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[ F I F T H S E R I E S . ]

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ART. VI.—*The Thermal Conductivity and Compressibility of several Rocks under High Pressures*; by  
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## INTRODUCTION.

The effect of pressure on the thermal conductivity of rocks has apparently never been determined, although a knowledge of it is important in calculating the thermal equilibrium of the earth's crust. The only measurements we have that can give any idea of the possible influence of hydrostatic pressure are on the effect of one-sided compression on the thermal conductivity of granite and marble.<sup>1</sup> It is of course well known that a one-sided compression need not be at all like a hydrostatic pressure in its effects, and the only conclusion that we can draw from the experiments of Lees as applied to conditions within the crust of the earth is that the effect of pressure is probably not very large.

In connection with my work on the effect of high pressures on general physical properties of various substances, I have recently been engaged in measuring the thermal conductivity of a number of liquids up to 12000 kg/cm<sup>2</sup>. Since the apparatus could be adapted without much trouble to measure the effect of pressure on the thermal conductivity of those solids whose conductivity is not too high, such as the rocks, I thought it of interest to make a few measurements from which at least the general trend of the effects to be expected could be inferred. I have been encouraged in this attempt by the interest and advice of Professor R. A. Daly, who also was so kind as to obtain several of the materials for me. I also have to thank my assistant, Mr. H. B. Curry, for making many of the readings.

<sup>1</sup> For references see the end of this article.

In addition to the thermal conductivity I have determined the compressibility of the materials used. It was comparatively easy to do this as the apparatus had already been developed for another purpose. When I began these measurements the important paper of Adams and Williamson<sup>2</sup> on the compressibility of rocks had not appeared, and such questions as that of the change with pressure of the compressibility of a rock had not received an answer. Since the appearance of that paper, my compressibility results lose some of their novel interest, but there are still a number of points in connection with them which I think may suggest new lines of thought. In the first place, I have measured the compressibility of several of my materials at two temperatures, neither very high it is true, 30° and 75°, but nevertheless sufficiently far apart to give an idea of the variation of compressibility with temperature. This of course is important when applying results obtained in the laboratory at room temperatures to actual conditions. The necessary information can not be obtained from the results of Adams and Williamson because their compressibilities were measured at only one temperature. In the second place, my method, which measures the linear as opposed to the cubical compressibility, brings out the importance of non-isotropy, as the measurements of Adams and Williamson on cubical compressibility cannot do. I have found surprisingly large differences of compressibility in different directions, and also entirely unexpected and abnormal hysteresis relations. Both these factors are of importance in calculating the velocity of propagation of earthquake waves or other deformational effects in the earth's crust.

#### *Thermal Conductivity.*

*The Method.*—The method used for measuring thermal conductivity is essentially the same as that used for liquids, and is described in detail elsewhere;<sup>3</sup> a brief summary here will suffice. The material whose conductivity is to be measured is worked to the form of a cylindrical shell, which is slipped between an outer and an inner cylinder of copper, each of which it fits accurately. An insulated wire stretched along the axis of the inner copper cylinder carries an electric current, so that the

axis is a source of heat. This heat flows out radially through the inner copper cylinder, through the shell of rock, through the outer copper cylinder, and into the surrounding temperature bath through the walls of the pressure cylinder into which the outer copper cylinder closely fits. The difference of temperature between the inner and outer copper cylinders is measured with thermo-couples. From a knowledge of the heat input, the temperature difference and the geometrical configuration, the thermal conductivity may be computed.

The interior of the pressure cylinder, in which the thermal conductivity apparatus is contained, is entirely filled with an electrically insulating liquid (petroleum ether for 30° and kerosene for 75°) in which the conductivity cylinders are completely bathed. Pressure may be applied to this liquid by means of the intensifier which I have often described.<sup>4</sup> The pressure is transmitted by the liquid equally to all parts of the conductivity apparatus. The thermal conductivity may now be determined as a function of the pressure in the liquid.

The most important correction to be applied to the readings directly obtained is that due to the lack of perfect fit between the cylinder of rock and the inner and outer copper cylinders. It was of course not possible to make the fit perfect, so that in addition to the rock in which we are interested there was always a film of liquid between the two copper cylinders. What was actually measured was the resultant conductivity of layers of rock and liquid in series. Since the thermal conductivity of the liquid is much less than that of the rock, it is necessary to know the geometrical dimensions accurately. The cylinders were formed by machining or grinding, and were of a gratifying geometrical regularity. The outside diameter was uniform within a few ten-thousandths of an inch, as was also the length. The outside measurements could be made with sufficient accuracy with ordinary micrometers, and mean dimensions obtained from a large number of readings. The inside diameter could not be measured directly. This was obtained from the total weight of the specimen, its density (which was determined by weighing in water in the regular way), and the outside dimensions. The copper cylinders could be measured directly with sufficient

accuracy. From the dimensions of the solid parts, the dimensions of the liquid layer could be determined. The dimensions of the rock cylinders were approximately 0.5000 inches (1.27 cm.) outside diameter and 0.4000 (1.02 cm.) inside diameter and 1 inch (2.5 cm.) long.

