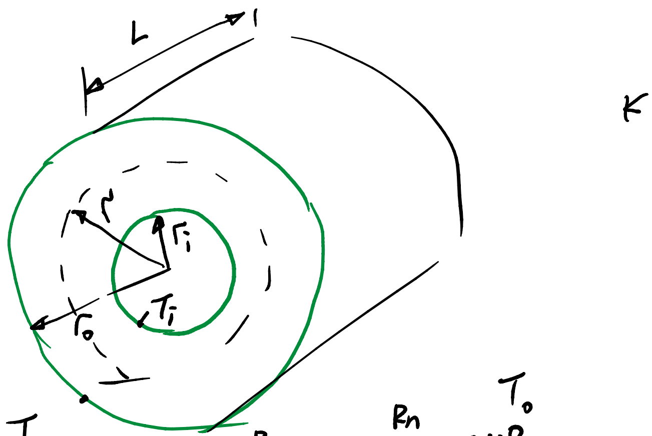
Thermal Circuits Cont'd



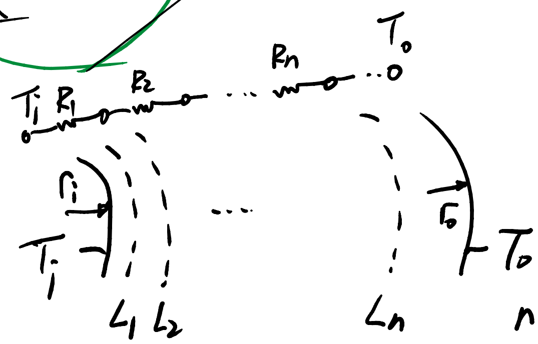
Challenge 1:

$A(r) = 2\pi r \cdot L$

$r_i \le r \le r_o$

Challenge 2:

T_o



n is very large

$q(r) = f(r)$?

$q \ne f(r)$

$$q = \Delta q = -K \cdot 2\pi r L \cdot \frac{\Delta T}{\Delta r}$$

$$\lim_{\Delta r \to 0} \Rightarrow \quad q = -K \cdot 2\pi r L \cdot \frac{dT}{dr} \quad \checkmark$$

Boundary conditions

$r = r_i \qquad T = T_i$

$r = r_o \qquad T = T_o$

$$\Rightarrow -\frac{q}{2\pi K L} = r \cdot \frac{dT}{dr} \qquad \text{Let } C = -\frac{q}{2\pi K L}$$

$$\Rightarrow \quad C = r \cdot \frac{dT}{dr} \Rightarrow \frac{C}{r} dr = dT$$

$$C \cdot \int_{r_i}^{r_o} \frac{1}{r} dr = \int_{T_i}^{T_o} dT$$

$$C \cdot (\ln r_o - \ln r_i) = T_o - T_i$$

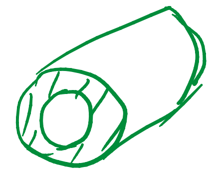
$$\Rightarrow C \cdot \ln \frac{r_o}{r_i} = T_o - T_i$$

$q = ?$ $\qquad q \quad \ln \frac{r_o}{\ } = T_o - T_i$

$q_{cyl} = ?$

$$-\frac{q}{2\pi k L}\cdot \ln\frac{r_o}{r_i} = T_o - T_i$$

$$\Rightarrow \boxed{q_{cyl} = \frac{T_i - T_o}{\frac{\ln r_o/r_i}{2\pi k L}}} \Rightarrow$$

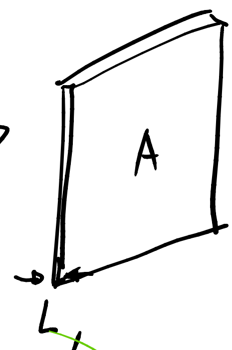


Cond. H.T. is a function of shape.

