3450:427/527 Applied Numerical Methods I, Kreider, Fall 2012

Homework Set 1

Each problem is worth 10 points.

Due date: Tuesday 11 September

1. Write .000304569432608 in normalized form in base 10 (scientific notation) with a precision of 9 digits using rounding.

2. Construct a floating point system (by specifying the mantissa precision t and how many binary digits m should be used in the exponent) for which machine epsilon is less than 10^{-22} and the largest positive machine number is greater than 10^{300}. Many answers are possible, but try to keep t and m as small as possible and still satsify the conditions.

3. Let $y = 77302$ be approximated by $\hat{y} = 77000$, and let $z = .00023$ be approximated by $\hat{z} = .0002$. For each pair, compute the absolute error and the relative error. Make a one or two sentence observation about the results – how does the magnitude of the numbers involved in the calculation affect the absolute error and the relative error? MAKE SURE THIS PROBLEM WORKS FOR ABSERR AND RELERR

4. Consider the calculation $(A + B)/C$ on a machine that uses 6 digit rounding. Let $A = 1.24854E0$, $B = 1.40032E - 5$, and $C = 1.20668E - 1$. Perform the calculation as it would be done on the machine. Report your answer in normalized form.

5. Solve $x^2 + 1.234x + .3805 = 0$ by hand using 4 digit arithmetic with rounding using the most accurate formulas available. Then compute the roots more accurately (on your calculator, to 8 or 9 digits). If cancellation error is present in this problem, identify the exact spot where it occurs.

6. MATLAB BASICS – plotting. Write a script file that defines the inline function $y = \dfrac{x+3}{x^2+5}$ and plots it over the interval $[-5, 5]$ with a resolution of 0.01. Cut and paste the script file and the plot into a Word document on one page (this is practice for saving paper on future assignments).

7. MATLAB BASICS – function files. Write a function file, called fcnhw1n7.m, that accepts 3 inputs, x, y and z, and outputs the 2 quantities $a = x^2/y + 3z$ and $b = e^x \sin(y)/\sqrt{z^2+1}$. Write a script file, called hw1n7.m, that calls the function twice, first with $(x, y, z) = (1, 2, 3)$ and then with $(x, y, z) = (-4, 1, 7)$ and prints the results with at least 10 significant digits. Submit the code listings and the numerical results.

8. For 527 students only: build the recurrence relation for $T_n = \displaystyle\int_0^2 x^n e^{2x}\, dx$. Write a code to compute the value of T_{14}. Submit the recurrence relation derivation, the code and the value of T_{14} accurate to at least 10 significant digits.