$$y_i \sim N\left(\alpha + \beta x_i + \delta x_i^2, \frac{1}{\tau}\right)$$

$$\Rightarrow \text{likelihood} = \prod_{i=1}^{n} \sqrt{\frac{\tau}{2\pi}} \exp\left[-\frac{\tau}{2}\left(y_i - \alpha - \beta x_i - \delta x_i^2\right)^2\right]$$

$$= \left(\frac{\tau}{2\pi}\right)^{n/2} \exp\left[-\frac{\tau}{2}\sum_{i=1}^{n}\left(y_i - \alpha - \beta x_i - \delta x_i^2\right)^2\right]$$

$$\text{prior} \propto \frac{b^a}{\Gamma(a)} \tau^{a-1} e^{-b\tau}$$

$$\exp\left[-\frac{1}{2}\left(\frac{\alpha^2}{S_a^2} + \frac{\beta^2}{S_b^2} + \frac{\delta^2}{S_c^2}\right)\right]$$

$$\propto \tau^{a-1} \exp\left[-b\tau - \frac{1}{2}\left(\frac{(\alpha-\bar{\alpha})^2}{S_a^2} + \frac{(\beta-\bar{\beta})^2}{S_b^2} + \frac{(\delta-\bar{\delta})^2}{S_c^2}\right)\right]$$

$$\text{posterior} \propto$$

$$\tau^{n/2+a-1} \exp\left[-\frac{\tau}{2}\sum_{i=1}^{n}\left(y_i - \alpha - \beta x_i - \delta x_i^2\right)^2 - b\tau\right.$$

$$\left. - \frac{1}{2}\left(\frac{(\alpha-\bar{\alpha})^2}{S_a^2} + \frac{(\beta-\bar{\beta})^2}{S_b^2} + \frac{(\delta-\bar{\delta})^2}{S_c^2}\right)\right]$$

$$\cdot\, \pi(\tau \mid \alpha, \beta, \delta) \propto$$

$$\tau^{n/2+a-1} \exp\left[-\tau\left(\frac{1}{2}\sum^{n}\left(y_i - \alpha - \beta x_i - \delta x_i^2\right)^2 + b\right)\right]$$

$$\sim \text{Gamma}\left(\frac{n}{2}+a, \ \frac{1}{2}\sum(y_i-\alpha-\beta x_i-\delta x_i^2)^2+b\right)$$

$$\pi(\alpha \mid \beta, \delta, \tau) \propto$$

$$\exp\left[-\frac{\tau}{2}\sum_{i=1}^{n}\left((y_i-\beta x_i-\delta x_i^2)-\alpha\right)^2-\frac{(\alpha-\bar{\alpha})^2}{2S_a^2}\right]$$

$$\exp\left[-\frac{\tau}{2}\left(\sum (y_i-\beta x_i-\delta x_i^2)^2-2\alpha(y_i-\beta x_i-\delta x_i^2)+\alpha^2\right)-\frac{(\alpha-\bar{\alpha})^2}{2S_a^2}\right]$$

$$\propto \exp\left[-\frac{\tau}{2}(-2\alpha)\sum y_i-\beta x_i-\delta x_i^2-\frac{\tau}{2}\sum \alpha 2-\frac{(\alpha-\bar{\alpha})^2}{2S_a^2}\right]$$

$$\propto \exp\left[\tau\alpha\sum y_j-\beta x_i-\delta x_i^2-\frac{\tau}{2}\sum\alpha^2-\frac{\alpha^2}{2S_a^2}+\frac{\alpha\bar{\alpha}}{S_a^2}\right]$$

$$=\exp\left[\alpha\left(\frac{\bar{\alpha}}{S_a^2}+\tau\sum(y_j-\beta x_i-\delta x_i^2)\right)-\alpha^2\left(\frac{n\tau}{2}+\frac{1}{2S_a^2}\right)\right]$$

$$=\exp\left[-\frac{1}{2}\left(n\tau+\frac{1}{S_a^2}\right)\left(\alpha^2-2\alpha\frac{\frac{\bar{\alpha}}{S_a^2}+\tau\sum(y_j-\beta x_i-\delta x_i^2)}{n\tau+\frac{1}{S_a^2}}\right)\right.$$

$$= \exp\left[\alpha\tau \sum (y_i - \beta x_i - \delta x_i^2) - \alpha^2\left(\frac{n\tau}{2} + \frac{1}{2 S_a^2}\right)\right]$$

$$= \exp\left[-\frac{1}{2}\left(n\tau + \frac{1}{S_a^2}\right)\left[\alpha^2 - 2\alpha\left[\frac{\tau\sum(y_i - \beta x_i - \delta x_i^2)}{n\tau + \frac{1}{S_a^2}}\right]\right]\right]$$

$$\Rightarrow \alpha \sim N\left(\varepsilon^2\left(\frac{\bar{\alpha}}{S_a^2} + \tau\sum y_i - \beta x_i - \delta x_i^2\right),\ \varepsilon^2\right)$$

where $\varepsilon^2 = \left(n\tau + \frac{1}{S_a^2}\right)^{-1}$

by symmetry

$$\beta \sim N\left(\varepsilon^2\left(\frac{\bar{\beta}}{S_b^2} + \tau\sum x_i(y_i - \alpha - \delta x_i^2)\right),\ \varepsilon^2\right)$$

where $\varepsilon^2 = \left(\tau\sum x_i^2 + \frac{1}{S_b^2}\right)^{-1}$

$$\delta \sim N\left(\varepsilon^2\left(\frac{\bar{\delta}}{S_c^2} + \tau\sum x_i^2(y_i - \alpha - \beta x_i)\right),\ \varepsilon^2\right)$$

$$\varepsilon^2 = \left(\tau\sum x_i^4 + \frac{1}{S_c^2}\right)^{-1}$$