EMCH361

Mechanical Engineering Lab I

Professor Yu

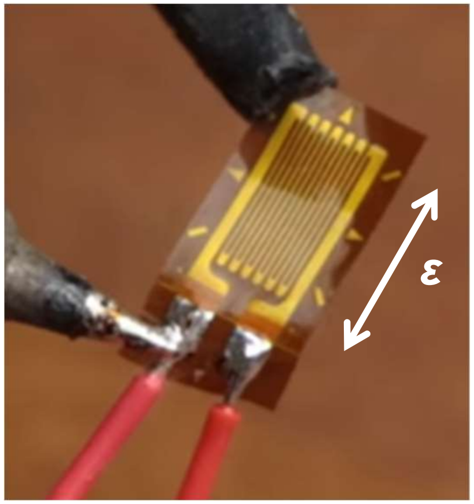
# Lecture #10
# Spring 2022

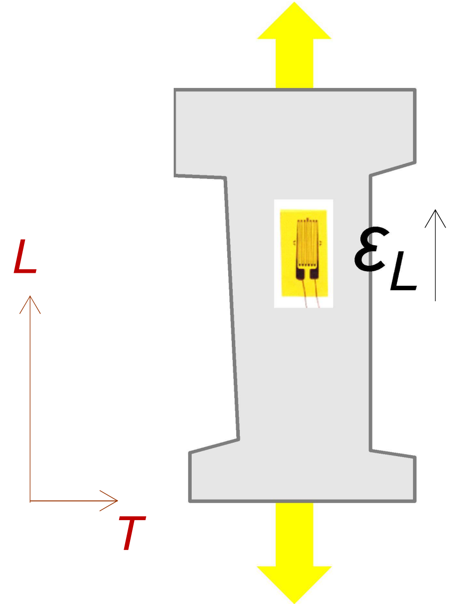
# Recap

- Gage factor (GF) and base resistance (R)

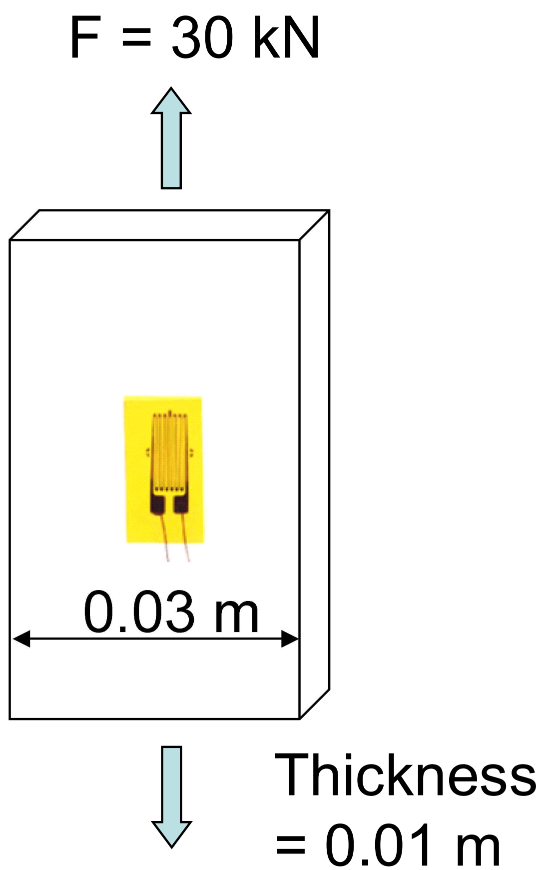
$$\varepsilon = \frac{1}{GF}\frac{\Delta R}{R}$$

- Strain gauge installation
  – Bonding adhesive
- Strain measurement
  – Point measurement
  – Directional



A strain gauge with GF=2.0 is mounted on a steel bar (E = 200 x $10^6$ kN/m$^2$) the tensile force of 30 kN. Determine the resistance change ($\Delta R$) of the strain gauge. The resistance of the Gauge was 120 Ω in the absence of the load.