FIG. 1.

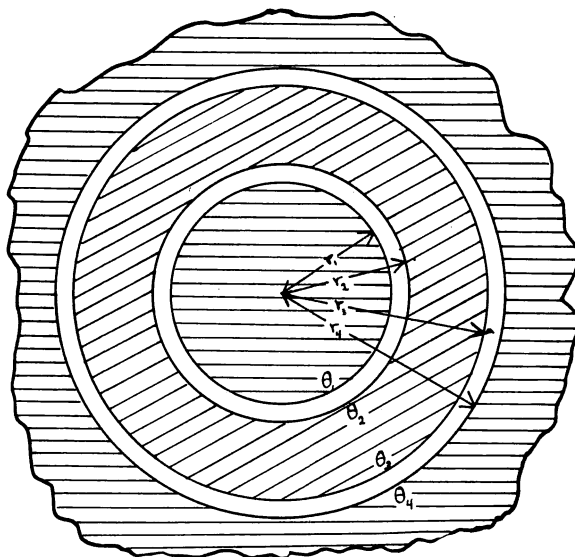


FIG. 1.—Cross section of the thermal conductivity measuring apparatus. Horizontal shading represents copper, diagonal shading the rock under measurement, and the unshaded part the pressure transmitting liquid.

In fig. 1 is shown a cross section of the thermal conductivity apparatus, in which the cylinder of rock is shown shaded diagonally between the outer and inner cylinders of copper (horizontally shaded) separated from them by the unshaded layers of liquid. If we call  $Q$  the radial heat flow per unit length along the axis, then we have the following formulas for the temperature differences across the inner film of liquid, the cylinder of rock, and the outer film of liquid:

$$\theta_1 - \theta_2 = \frac{Q}{2\pi} \cdot \frac{1}{K_1} \log \frac{r_2}{r_1},$$

$$\theta_2 - \theta_3 = \frac{Q}{2\pi} \cdot \frac{1}{K_s} \log \frac{r_3}{r_2},$$

$$\theta_3 - \theta_4 = \frac{Q}{2\pi} \cdot \frac{1}{K_l} \log \frac{r_4}{r_3},$$

Here  $K_l$  is the thermal conductivity of the liquid,  $K_s$  that of the solid, both measured in ordinary C. G. S. units, and the  $\theta$ 's are the respective temperatures. Adding these expressions gives

$$\theta_1 - \theta_4 = \frac{Q}{2\pi} \left[ \frac{1}{K_l} \log \frac{r_2 r_4}{r_1 r_3} + \frac{1}{K_s} \log \frac{r_3}{r_2} \right]$$

for the temperature difference between the outer and inner copper cylinders. This difference,  $\theta_1 - \theta_4$ , is the quantity measured experimentally.

It is to be noticed that under hydrostatic pressure the geometrical factors which enter this expression remain unaltered, since the ratio of the radii of the copper cylinders ( $r_4/r_1$ ) and the cylinder of rock ( $r_2/r_3$ ) are unaffected by pressure. There is, however, a geometrical correction due to the change with pressure of the length of the axis of the cylinder, but this correction is so small that it was neglected in the present work.

The formula gives at once the means of calculating the conductivity of the rock if that of the surrounding liquid is known. This was determined as a function of pressure by the experiments on liquids, and the numerical values may be found in the paper describing those experiments. The correction for the liquid was of the order of one-third or one-quarter of the total effect at atmospheric pressure. Under the maximum pressure the conductivity of the liquid increases by a factor of 2.7, so that the correction is correspondingly reduced at the high pressures. The results are, therefore, better at the high than at the low pressures.

*Choice of Materials.*—The chief restriction on the choice of materials was the necessity of being able to work them into accurate cylindrical shells. Pipestone and talc at once suggested themselves, as they can be readily machined in the lathe with high precision, and they were prepared in this manner. The other materials

I chose after conference with Professor Daly, the object being to obtain a few materials which could be worked with some degree of ease, and which at the same time, in conjunction with the talc and pipestone already selected, should fairly represent the range of material to be encountered in the earth's crust. It was at his suggestion that Solenhofen limestone and a diabasic basalt were selected. Both of these were too hard to machine, and had to be worked to shape by grinding. I am indebted to Mr. David Mann for devising a method of doing this, and to my mechanic Mr. T. R. Richardson for skillfully preparing the specimens. In addition to these, pyrex glass was also measured at the suggestion of Professor Daly, who recognized the importance of an example of a glassy non-crystalline material. This was not prepared by grinding, but was cut into the proper length from a piece of commercial tubing of such size as to fit closely the copper cylinders both inside and outside. This specimen was selected personally by Professor Daly after detailed examination of an extensive commercial stock. Even with this care, the glass was not as perfect geometrically as the ground specimens, but showed eccentricities of 1.5%. Mean dimensions of the glass were used in the calculations.

In addition to the rocks I have measured the thermal conductivity of rock salt under pressure, chiefly for its physical interest, as we are now becoming able to explain a good many of the physical properties of simple crystals, and of rock salt in particular, in terms of the fine structure, and the thermal conductivity of no simple crystal has been measured under pressure.

