

# Thermal conductivity of base-course materials

Jean Côté and Jean-Marie Konrad

**Abstract:** This paper presents the results of a comprehensive laboratory study on the thermal conductivity of dense and broadly graded coarse base-course materials used in pavements. Materials were selected from eight quarries along the axis of the St. Lawrence River to include a variety of samples of different geological origins. Nearly 200 tests were performed in a thermal conductivity cell using Pyrex heat flux meters to characterize the relationships between the thermal conductivity of unfrozen and frozen samples and the water–ice content. Sixteen tests were also performed on solid rock cylinders to characterize the influence of mineralogy on the thermal conductivity of solid particles from the selected quarries. The most widely used empirical prediction models for thermal conductivity of soils from the literature were found inappropriate to estimate the thermal conductivity of base-course materials. An improved model using the geometric mean method to compute the thermal conductivity for the solid particles and the saturated materials, a modified form of the geometric mean method to predict the thermal conductivity of dry materials, and empirical relationships to assess the normalized thermal conductivity of unfrozen and frozen base-course materials are presented. This new model predicted well the thermal conductivity for more than 150 unfrozen and frozen coarse sand and gravel samples from the literature. A step by step methodology is proposed to assess the thermal conductivity of base-course materials.

**Key words:** base course, porosity, degree of saturation, mineralogy, unfrozen–frozen, thermal conductivity.

**Résumé :** Cette article présente les résultats d'une étude détaillée en laboratoire sur la conductivité thermique de matériaux denses et grossiers à granulométrie étalée utilisés pour les fondations de chaussées. Des matériaux ont été sélectionnés dans huit carrières dans l'axe du fleuve Saint-Laurent pour inclure une variété d'échantillons d'origines géologiques différentes. Près de 200 essais ont été réalisés dans une cellule de conductivité thermique au moyen de fluxmètres en pyrex pour caractériser les relations entre la conductivité thermique d'échantillons gelés et non gelés et la teneur en eau–glace. On a aussi fait 16 essais sur des cylindres de roc solides pour caractériser l'influence de la minéralogie sur la conductivité thermique des particules solides des carrières sélectionnées. On a trouvé que les modèles empiriques les plus généralement utilisés pour la prédiction de la conductivité thermique des sols dans la littérature sont impropres à l'estimation de la conductivité des matériaux de fondations de chaussées. On présente un modèle amélioré utilisant la méthode de la moyenne géométrique pour calculer la conductivité thermique des particules solides et des matériaux saturés, une forme modifiée de la méthode de la moyenne géométrique pour prédire la conductivité thermique des matériaux secs, et des relations empiriques pour évaluer la conductivité thermique normalisée des matériaux gelés et non gelés de fondations de chaussées. Le nouveau modèle prédit bien la conductivité thermique pour plus de 150 échantillons gelés et non gelés de sable et graviers grossiers prélevés dans la littérature. On propose une méthodologie étape par étape pour évaluer la conductivité thermique des matériaux de fondations de chaussées.

**Mots clés :** fondations de chaussées, porosité, degré de saturation, minéralogie, gelé – non gelé, conductivité thermique.

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## Introduction

Heat transfer and frost action analyses in pavements require the knowledge of the thermal properties of each layer of the pavement structure including subgrade soils. Among the various thermal properties, thermal conductivity is one of the most important input parameters in heat transfer modelling. It is well known that the thermal conductivity of a

soil is strongly influenced by its density and water content because of contrasting values of its basic components. For instance, the thermal conductivity of solid particles generally varies from 1 to 5 W/m°C, and those for water, ice, and air are 0.600, 2.240, and 0.024 W/m°C, respectively. Moreover, several other factors such as grain mineralogy and fabric also need to be considered.

In the last number of decades, many studies were conducted to quantify the effects of these factors on the thermal conductivity,  $k$ , of soils. Theoretical models were developed for the calculation of the thermal conductivity of dry soils (Smith 1942) and moist soils (Mickley 1951; De Vries 1963). Empirical models were also proposed to fit experimental data for unfrozen and frozen moist soils (Kersten 1949; Van Rooyen and Winterkorn 1959; Johansen 1975). These models provide estimates of the thermal conductivity of soils using basic geotechnical index properties that can be

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readily obtained compared to direct measurements of thermal conductivity.

As pointed out by Côté and Konrad (2003), mass transfer characteristics of pavement base-course materials were not systematically studied in the past. Unfortunately, the same applies to heat transfer characteristics of these base-course materials. This has resulted in a lack of experimental data to evaluate the reliability of existing thermal conductivity models and, if needed, to establish reliable empirical models for the prediction of thermal conductivity. The grain-size distribution of pavement base-course materials typically ranges from 0 to 20 mm, inclusively. This broad grading generally leads to high dry densities, ranging from 1800 up to 2350 kg/m<sup>3</sup>. The fabric of soils, which refers to the size and arrangement of particles and the pore space distribution (Mitchell 1993), has an undisputable influence on the thermal conductivity of soils (Kersten 1949; Van Rooyen and Winterkorn 1957; Johansen 1975; Farouki 1981; Mitchell 1993). It is thus expected that prediction of the thermal conductivity of compacted base-course materials should be different from that of the well-documented sands and fine-grained soils.

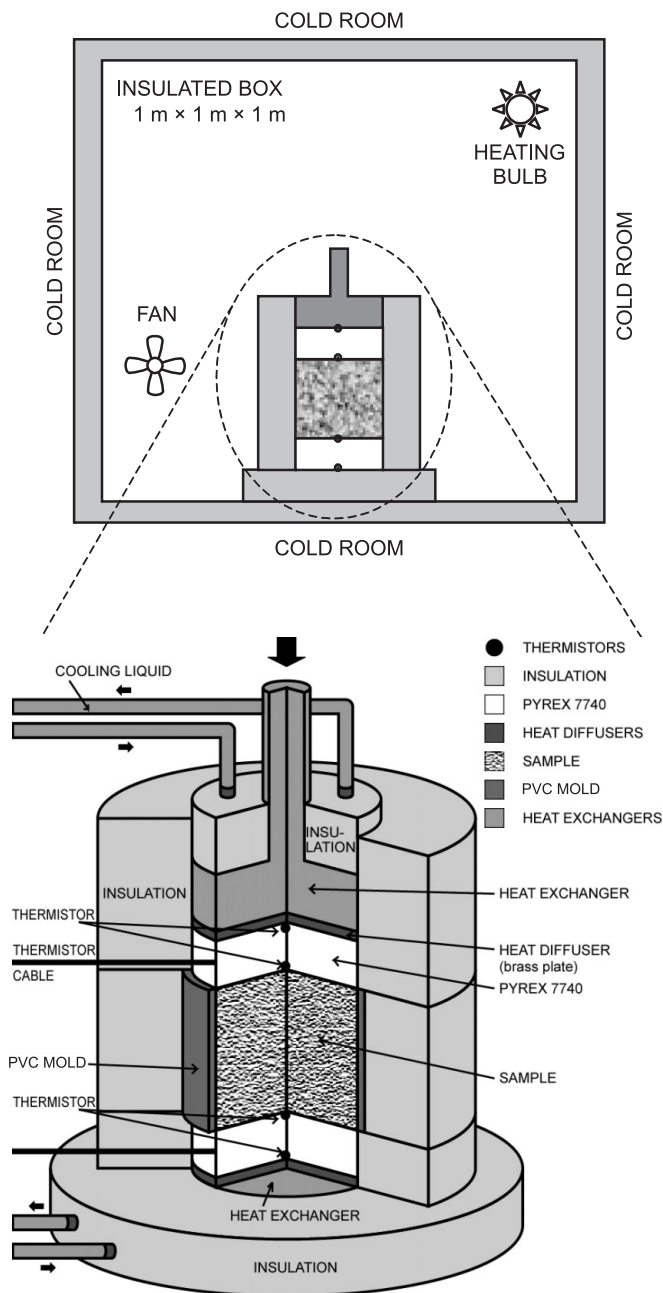
The objectives of this paper are to characterize the thermal conductivity of pavement base-course materials and to develop an adequate and reliable empirical model adapted to these materials. The paper presents the experimental setup for the measurement of thermal conductivity using Pyrex heat flux meters and the experimental results for unfrozen and frozen base-course materials. These observations are compared with predicted values obtained from current empirical models developed by Kersten (1949) and Johansen (1975), generally used by pavement engineers, to show their limitations for dense, well-compacted pavement materials. A new empirical model using the normalized thermal conductivity concept is subsequently proposed for the prediction of thermal conductivity of base-course materials. The new model is tested with a set of data from Kersten for crushed rocks similar to the materials used in this study. Lastly, the paper describes a step by step methodology for the assessment of thermal conductivity of base-course materials.

## Experimental program

### Experimental setup

Figure 1 shows the experimental setup used in this study. It comprises a thermal conductivity cell surrounded by an insulated and temperature-controlled box inside a large cold room maintained at a constant temperature of about 4 °C below the average temperature used in the test cell. Base-course materials are compacted into a cylindrical PVC mold with an inside diameter and height of 101.6 and 75 mm, respectively. The sample and the mold are placed between two Pyrex disks 101.6 mm in diameter and 30 mm high. Each Pyrex disk is instrumented with two thermistors embedded in the centre, at a few tenths of a millimetre from the planar faces. The temperature boundary conditions at the top and bottom of this three-layer system are maintained constant with two independent heat exchangers (Fig. 1) to create a constant vertical heat flow through both the sample and the Pyrex disks. The sample and the Pyrex disks are tightly surrounded with a 50 mm thick polystyrene jacket to reduce ra-

Fig. 1. Experimental setup.



dial heat losses. The ambient temperature of the insulated box is maintained equal to the mean value of the temperatures applied at both extremities of the system.

The heat flux in the thermal conductivity cell is measured through the Pyrex disks (Corning® Pyrex 7740), as the thermal conductivity of this standardized borosilicate glass is known (Powell et al. 1966) and as the temperature is measured at each end of both Pyrex disks. The thermal conductivity of the Pyrex 7740 varies little with temperature: the thermal conductivity is 1.015 W/m°C at -20 °C and 1.090 W/m°C at +20 °C, and it can be assessed from the relationship given by Powell et al. (1966). As the thermal conductivity of the materials in this study is expected to range from 0.2 to 5.0 W/m°C, the thermal conductivity contrast

between the samples and the Pyrex heat flux meters remains much lower than that with conventional copper heat flux meters having thermal conductivities close to 400 W/m°C.

The temperatures of the top and bottom of each Pyrex heat flux meter are recorded every 15 min through an acquisition system and plotted as a function of time. When temperatures become constant with time, steady state heat flow is reached. The thermal gradients in the Pyrex disks are readily determined when the distance between the centre of each thermistor for both disks and the temperature measurements are known. The thermal gradient in the sample is determined from the Pyrex-sample interface temperatures, which are extrapolated from the thermistor measurements in both Pyrex disks. The thermal conductivity (in W/m°C) of the tested sample is approximated as

$$[1] \quad k = \frac{q_{uf} + q_{lf}}{2} \frac{\Delta h}{\Delta T} = \left[ \left( k_{uf} \frac{\Delta T_{uf}}{\Delta h_{uf}} + k_{lf} \frac{\Delta T_{lf}}{\Delta h_{lf}} \right) \frac{\Delta h}{\Delta T} \right] / 2$$

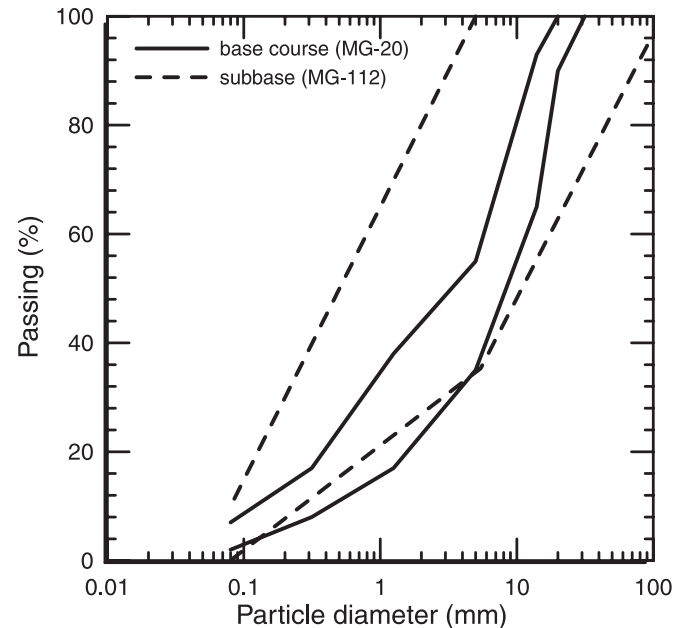
where  $q$  is the heat flux (W/m°C);  $\Delta h$  is the distance between two temperature measurements (m);  $\Delta T$  is the temperature difference (°C); and the subscripts "uf" and "lf" refer to the upper and lower heat flux meters, respectively.

