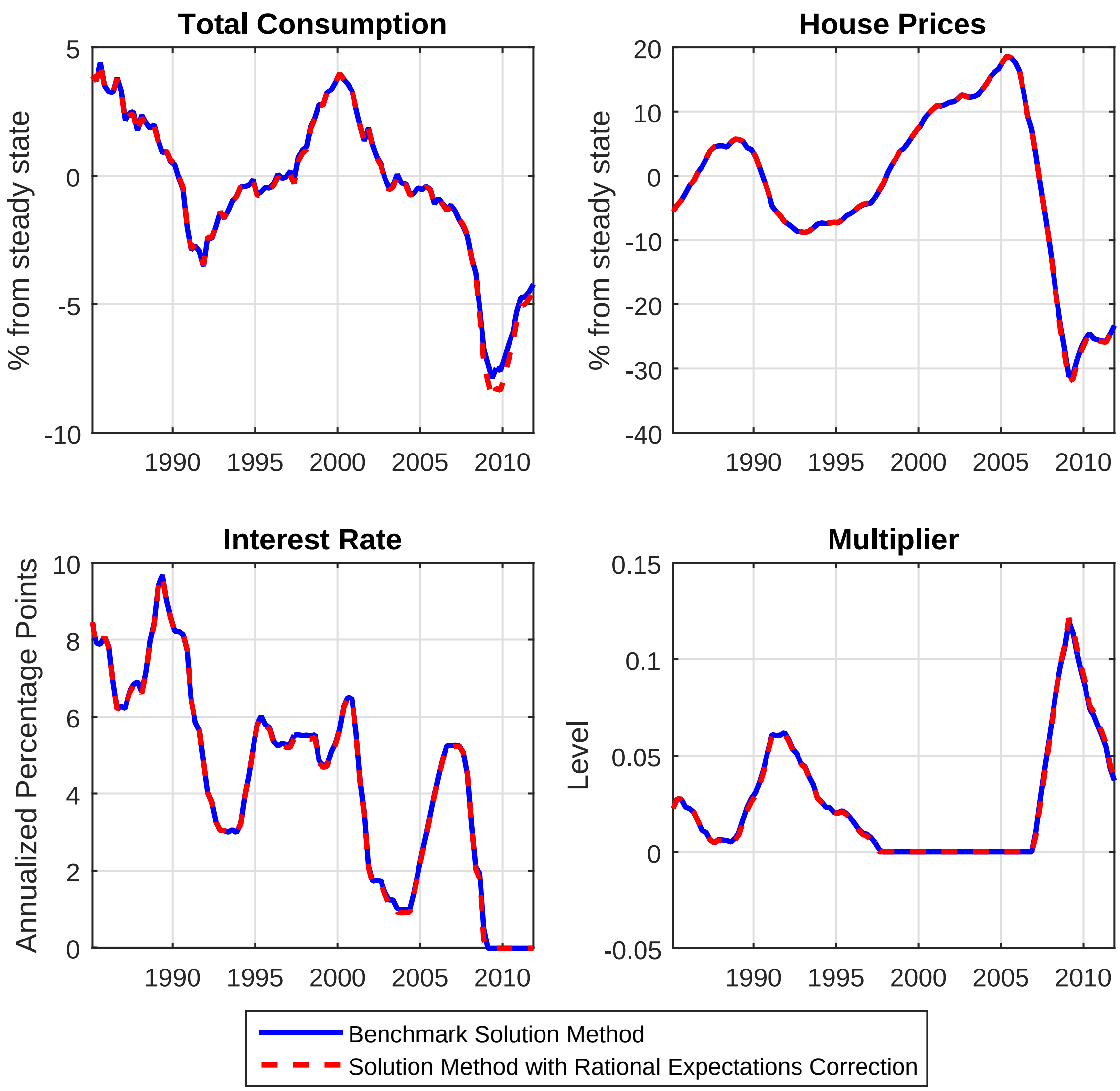
Note: Horizontal axes: horizon in quarters. The panels show the impulse responses of house prices, consumption, interest rate and inflation to an estimated one standard deviation shock in the estimated model.