*Analysis of the Materials.*—The pipestone was obtained from the Ward Natural History Establishment at Rochester, N. Y., and is the material described by Dana as "catlinite." It is an iron-rich clay stone; a typical analysis gives:  $\text{SiO}_2$  57.4,  $\text{Al}_2\text{O}_3$  25.9,  $\text{Fe}_2\text{O}_3$  8.7,  $\text{H}_2\text{O}$  7.4,  $\text{MgO}$  and  $\text{CaO}$  trace. It is without gritty inclusions of any kind, and may be turned in the lathe without damage to the keen edge of a steel tool. The material was not perfectly homogeneous in appearance, but the color varied from a deep red to a much paler and yellowish hue. The thermal conductivity sample was a deep red throughout. Its density at  $20^\circ$  was 2.840 (corrected for vacuum, as are all the densities given in the following).

The talc was from the Hewitt mine, Hewitt, N. C. and was very soft without any gritty inclusions. The analysis is given as follows in Economic Paper No. 3, 1900, of the N. C. Geological Survey: Water 5.10; Silica ( $\text{SiO}_2$ ) 61.35;  $\text{Al}_2\text{O}_3$  4.42;  $\text{FeO}$  1.68;  $\text{CaO}$  0.82;  $\text{MgO}$  26.03;  $\text{Na}_2\text{O}$  0.62; and  $\text{K}_2\text{O}$  trace.

The material is not perfectly homogeneous, but there is a certain amount of schistosity evident, the material having apparently been laid down in layers. The density at  $20^\circ$  is 2.751.

The Solenhofen limestone is very nearly pure  $\text{CaCO}_3$ . It is of a light brownish color, and was deposited in layers from one-half to three-quarters of an inch thick. The thermal conductivity and the compressibility samples were cut from the same layer. The density at  $20^\circ$  is 2.602.

The diabasic basalt is described as follows by Professor E. S. Larsen of the University Museum; I am much indebted to him for procuring the analysis.

"The sample is made up in large part of labradorite laths and augite grains, the latter moulding about and between the former. There is about 10% of interstitial brown matrix that was originally mostly glass but is now partly devitrified and carries microlites of augite and is clouded with magnetite grains. Olivine in grains now altered to a serpentine-like mineral makes up nearly 10% of the rock. The feldspars are partly altered to a mineral that resembles sericite.

The plagioclase laths average about 0.1 mm. in length, the olivine grains about 0.1 mm. in diameter and the augite grains a little larger. The serpentine and sericite are hydrous minerals and might lose some of their water even below  $100^\circ \text{C.}$ "

The density at room temperature was 2.924.

The pyrex glass was approximately:  $\text{SiO}_2$  81.35,  $\text{B}_2\text{O}_3$  11.50,  $\text{Na}_2\text{O}$  4.00;  $\text{Al}_2\text{O}_3$  2.1,  $\text{CaO}$  0.2,  $\text{MgO}$  0.28. The density at room temperature was 2.234.

The rock salt was a natural crystal, perfectly clear and flawless. It was machined to shape in the lathe with cutting tools, and I owe it to the skill of Mr. David Mann.

*Results.*—The regularity of the readings was not as great as that possible to obtain with the liquids. The reason is that the apparatus is best adapted to materials with low conductivity, and the conductivity of liquids is lower than that of those solids measured here. Nevertheless a degree of accuracy was obtained amply sufficient for the purpose in hand, and doubtless the accuracy of

these measurements on a single specimen is greater than the accuracy with which different specimens may be reproduced. A typical set of readings is shown in fig. 2, in which are plotted against pressure the uncorrected and directly observed values of the resistance in the electrical measuring circuit. This resistance is inversely

FIG. 2.

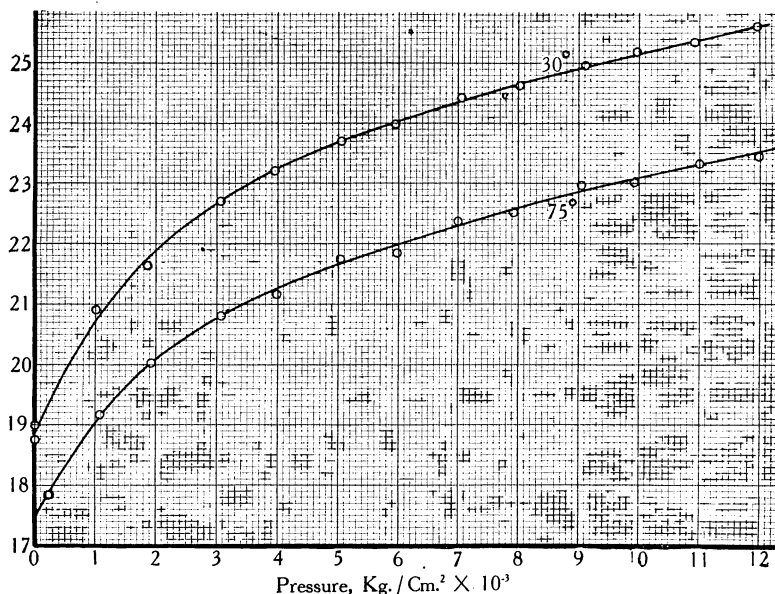


FIG. 2.—Observed values of relative conductivity as ordinates, against pressure as abscissæ. The curves show the effect of pressure on combined conductivity of the rock and the concentric shells of liquid shown in Figure 1. The effect of pressure on the liquid alone is independently measured, and hence the effect on the rock alone is calculated.