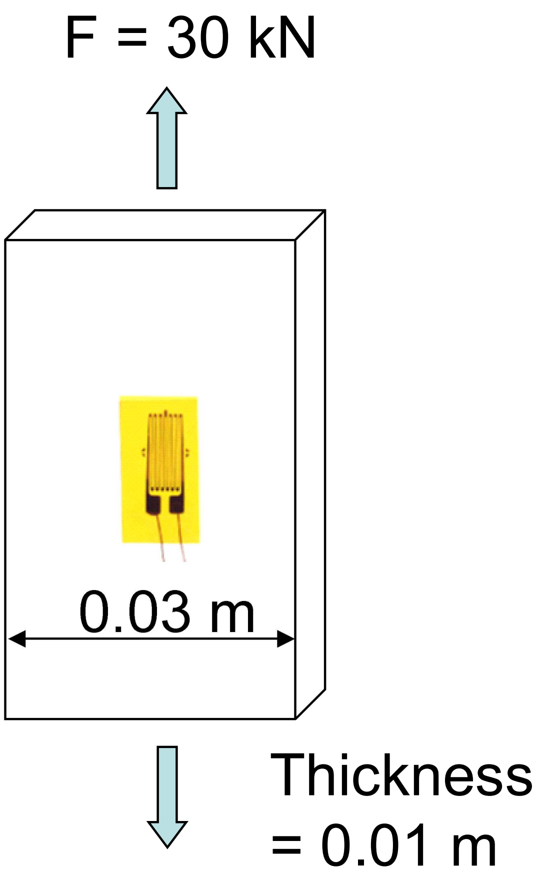



$$\sigma = \frac{F}{A} = \frac{30kN}{(0.03m \times 0.01m)} = 1\times 10^5 kN/m^2$$

$$\varepsilon = \frac{\sigma}{E} = \frac{1\times 10^5 kN/m^2}{200\times 10^6 kN/m^2} = 5\times 10^{-4}$$

$$\Delta R = GF \cdot \varepsilon \cdot R$$

A strain gauge with GF=2.0 is mounted on a steel bar ($E = 200 \times 10^6$ kN/m$^2$) the tensile force of 30 kN. Determine the resistance change of the strain gauge. The resistance of the Gauge was 120 Ω in the absence of the load.




$$\sigma = \frac{F}{A} = \frac{30kN}{(0.03m \times 0.01m)} = 1 \times 10^5 kN/m^2$$

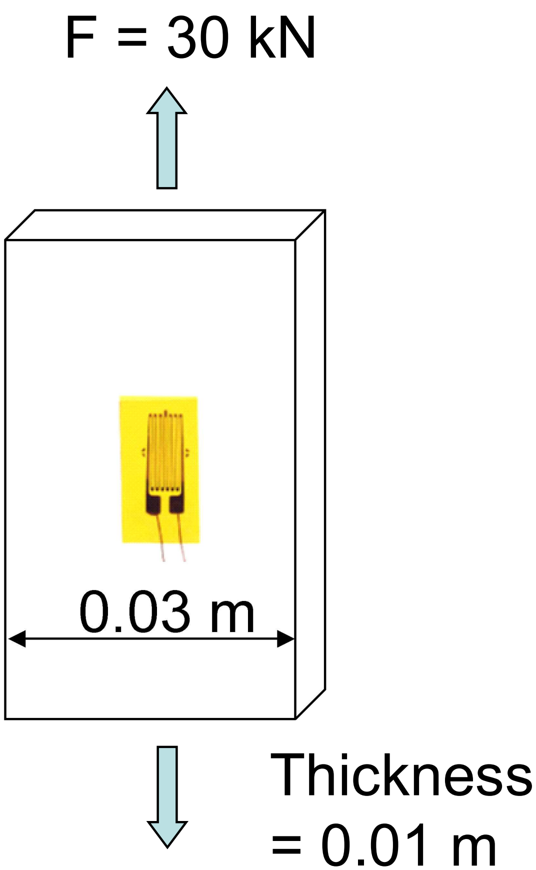
$$\varepsilon = \frac{\sigma}{E} = \frac{1 \times 10^5 kN/m^2}{200 \times 10^6 kN/m^2} = 5 \times 10^{-4}$$

