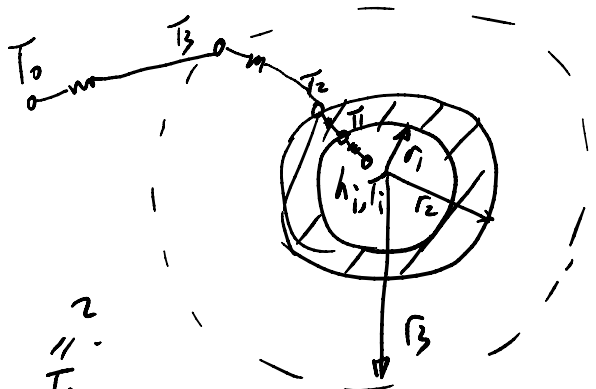
Solution: (a) $r_{crit} = \frac{k_{in}}{h_o} = \frac{0.073}{8} = 0.913\ cm$

$r_3 >> r_{crit} \Rightarrow$ Good design

$r_3 = 10\ cm$



(b) q_{loss}

$T_i = 500K$ T_1 T_2 $T_3 = 2$ $T_0 = 300K$

R_i R_s R_{in} R_o

$$q_{loss} = \frac{T_i - T_o}{R_i + R_s + R_{in} + R_o}$$

$P = 2\pi r$

$A = P \cdot L$

where, $R_i = \frac{1}{h_i \cdot A_i} = \frac{1}{h_i \cdot 2\pi r_1 \cdot L} = 0.023\ \frac{K}{W}$

$R_s = \frac{\ln r_2/r_1}{2\pi K_s \cdot L} = 1.021 \times 10^{-4}\ \frac{K}{W}$

$R_{in} = \frac{\ln r_3/r_2}{2\pi K_{in} \cdot L} = 0.175\ \frac{K}{W}$

$R_o = \frac{1}{h_o \cdot A_o} = \frac{1}{h_o \cdot 2\pi r_3 \cdot L} = 9.95 \times 10^{-3}\ \frac{K}{W}$

$R_{total} = 0.208\ \frac{K}{W}$

$\Rightarrow q_{..} = \frac{500 - 300}{0.208} = 960.5\ W$

$$\Rightarrow q_{loss} = \frac{500 - 300}{0.208} = 960.5\ W$$

(c) $T_3 = ?$

$$q_{loss,\ w/o\ in} = h_o \cdot A_o \cdot (T_{wall} - T_o)$$

$$= 8 \cdot 2\pi \cdot r_2 \cdot L \cdot (500 - 300)$$

$$= 8 \cdot 2\pi \cdot 2\times10^{-2} \cdot 20 \cdot 200$$

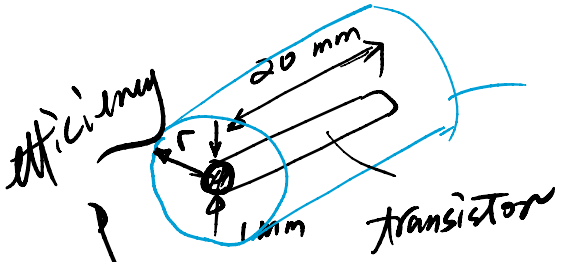
$$= 8 \cdot 2\pi \cdot 80 = 640 \cdot 2 \cdot \pi = 1280 \cdot 3.1415$$

$$= 4019\ W$$

$$q_{loss} = \frac{T_3 - T_o}{R_o} \Rightarrow T_3 = q_{loss} \cdot R_o + T_o$$

$$= 309.55\ K$$

Example.



efficiency

$q = 0.5\ W$. $\eta = 50\%$

$K_{in} = 0.1\ \frac{W}{m \cdot K}$

$T_e = 300\ K$

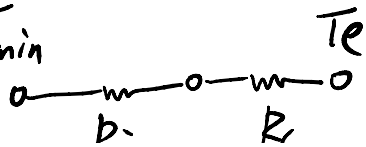
$h_e = 16\ \frac{W}{m^2 \cdot K}$

i) What's the optimal r of insulation to enable $T_{min} = ?$

ii) $T_{min} = ?$

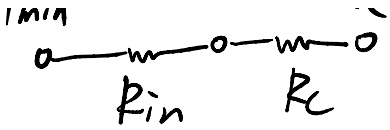
Solution: i) $r_{crit} = r = \frac{K_{in}}{h_e} = 6.25\ mm$

ii)



$q_{loss} = 0.25\ W$

ii)



$q_{loss} = 0.25\ W$

$K_{Si} = 140 \frac{W}{m \cdot K}$

$K_{AlN} = 400 \frac{W}{m \cdot K}$

$$q_{loss} = \frac{T_{min} - T_e}{R_{in} + R_c}$$

$$\Rightarrow T_{min} = q_{loss} \cdot (R_{in} + R_c) + T_e = 370.1K$$