3450:427/527 Applied Numerical Methods I, Kreider, Fall 2012

Homework Set 3

Each problem is worth 10 points.

Due date: Tuesday 9 October

1. Implement the back-substitution algorithm (backsub) in Section 3.3 and apply it to the system

$$\begin{bmatrix} 5 & -2 & 1 & 15 \\ 0 & -5 & 1607 & -3 \\ 0 & 0 & 4 & 4 \\ 0 & 0 & 0 & 21 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

Turn in a listing of the code, as well as the output.

2. Implement the algorithm on page 136-7, called uptrbk.m, which employs upper triangularization with partial pivoting. Apply it to the system

$$\begin{bmatrix} 5 & -2 & 1 & 15 \\ -3 & -5 & 1607 & -3 \\ 10 & 2 & 4 & 4 \\ 4 & 1.2 & 9.8 & 21 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 7 \\ -2 \\ 33 \\ 4 \end{bmatrix}$$

3. Let's play with the idea of ill-conditioning for a 9×9 system $Ax = B$. First, create A that is all ones except for 1.01's on the main diagonal:

```
A = ones(9,9);
for j=1:9
  A(j,j) = 1.01;
end
```

Let $B = (1:9)'$.

(a) Solve the system using the backslash command and call the result $x1$.

(b) Perturb the matrix A slightly by multiplying $A(5,3)$ by 1.02 (a 2% change):

```
A(5,3) = A(5,3)*1.02
```

Solve the system using the backslash command and call the result $x2$.

(c) Compare $x1$ and $x2$ by printing each value and the absolute value of the difference for each $j = 1:9$. Then check the overall effect by computing the relative error:

```
norm(x1-x2)/norm(x1)
```

Answer the question: A 2% change in one element causes a —% change in the overall solution. Is there evidence of ill-conditioning? If there is, how would you rate the severity of the problem?

(d) Now consider $Cx = B$, where B is the same, but C is a 9×9 matrix that is all ones except for 10's on the main diagonal. Solve the system and call the solution $y1$.

(e) Perturb the matrix by multiplying $C(5, 3)$ by 1.02. Solve the system and call the solution $y2$.

(f) Repeat part c. A 2% change in one element causes a —% change in the overall solution. Is there evidence of ill-conditioning? If there is, how would you rate the severity of the problem?

(g) Now compute the condition numbers of the unperturbed matrices A and C to see how they compare.

4. Use the Jacobi iteration algorithm to solve the system

$$\begin{aligned} x - 5y - z &= -8 \\ 4x + y - z &= 13 \\ 2x - y - 6z &= -2 \end{aligned}$$

Remember that you might need to modify the system to put it into the appropriate form. Use 2 different starting guesses, $(0, 0, 0)'$ and $(10, 20, -30)'$, to see how many iterations it takes to converge. Use the error tolerance tol $= 1e - 10$. Turn in a listing of the code, as well as the output. Include the number of iterations in your output.

5. Now use the Gauss-Seidel algorithm on the problem above. Use the same 2 starting guesses. Use the error tolerance tol $= 1e - 10$. Turn in a listing of the code, as well as the output. Include the number of iterations in your output.

6. For 527 students. Let's check timing information on different algorithms – the routine uptrbk and the built-in backslash command. First, build a random 1000×1000 system:

```
A = rand(1000,1000);
B = rand(1000,1);
```

To get timing information, use the cputime command:

```
z1 = cputime;
X1 = uptrbk(A,B);
z2 = cputime;
z2-z1
```

Do the same thing using the backslash command (call the solution X2) and compare the times. Clearly, the MATLAB command is much more efficient. Finally, display norm(X1-X2) to see that the 2 algorithms give essentially the same answer.

Submit a copy of your script file (but not another copy of uptrbk), the 2 CPU times and the norm.