From previous,

$$\Delta R = GF \cdot \varepsilon \cdot R$$

$$\Delta R = \ ?$$

A strain gauge with GF=2.0 is mounted on a steel bar ($E = 200 \times 10^6$ kN/m$^2$) the tensile force of 30 kN. Determine the resistance change of the strain gauge. The resistance of the Gauge was 120 Ω in the absence of the load.




$$\sigma = \frac{F}{A} = \frac{30kN}{(0.03m \times 0.01m)} = 1 \times 10^5 kN/m^2$$

$$\varepsilon = \frac{\sigma}{E} = \frac{1 \times 10^5 kN/m^2}{200 \times 10^6 kN/m^2} = 5 \times 10^{-4}$$

From previous,

$$\Delta R = GF \cdot \varepsilon \cdot R$$

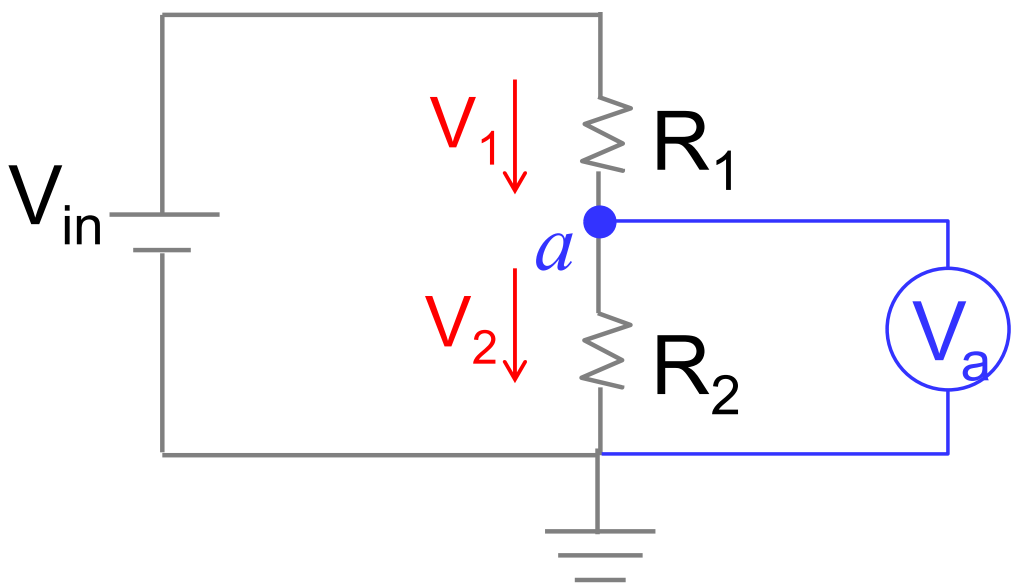
$$\Delta R = 2 \cdot 5 \times 10^{-4} \cdot 120\Omega = 0.12\Omega$$

# Practical strain measurement

- Therefore, you have to accurately measure very small changes in resistance. Issue! →
- In practice strain gage measurement configurations are based on the concept of a **Wheatstone bridge**, a classic circuit designed to measure very small resistive change using a network of four resistive arms, with very **high accuracy and precision**

# Recall: voltage divider

(and Ohm's law: $I = V/R$)