The tests were carried out at a mean temperature of about  $5 \pm 1$  °C for the unfrozen samples and about  $-5 \pm 1$  °C for the frozen samples. The temperature difference between the top of the upper Pyrex disk and the bottom of the lower Pyrex disk is generally equal to 8 °C for both testing temperatures. Temperature gradients of 0.2–0.6 °C/cm were obtained across the soil samples. King (1979) asserts that the thermal conductivities of saturated crushed rocks are between 1% and 2% lower than the true values when measured with thermal gradients from 0.4 to 3.0 °C/cm. The effect of the thermal gradient is thus neglected in this study. It is noted that the relative error on the thermal conductivity measurements due to the precision of the thermistors ( $\pm 0.025$  °C) and the position of each Pyrex disks ( $\pm 0.5$  mm) is less than  $\pm 5\%$ .

Another factor that can influence the thermal conductivity measurements is the thermal resistance that may occur at the contact between the heat flux meters and the sample. This effect has been discussed by Birch and Clark (1940) and studied more systematically by King (1979), in which the reported contact resistance, expressed as an equivalent thickness of materials, was about 0.6 mm for limestone ( $k = 2.9$  W/m°C) and 3.0 mm for sandstone ( $k = 5.4$  W/m°C) for sample thicknesses varying from 5 to 20 mm. Considering that the height of samples tested in the present study is 75 mm, contact resistances, expressed as an equivalent thickness, of 0.6 mm for the materials with the lower conductivities and 3.0 mm for those with higher conductivities will lead to an overestimation of the thermal conductivity of less than 1% and less than 4%, respectively. The contact resistance was thus neglected in this study.

The thermal conductivity measurement system has also been tested with a reference material which consisted of two stacked Pyrex disks similar to those used as heat flux meters. The measured thermal conductivities agreed with theoretical values within less than 5% of deviation, which was considered sufficiently accurate for the purpose of this study

Fig. 2. Grain-size distribution of base and subbase materials.



and which also confirms that the contact resistance can indeed be neglected.

### Materials tested

The main focus of this study was to establish reliable thermal conductivity data of unfrozen and frozen base-course materials that are used in pavements as base and subbase courses. These materials are referred to as MG-20 and MG-112, respectively, in the specifications of the Ministry of Transportation of Quebec (MTQ). The grain-size distribution of MG-20 is shown in Fig. 2 by the solid lines and that of MG-112 is represented by the broken lines. The materials studied herein had a maximum grain size of 20 mm.

The materials were selected from eight different quarries along the axis of the St. Lawrence River to include a variety of samples with different geological origins to characterize the influence of mineralogy on thermal conductivity. The materials studied were all crushed rocks: (i) gabbro from St-Joseph-de-Beauce, which is located about 50 km south of Québec City; (ii) dolostone from St-Eustache, about 15 km northwest of Montréal; (iii) granite A from Québec City (Valcartier); (iv) granite B from Québec City (Lac-St-Charles); (v) limestone from St-Marc-des-Carières, about 50 km west of Québec City; (vi) quartzite from Rimouski; (vii) sandstone from St-Anaclet, 20 km east of Rimouski; and (viii) syenite from Mont-St-Hilaire, about 60 km east of Montréal.

The mineralogy of the samples was not systematically determined in the laboratory, but typical compositions are given in the following according to general data from Dietrich and Skinner (1979) and Blatt and Tracy (1996) and data available from some of the quarries studied. It is noted that the descriptive mineralogical composition of the limestone was determined by Côté (1997) for the purpose of the study of hydraulic characteristics of base-course materials. Gabbros are volcanic rocks generally composed of calcic plagioclase (labradorite) and calcic pyroxene (augite); in

**Table 1.** Typical mineral composition for the materials tested.

| Material  | Typical mineralogical composition                      |                                |                     |
|-----------|--|--------------------------------|---------------------|
|           | Major phase  | Minor phase                    | Trace               |
| Dolostone | Dolomite   | Calcite                        |                     |
| Gabbro    | Plagioclase (labradorite), pyroxene (augite), olivine  | Olivine, quartz                |                     |
| Granite   | Quartz, feldspar, plagioclase, many secondary minerals |                                |                     |
| Limestone | Calcite  | Dolomite                       | Aragonite, siderite |
| Quartzite | Quartz grains in quartz cement                         |                                |                     |
| Sandstone | Quartz   | Feldspar, plagioclase          |                     |
| Syenite   | Feldspar, plagioclase                                  | Nepheline, pyroxene, amphibole |                     |

these rocks olivine is sometimes an important constituent, and quartz may be present as a minor constituent. The mineralogical composition of the gabbro in this study was as follows: 68% labradorite (plagioclase), 20% calcite, 10% quartz, and 2% chlorite. Limestones are sedimentary rocks mainly made of calcite (calcium carbonate), whereas dolostones, also sedimentary rocks, are made of dolomite (magnesium–calcium carbonate); calcite is also sometimes present in dolostones. Granites are plutonic rocks principally made of quartz, feldspar, and plagioclase, with minor mafic minerals such as biotite and hornblende. The granites found in the Québec City area generally have less than 25% quartz. Sandstones are detrital sedimentary rocks and are mainly composed of grains of quartz with feldspar and plagioclase. The sandstone studied here has a relatively large proportion (~20%) of albite, which is a plagioclase mineral. The quartzite is a variety of sandstone that is made of quartz grains well cemented in neogenic quartz films. The mineralogical composition of the Rimouski quartzite is as follows: 76% quartz, 20% plagioclase, 2% calcite, and 2% mica. Lastly, the type of syenite (plutonic rock) studied here was a nepheline syenite, which is made of feldspar, plagioclase, nepheline, and minor mafic minerals such as pyroxene and amphibole. The very low quartz content distinguishes syenite from granite. These typical mineralogical compositions are summarized in Table 1.

The dolostone, granite B, quartzite, sandstone, and syenite samples were tested using their original grain-size distributions and compacted using the 4.54 kg Proctor hammer and a water content of 8.8% for granite B and 5.6% for the other samples. Gabbro, limestone, and mostly granite A samples were compacted with varying compacting energies (Proctor hammer, vibrating hammer), different grain-size distributions, and water contents varying from 3.0% to 7.7% to characterize the thermal conductivity as a function of dry density. After compaction, the samples were placed in the thermal conductivity measurement system and a temperature of about  $-9^{\circ}\text{C}$  was applied to the bottom of the lower Pyrex disk and a temperature of about  $-1^{\circ}\text{C}$  to the top of the upper disk, resulting in a mean temperature of about  $-5^{\circ}\text{C}$  in the sample. The freezing of the samples took about 1–5 h depending on water content, and thermal steady state was usually reached after 12 h of testing. The temperature distribution used to compute the thermal conductivity was taken after about 24 h. The temperatures were then increased to about  $1^{\circ}\text{C}$  at the top of the upper disk and about  $9^{\circ}\text{C}$  at the bottom of the lower disk, giving a mean temperature of about  $5^{\circ}\text{C}$  in the sample. The melting of the samples took

about 1–5 h depending on water content. Temperatures at thermal steady state were used to compute the thermal conductivity of the unfrozen samples. The testing time for the measurement of thermal conductivities for a sample at a single water content in both the unfrozen and frozen states was about 48 h. After this period, the sample was partially dried to obtain lower water contents and the thermal conductivity measurements in both the frozen and unfrozen states were undertaken using the same procedure. This was repeated until the sample was almost completely dry. The experimental procedure yielded the relationship between thermal conductivity and degree of saturation at a constant dry density for unfrozen and frozen materials.

The thermal conductivity of solid particles was also measured using cylindrical rock samples drilled out of blocks taken from the quarries. The surfaces at both ends of the cylinders were squared and polished, and a thin film of silicon grease was applied to minimize any thermal resistance at the rock–Pyrex interface.

The characteristics of each material tested herein are given in Table 2.

## Experimental results

### Typical results for unfrozen–frozen base-course and solid rock samples

Typical experimental data are shown in Fig. 3 for one granite A sample, with a porosity of 0.18 and water content of 3%, tested in a frozen and an unfrozen state. The sample was subjected to closed-system freezing, i.e., both ends freeze simultaneously without access to water. The temperature at the top of the Pyrex disk was maintained constant at  $-8.25^{\circ}\text{C}$  while the bottom of the lower Pyrex disk was at  $-1.40^{\circ}\text{C}$ . The left-hand part of Fig. 3a shows the evolution of temperature with time for the thermistors embedded in the Pyrex disks. As the moist sample begins to freeze, the temperature threshold observed between 2 and 5 h is caused by the release of the latent heat of fusion of pore water during phase change. The sample was completely frozen after about 5 h, and thermal steady state was reached after 12 h of testing. The temperature profile at 22 h is shown in the left-hand part of Fig. 3c. The temperature gradients were  $68.4^{\circ}\text{C/m}$  in the upper Pyrex disk,  $35.8^{\circ}\text{C/m}$  in the sample, and  $69.9^{\circ}\text{C/m}$  in the lower disk. The thermal conductivity in each Pyrex disk is  $1.046\text{ W/m}^{\circ}\text{C}$  for the upper disk (mean temperature of  $-7.18^{\circ}\text{C}$ ) and  $1.055\text{ W/m}^{\circ}\text{C}$  for the lower disk (mean temperature of  $-2.53^{\circ}\text{C}$ ) according to Powell et



**Table 2.** Mass–volume properties of the materials tested and details of the testing program.

| Material  | Location             | $\rho_s$ (kg/m <sup>3</sup> ) | $\rho_d$ (kg/m <sup>3</sup> ) | $n$  | Range of $w$ (%) | No. of tests <sup>a</sup> |        | Rock cylinders |
|-----------|----------------------|-------------------------------|-------------------------------|------|------------------|---------------------------|--------|----------------|
|           |                      |                               |                               |      |                  | Unfrozen                  | Frozen |                |
| Dolostone | St-Eusatche          | 2850                          | 2405                          | 0.16 | 0.04–5.66        | 4                         | 4      | 2              |
| Gabbro    | St-Joseph-de-Bauce   | 2950                          | 2231                          | 0.24 | 0.27–7.91        | 4                         | 4      | 2              |
|           |                      |                               | 2274                          | 0.23 | 0.12–4.88        | 4                         | 4      |                |
| Granite A | Valcartier           | 2750                          | 2104                          | 0.24 | 0.17–4.84        | 4                         | 4      | 2              |
|           |                      |                               | 2191                          | 0.20 | 0.16–4.73        | 4                         | 4      |                |
|           |                      |                               | 2265                          | 0.18 | 0.16–2.98        | 3                         | 3      |                |
|           |                      |                               | 2051                          | 0.25 | 0.16–4.73        | 4                         | 4      |                |
|           |                      |                               | 1921                          | 0.30 | 0.02–7.65        | 6                         | 6      |                |
|           |                      |                               | 1834                          | 0.33 | 0.09–7.66        | 6                         | 6      |                |
|           |                      |                               | 2204                          | 0.20 | 0.12–5.90        | 7                         | 7      |                |
|           |                      |                               | 2261                          | 0.18 | 0.12–5.37        | 6                         | 6      |                |
| Granite B | Lac-St-Charles       | 2650                          | 1854                          | 0.30 | 0.00–8.80        | 4                         | 4      | 2              |
| Limestone | St-Marc-des-Carières | 2670                          | 2313                          | 0.13 | 0.10–5.40        | 6                         | 6      | 2              |
|           |                      |                               | 2320                          | 0.13 | 0.13–5.60        | 4                         | 4      |                |
| Quartzite | Rimouski             | 2650                          | 2263                          | 0.15 | 0.41–5.43        | 4                         | 4      | 2              |
|           |                      |                               | 1904                          | 0.28 | 0.01             | 1                         | 1      |                |
|           |                      |                               | 1706                          | 0.36 | 0.08             | 1                         | 1      |                |
|           |                      |                               | 1545                          | 0.42 | 0.11             | 1                         | 1      |                |
| Sandstone | St-Anaclet           | 2670                          | 2276                          | 0.15 | 0.15–5.70        | 4                         | 4      | 2              |
|           |                      |                               | 1886                          | 0.29 | 0.03             | 1                         | 1      |                |
|           |                      |                               | 1720                          | 0.35 | 0                | 1                         | 1      |                |
| Syenite   | Mont-St-Hilaire      | 2730                          | 2323                          | 0.15 | 0.12–5.49        | 4                         | 4      | 2              |
|           |                      |                               | 2254                          | 0.18 | 0.46             | 1                         | 1      |                |
|           |                      |                               | 2004                          | 0.27 | 0.14             | 1                         | 1      |                |
|           |                      |                               | 1792                          | 0.34 | 0.06             | 1                         | 1      |                |

<sup>a</sup>Total number of tests = 190.

al. (1966). These values used in eq. [1] give a thermal conductivity of 1.87 W/m°C for the frozen sample as follows:

$$k_f = [(1.046 \times 62.6 + 1.055 \times 64.9)/(1/35.9)]/2 = 1.87$$

After 24 h of testing in the frozen state, the temperatures at the top and bottom of the Pyrex disks were then changed to about 1.60 and 10.15 °C, respectively, as shown in Fig. 3a. The temperature threshold between 24 and 28 h of elapsed testing was caused by the energy absorbed by the sample as pore ice melts. Figure 3c shows the steady state temperature profile measured at 46 h for the unfrozen sample. The temperature gradients were equal to 79.1 °C/m in the upper disk, 48.8 °C/m in the unfrozen sample, and 80.8 °C/m in the lower disk. The thermal conductivity was 1.065 W/m°C (mean temperature of 2.77 °C) in the upper Pyrex disk and 1.075 W/m°C in the lower Pyrex disk (mean temperature of 8.93 °C). From these data, the computed thermal conductivity of the unfrozen sample was thus  $k_u = 1.75$  W/m°C.