proportional to the temperature difference between the inner and outer copper cylinders. Since the heat input is kept constant, the plotted points are proportional to the resultant conductivity of the film of liquid and the shell of rock (in this case basalt) in series. It will be noticed that the observed points lie on a line of considerable curvature, the curvature being much greater at the lower pressures. This curvature is due to the rapid change with pressure of the thermal conductivity of the



transmitting liquid; when correction is applied for it we are left with a conductivity for the basalt alone which increases linearly with the pressure.

The method of computation was to pass a smooth curve through the observed points, read off from this curve the points at intervals of 2000 kg. (thus giving seven points covering a pressure range of 12000 kg/cm<sup>2</sup>), apply to these points the various corrections (chiefly for the effect of the transmitting liquid), and then finally to smooth these computed results. In this final smoothing the only point which usually needed any further adjustment was the point at atmospheric pressure, which, as has already been explained, was the most difficult to obtain accurately.

In all cases the thermal conductivity is found to increase with increasing pressure, and for all of the materials except pipestone the increase with pressure is linear. The results are contained in Table I. In this

TABLE I  
Summary of Thermal Conductivity Measurements.

Substance	Thermal Conductivity as Function of Pressure	Per Centage Change of Conductivity for 1000 kg/cm <sup>2</sup>
Pyrex Glass.....	$0.00261 + 0.00001 \left( \frac{p}{1000} \right)$ , 30° and 75°	+ 0.38
Basalt.....	$\left\{ \begin{array}{l} 0.00404 + 0.000019 \left( \frac{p}{1000} \right), \text{ 30°} \\ 0.00414 + 0.0000089 \left( \frac{p}{1000} \right), \text{ 75°} \end{array} \right.$	 + 0.47 + 0.22
Solenhofen ..... Limestone.....	$\left\{ \begin{array}{l} 0.00523 + 0.000005 \left( \frac{p}{1000} \right), \text{ 30°} \\ 0.00451 + 0.000030 \left( \frac{p}{1000} \right), \text{ 75°} \end{array} \right.$	 + 0.1 + 0.67
Talc.....	$0.00733 + 0.000115 \left( \frac{p}{1000} \right)$ , 30°	+ 1.57
NaCl.....	$\left\{ \begin{array}{l} 0.00880 + 0.000317 \left( \frac{p}{1000} \right), \text{ 30°} \\ 0.00756 + 0.00027 \left( \frac{p}{1000} \right), \text{ 75°} \end{array} \right.$	 + 3.6 + 3.6
Pipestone	<div>Pressure Conductivity</div> <div> 0      0.00438  2000    .00506  4000    .00544  6000    .00563  8000    .00574  10000   .00587  12000   .00596 </div>	 30°       Average + 3.0

is given the absolute conductivity in gm. cal. per sec. per  $\text{cm}^2$  (per degree Centigrade per cm.) as a function of pressure (measured in  $\text{kg}/\text{cm}^2$ ) at  $30^\circ$  and  $75^\circ$ , and also the percentage change of conductivity at each of these temperatures for a pressure change of 1000 kg. (corresponding to a depth in the earth's crust of about 2.5 miles). The results for pipestone, which do not vary linearly with pressure, are given as a function of pressure at 2000 kg. intervals.

For the harder rocks, basalt and Solenhofen limestone, which presumably correspond more nearly to the average to be found in the crust, the change per 1000 kg. is 0.5% or less. It is greater for the softer substances, talc and pipestone, being 1.57 and 3.0% respectively, and is largest for NaCl, for which it is 3.6%. (It should be remarked with regard to NaCl that my value of the absolute conductivity is of the order of one-half that which Eucken<sup>5</sup> has recently found. The absolute conductivity of this is the highest of any of the solids measured here, and furthermore this apparatus is least well adapted to measuring it.)

In general, the change of conductivity with pressure does not appear to be so large that it will introduce any important change into previous geological speculations which involve only the conductivity near the surface of the crust.

The effect of temperature on thermal conductivity, unlike that of pressure, does not seem to be always of the same sign. The conductivity of limestone and NaCl decrease with rising temperature by fairly large amounts, but that of basalt increases slightly. A positive temperature coefficient of conductivity for basalt was also found by Poole,<sup>6</sup> as opposed to a numerically much larger negative coefficient for granite. The numerical value of Poole's coefficient agrees as nearly with mine as can be expected for material from different sources. For ordinary liquids the thermal conductivity usually decreases with rising temperature, and this may perhaps be taken as the normal effect in rocks. But there is not any great certainty here, and the combined effect of temperature and pressure at depths of several hundred miles, where the pressure is of the order of a hundred thousand atmospheres, may be such that we must be prepared to

entertain the possibility of the conductivity being several fold greater than the accepted value.