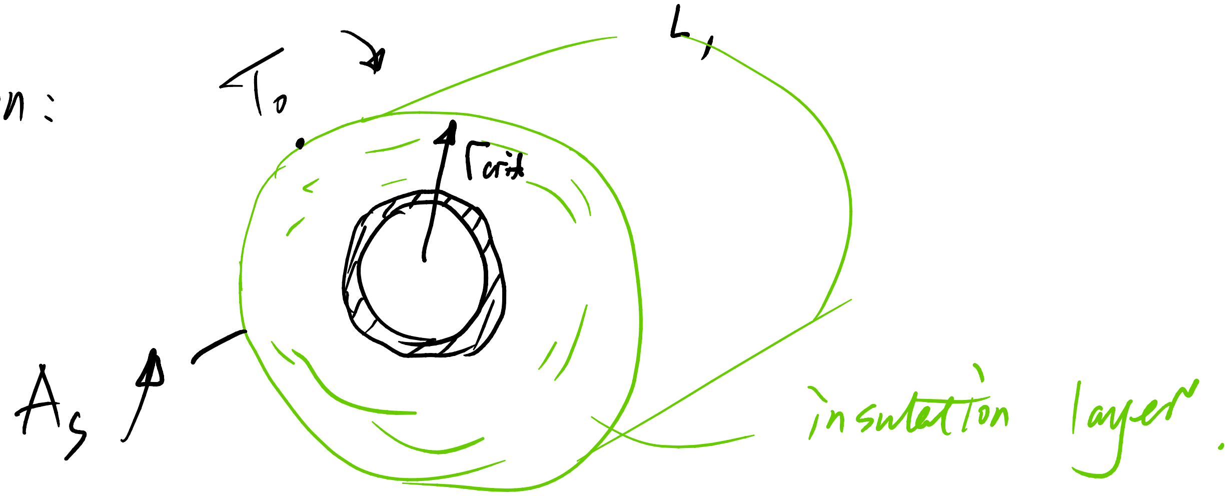
$$q = \frac{\Delta T}{R} \Rightarrow R = \frac{\Delta T}{q}$$

$$\Rightarrow \quad \frac{T_i - T_o}{q_{cyl}} = \boxed{\frac{\ln r_o/r_i}{2\pi k L} = R_{cyl}}$$

$$R_{plate} = \frac{L}{k\cdot A} \Rightarrow$$



Question:

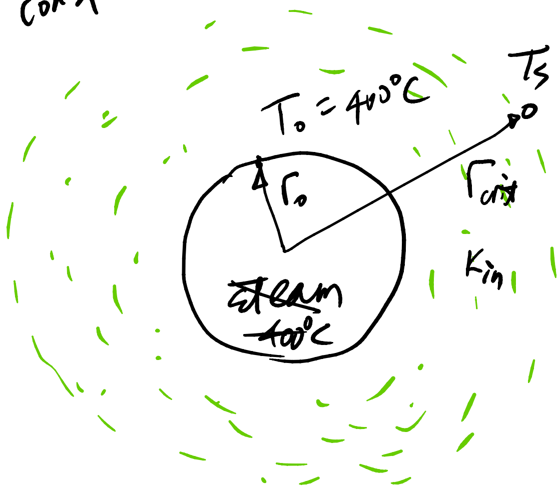


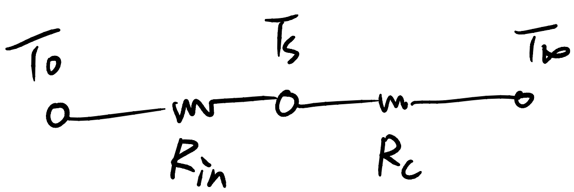
$$q_{loss} = \underset{\downarrow\ \text{const}}{h_c} \cdot \underset{\uparrow}{A_s} \cdot (\underset{\downarrow}{T_o} - \underset{\downarrow\ \text{const}}{T_\infty})$$

const

const

h_∞, $T_\infty = -20°C$





$$q = \frac{\Delta T}{R} = \frac{T_0 - T_\infty}{R_{in} + R_c}$$

where, $$R_{in} = \frac{\ln \frac{r_{crit}}{r_o}}{2\pi k_{in} \cdot L}$$

$$R_c = \frac{1}{h_\infty \cdot A_s} = \frac{1}{h_\infty \cdot 2\pi r_{crit} \cdot L}$$

$$\Rightarrow q(r_{crit}) = \frac{T_0 - T_\infty}{\underbrace{\frac{\ln \frac{r_{crit}}{r_o}}{2\pi k_{in} L} + \frac{1}{h_\infty 2\pi r_{crit} \cdot L}}_{R_{total}}}$$

$$\frac{dq}{dr_{crit}}$$

Let $\dfrac{d R_{total}(r_{crit})}{d r_{crit}} = 0 \Rightarrow \boxed{r_{crit} = \dfrac{k_{in}}{h_o}}$

e.g. A $L = 20\,m$ long steam pipe, $T_i = 500K$, $h_i = 35 \frac{W}{m^2 \cdot K}$.

$k_s = 54 \frac{W}{m \cdot K}$; wrapped with $k_{gf} = 0.073 \frac{W}{m \cdot K}$.

$r_1 = 1cm$, $r_2 = 2cm$, $r_3 = 10\,cm$, $T_0 = 300\,K$, $h_o = 8 \frac{W}{m^2 \cdot K}$

i) Good or bad?

ii) $q_{loss} = ?$