Figure 4 shows a typical evolution of temperature with time for a solid rock cylinder (granite A) at two testing temperatures. In the left-hand part of Fig. 4, the sample was tested at temperatures below 0 °C, and the right-hand part of Fig. 4 shows temperature variations when the sample temperature was raised above 0 °C. As the rock cylinders are expected to have no or very little pore water, no temperature threshold was observed. The thermal conductivities mea-

sured for this sample were 2.61 W/m°C at the mean temperature of –4.6 °C and 2.59 W/m°C at the mean temperature of 6.2 °C.

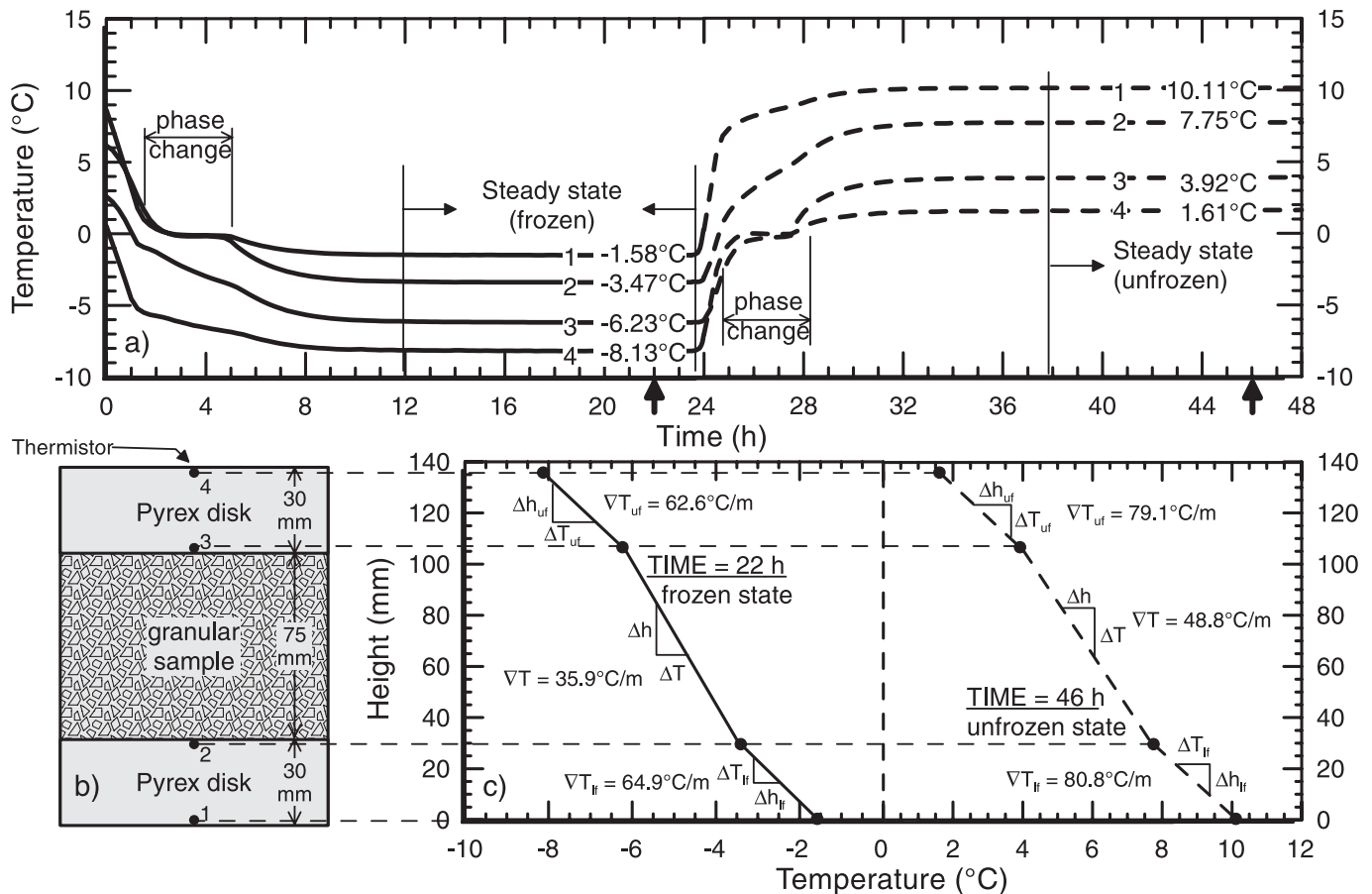
A total of 188 thermal conductivity tests were performed in this study, with 16 tests on solid rock cylinders and 172 tests on 24 compacted base-course material samples as outlined in Table 2.

### Thermal conductivity of solid particles, $k_s$

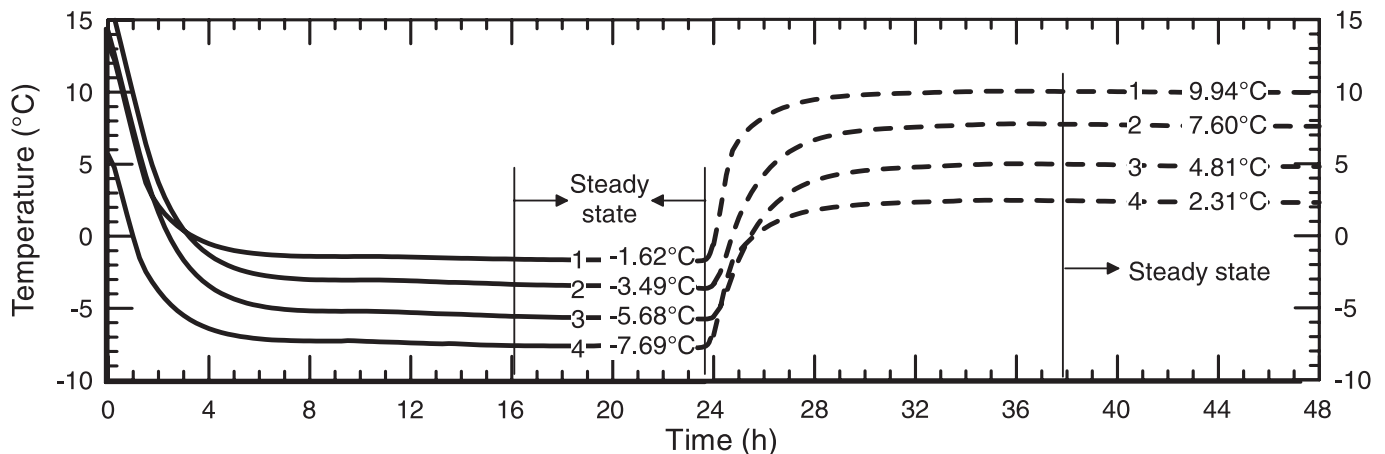
The thermal conductivity of each rock cylinder, presented in Table 3, is the mean of two measurements made at mean temperatures of about 5 °C and –5 °C. Typical variations of less than 5% were observed between the values obtained at both temperatures. The thermal conductivity of solid rock varied from 1.6 W/m°C for the syenite sample to 5.0 W/m°C for the quartzite sample. Intermediate values of 2.2, 2.3, 2.6, 3.3, and 3.4 W/m°C were obtained for gabbro, limestone, granites A and B, dolostone, and sandstone, respectively.

Table 4 gives the mean values of thermal conductivity computed from data published by Horai (1971) for different families of minerals that form the rocks studied herein, where quartz mineral has the highest thermal conductivity value of 7.69 W/m°C and plagioclase mineral (labradorite) has the lowest value of 1.53 W/m°C. From this study, it follows that the highest thermal conductivities should be obtained for a rock with the highest quartz content, as mentioned by Johansen (1975). Clearly, the highest thermal conductivity (5.0 W/m°C) was obtained for the highest quartz

**Fig. 3.** Typical data for a sample of granite A: (a) evolution of temperature with time, in which lines 1–4 represent data from thermistors 1–4 as shown in b; (b) sample layout; and (c) temperature distributions at the frozen and unfrozen steady states.  $h_{lf}$ , height of lower heat flux meter;  $h_{uf}$ , height of upper heat flux meter.



**Fig. 4.** Typical data for the granite A solid rock sample, showing temperature variations below and above 0 °C.



content of 76% in quartzite. The second highest value of 3.4 W/m°C was obtained for the sandstone cylinder, which is also quartz rich. Quartz content is not the only factor that may explain this ranking. In fact, high contents of chlorite, dolomite, olivine, and pyroxene can also lead to above-average thermal conductivity of rocks. For example, the dolostone studied here has the third highest value of 3.3 W/m°C, which is primarily due to its high content of do-

lomite mineral with a thermal conductivity of 5.5 W/m°C. Intermediate values of thermal conductivity were obtained for granites (granites A and B, 2.6 W/m°C), limestone (2.3 W/m°C), and gabbro (2.2 W/m°C). Syenite had the lowest thermal conductivity, with a value of 1.6 W/m°C, mainly due to the total absence of quartz mineral. All of these values compare well with the typical range of thermal conductivities of rocks as shown in Table 5, which lists

**Table 3.** Thermal conductivity of solids particles of the materials tested.

| Material  | $k_s$ (W/m°C) |
|-----------|---------------|
| Dolostone | 3.3           |
| Gabbro    | 2.2           |
| Granite A | 2.6           |
| Granite B | 2.6           |
| Limestone | 2.3           |
| Quartzite | 5.0           |
| Sandstone | 3.4           |
| Syenite   | 1.6           |

**Table 4.** Thermal conductivity of some rock-forming minerals (mean values calculated and converted to SI units from Horai 1971).

| Mineral                   | $k$ (W/m°C) |
|---------------------------|-------------|
| Amphibole                 | 3.46        |
| Calcite                   | 3.59        |
| Chlorite                  | 5.15        |
| Dolomite                  | 5.51        |
| Feldspar                  | 2.25        |
| Mica                      | 2.03        |
| Olivine                   | 4.57        |
| Plagioclase               | 1.84        |
| Plagioclase (labradorite) | 1.53        |
| Pyroxene                  | 4.52        |
| Quartz                    | 7.69        |

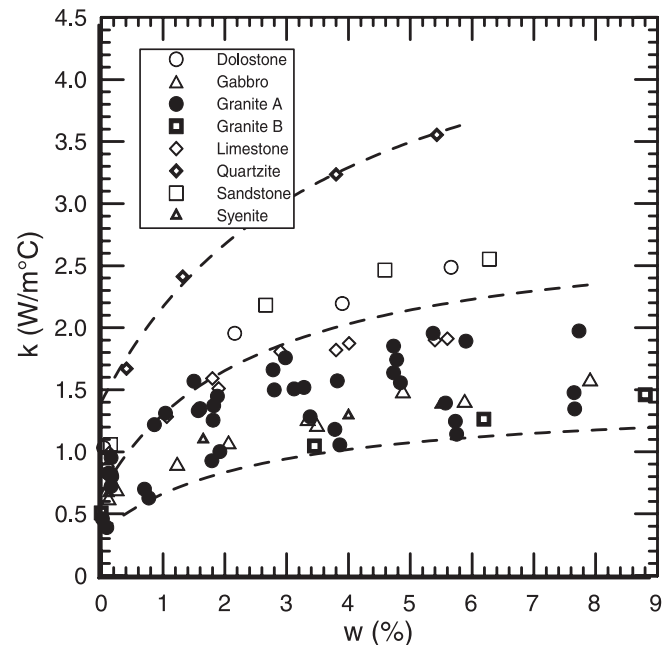
mean thermal conductivity values for different types of rocks obtained from Andersland and Anderson (1978), Birch and Clark (1940), Goguel (1975), Johnston (1981), Jumikis (1977), Missenard (1965), and Sass et al. (1971).