Pyrex glass does not depart from the average crystalline rock in the effect of pressure on conductivity, although the absolute conductivity is lower, as is usually the case for a glass.

*Compressibility.*

The normal behavior of compressibility is as follows. When a homogeneous isotropic solid is subjected to hydrostatic pressure it experiences a uniform change of dimensions in all directions, so that the geometrical shape of any part of the solid is unaltered; this change of dimensions is a function of the pressure and temperature only and does not depend in any way on the past pressure history of the specimen. In other words, there are no hysteresis, elastic-after-effects, or allied phenomena, but the strain is a single valued function of the pressure. The percentage change of volume for a given increment of pressure in all cases hitherto examined becomes less at the higher pressures (that is, the compressibility decreases at high pressures), and in nearly all cases, although not universally, the compressibility is greater at higher temperatures.

For the materials of this paper, this simple type of behavior is not always followed, but there are significant departures. To exhibit these, it will be best to describe the results in detail.

*Pipestone.*—The first sample of this which I measured behaved in a way which was in one respect the most significant and striking of any of this series. The experimental results are plotted in fig. 3, which shows the observed fractional changes of length against the corresponding pressures. In the first place the change of length is not a single valued function of the pressure but there is a very marked loop, in shape like the well known hysteresis loops of magnetism or the stress-strain loops of a substance stressed beyond the elastic limit. The greatest width of this loop was 18.2% of the maximum change of length. This loop is not a temporary phenomenon obtained only on the first application of pressure, such as I have sometimes found, and which may be explained by flaws or crevices which are squeezed tight

by a high pressure, but it is a legitimate and permanent effect, which may be repeated, as this specimen had been subjected to a previous pressure seasoning. The two points in fig. 3 which appear to be off the curve, at 1000 and 10000 kg., were obtained by a slightly different procedure than the others, the 1000 point after a release of

FIG. 3.

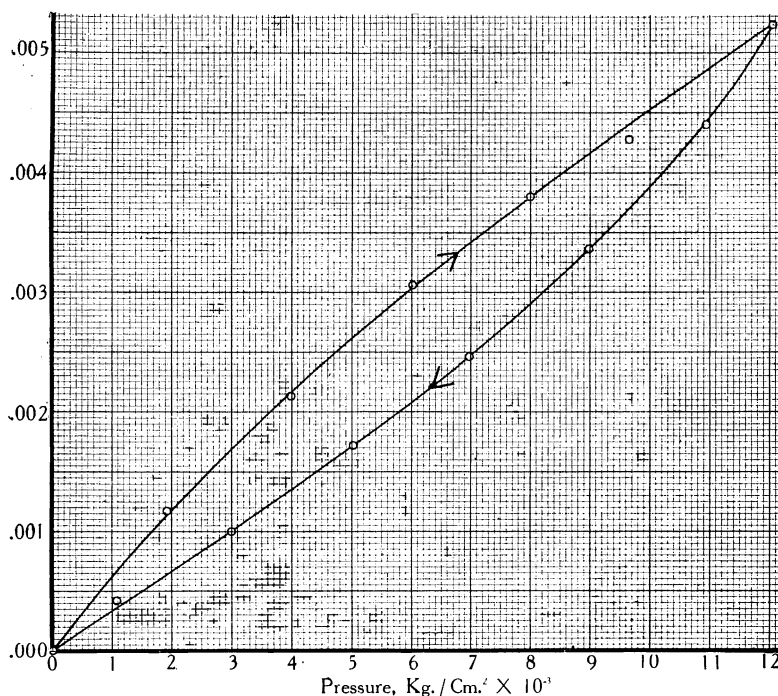


FIG. 3.—The compressibility cycle of pipestone, fractional change of length against pressure. The direction in which the hysteresis loop is described is abnormal.

pressure beyond 1000 and subsequent increase back to 1000, and the 10000 point after an increase of pressure above 10000 and subsequent release back to 10000. Experience with other lag effects would lead one to expect these points to lie off the curve in exactly the way found, and the fact that they were so found is confirmation of the legitimacy of the effect.

Apart from the existence of the loop, the second surprising feature is the direction in which it is described. "Hysteresis" means a lagging behind, and this suggests in general the physical reason for the effect. Normal hysteresis would demand that when pressure is released, coming from a higher value, for example, the change of volume does not assume the value appropriate to that pressure, but lags behind, assuming a value to be found at some higher pressure on the ascending branch of curve. The physical reason is usually sought in internal friction or in some inherent small-scale instability except when in one of two extreme configurations, as is best illustrated in Ewing's theory of ferro-magnetism.

