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Assignment 2

I have a tree in my front yard. It is 20 feet tall and rooted at PR(50,0,100). The origin of the coordinate system is at the bottom-front-left corner of my house. X axis of the coordinate system extends from the bottom-front-left corner to the bottom-front-right corner, the Y axis goes up, and the Z axis points to the street. I am hiring someone to cut the tree. When it falls, I want it to rotate around its root and falls to the right. I am watching the fall of the tree in my house at PE(50,0,-10) through the huge window in the front of my house.

1.1 List the geometric transformations needed to show the falling of the tree.

1. Translate the tree to the z-axis.
2. Rotation transformation about the z-axis.
3. Translate the tree back to its former position.

1.2 Write down the matrixes for the geometric transformations in homogeneous coordinate.

Translation to the x-axis:

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \partial x \\ 0 & 1 & 0 & \partial y \\ 0 & 0 & 1 & \partial z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -\partial x \\ 0 & 1 & 0 & -\partial y \\ 0 & 0 & 1 & -\partial z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Where

$$rotation = -\pi/2$$

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Position of the top of the tree:

$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \rightarrow \begin{bmatrix} 50 \\ 20 \\ 100 \\ 1 \end{bmatrix}$$

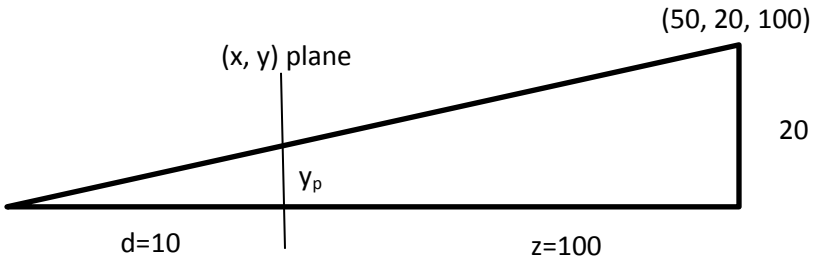
Translation:

$$\begin{bmatrix} dx \\ dy \\ dz \\ 1 \end{bmatrix} \rightarrow \begin{bmatrix} 50 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Solved:

$$\begin{bmatrix}1 & 0 & 0 & 50\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}0 & -1 & 0 & 0\\ 1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 0 & -50\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}50\\ 20\\ 100\\ 1\end{bmatrix} = (70, 0, 100, 1)$$

1.3 Write down the matrix to project the image of the tree on the window as seen by me (ignore my height).



$$\begin{bmatrix}x'\\ y'\\ z'\\ w'\end{bmatrix} = \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 1/d & 1\end{bmatrix}\begin{bmatrix}x\\ y\\ z\\ 1\end{bmatrix}$$

Where

$$d = 10$$

$$z = 100$$

$$w' = \left(z/d\right) + 1$$

$$y_p = y'/w'$$

$$x_p = x'/w'$$

Solved:

$$\begin{bmatrix}x'\\ y'\\ z'\\ w'\end{bmatrix} = \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 1/10 & 1\end{bmatrix}\begin{bmatrix}50\\ 20\\ 100\\ 1\end{bmatrix} = \begin{bmatrix}50\\ 20\\ 0\\ 11\end{bmatrix}$$

$$x_p = 50/11$$

$$y_p = 20/11$$

1.4 Apply the above matrixes to the tip of the tree, and show your results after each matrix is applied.

(See above problems)

In order to pull the tree down, the lumberjack ties a rope to the tip of the tree. He pulls the tree while standing at PL(70,0,100).

2.1. Compute the angle between the tree and the pulling force using vector-dot product. Ignore the height of the lumberjack and note the tree points from its root to the tip.

$$\cos\theta = \frac{A \cdot B}{|A||B|}$$

Where A and B are relative to a local origin of (50, 20, 100)

$$A = (0, -20, 0)$$

$$B = (20, -20\ , 0)$$

$$\cos\theta = \frac{(0 * 20) + (-20 * -20) + (0 * 0)}{20 * \sqrt{20^2 + 20^2}}$$

$$\theta = \ .7854\ Rad = 45°$$

2.2. Write down the line equation for the rope in parametric form, assuming the rope starts from the tip of the tree and ends where the lumberjack stands.

$$x(t) = 50\ +\ t * 20$$

$$y(t) = 20 - t * 20$$

$$z(t) = 100$$

Where

$$t \in (0, 1)$$

A bird nesting in the tree is disturbed by the activity and flies around the tree trying to protect its home.

3.1 Define the line equation for the rope in f(x,y,z)=0 form. (Hints: you can derive it from the slope equation and you can ignore z).

$$0 = (y - y_0) - m(x - x_0)$$

$$m = \frac{20 - 0}{50 - 70} = -1$$

$$0 = (y - 0) + 1(x - 70)$$

$$x \in (50, 70)$$

3.2 For the following positions of the bird, find out (using the f formula from 3.1) if the bird is above the rope, below the rope or on the rope. If bird is on the rope, find out its relative location on the rope measuring from the tip of the tree.

PB1(60,10,100),

$$0 = (10 - 0) + 1(60 - 70)$$

On the line

PB2(55, 10,100)

$$-5 = (10 - 0) + 1(55 - 70)$$

Below the line

PB3(65,15,100)

$$10 = (15 - 0) + 1(65 - 70)$$

Above the line