#### Thermal conductivity of compacted base-course materials

The bulk results of the thermal conductivity measurements are shown as a function of water content by weight in Fig. 5 for unfrozen compacted base-course materials and as a function of ice content by weight in Fig. 6 for the frozen state. Since water content and ice content by weight are equal, the symbol  $w$  and the term water content are used for both unfrozen and frozen states. It is noted that the apparent data scattering for granite A is primarily due to variations of dry density in the samples. This is discussed in more detail later in the paper. As anticipated, the thermal conductivity has a general tendency to increase with an increase in water content, as water-ice is a much better heat conductor than air. This is schematically shown by the broken lines. For example, at water contents of 0.4%, 1.3%, 3.8%, and 5.4%, the quartzite sample displayed thermal conductivities of 1.67, 2.25, 3.26, and 3.56 W/m°C, respectively, in the unfrozen state and 1.67, 2.23, 3.35, and 4.32 W/m°C in the frozen state. These data also reveal that at water contents higher than 2%, the thermal conductivity of the materials in the frozen state is higher than that in the unfrozen state, which is consistent with the fact that ice has a thermal conductivity almost four times higher than that of water. It is also well known that the thermal conductivity of soil increases with

**Table 5.** Average thermal conductivity of rocks computed from various sources.

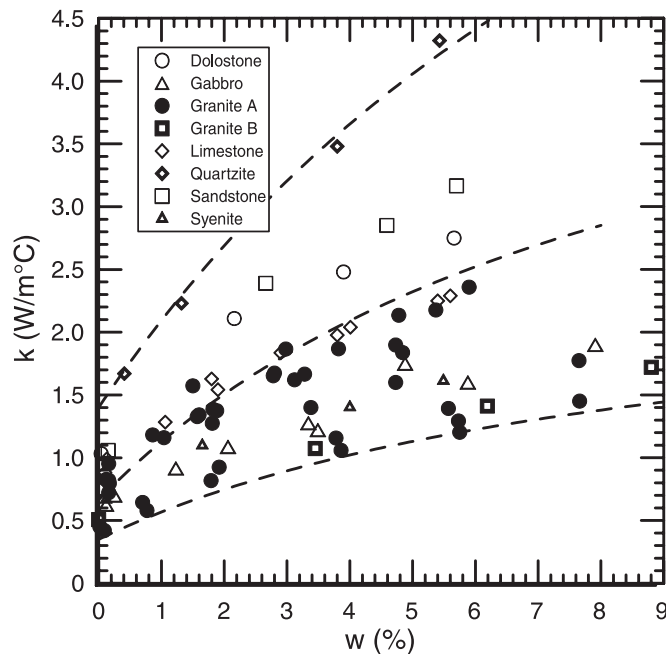
| Material    | $\rho_s$ (kg/m <sup>3</sup> ) | $k_s$ (W/m°C) |
|-------------|-------------------------------|---------------|
| Anorthosite | 2.73                          | 1.8           |
| Basalt      | 2.90                          | 1.7           |
| Diabase     | 2.98                          | 2.3           |
| Dolostone   | 2.90                          | 3.8           |
| Gabbro      | 2.92                          | 2.2           |
| Gneiss      | 2.75                          | 2.6           |
| Granite     | 2.75                          | 2.5           |
| Limestone   | 2.70                          | 2.5           |
| Marble      | 2.80                          | 3.2           |
| Quartzite   | 2.65                          | 5.0           |
| Sandstone   | 2.80                          | 3.0           |
| Schist      | 2.65                          | <1.5          |
| Shale       | 2.65                          | 2.0           |
| Syenite     | 2.80                          | 2.0           |
| Trapp rock  | 2.90                          | 2.0           |

**Fig. 5.** Thermal conductivity of unfrozen base-course materials as a function of water content.

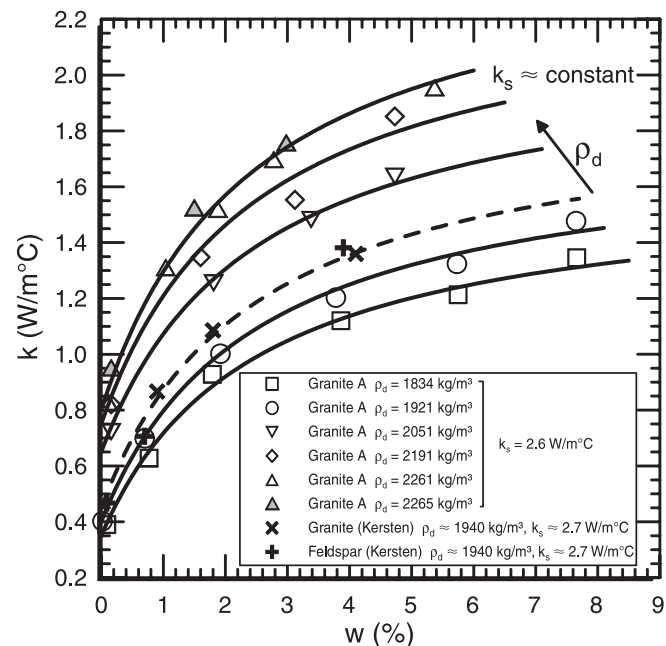
increasing dry density and with increasing thermal conductivity of solid particles, as outlined in more detail later in the paper.

To study the effect of dry density on thermal conductivity, several samples of material labelled granite A were compacted with various compaction energies and different grain-size distributions (from gravel to coarse sand). Dry densities ranging from 1834 to 2265 kg/m<sup>3</sup> were thus obtained. It is obvious from Fig. 7 that for a constant water content and a constant thermal conductivity of solid particles, the thermal conductivity of unfrozen material increased systematically with increasing dry density. At a constant water content of 4%, samples with dry densities of 1834 and 2265 kg/m<sup>3</sup> have thermal conductivities of about 1.1 and 1.9 W/m°C, respectively. Although not shown here, the same trend was

**Fig. 6.** Thermal conductivity of frozen base-course materials as a function of water content.

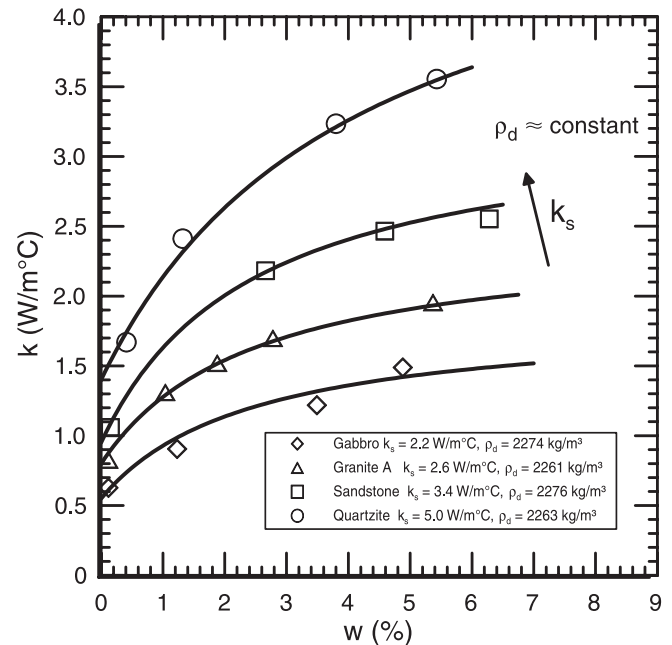


**Fig. 7.** Thermal conductivity of unfrozen base-course materials as a function of water content and dry density.



observed for the frozen samples. These results compare well with data from Kersten (1949) for crushed granite and crushed feldspar, with estimated thermal conductivities of 2.7 and 2.6 W/m°C, respectively, based on the mineralogical composition as computed with the geometric mean method, which is discussed in the next section. Both materials used by Kersten had a mean dry density of about 1940 kg/m<sup>3</sup>, and the relationship between  $k$  and  $w$  lies between those of the granite A samples having dry densities of 1921 and 2051 kg/m<sup>3</sup>, which indicates the consistency of the results

**Fig. 8.** Thermal conductivity of unfrozen base-course materials as a function of water content and thermal conductivity of solid particles.



obtained herein. It is also noted that results for the granite A samples having the same dry density (2261 and 2265 kg/m<sup>3</sup>) display about the same results, thus confirming that the measurements are repeatable.

Results for materials of various mineralogical compositions enable the study of the effect of thermal conductivity of solid particles on the variations of thermal conductivity of soils. Figure 8 clearly shows that for constant water contents and a dry density value of about 2270 kg/m<sup>3</sup>, the thermal conductivity of unfrozen base-course materials increases with increasing thermal conductivity of solid particles,  $k_s$ . For instance, at a water content of 4%, crushed gabbro ( $k_s = 2.2$  W/m°C) has a thermal conductivity of about 1.6 W/m°C, crushed granite A ( $k_s = 2.6$  W/m°C) has a value of about 1.9 W/m°C, crushed sandstone ( $k_s = 3.4$  W/m°C) has a value of about 2.4 W/m°C, and crushed quartzite ( $k_s = 5.0$  W/m°C) has a value of 3.3 W/m°C. Again, this trend was also observed for the materials in the frozen state.

## Evaluation of current prediction models

The measurement of thermal conductivity of soils in the laboratory is often time consuming and necessitates very precise instrumentation. Prediction models of the thermal conductivity using basic soil properties such as water content, degree of saturation, dry density, or porosity are thus particularly useful to perform thermal analyses of earth structures. At this point, it is of value to evaluate the ability to predict the thermal conductivity of unfrozen–frozen base-course materials of two of the most used empirical models, which were developed by Kersten (1949) and Johansen (1975). Since Johansen's model requires  $k_s$  as input, the geometric mean method for estimating  $k_s$  using mineralogical



composition is evaluated with the results obtained for the solid quartzite and solid gabbro samples from this study.

### Geometric mean method for estimating $k_s$

The wide variety of thermal conductivity values for the rock-forming minerals found in Table 4 and the results obtained in this study reveal that the knowledge of the complete mineralogy of the rock is necessary for an accurate assessment of the rock's thermal conductivity. Based on the mineralogical composition, the value of  $k_s$  (in W/m°C) for a rock sample can be computed from the generalized geometric mean method as expressed by eq. [2], a method that was successfully used by Sass et al. (1971) and Woodside and Messmer (1961):

$$[2] \quad k_s = \prod_{j=1}^z k_{mj}^{x_j} \quad \text{with} \quad \sum_{j=1}^z x_j = 1$$

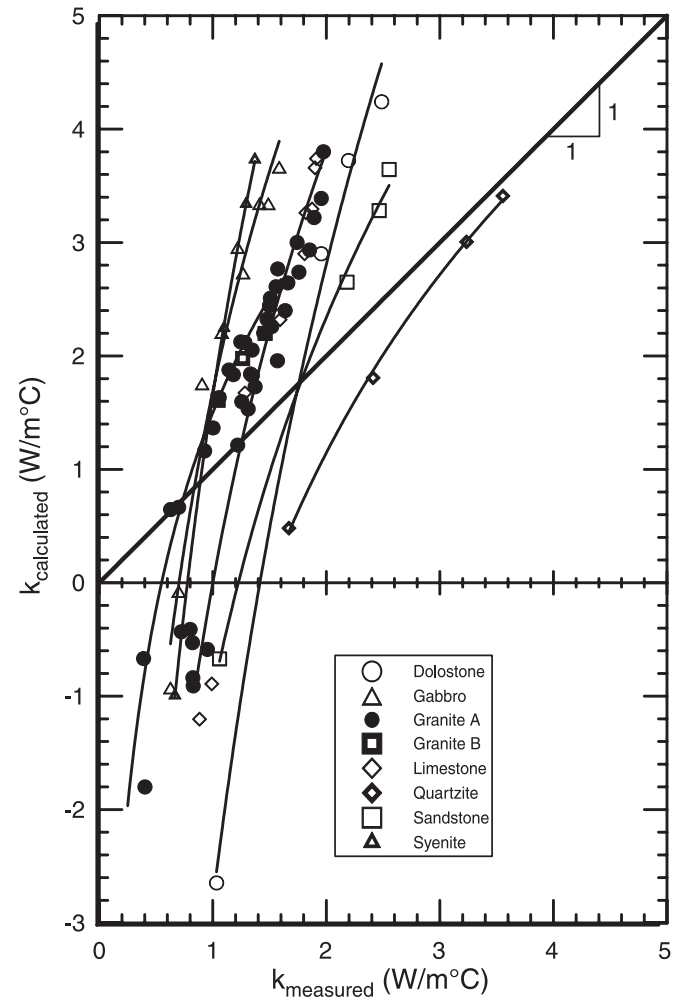
where  $\prod$  represents the product of the thermal conductivity of the minerals  $k_m$  raised to the power of their volumetric proportion  $x$ , and the sum of the volumetric proportion of the minerals is equal to 1. The subscript  $j$  refers to the  $j$ th mineral, there being  $z$  minerals altogether. Equation [2] gives the best results when the thermal conductivity of each mineral does not contrast by more than one order of magnitude.