The phenomena found here are the exact reverse of normal. On the descending branch of the curve the change of volume is that to be found at a lower instead of at a higher pressure on the ascending branch. In other words, instead of this being a lag phenomenon, it is a phenomenon of advance, and the loop is a reverse loop. I have sometimes found effects like this, but only in complicated systems in which the explanation can be found in the interaction of a number of unlike phenomena, and never in such a marked degree as here. It is probable, I think, that there must be a complicated state of internal strains to account for such a state of affairs. An artificial mechanism may be imagined, which may have some relation to the actual mechanism, which will give effects of this kind. Suppose the solid contains a number of lenticular cavities provided with a valve mechanism such that liquid can get from the outside into the interior only when the pressure outside is in excess of that inside by a definite amount, and similarly liquid can pass from the inside out only when the pressure inside rises a definite amount above that outside. When fluid pressure is applied to such a cavity, the first effect is to squeeze it flat, but when pressure rises high enough, liquid penetrates to the inside and the cavity swells, tending to resume its natural lenticular shape. When the external pressure is now removed, the cavity first swells because of the volume expansion of the liquid inside, but when pressure outside has decreased to a certain value, liquid comes out from the interior and the cavity shrinks. That is, such a mechanism has an abnor-

mally large compressibility during the initial stage and an abnormally small compressibility during the final stage of a pressure increase, with the reversed behavior during decrease, which is exactly the behavior of the reversed loop found above.

The importance of this effect in speculation is evident, for it is a consequence of it that the initial elastic constants for small stress differences of a substance like pipestone may vary by a factor of more than two-fold, depending on the past stress history of the material. Furthermore, the mean elastic constant will vary greatly with the magnitude of the deforming stress or the deformation, so that it may be quite different for the strains which we have in tidal deformation and those found in mountain building, for example.

The effect is so important that it seemed worth while to verify it on other samples. Accordingly another specimen was tried in another apparatus, in order to eliminate any possibility of instrumental error. This second specimen was measured at 30° and 75°. The effect was again found, but much less in amount, the width of the reverse loop being at 30° only 3.1% of the maximum effect, and at 75° 2.3%. The sample used with the second apparatus was a short rod about 2.5 cm. long, whereas that used with the first was 15 cm. long. To further make certain freedom from instrumental error, a piece 2.5 cm. long was now cut from the first sample and measured in the second apparatus at 30°. This endeavor to ensure freedom from instrumental error was really quite superfluous because the identical apparatus had been used in measuring the compressibility of metals,<sup>7</sup> whose changes of dimensions are much smaller, with no hysteresis or similar effects of as much as 0.1%. The effect was again found; the loop was again reverse, and the width was 11.4% of the maximum. The shape of the loop was not as regular as that found before, but was as indicated in fig. 4.

It is evident that the pipestone is not at all a homogeneous material but that different parts differ greatly in hysteresis effects, and presumably in internal strain. Along with differences in hysteresis goes a difference in the absolute compressibility; the compressibility was considerably less for the piece that showed large reverse hysteresis. The following results were found:

For the long rod with 18% reverse hysteresis the average linear compressibility between 0 and 12000 kg/cm<sup>2</sup> was at the rate of  $4.34 \times 10^{-7}$  per kg. For the short piece cut from one end of the long rod the average change of length between 0 and 12000 was at the rate of  $3.87 \times 10^{-7}$ . The reverse loop of the other specimen was so narrow that it

FIG. 4.

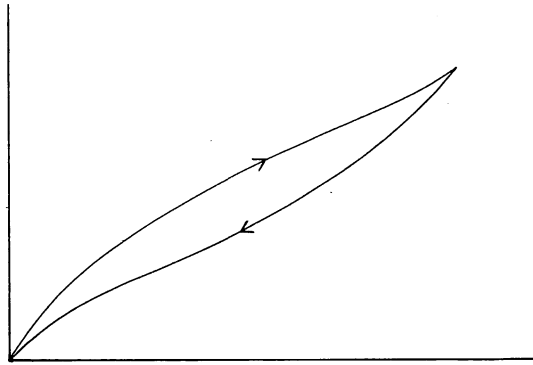


FIG. 4.—The compressibility cycle of another sample of pipestone, change of length against pressure. Again the direction of the loop is abnormal.

was possible to draw an average line half way between the ascending and descending branches of the curve. The change of length as a function of pressure on this average line is given by the formulas:

$$\frac{\Delta l}{l_0} = 6.50 \times 10^{-7} p - 5.81 \times 10^{-12} p^2 \quad \text{at } 30^\circ,$$

$$\frac{\Delta l}{l_0} = 6.89 \times 10^{-7} p - 5.81 \times 10^{-12} p^2 \quad \text{at } 75^\circ.$$

The average linear compressibility of the specimen with little reverse hysteresis is thus 50% greater than that of the other. It is not to be expected that the compressibility of material of this kind is the same in every direction, and therefore the volume compressibility cannot be computed from the linear compressibility in a single direction.

It is to be emphasized that the abnormal behavior of

pipestone may be expected to represent a lower rather than an upper limit to the abnormalities of the materials of the earth's crust *in situ*. For this piece of pipestone, measured in a laboratory, was a small piece whose external surfaces were free from stress, and in which any internal strains must have been fairly small, because due to the differential strains in a small mass. The rocks in the crust, however, are subjected to shearing stresses up to the limit of their strength, and the stresses vary greatly from place to place, so that the internal strains, and therefore probably elastic abnormalities, must be high.