Figure A.4: Accuracy of Solution Method: Intertemporal Errors for the DSGE Model



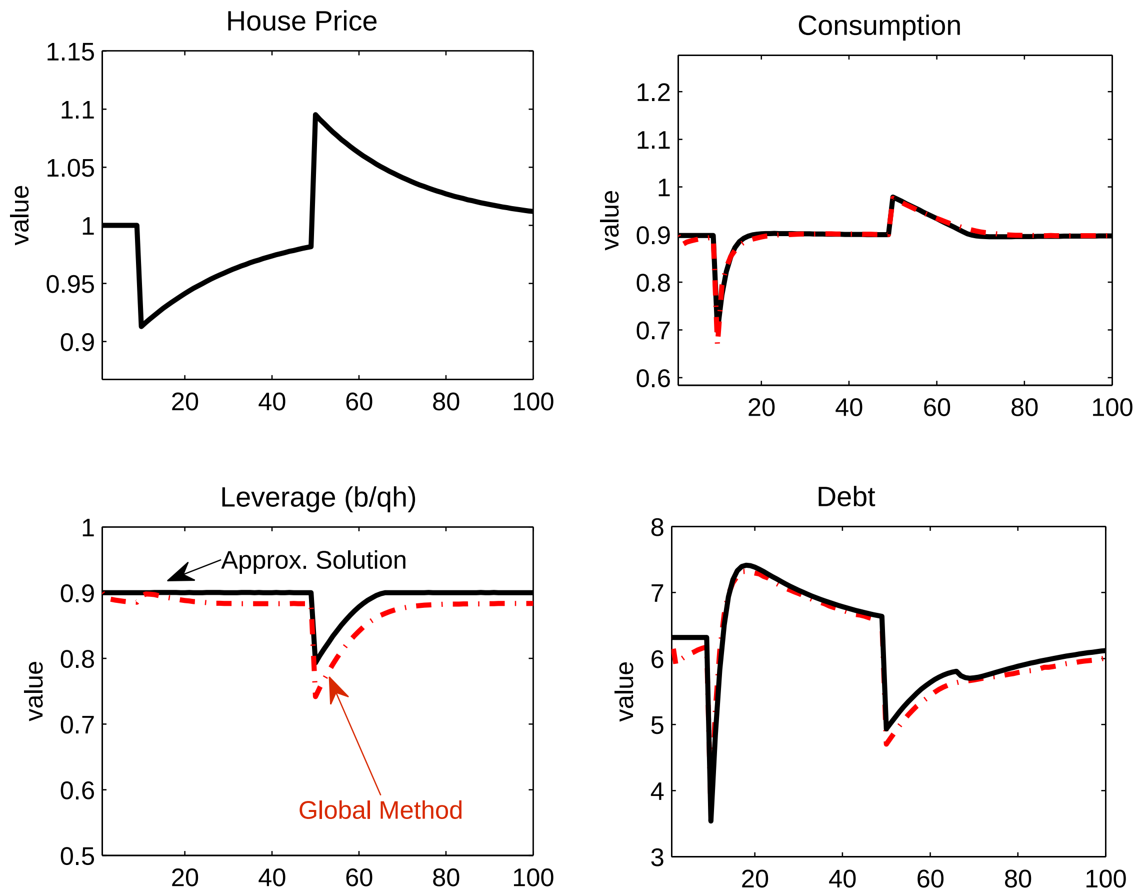
Note: For each Euler equation, the histograms report residual equation errors in decimal log basis. The dotted lines mark the mean residual equation error.