The complete mineralogical compositions for the quartzite and the gabbro in this study were available from their respective quarries. The quartzite consisted of 76% quartz, 20% plagioclase, 2% calcite, and 2% mica, and the gabbro consisted of 68% labradorite (plagioclase), 20% calcite, 10% quartz, and 2% chlorite. Using eq. [2] and the thermal conductivities for rock-forming minerals given in Table 4, the computed thermal conductivity is  $k_s = 7.69^{0.76} \times 1.84^{0.20} \times 3.59^{0.02} \times 2.03^{0.02} = 5.54$  W/m°C for the quartzite and  $k_s = 1.53^{0.68} \times 3.59^{0.20} \times 7.69^{0.10} \times 5.15^{0.02} = 2.18$  W/m°C for the gabbro. These computed values compare well with the measured thermal conductivities for the quartzite and gabbro of 5.0 and 2.2 W/m°C, respectively, and tend to confirm the validity of the geometric mean method for the assessment of the thermal conductivity of solid particles of crushed rock. The computed and measured values also compare well with data from the literature as given in Table 5 where the mean values for quartzite and gabbro are 5.0 and 2.2 W/m°C, respectively.

### Kersten's model

Kersten's (1949) work on the thermal conductivity of soils still remains the most extensive study of its kind. Over 1000 measurements were made on 19 different soil types, including gravels, crushed rocks, sands, silts, clays, and peat. However, the results from only nine of these 19 soils were used to develop the empirical relationships between thermal conductivity, water content, and dry density for both unfrozen and frozen states. The influence of fabric is highlighted because different relationships were necessary to fit the experimental results for silt and clay and for sandy soils. The model for sandy soils which is generally used by pavement engineers to estimate the thermal conductivity for base-course and subbase course materials was calibrated with the data obtained from Chena River gravel, Fairbanks (coarse) sand, Lowell (medium) sand, and Dakota sandy

**Fig. 9.** Predicted thermal conductivity ( $k_{\text{calculated}}$ ) of unfrozen base-course materials using Kersten's (1949) model versus actual values ( $k_{\text{measured}}$ ).



loam. These materials had relatively high quartz contents ranging from 43% to 72%. The textures of these soils were very different, from a poorly graded gravel to a sandy loam with porosity ranging from 0.21 to 0.37 for Chena River gravel, 0.29 to 0.37 for Fairbanks sand, 0.33 to 0.44 for Lowell sand, and 0.20 to 0.40 for Dakota sandy loam.

The relationships proposed by Kersten (1949) for the prediction of thermal conductivity of sandy soils for unfrozen and frozen states were converted to SI units by Andersland and Anderson (1978) and are given by eqs. [3] and [4], respectively:

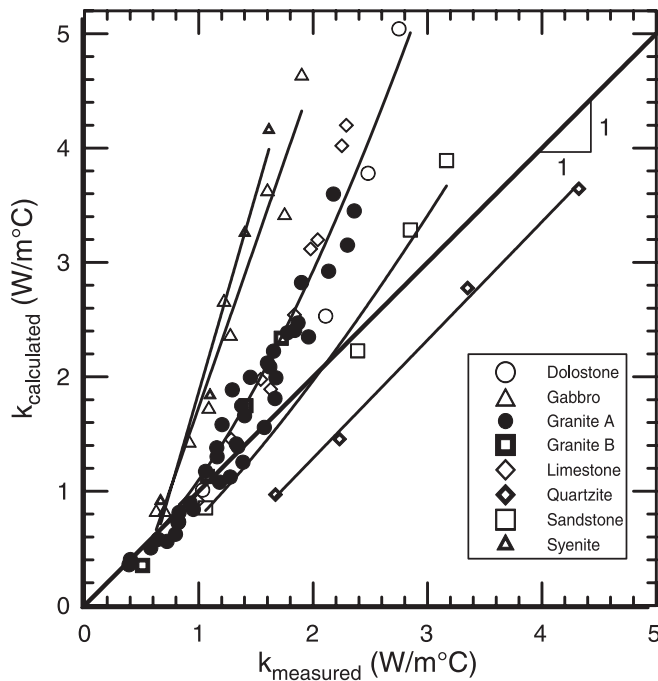
$$[3] \quad k_u = 0.1442[0.9 \log(w) - 0.2] \times 10^{0.6243 \rho_d}$$

$$[4] \quad k_f = 0.001442 \times 10^{1.373 \rho_d} + 0.01226w \times 10^{0.4994 \rho_d}$$

where  $k_u$  and  $k_f$  are the thermal conductivity for unfrozen and frozen conditions (W/m°C),  $w$  is the water content (%), and  $\rho_d$  is the dry density (g/cm<sup>3</sup>).

Figures 9 and 10 compare the predicted values of thermal conductivities ( $k_{\text{calculated}}$ ) using these equations with the experimental results ( $k_{\text{measured}}$ ) obtained for the materials of this study in both unfrozen and frozen states. It is worth recalling that for each material, low thermal conductivities in-

**Fig. 10.** Predicted thermal conductivity of frozen base-course material using Kersten's (1949) model versus actual values.



indicate low water contents and higher thermal conductivities indicate higher water contents. As seen in Fig. 9, predictions for the unfrozen samples made with Kersten's model generally lead to values that are higher than the actual values. The predicted thermal conductivities for crushed gabbro can be up to four times higher than the measured values. For dolostone, granites A and B, limestone, and syenite, the calculated  $k$  values for the highest water content are about twice as high as the measured values. Negative values are also obtained in the low water content domain. The model predicted the values for crushed sandstone and quartzite quite well, however. For actual  $k$  values of less than 1 W/m°C, Kersten's model predicts negative thermal conductivities, which are physically unacceptable.

#### Johansen's model

Johansen (1975) developed his model using data from Kersten (1949) with the normalized thermal conductivity concept  $k_r$ . As Johansen's model considered the mineralogy of soils, he used Kersten's data in a more systematic manner than Kersten himself. All natural soils were used to develop relationships between  $k_r$  and the degree of saturation  $S_r$ , one for unfrozen fine-grained soils (silty and clayey soils), one for unfrozen coarse-grained soils (medium and fine sands), and a unique relationship for both frozen fine- and coarse-grained soils. Crushed rock and Chena River gravel were not considered in the coarse-grained soil family. Relationships for the coarse-grained soils which again are used by pavement engineers to estimate the thermal conductivity of base-course and subbase materials were given as

$$[5] \quad k_{ru} = 0.7 \log(S_r) + 1$$

$$[6] \quad k_{rf} = S_r$$

where  $k_{ru}$  and  $k_{rf}$  represent the normalized thermal conductivity for the unfrozen and frozen states, respectively.

The thermal conductivity is normalized with respect to the thermal conductivities of the saturated and fully dried soils:

$$[7] \quad k_r = (k - k_{dry}) / (k_{sat} - k_{dry})$$

where  $k$  is the thermal conductivity of the soil for a given water content; and  $k_{sat}$  and  $k_{dry}$  are the thermal conductivities of the saturated and dry soil, respectively.

The degree of saturation is obtained from eq. [8]:

$$[8] \quad S_r = w \rho_d / n \rho_w$$

where  $w$  is the water content (fraction),  $\rho_w$  is the density of water,  $\rho_d$  is the dry density of the sample, and  $n$  is the porosity.

Johansen (1975) obtained single relationships between the normalized thermal conductivity and the degree of saturation, which can be viewed as normalized water content. Moreover, he suggested that empirical relationships between  $k_r$  and  $S_r$  must satisfy the following conditions:

$$[9] \quad \begin{aligned} S_r = 0 & : k = k_{dry} \Rightarrow k_r = 0 \\ S_r = 1 & : k = k_{sat} \Rightarrow k_r = 1 \end{aligned}$$

It is noted that eq. [5] does not satisfy the first condition given in eq. [9], since  $\log(0)$  is equal to  $-\infty$ .

The thermal conductivities (W/m°C) in the unfrozen state  $k_u$  and in the frozen state  $k_f$  are then computed when rearranging eq. [7] as

$$[10] \quad k_u = (k_{sat(u)} - k_{dry})k_{ru} + k_{dry}$$

$$[11] \quad k_f = (k_{sat(f)} - k_{dry})k_{rf} + k_{dry}$$

To compute the thermal conductivity of the saturated soil, Johansen (1975) proposed using the well-accepted method of the geometric mean. It is noted that this equation gives satisfying results when the ratio of the thermal conductivity of the solid particles to that of the water (or the ice) is lower than 10:

$$[12] \quad \begin{aligned} k_{sat(u)} &= k_s^{1-n} k_w^n \quad T \geq 0^\circ \text{C} \\ k_{sat(f)} &= k_s^{1-n} k_i^n \quad T < 0^\circ \text{C} \end{aligned}$$

where  $k_s$ ,  $k_w$ , and  $k_i$  are the thermal conductivity of the solid particles (in W/m°C), water (0.6 W/m°C), and ice (2.24 W/m°C), respectively; and  $n$  is the porosity of soil and can be computed using

$$[13] \quad n = 1 - \rho_d / \rho_s$$

where  $\rho_s$  is the density of solid particles (kg/m³).

Johansen (1975) did not consider the effect of volume change from water to ice on the sample's porosity and on the degree of saturation of frozen soils.

The thermal conductivity (in W/m°C) of solid particles is inferred from the quartz content using

$$[14] \quad k_s = \begin{cases} 2.0^{1-q} \times 7.7^q & \text{for } q > 0.2 \\ 3.0^{1-q} \times 7.7^q & \text{for } q \leq 0.2 \end{cases}$$

where  $q$  is the volume fraction of quartz in the solid particles.

Johansen (1975) stressed that the thermal conductivity of dry soil displays high sensitivity to dry density (or porosity) and microstructure variations, whereas the mineralogical influence is rather negligible. He experimentally observed that dry crushed rocks displayed systematically higher thermal conductivities than natural soils; this was also pointed out by Smith (1942). On the basis of his experimental observations, Johansen proposed two empirical relationships to assess the thermal conductivity (in  $\text{W/m}^\circ\text{C}$ ) of (i) dry natural soils and (ii) dry crushed rocks. The relationship for crushed rocks is given by

$$[15] \quad k_{\text{dry}} = 0.039n^{-2.2}$$

The predicted values of thermal conductivity using Johansen's (1975) model for the materials studied in this paper are presented in Figs. 11 and 12 for the unfrozen and frozen states, respectively. These predictions are generally more accurate than those obtained with Kersten's (1949) model because Johansen's model considers the thermal conductivity of solid particles, especially for water contents near saturation. A few negative values at low water contents for unfrozen materials are still obtained in Fig. 11, however. Moreover, predicted thermal conductivities for the nearly dry limestone and dry syenite samples are about four times higher than the measured values. For dolostone and sandstone, the predicted thermal conductivity in the dry state is approximately equal to that for the nearly saturated state. Furthermore, significant differences were observed for the limestone and syenite samples.

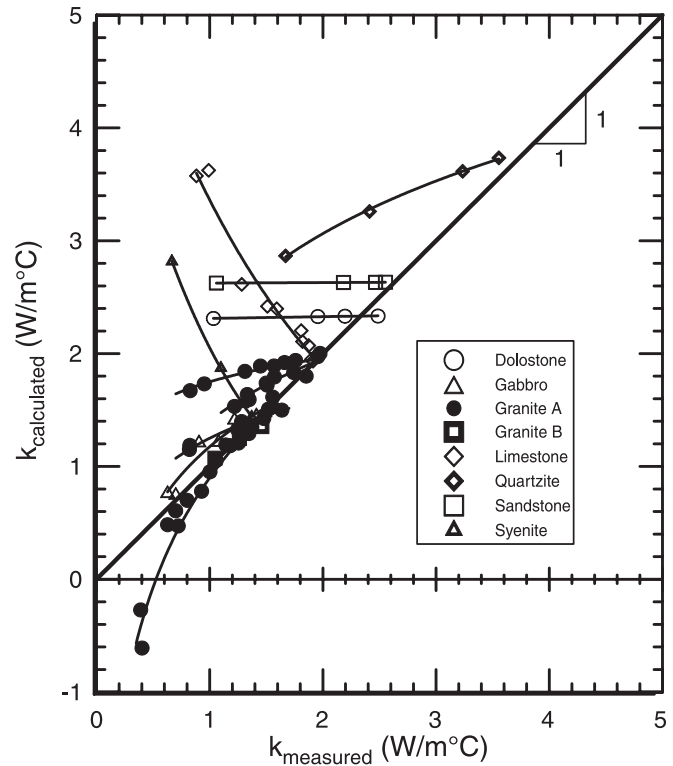
As the  $k_r$ - $S_r$  relationship is linear for the frozen state, no negative or underpredicted values are observed in Fig. 12. The model still leads to overpredicted values for the nearly dry states, however.