Voltage drops:

$$V_1 = IR_1$$

$$V_2 = IR_2$$

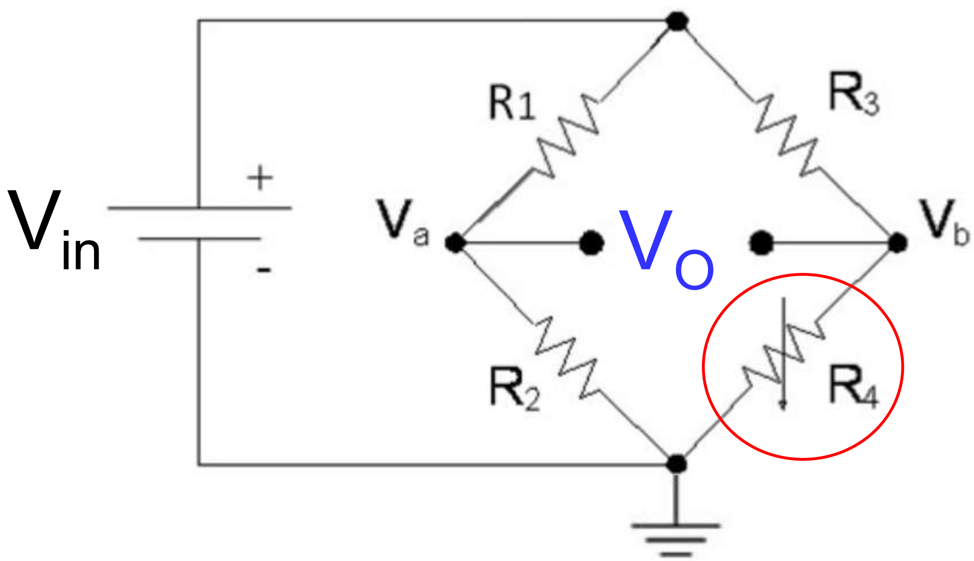
With $\boxed{I = \frac{V_{in}}{R_1 + R_2}}$

Voltage across $R_2$ w.r.t. ground:

$$V_a = V_2 = V_{in}\frac{R_2}{R_1 + R_2} = V_{in}\frac{1}{\frac{R_1}{R_2} + 1}$$

# Wheatstone bridge

## a "**differential resistance measurer**"

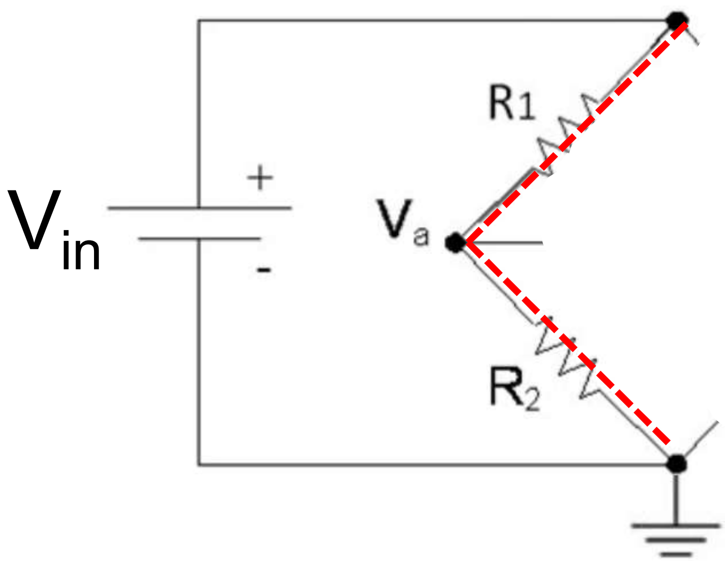



Classic Wheatstone bridge configuration with one variable resistor (this is also called "quarter bridge")

- Four resistors, 3 are of fixed value(s) and 4th is variable known as the sensing element whose resistance changes due to stress, pressure, or temperature
- Externally supplied voltage $V_{in}$
- To be measured output voltage $V_O$