Figure A.5: Accuracy of Solution Method: Simulated Time Series



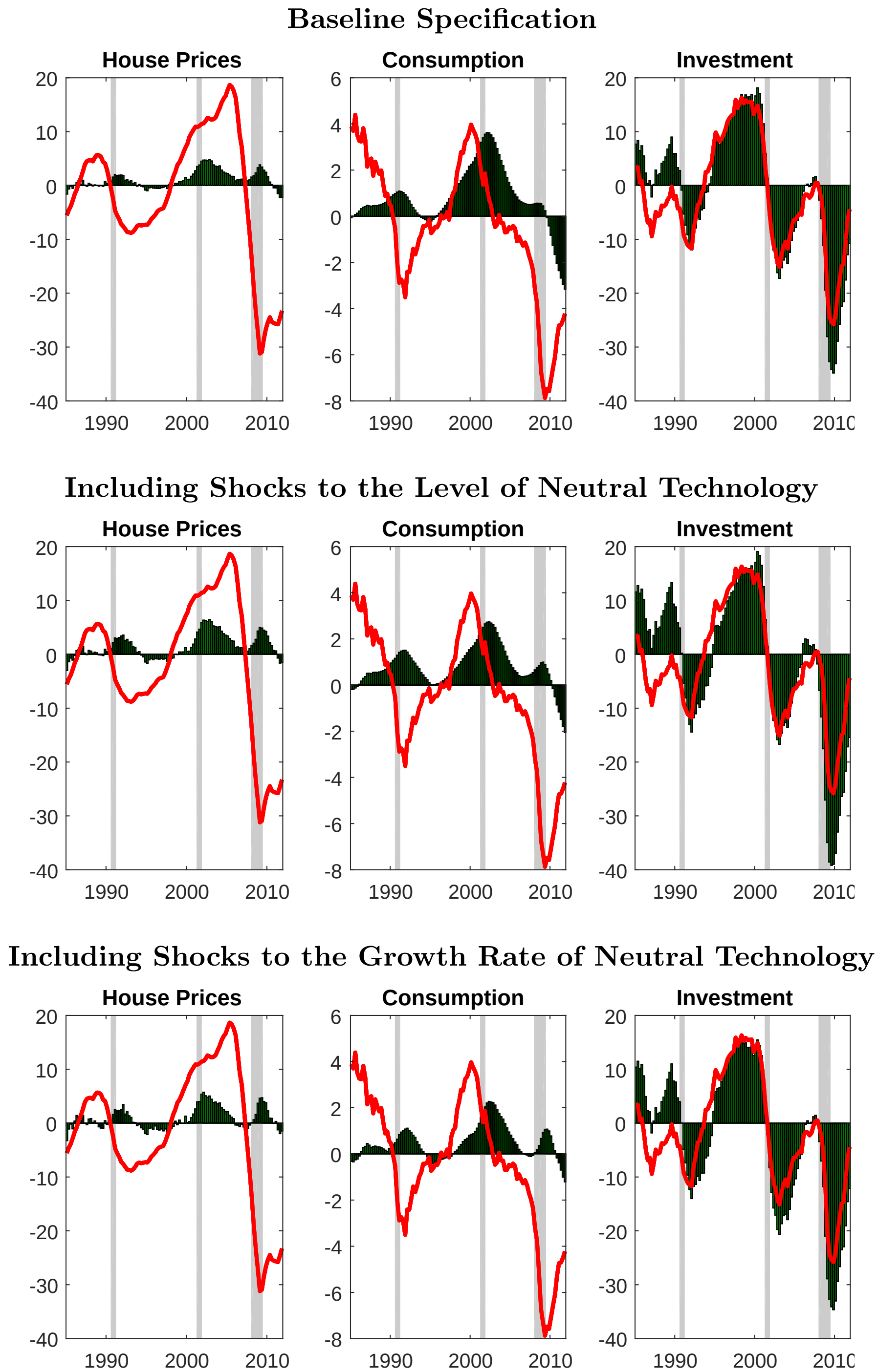
Note: Comparing simulated time series with and without rational expectations correction.

Figure A.6: Accuracy of Solution Method: Comparison of Impulse Responses for the Basic Model



Note: The units show in the horizontal axes are quarters. Impulse responses of the basic model to a negative house price shock in period 10 and a positive house price shock in period 50. The economy is at the nonstochastic steady state in period 1. The solid lines plot the responses using the approximate solution method used for the estimation of the DSGE model. The dashed lines plot the impulse responses using the global solution method (value function iteration).

Figure A.7: The Contribution of Technology Shocks To the Evolution of Key Observed Variables Under Alternative Model Specifications



The only technology shocks in the baseline specification are shocks to the level of investment-specific technology. The alternative specifications layer on neutral technology shocks. In each panel, the red solid lines denote observed variables (detrended), while the black bars show the evolution of those same variables when only the estimated technology shocks are turned on – both investment-specific and neutral technology shocks. (Recall that, by construction, the model matches the observations when all shocks are turned on.) Finally, the shaded areas denote recession periods, following the chronology determined by the NBER Business Cycle Dating Committee.