### Discussion of the models

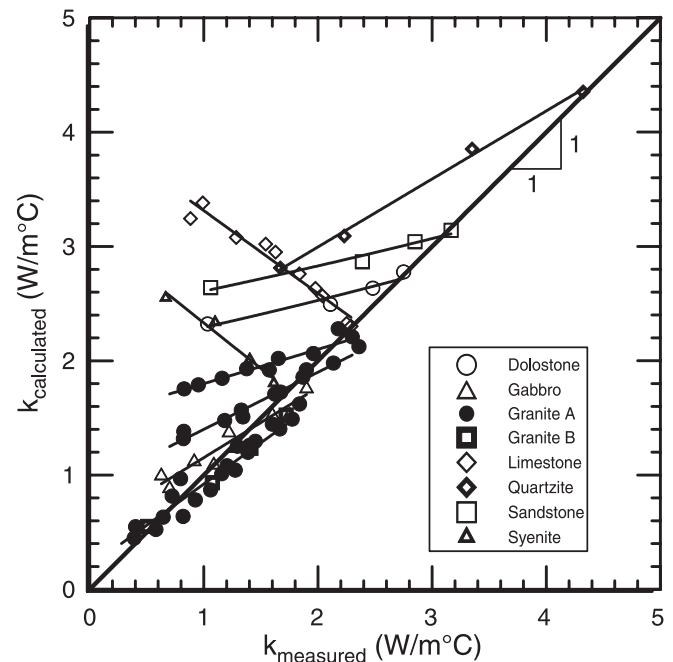
Given that the evaluated models were developed for sandy soils, it is not surprising that they are less suited to the coarser base-course materials studied in this paper. Moreover, the use of logarithmical functions of water content (Kersten 1949) or degree of saturation (Johansen 1975) can lead to predictions of negative thermal conductivities, which are not desirable for automated numerical applications. Furthermore, Kersten's (1949) model does not consider the thermal conductivity of solid particles, which has a significant influence on the thermal conductivity of base-course materials as shown by the results of this study. Johansen (1975) took this parameter partially into account, since he considered solely quartz content in his equations. This simplified method can lead to significant errors for given types of materials like the syenite sample, for which the predicted thermal conductivity of solid particles using eq. [14] would be  $3.0 \text{ W/m}^\circ\text{C}$ , whereas the measured value is only  $1.6 \text{ W/m}^\circ\text{C}$ . It was also noticed that the relationship for the prediction of  $k_{\text{dry}}$  using Johansen's model seems to be inadequate for very dense base-course materials and leads to unrealistic values.

Despite these limitations, the concept of the normalized

**Fig. 11.** Predicted thermal conductivity of unfrozen base-course materials using Johansen's (1975) model versus actual values.



**Fig. 12.** Predicted thermal conductivity of frozen base-course materials using Johansen's (1975) model versus actual values.



thermal conductivity proposed by Johansen (1975) looks appropriate for the development of an improved model to assess the thermal conductivity of base-course materials.

## Development of an improved model for base-course materials

As discussed previously, prediction of the thermal conductivity of base-course materials using the normalized thermal conductivity model (Johansen 1975) leads to less dispersed values than Kersten's (1949) model. Farouki (1981, 1982) evaluated many theoretical and empirical models for the computation of thermal conductivity of soils. In general, the model proposed by Johansen (1975) gave the best results in the widest range of soils and degrees of saturation.

Improvements to the  $k_r$  model are thus proposed to adapt it to dense and broadly graded base-course materials. An existing method for determination of the thermal conductivity of solid particles which considers the complete mineralogy is reviewed. A modified form of the geometric mean method is proposed to assess  $k_{dry}$ , and mathematical equations that relate  $k_r$  to  $S_r$  which fully satisfy the conditions given by eq. [9] are developed for unfrozen and frozen base-course materials.

### Porosity and degree of saturation of frozen soil

The freezing of pore water may induce a volume change of the sample, as pore water expands by about 9% when it turns into ice. This will then lead to a higher porosity in the frozen state than in the unfrozen state.

For saturated soils freezing in a closed system, the volume of voids will increase by 9%, leading to the porosity of saturated frozen soil,  $n_f$ :

$$[16] \quad n_f = 1.09n_u / (1 + 0.09n_u)$$

where  $n_u$  is the porosity of the unfrozen soil as expressed by eq. [13].

As no volume change is expected in dry soil when its temperature goes below 0 °C, its porosity is equal to that of the unfrozen soil ( $n_f = n_u$ ). Considering that the volume of voids in the frozen soil increases linearly from  $n_u$  (dry state) to  $n_f$  (saturated state) as a function of water content, the degree of saturation of frozen soil  $S_{rf}$  can be obtained from the degree of saturation of the unfrozen soil  $S_{ru}$  (eq. [8]) as follows:

$$[17] \quad S_{rf} = 1.09S_{ru} / (1 + 0.09S_{ru})$$

Although it is important to consider volume change in the laboratory, especially when samples are freezing in a closed system, field conditions often lead to open system freezing, and porosity change may be negligible in relatively highly pervious soils such as clean sands and gravels.

### Thermal conductivity of solid particles, $k_s$

The thermal conductivity of solid particles has an important influence on the thermal conductivity of base-course materials. This parameter is used for the assessment of thermal conductivity of saturated soils. Johansen (1975) computes the thermal conductivity of solid particles using eq. [14], in which the thermal conductivities of the minerals other than quartz are equal to 2 W/m°C. When the quartz content is lower than 20%, the thermal conductivity of minerals other than quartz is equal to 3 W/m°C. A comparison of Johansen's simplified method and the generalized method was made with the measurements by Birch and Clark (1940) on various types of rocks. Table 6 shows the mineralogical

compositions and the measured thermal conductivities compared with the computed thermal conductivities using eqs. [2] and [14] in conjunction with values from Horai (1971).

All the computed values using the generalized method (eq. [2]) show differences of less than 10% when compared with the measured values. The differences obtained using Johansen's (1975) method are generally below 15% for rock with a quartz content higher than 20% which is considered acceptable, but for materials with no quartz content the differences increase up to 50% from the measured values.

It is thus proposed to use eq. [2] to compute the thermal conductivity of solid particles using the thermal conductivity of the forming minerals. Mean values of thermal conductivity for the major families of rock-forming minerals are found in Table 4. Complete data for most rock-forming minerals are given in Horai (1971). When the mineralogical composition of rocks is not available, thermal conductivity tests can be performed on solid rock cylinders or saturated crushed rock samples (Birch and Clark 1940; King 1979; Sass et al. 1971). Otherwise, typical thermal conductivity values taken from Table 5 can be used directly in eq. [12] to compute the thermal conductivity of saturated materials.

### Thermal conductivity of dry materials, $k_{dry}$

Thermal conductivity measurements were made on samples dried in an oven at a temperature of 105 °C for 24 h. Since most of the samples still contained small amounts of water ( $0\% < S_r < 2.3\%$ ), thermal conductivity at the dry state was extrapolated from the relationships obtained between  $k_u$  and  $S_{ru}$  and between  $k_f$  and  $S_{rf}$  for each sample as indicated in Fig. 13 by the left-hand arrow. The theoretical value of thermal conductivity at the saturated state (eq. [12]) was also used to improve curve fitting.

Values of  $k_{dry}$  are shown in Fig. 14, where four distinct relationships are obtained: one for the quartzite materials ( $k_s = 5.0$  W/m°C), one for the sandstone and dolostone materials ( $k_s = 3.4$  and  $3.3$  W/m°C), one for the gabbro, limestone, granite A, and granite B materials ( $k_s = 2.2$  to  $2.6$  W/m°C), and one for the syenite materials ( $k_s = 1.6$  W/m°C). The fairly good correlations obtained between  $k_{dry}$  values and porosity confirm the high sensitivity of thermal conductivity of dry materials to variations in porosity observed by Smith (1942) and Johansen (1975). From the four general correlations obtained in Fig. 14, it can also be concluded that mineralogical composition plays an important role in the thermal conductivity of dry crushed rock. For example, at a porosity of 0.15, the thermal conductivity of dry syenite ( $k_s = 1.6$  W/m°C) is equal to 0.64 W/m°C, that of dry sandstone ( $k_s = 3.4$  W/m°C) is equal to 1.1 W/m°C, and that of quartzite ( $k_s = 5.0$  W/m°C) is equal to 1.4 W/m°C. This effect tends to decrease with increasing porosity, however, and values of thermal conductivity seem to converge for a porosity of about 0.4. It is noted that Smith and Johansen did not observe significant variations in the thermal conductivity of dry crushed rocks and fragmented soils with porosities ranging from 0.3 to 0.7.

The relationship proposed by Johansen (1975) for dry crushed rocks is also shown in Fig. 14 (eq. [15], broken line). Even if this function tends to similar values at higher porosities, it is clear that the use of eq. [15] gives values that



Table 6. Measured and computed thermal conductivity of rocks (from Birch and Clark 1940).

| Rock                  | Mineral (%) |                       |                      |                         |                |                        |                   |         |       |          | Thermal conductivity, $k_s$ (W/m°C) <sup>a</sup> |                            |  |
|-----------------------|-------------|-----------------------|----------------------|-------------------------|----------------|------------------------|-------------------|---------|-------|----------|--|----------------------------|--|
|                       | Quartz      | Feldspar (orthoclase) | Plagioclase (albite) | Plagioclase (bytownite) | Mica (biotite) | Amphibole (hornblende) | Pyroxene (augite) | Olivine | Other | Measured | Geometric mean (eq. [2])                         | Johansen (1975) (eq. [14]) |  |
|                       |             |                       |                      |                         |                |                        |                   |         |       |          |  |                            |  |
| Granite (Rockport)    | 28          | 64.0                  |                      |                         |                | 6.0                    |                   |         | 2.0   | 3.39     | 3.30 (2.7)                                       | 2.92 (13.9)                |  |
| Granite (Barre)       | 26          | 25.0                  | 37                   |                         | 9.0            |                        |                   |         | 3.0   | 2.80     | 2.96 (5.7)                                       | 2.84 (1.4)                 |  |
| Granite (Westerley)   | 19          | 33.0                  | 40                   |                         | 6.0            |                        |                   |         | 2.0   | 2.47     | 2.72 (10.1)                                      | 3.59 (43.5)                |  |
| Quartz monzonite      | 34          | 27.0                  |                      | 33                      | 4.8            | 0.2                    |                   |         | 1.0   | 3.04     | 3.05 (0.3)                                       | 3.16 (3.9)                 |  |
| Tonalite              | 28          |                       |                      | 50                      | 15.0           | 7.0                    |                   |         |       | 2.63     | 2.68 (1.9)                                       | 2.92 (11.0)                |  |
| Syenite               |             | 84.3                  |                      |                         | 0.4            |                        | 12.9              | 1.6     |       | 2.38     | 2.50 (5.0)                                       | 3.00 (26.1)                |  |
| Diabase (Maryland)    |             |                       |                      | 48                      | 1.0            |                        | 49.0              | 1.0     |       | 2.32     | 2.47 (6.4)                                       | 3.00 (29.3)                |  |
| Diabase (Vinal Haven) |             |                       |                      | 61                      | 4.0            |                        | 19.0              | 13.0    |       | 2.17     | 2.19 (0.9)                                       | 3.00 (38.2)                |  |
| Diabase (Westfield)   |             |                       |                      | 66                      |                |                        | 32.0              |         | 2.0   | 2.11     | 2.11 (0.0)                                       | 3.00 (42.2)                |  |
| Gabbro (French Creek) |             |                       |                      | 51                      | 0.5            |                        | 47.0              |         | 1.5   | 2.29     | 2.40 (4.8)                                       | 3.00 (31.0)                |  |
| Gabbro (Mellen)       |             |                       |                      | 72                      | 1.2            |                        | 14.1              | 11.0    | 1.3   | 1.99     | 2.02 (1.5)                                       | 3.00 (50.8)                |  |

<sup>a</sup>Percent error is given in parentheses.

Fig. 13. Determination of the thermal conductivity of dried materials from experimental data and theoretical values of the thermal conductivity of saturated materials.