*Solenhofen Limestone.*—This material was in the form of a rod 10 cm. long and roughly 6 mm. in diameter, cut with the diamond saw from the same piece as the thermal conductivity cylinder. Its compressibility was measured at 30° and 75°. At both these temperatures the same reverse hysteresis was found as for pipestone, but much less in amount. At 30° the width of the loop was 6.4% of the maximum effect, and at 75° 2.4%. The effect decreases with rising temperature, as it did also for pipestone, but by a much larger factor. A decrease with rising temperature is what would be expected if it is an internal strain effect.

At 30° the average *volume* compressibility between 0 and 12000 kg/cm<sup>2</sup> is 0.0<sub>5</sub>1338 per kg., and at 75° 0.0<sub>5</sub>1392, assuming equal compressibility in all directions. If the mean compressibility is taken from a curve drawn half-way between the ascending and descending branches of the hysteresis loop, then it will be found that the mean compressibility decreases with rising pressure, as is normal. The departure from linearity, which is about the same at both temperatures, is such that at 6000 kg. the actual change of volume is about 4% greater than would be calculated by using the average compressibility between 0 and 12000.

. Adams, Williamson, and Johnston<sup>8</sup> found for the mean compressibility of CaCO<sub>3</sub> between 2000 and 10000 kg.  $13.6 \times 10^{-7}$ , and found no variation with pressure. Their average figure agrees surprisingly well with the mean value found above for limestone.

*Basalt.*—The sample used was about 2.5 cm. long, and was the core of the thermal conductivity cylinder. It



was measured at 30° and 75°. Again reverse hysteresis was found, but this time the greatest width of the loop was only 1.5% of the maximum effect. Within the limits of error (not over 0.07%) the compressibility was the same at the two temperatures. If the mean of the ascending and descending branches of the loop is taken, and the compressibility is assumed equal in all directions, then the relation between pressure and change of volume is given by the formula:

$$\frac{\Delta V}{V_0} = 15.60 \times 10^{-7} p - 13.4 \times 10^{-12} p^2.$$

Adams and Williamson<sup>9</sup> found for two diabases of densities 3.002 and 2.975,  $12.4 \times 10^{-7}$  and  $14.1 \times 10^{-7}$  respectively for the average compressibility between 2000 and 10000, with no perceptible variation with pressure. My basalt, which is diabasic in character, was not as dense; the agreement is as good as to be expected.

*Talc.*—Measurements were made on four samples. The first sample was a rod 10 cm. long, not from the identical piece as the thermal conductivity sample, but from the same lot. It was without evident schistosity or other structure. It was measured at 30° and 75°. The smoothed results at these two temperatures were sensibly the same, although at 75° there were perfectly consistent small departures from the usual course consisting of a somewhat accelerated volume change at low pressures followed by a retardation, such that the total volume change at 12000 was the same as at 30°. There was no reverse hysteresis in perceptible amount. The relation between volume change and pressure for this sample, assuming equal compressibility in all directions, is given by the formula:

$$\frac{\Delta V}{V_0} = 10.4 \times 10^{-7} p - 6.3 \times 10^{-12} p^2.$$

This compressibility is so much less than I expected from the general softness of the substance, that the question arose whether it was really equally compressible in all directions. Professor Palache informed me that a structure would be expected from the manner of deposi-

tion, and that, therefore, there might be differences of compressibility in different directions. He was so kind as to select for me a sample from the same lot as the other samples which did show schistosity, the material evidently having been laid down on a series of planes. From this block three short rods were cut in directions mutually at right angles, and the compressibility of each of these rods was measured at 30°.

The results obtained for these three samples were all perfectly regular, and without hysteresis, direct or reverse. The relation between change of length and pressure could in each case be represented by an expression of the second degree in the pressure. The following values were found:

Sample # I

$$\frac{\Delta l}{l_0} = 8.76 \times 10^{-7} p - 16.6 \times 10^{-12} p^2$$

Sample # II

$$\frac{\Delta}{l_0} = 5.69 \times 10^{-7} p - 5.49 \times 10^{-12} p^2$$

Sample # III

$$\frac{\Delta l}{l_0} = 3.84 \times 10^{-7} p - 4.93 \times 10^{-12} p^2.$$

Samples I and III were cut parallel to the plane of schistosity, while II was perpendicular to it. The results show in the first place a variation with direction of 2.3 fold, which is greater than I had expected in material of this character. In the second place, there is no immediate connection evident with the plane of schistosity, the linear compressibility of II being intermediate between the other two.

The average initial volume compressibility, obtained by adding the three initial linear compressibilities, is  $18.3 \times 10^{-7}$ . The sample first measured was evidently

cut by accident along a direction of very low compressibility.

It is evident enough that a distortional wave through material of this character will not be propagated as through a homogeneous isotropic substance.