# Wheatstone bridge

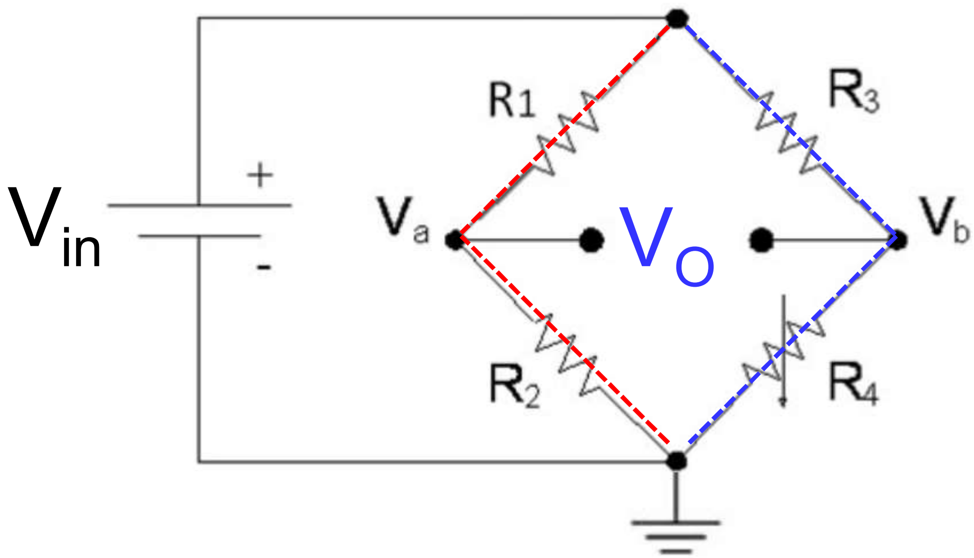
## a "**differential resistance measurer**"




<u>Classic Wheatstone bridge configuration with one variable resistor (this is also called "quarter bridge")</u>

- Four resistors, 3 are of fixed value(s) and 4th is variable known as the sensing element whose resistance changes due to stress, pressure, or temperature
- Externally supplied voltage $V_{in}$
- To be measured output voltage $V_O$

The two dividers in Wheatstone bridge



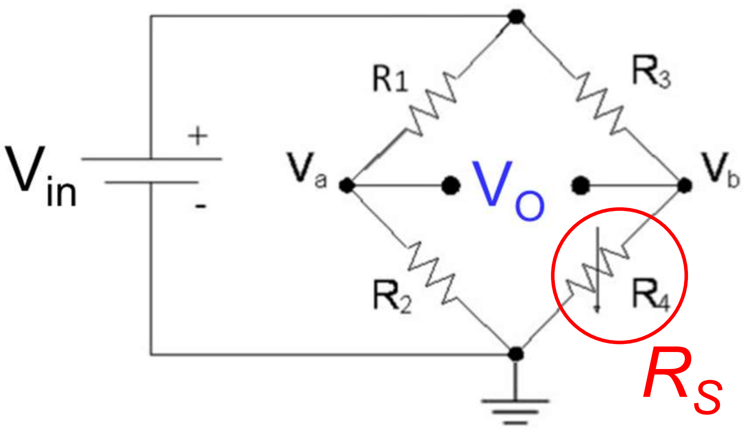

$$V_a = V_{in} \frac{1}{\frac{R_1}{R_2} + 1}$$

$$V_b = V_{in} \frac{1}{\frac{R_3}{R_4} + 1}$$

Therefore,

$$V_{ab} = V_{in} \left( \frac{1}{\frac{R_1}{R_2} + 1} - \frac{1}{\frac{R_3}{R_4} + 1} \right) = V_{in} \left( \frac{R_2}{R_1 + R_2} - \frac{R_4}{R_3 + R_4} \right)$$