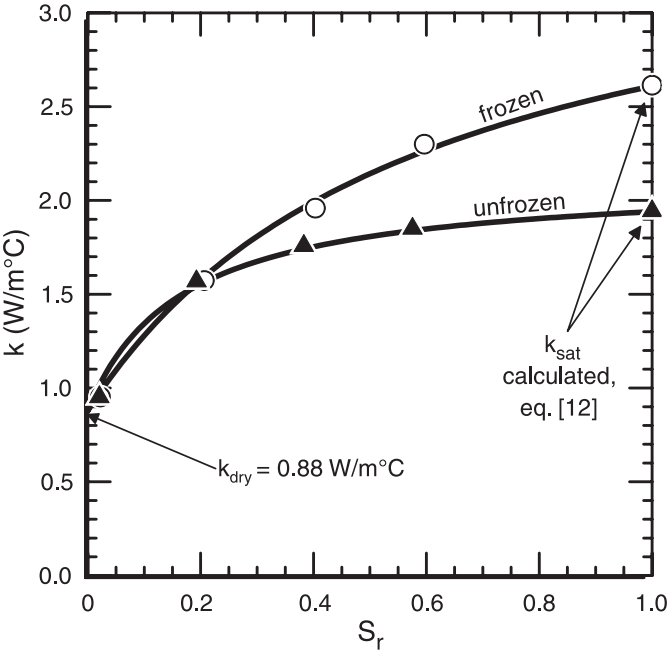
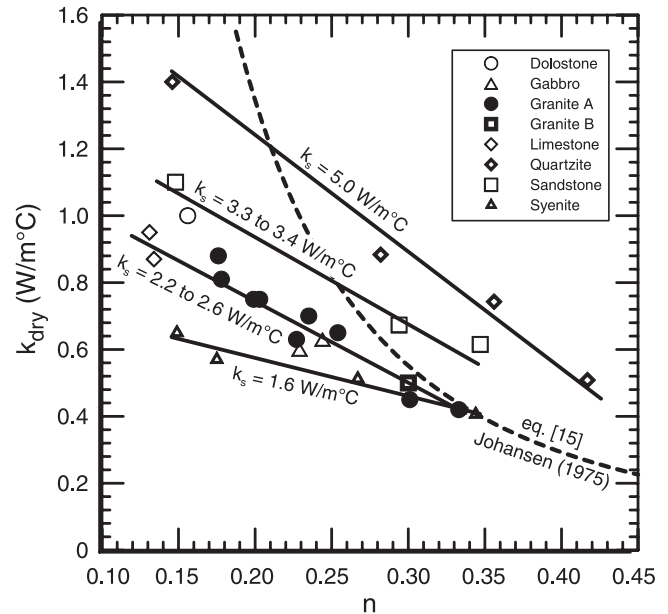


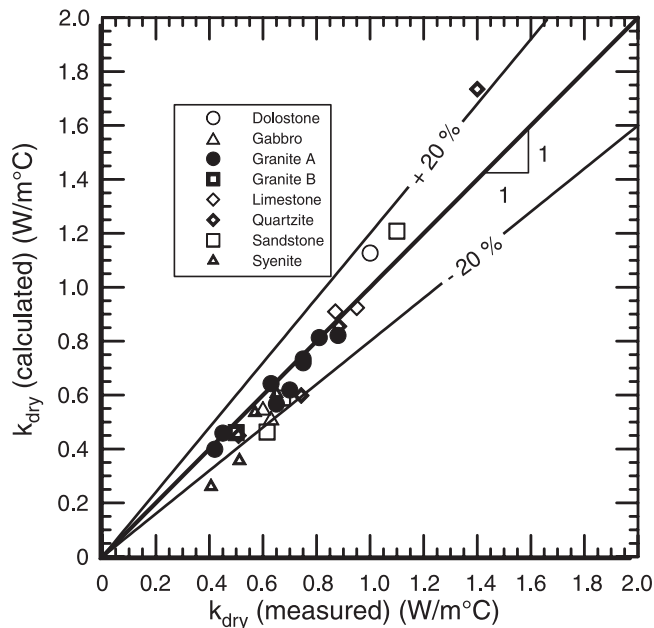
Fig. 14. Influence of porosity on the thermal conductivity of dry base-course materials.



are far overpredicted for materials other than quartzite at porosities between 0.10 and 0.25.

The effect of mineralogy on the thermal conductivity of dry crushed rocks can be taken into account by using the geometric mean method as expressed by eq. [12], where  $k_a$  (thermal conductivity of air) replaces  $k_w$  or  $k_i$ . It is generally accepted that the geometric mean method gives satisfactory results when the ratio of thermal conductivity of solids to that of fluids,  $k_s/k_f$ , is lower than 15. With air being the fluid,  $k_s/k_f$  is between 80 and 200 for the materials studied

**Fig. 15.** Predicted versus measured thermal conductivity of dry materials using eq. [18].



herein, leading to predicted values 15%–66% higher than the values obtained from laboratory results. A modified form of the geometric mean method was thus developed as expressed by eq. [18] (in W/m°C), where fitting parameters  $\alpha$  and  $\beta$  were set to 0.59 and 0.73 to fit experimental data:

$$[18] \quad k_{\text{dry}} = k_s^{(1-n)\alpha} k_a^{n\beta}$$

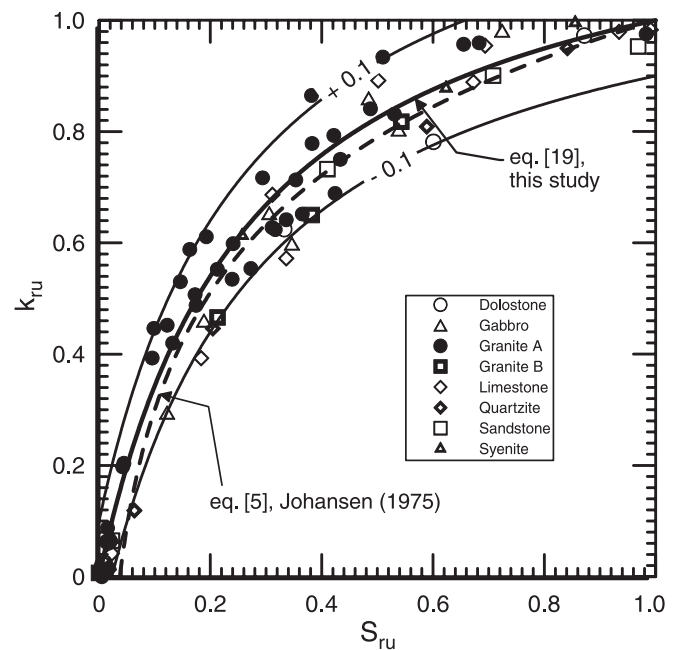
This modified form of the geometric mean method respects the physical limits of thermal conductivity for the full range of porosities ( $n = 0$ :  $k_{\text{dry}} = k_s$ ;  $n = 1$ :  $k_{\text{dry}} = k_a$ ). Calculated values of  $k_{\text{dry}}$  using eq. [18] for the materials of this study are generally within  $\pm 20\%$  of error compared with measured values as shown in Fig. 15.

#### Normalized thermal conductivity, $k_r$

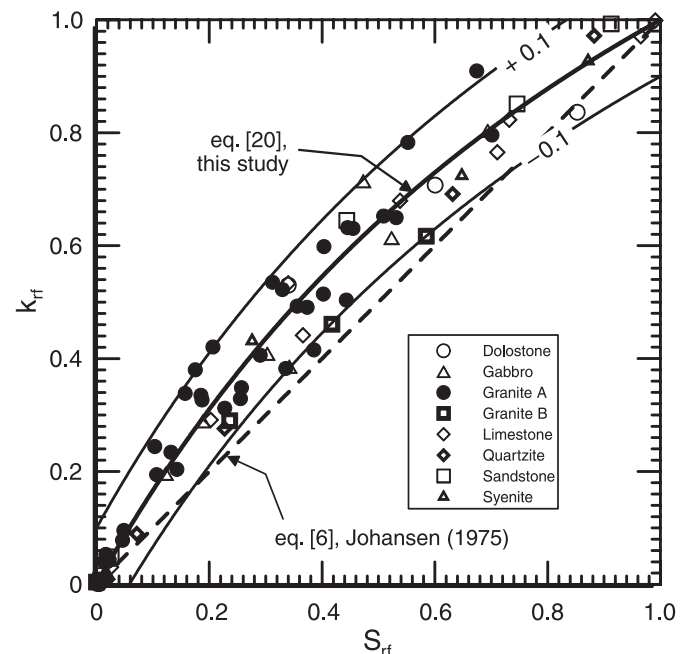
The thermal conductivity measured for each sample was normalized according to eq. [7] using the thermal conductivities in the saturated state derived from eq. [12] in the dry state as extrapolated from the  $k$ – $S_r$  relationships. The results are presented in Fig. 16 for the unfrozen state and in Fig. 17 for the frozen state. Similar to the model of Johansen (1975), the normalized thermal conductivity can be related to the degree of saturation to obtain single relationships for the frozen and unfrozen states, except that the degree of saturation is now defined according to eqs. [8] and [17]. Simple mathematical equations that satisfy the conditions of eq. [9] are proposed to express the normalized thermal conductivity of unfrozen base-course materials (eq. [19]) and frozen materials (eq. [20]) as functions of the degree of saturation. It is noted that these equations fit most of the experimental data within a range of normalized thermal conductivity of  $\pm 0.1$  as outlined by the thinner lines in Figs. 16 and 17:

$$[19] \quad k_{\text{ru}} = 4.7S_{\text{ru}}/(1 + 3.7S_{\text{ru}})$$

**Fig. 16.** Normalized thermal conductivity of unfrozen base-course materials versus the degree of saturation.



**Fig. 17.** Normalized thermal conductivity of frozen base-course materials versus the degree of saturation.



$$[20] \quad k_{\text{rf}} = 1.8S_{\text{rf}}/(1 + 0.8S_{\text{rf}})$$

The relationship for the frozen materials has a less curved shape than that observed for the unfrozen materials. This is mainly due to the fact that, in the frozen state, ice has a higher thermal conductivity, which gives higher saturated thermal conductivities in the frozen state than in the unfrozen state. The relationships obtained by Johansen (1975) for medium and fine sands are also shown using broken lines in Figs. 16 and 17. It appears that the use of Johansen's rela-

**Table 7.** Mineralogical composition and computed thermal conductivity of solid particles for gravels and coarse sands (Kersten 1949).

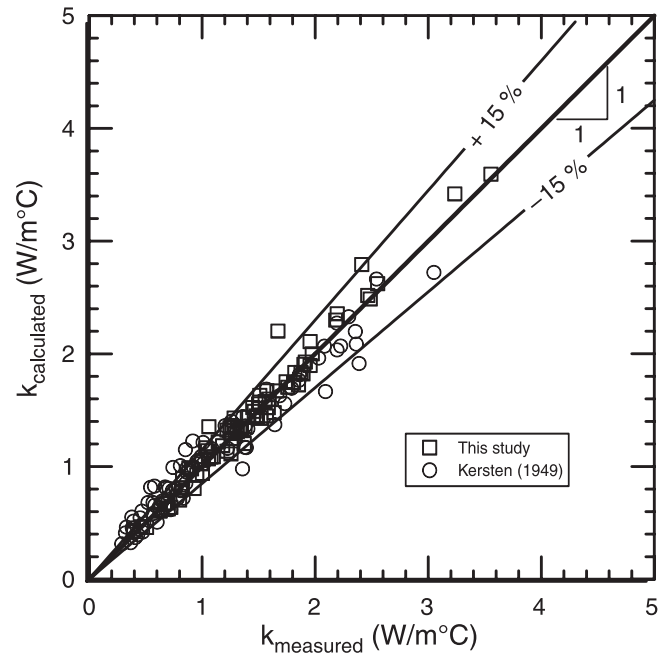
| Material           | Mineral (%) |            |             |                                    |                        |      |       | $k_s$ (computed, eq. [2]; W/m°C) |
|--------------------|-------------|------------|-------------|------------------------------------|------------------------|------|-------|----------------------------------|
|                    | Quartz      | Orthoclase | Plagioclase | Olivine–<br>amphibole–<br>pyroxene | Magnetite–<br>hematite | Mica | Other |                                  |
| Chena River gravel | 43.1        | 11.6       | 12.9        | 27                                 |                        | 2.1  | 3.3   | 4.4                              |
| Fairbanks sand     | 59.4        | 8.6        | 6.3         | 18                                 | 2.5                    | 0.1  | 5.1   | 5.5                              |
| Crushed quartz     | 100.0       |            |             |                                    |                        |      |       | 7.7                              |
| Crushed trapp rock | 3.0         | 10.0       | 50.0        | 34                                 | 2.0                    |      | 1.0   | 2.7                              |
| Crushed feldspar   | 15.0        | 55.0       | 30.0        |                                    |                        |      |       | 2.6                              |
| Crushed granite    | 20.0        | 30.0       | 40.0        |                                    |                        |      | 10.0  | 2.7                              |

tionship for unfrozen base-course materials would slightly underpredict  $k_r$  values and give negative values at the nearly dry state. The linear relationship that Johansen obtained between  $k_r$  and  $S_r$  will also underpredict the  $k_r$  values in the frozen state. The differences between his relationships and those proposed herein can be attributed to a fabric effect, as the materials studied here are of a coarser nature than the medium and fine sand that Johansen used to develop his model for coarse soil.

#### Evaluation of the model

More than 150 experimental data measured by Kersten (1949) were compared with values computed using the proposed model. These data were obtained for gravels (Chena River gravel, 32 test results) and coarse sands (Fairbanks sand, 34 tests results; crushed quartz, 34 test results; crushed feldspar, 24 test results; crushed trap rock, 16 test results; and crushed granite, 18 test results). The proportion of grains over 2 mm in size was equal to 85% for the Chena River gravel and ranged from 15% to 30% for the coarse sands compared with proportions between 20% and 75% for the materials studied herein. The samples were tested in the unfrozen and frozen states at the compaction water content only, resulting in nearly 80 different samples with porosity ranging from 0.21 to 0.45 and degrees of saturation from 0% to 70%. It is noted that the experimental data for the crushed rocks were not used, either by Kersten (1949) or by Johansen (1975), to develop their respective models. No tests were made to determine the thermal conductivity of the solid particles, but complete mineralogical compositions were given by Kersten, making it possible to estimate the  $k_s$  values using eq. [2] and the thermal conductivities of minerals from Horai (1971). Table 7 presents the mineralogical compositions and the computed  $k_s$  values for the gravels and coarse sands.