The great variation found in the linear compressibility of pipestone and talc probably has an important bearing on a suggestion in the recent paper of L. H. Adams and Williamson.<sup>10</sup> These authors measured the cubic compressibility of rocks between 2000 and 12000 kg. by a direct method, but did not carry their results below 2000. F. D. Adams and E. G. Coker,<sup>11</sup> on the other hand, had measured the compressibility at low stresses by an indirect method which involved the assumption of the homogeneity and isotropy of the material. They found compressibilities which in some cases were twice as high as those found by L. H. Adams and Williamson. Adams and Williamson reconcile the two sets of results by supposing below 2000 kg. a very rapid increase of compressibility, which is perhaps connected in some way with a looseness of structure of the rocks which disappears at high pressures. Of course it is possible that this may be the correct explanation, and in any event the ultimate answer can only be obtained by a direct measurement of the compressibility at low pressures, but in view of my results above it seems to me that a much more likely explanation is that the values of F. D. Adams and Coker are in error because of failure of the necessary isotropy. Differences of elastic properties with direction of the magnitude which I have found above would be more than sufficient to explain the divergent results.

*Pyrex Glass.*—This material was in the form of a rod about 15 cm. long and 6 mm. in diameter. It was from the same length of tubing as the thermal conductivity sample, and was made from it by working in the fire into a solid rod and drawing down. It was not perfectly homogeneous, but there were a few minute bubbles in it.

The compressibility was measured at 30° and 75°. The results were very regular and self-consistent. The maximum departure of any point from a smooth curve was 0.36% of the maximum effect. There was no hysteresis of any kind. Within the limits of error there was no difference between the compressibility at the two temperatures. Assuming equal compressibility in all

directions, the relation between pressure and change of volume is given by the formula:

$$\frac{\Delta V}{V_0} = 30.27 \times 10^{-7} p + 9.9 \times 10^{-12} p^2.$$

This glass shows the abnormal feature, the first time that I have found it, of a compressibility which *increases* at high pressures. The effect is comparatively large, and much beyond the possibility of any experimental error. As I have already said elsewhere in discussing the possibility of such an effect,<sup>12</sup> there seems to be no fundamental reason why the effect should not exist, but it certainly is not what we expect. One might be inclined to search for a connection between the effect and the minute bubbles in the glass, but the effect of bubbles would be expected to be in the opposite direction, giving too high a compressibility at low pressures before the bubbles close up, and furthermore if the bubbles are numerous enough to have a perceptible effect on compressibility, one would look for hysteresis arising from non-hydrostatic internal strains in the neighborhood of the bubbles.\*

Evidence confirming the reality of this effect may be drawn from the work of Adams, Williamson,<sup>13</sup> and Johnston on compressibility. They found for all their materials whose compressibility was as high as this a decrease of compressibility with rising pressure with the exception of silica glass, which was constant between 2000 and 8000 kg., in spite of the comparatively high value of its compressibility,  $3.1 \times 10^{-6}$ . Now the experiments of Adams, Williamson and Johnston are not as sensitive as these to changes of compressibility with pressure, and it may well be that if they did not find the effect it is really negative. At any rate it is abnormally small.

The compressibility of the pyrex found above,  $3.03 \times 10^{-6}$ , is unusually high for a glass which for ordinary compositions is in the neighborhood of 2.3. The

\* Note added on reading proof. The abnormal change of compressibility with pressure has now been verified on a sample of pyrex glass free from flaws and thoroughly annealed, prepared for me by the Corning Glass Works for this express purpose.

value for pyrex is close to the value of Adams, Williamson, and Johnston for silica glass, suggesting a similarity of effect.

This increase of compressibility at high pressures deserves to be more extensively investigated with other materials.

*Summary.*

*Thermal Conductivity.*—The thermal conductivity of pipestone, talc, Solenhofen limestone, basalt, pyrex glass and rock salt has been measured up to 12000 kg/cm<sup>2</sup> at 30° and in most cases at 75° also. The conductivity increases with rising pressure by an amount which may vary from 0.1% per 1000 kg. at 30° for Solenhofen limestone to 3.6% for rock salt. Probably for the ordinary rock of the earth's crust the effect is not over 0.5% at ordinary temperatures. The absolute conductivity and its pressure coefficient may either increase or decrease with rising temperature.

*Compressibility.*—A simple hydrostatic pressure applied to substances of the character of those measured here does not in general produce a uniform change of volume, but the effects are complicated. The relation between pressure and deformation may be affected by hysteresis in a direction the reverse of normal; I have found this in pipestone, Solenhofen limestone, and basalt in amounts varying from a few per cent to 18 per cent of the whole deformation. Talc and pipestone show large differences of linear compressibility in different directions; this effect was not investigated for the other materials. Pyrex glass has an abnormal increase of compressibility at high pressures.

*Geological Consequences.*—As far as the results on thermal conductivity go, only rough conclusions can be drawn, but it would seem that in view of the possible variations found with pressure and temperature one should be prepared in geological speculations to consider that thermal conductivity at a depth of several hundred miles may be several fold greater than the value now accepted.

That the compressibility is different in different directions suggests that deep in the earth's crust where there are large differential stresses, with the probability of

much greater variations with direction than those found here, it may be very far from the truth to represent the elastic behavior as that of a single homogeneous isotropic solid with two elastic constants. One must furthermore be prepared for the earth's crust to act with different effective elastic constants according as the strains are large or small, and depending on the past history.

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