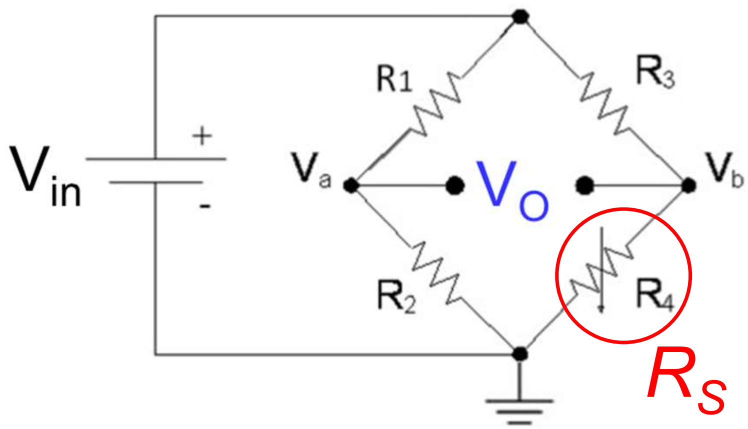
$V_o$

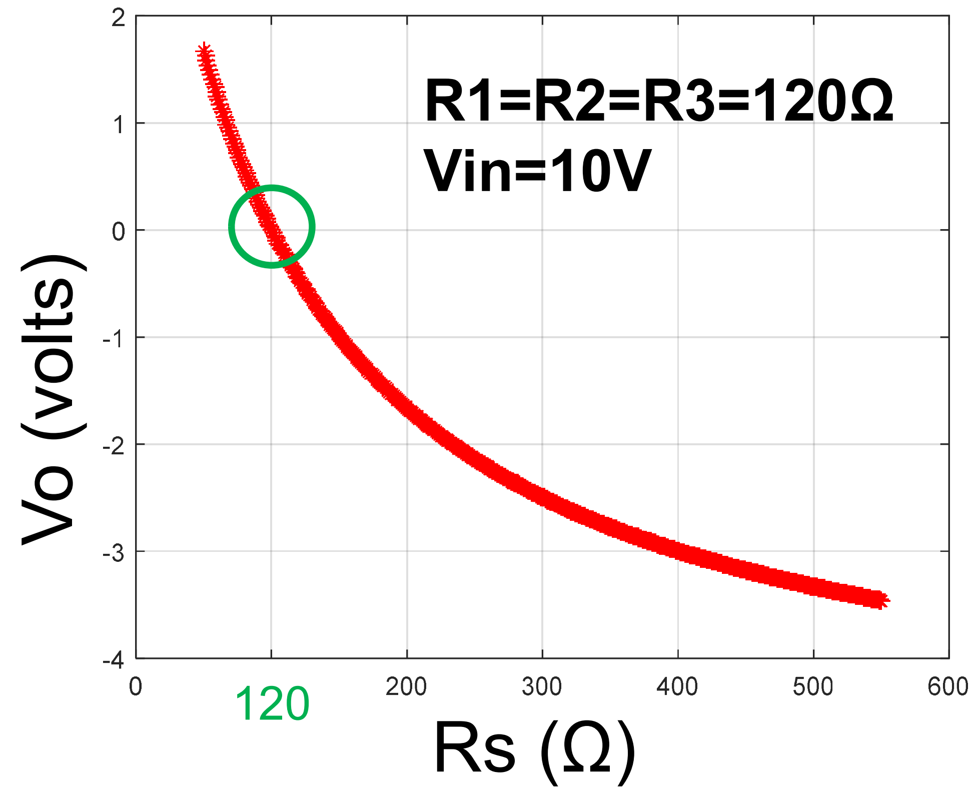



**R1=R2=R3=120Ω**
**Vin=10V**

$$V_O = V_{in}\left(\frac{R_2}{R_1+R_2} - \frac{R_S}{R_3+R_S}\right)$$=0 when Rs= 120 Ω

The bridge is balanced.



