The reaction from protein to repressor is a single enzyme-catalysed step whereas the other reactions are long & complex sequences. Therefore, M will reach equilibrium much quicker than Y or Z. If we want to look at the behaviour of Y or Z, we can safely assume that M has its equilibrium value always. Thus we set $\frac{dM}{dt} = 0$, giving

$$\frac{dY}{dt} = \frac{c}{a+lz} - kY$$

$$\frac{dz}{dt} = eY - fz$$

@ equilibrium

$$\frac{c}{a+lz_E} - kY_E = 0$$

$$eY_E - fz_K = 0$$

we let $Y = Y_E + y$ and $z = z_E + z$ where z, y small displacements, then

$$\frac{dY}{dt} = \frac{dy}{dt} = \frac{c}{a+lz_E+lz} - kY_E - ky$$

we proceed,

$$\frac{c}{a+lz_k+lz} = \frac{c}{a+lz_E} \times \frac{a+z_E}{a+lz_E+lz}$$

$$= \frac{c}{a+lz_E} \div \left(1+\frac{lz}{a+lz_E}\right)$$

since z is small $(1+Rz)^{-1} \simeq 1-Rz$

$$\frac{c}{a+lz_E+lz} = \frac{c}{a+lz_E}\left(1-\frac{lz}{a+lz_E}\right)$$

hence

$$\frac{dy}{dt} = \frac{-clz}{(a+lz_E)^2} - ky$$

$$\frac{dz}{dt} = ey - fz$$

or

$$\frac{dy}{dt} = -Kz - ky \quad (*), \quad \frac{dz}{dt} = ey - fz$$

we aim to eliminate z,

$$\frac{d^2y}{dt^2} = -K\frac{dz}{dt} - k\frac{dy}{dt}$$

$$= -k(ey - fz) - k\frac{dy}{dt}$$

but $z = -\frac{1}{k}\left(\frac{dy}{dt} + ky\right)$ from (*)

So we get

$$\boxed{\frac{d^2y}{dt^2} + (f+k)\frac{dy}{dt} + (fk+ke)y = 0}$$