3450:427/527 Applied Numerical Methods I, Kreider, Fall 2012

Homework Set 4

Each problem is worth 10 points.

Due date: Thursday 8 November

1. Implement the code for Horner's algorithm discussed in class, in the form y = horner(A,r). Verify that it is working correctly by testing it on a simple function, say $P(x) = x^2 - 3x$, for which you can easily compute $P(r)$ for various r values. Then use the routine to evaluate $P(x) = 3x^5 - 2x^2 + 7x - 2$ at $x = 3.6$ and $x = -2.3$.

2. Use Lagrange approximation (routine lagran on page 217) to interpolate $f(x) = \cos(\exp(x/3))$ at $x_0 = 0, x_1 = 1, \ldots, x_9 = 9$. Then use your horner function to plot the interpolating polynomial on [0,9] along with the data points to see how good or bad it is. To do this, put your data in vectors X and Y, and then type

```
[C,L] = lagran(X,Y);
C = fliplr(C);  % the coefficients come out backwards, so flip them around
r = 0:.01:9;
for i = 1:length(r)
   f(i) = horner(C,r(i));
end
Y2 = cos(exp(r/3));
plot(X,Y,'o',r,f,'k',r,Y2,'r:')
```

   Notice how the evaluation of the interpolating polynomial is completely independent of its construction. Turn in the code listings and the plot.

3. Let $f(x) = x \sin x$. Consider the interpolating polynomial $P_4(x)$ that interpolates f at the points $x_0 = 0, x_1 = 0.1, x_2 = 0.2, \ldots, x_4 = 0.4$. Write the expression for the error $E_4(x)$ and use it to find the maximum possible error of interpolation in $[0, 0.4]$. Use graphs to estimate the maximum values of $f^{(N+1)}(x)$ and $w(x) = (x - x_0) \cdots (x - x_4)$. Do not compute $P_4(x)$, just do the error analysis.

4. Implement the algorithms divdiff and newtval discussed in class. Apply them to the data set

.1	1
.2	4
.3	2
.4	3
.45	6
.5	12
.62	15
.7	20
.8	12
.9	7
1.1	2

(a) Plot the data and the interpolating polynomial using the MATLAB commands

```
X = x values of data
Y = y values of data
D = divdiff(X,Y);
x = 0: 0.001: 1.2; n = length(x); y = zeros(1,n);
for k=1:n
   y(k) = newtval(D,X,x(k));
end
plot(X,Y,'o',x,y), axis([0 1.2 -100 100])
```

(b) Write a brief statement indicating whether or not the interpolating polynomial is satisfactory. (c) If there were a data point at $x = 0.85$, what would you expect the y value to be, roughly (based only on the data set given above)? Now evaluate the interpolating polynomial at $x = 0.85$. Does that value match your expections?

5. For 527 students. For $f(x) = e^{2x}$, find the Pade Approximant $R_{22}(x)$. It is easiest to compute this by hand.