Comparative results of computed thermal conductivity using the proposed model with the experimental data from this study and from Kersten (1949) are shown in Figs. 18 and 19 for the unfrozen and frozen state, respectively. The parameters  $k_{sat(u)}$ ,  $k_{sat(f)}$ ,  $k_{dry}$  and  $k_{ru}$ , and  $k_{rf}$  were computed using eqs. [12], [15], [19], and [20], respectively. As anticipated, the improved model accurately predicts the thermal conductivity for the materials in this study, with differences from the experimental data of less than 10% for the unfrozen state and less than 15% for the frozen state. The model also provided fairly good predictions for Kersten's coarse sands and gravels, with errors of generally less than 15% for the un-

**Fig. 18.** Predicted thermal conductivity of unfrozen base-course materials using the proposed model versus actual values.

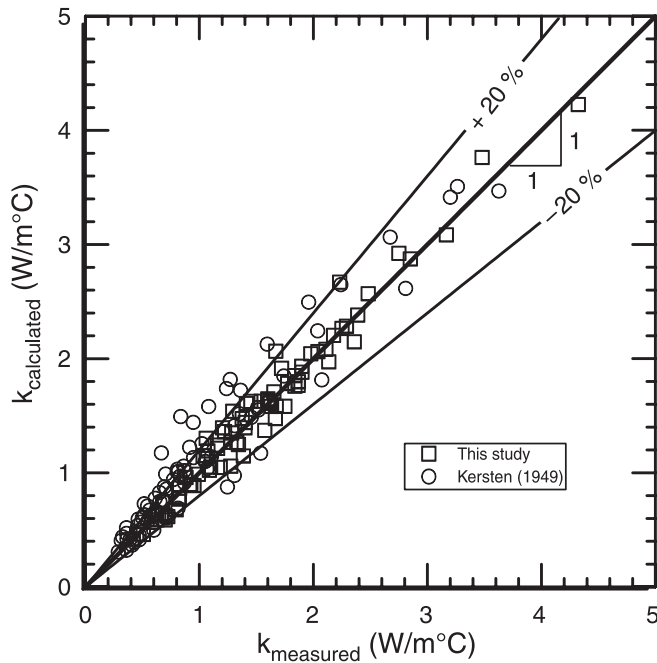
frozen state and generally less than 20% for the frozen state. These larger but acceptable differences from an engineering point of view probably reflect the differences in fabric between Kersten's materials and the materials studied here.

#### Methodology to assess the thermal conductivity of base-course materials

The thermal conductivity of base-course materials can be very simply assessed using the relationships developed in this paper in a four-step method outlined as follows.

- (1) Material characterization — Knowledge is needed of basic soil parameters such as the degree of saturation  $S_r$  and the porosity  $n$ . In the unfrozen state, the porosity  $n_u$  and degree of saturation  $S_{ru}$  can be calculated using eqs. [13] and [8], respectively. To consider the volume change from water to ice occurring during freezing periods, the porosity  $n_f$  and degree of saturation  $S_{rf}$  in the frozen state are obtained from eqs. [16] and [17], respectively. When freezing occurs in an open system, allowing for drainage of unfrozen water, the volume-

**Fig. 19.** Predicted thermal conductivity of frozen base-course materials using the proposed model versus actual values.



change effect can be neglected,  $n_f$  is equal to  $n_u$ , and  $S_{rf}$  is equal to  $S_{ru}$ . The origin of the material must also be known to determine the type of rock (granite, gabbro, limestone, etc.) that composes the base-course material. This information is generally found in the quarries that provide the material. Although not absolutely necessary, a mineralogical analysis of the rocks can be useful to obtain more accurate predictions.

- (2) Thermal conductivity of saturated and dried materials — The thermal conductivity of unfrozen and frozen saturated soils  $k_{sat}$  can be assessed using the geometric mean method given by eq. [12] and the thermal conductivity of the solid particles  $k_s$ . The porosity  $n$  becomes equal to  $n_u$  for the unfrozen state and  $n_f$  for the frozen state. The conductivity of solid particles can be taken from average values for different types of rocks given in Table 5. When complete information on the mineralogical composition of the materials is available, the thermal conductivity of solid particles can be computed with the generalized geometric mean method (eq. [2]) and the mean thermal conductivity values for various families of minerals found in Table 4 or Horai (1971). The thermal conductivity of the dry materials,  $k_{dry}$ , is obtained using eq. [18] as a function of porosity.
- (3) Unfrozen and frozen normalized thermal conductivity — The normalized thermal conductivity allows the characterization of the evolution of thermal conductivity between the limit values obtained from the saturated and dry states as a function of degree of saturation. The unfrozen normalized thermal conductivity value,  $k_{ru}$ , is computed using eq. [19] and that of the frozen state,  $k_{rf}$ , using eq. [20].
- (4) Unfrozen and frozen thermal conductivity — The thermal conductivity  $k$  in the unfrozen and frozen states is directly computed from eqs. [10] and [11], respectively,

**Fig. 20.** Method for assessment of the thermal conductivity of base-course materials.

$$k = (k_{sat} - k_{dry}) \times \frac{k_r}{k_s^{1-n_u} \times k_w^{n_u}} + k_{dry}$$

unfrozen  
 $k_s^{1-n_u} \times k_w^{n_u}$   
 frozen  
 $k_s^{1-n_f} \times k_i^{n_f}$   
 mineralogy known  
 $\prod_{j=1}^N k_{mj}^{x,j}$  — values from Table 4
 

 unfrozen  
 $\frac{4.7S_{ru}}{1+3.7S_{ru}}$   
 frozen  
 $\frac{1.8S_{rf}}{1+0.8S_{rf}}$   
 mineralogy unknown  
 $k_s^{((1-n)^{0.59})} \times k_a^{(n^{0.73})}$   
 typical values from Table 5

using the values of  $k_{sat}$ ,  $k_{dry}$ , and  $k_r$  determined in steps 1–3.

The last three steps of the method are summarized in Fig. 20.

### Working example

The use of the new approach is illustrated with a working example in which the thermal conductivity of the unfrozen and frozen sample presented in Fig. 3 is computed. The sample is a crushed granite with a dry density of 2265 kg/m<sup>3</sup> and water content of 3%. The density of the solid particles is equal to 2750 kg/m<sup>3</sup>, and a typical mineralogical composition is used for the purpose of this example: 20% quartz, 50% plagioclase, and 30% feldspar.

The first step in the methodology is to determine the porosity and degree of saturation in the unfrozen state using eqs. [13] and [8], respectively. As the sample was tested in a closed system, the volume-change effect is taken into account for frozen conditions using eqs. [16] and [17], respectively: (i) for porosity,

$$n_u = n = 1 - 2265/2750 = 0.176$$

$$n_f = (1.09 \times 0.176)/(1 + 0.09 \times 0.176) = 0.189$$

and (ii) for degree of saturation,

$$S_{ru} = S_r = (0.03 \times 2265)/(0.176 \times 1000) = 0.39$$

$$S_{rf} = (1.09 \times 0.39)/(1 + 0.09 \times 0.39) = 0.41$$

In the second step, the thermal conductivity of the solid particles, the thermal conductivities of unfrozen and frozen saturated materials, and the thermal conductivity of the dry materials are determined using eqs. [2], [12], and [18], respectively: (i) for solid particles,

$$k_s = 1.84^{0.50} \times 2.25^{0.30} \times 7.69^{0.20} = 2.60 \text{ W/m}^\circ\text{C}$$

(ii) for saturated unfrozen material,

$$k_{sat(u)} = 2.60^{1-0.176} \times 0.6^{0.176} = 2.01 \text{ W/m}^\circ\text{C}$$

(iii) for saturated frozen material,

$$k_{sat(f)} = 2.60^{1-0.189} \times 2.24^{0.189} = 2.53 \text{ W/m}^\circ\text{C}$$



and (iv) for dry material,

$$k_{\text{dry}} = 2.60^{(1-0.176)^{0.59}} \times 0.024^{(0.176)^{0.73}} = 0.82 \text{ W/m}^\circ\text{C}$$

The third step gives the normalized thermal conductivity for unfrozen and frozen material using eqs. [19] and [20], respectively: (i) for unfrozen material,

$$k_{\text{ru}} = (4.7 \times 0.39)/(1 + 3.7 \times 0.39) = 0.75$$

and (ii) for frozen material,

$$k_{\text{rf}} = (2 \times 0.41)/(1 + 0.41) = 0.56$$

In the fourth and last step, the thermal conductivity (in  $\text{W/m}^\circ\text{C}$ ) of the moist sample in the unfrozen and the frozen states is given by eqs. [10] and [11], respectively: (i) for unfrozen material,

$$k_u = (2.01 - 0.84) \times 0.75 + 0.82 = 1.70$$

and (ii) for frozen material,

$$k_f = (2.53 - 0.84) \times 0.56 + 0.82 = 1.77$$

These computed values compare well with the measured values for the tests shown in Fig. 3, where  $k_u$  and  $k_f$  are 1.75 and 1.87  $\text{W/m}^\circ\text{C}$ , respectively.

## Conclusion

A comprehensive laboratory study of the thermal conductivity of dense and broadly graded base-course materials was conducted on materials selected from eight quarries along the axis of the St. Lawrence River. Nearly 200 tests on unfrozen and frozen compacted base-course samples and solid rock cylinders were performed in a thermal conductivity cell using Pyrex heat flux meters. As anticipated, the experimental results indicated that the thermal conductivity is strongly influenced by water content, porosity of the samples, and mineralogy of the solid particles of the materials. These results compared well with the limited data available from the literature.

Empirical models from Kersten (1949) and Johansen (1975) were evaluated to determine their ability to predict thermal conductivity of base-course materials. Unsatisfactory results were obtained with both models for most of the unfrozen and frozen samples studied. This was attributed to differences in the fabric of the soils used to calibrate the models and the fabric of the materials studied in this paper. The normalized thermal conductivity concept used by Johansen (1975), however, was found appropriate to develop a new model adapted to the prediction of the thermal conductivity of base-course materials.

The model developed in this study relies on (i) two relationships to compute the porosity and the degree of saturation that consider the effect of volume change as water turns to ice in frozen soils; (ii) the geometric mean model for the computation of the thermal conductivity of solid particles and for the saturated base-course materials in the unfrozen and frozen states; (iii) a modified form of the geometric mean model for the computation of the thermal conductivity of dry base-course materials; and (iv) two simple relationships between the normalized thermal conductivity and the

degree of saturation for base-course materials in the unfrozen and frozen states.

The new model was put to the test with more than 150 data available from Kersten (1949) for gravels and coarse sands. The predicted values generally agreed with the measured values within errors less than 15% for unfrozen materials and less than 20% for frozen samples. Lastly, a step by step method with a working example was proposed to assess the thermal conductivity of base-course materials.

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## List of symbols

$h$  height (m)  
 $h_{lf}$  height of lower heat flux meter (m)

$h_{uf}$  height of upper heat flux meter (m)  
 $k$  thermal conductivity (W/m°C)  
 $k_a$  thermal conductivity of air (W/m°C)  
 $k_{calculated}$  calculated thermal conductivity (W/m°C)  
 $k_{dry}$  thermal conductivity of dry soil (W/m°C)  
 $k_f$  thermal conductivity of frozen soil (W/m°C)  
 $k_{fl}$  thermal conductivity of fluid (W/m°C)  
 $k_i$  thermal conductivity of ice (W/m°C)  
 $k_m$  thermal conductivity of mineral (W/m°C)  
 $k_{measured}$  measured thermal conductivity (W/m°C)  
 $k_r$  normalized thermal conductivity  
 $k_{rf}$  normalized thermal conductivity of frozen soil  
 $k_{ru}$  normalized thermal conductivity of unfrozen soil  
 $k_s$  thermal conductivity of solid particles (W/m°C)  
 $k_{sat}$  thermal conductivity of saturated soil (W/m°C)  
 $k_u$  thermal conductivity of unfrozen soil (W/m°C)  
 $k_w$  thermal conductivity of water (W/m°C)  
 $n$  porosity  
 $n_f$  porosity of frozen soil  
 $n_u$  porosity of unfrozen soil  
 $q$  heat flux (W/m<sup>2</sup>) or volume fraction of quartz  
 $q_{lf}$  heat flux in lower heat flux meter (W/m<sup>2</sup>)  
 $q_{uf}$  heat flux in upper heat flux meter (W/m<sup>2</sup>)  
 $S_r$  degree of saturation  
 $S_{rf}$  degree of saturation of frozen soil  
 $S_{ru}$  degree of saturation of unfrozen soil  
 $T$  temperature (°C)  
 $T_{lf}$  temperature in lower heat flux meter (°C)  
 $T_{uf}$  temperature in upper heat flux meter (°C)  
 $w$  water content (% by weight)  
 $x$  volume fraction of mineral  
 $\alpha$  fitting parameter  
 $\beta$  fitting parameter  
 $\rho_d$  dry density (kg/m<sup>3</sup> or g/cm<sup>3</sup>)  
 $\rho_s$  density of solid particles (kg/m<sup>3</sup>)  
 $\rho_w$  density of water (kg/m<sup>3</sup>)