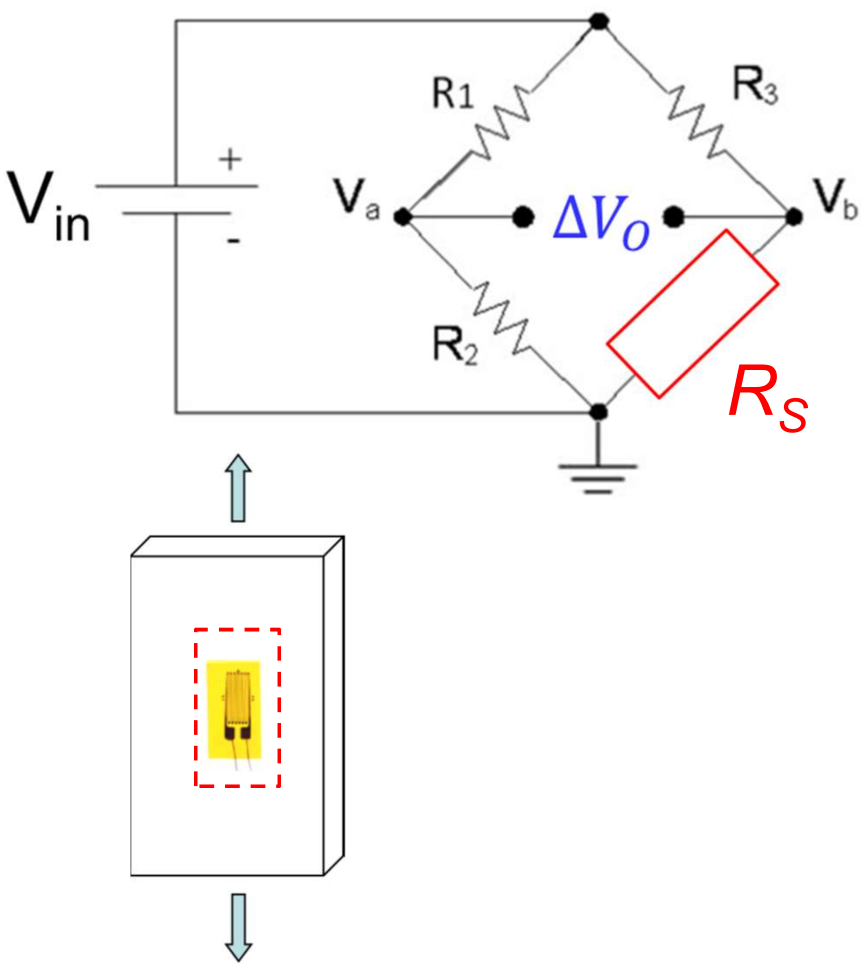

$$V_O = V_{in}\left(\frac{R_2}{R_1+R_2} - \frac{R_S}{R_3+R_S}\right)$$ =0 when Rs= 120 Ω

The bridge is balanced.

In applications, for example the strain gauge in Lab 5, the Rs=120Ω is from the gauge's base resistance and the other resistors will be set at this value. Any change in dimension will cause nonzero voltage output

Example (cont'd)

Now we place the strain gauge in our earlier example here in the quarter bridge arm, as shown below. Assume R1 = R2 = R3 = $R_S$ = R(=120 Ω). Calculate the change of output voltage when input voltage is 10.0 V.




$$V_O = V_{in}\left(\frac{R_2}{R_1 + R_2} - \frac{R_S}{R_3 + R_S}\right)$$

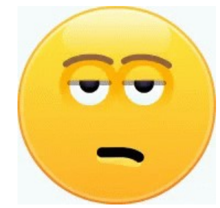
$$= V_{in}\left(\frac{R_2}{R_1 + R_2} - \frac{R + \varepsilon \cdot GF \cdot R}{R_3 + R + \varepsilon \cdot GF \cdot R}\right)$$

$$(\varepsilon = 5 \times 10^{-4})$$

$V_O$=-0.0025 volt

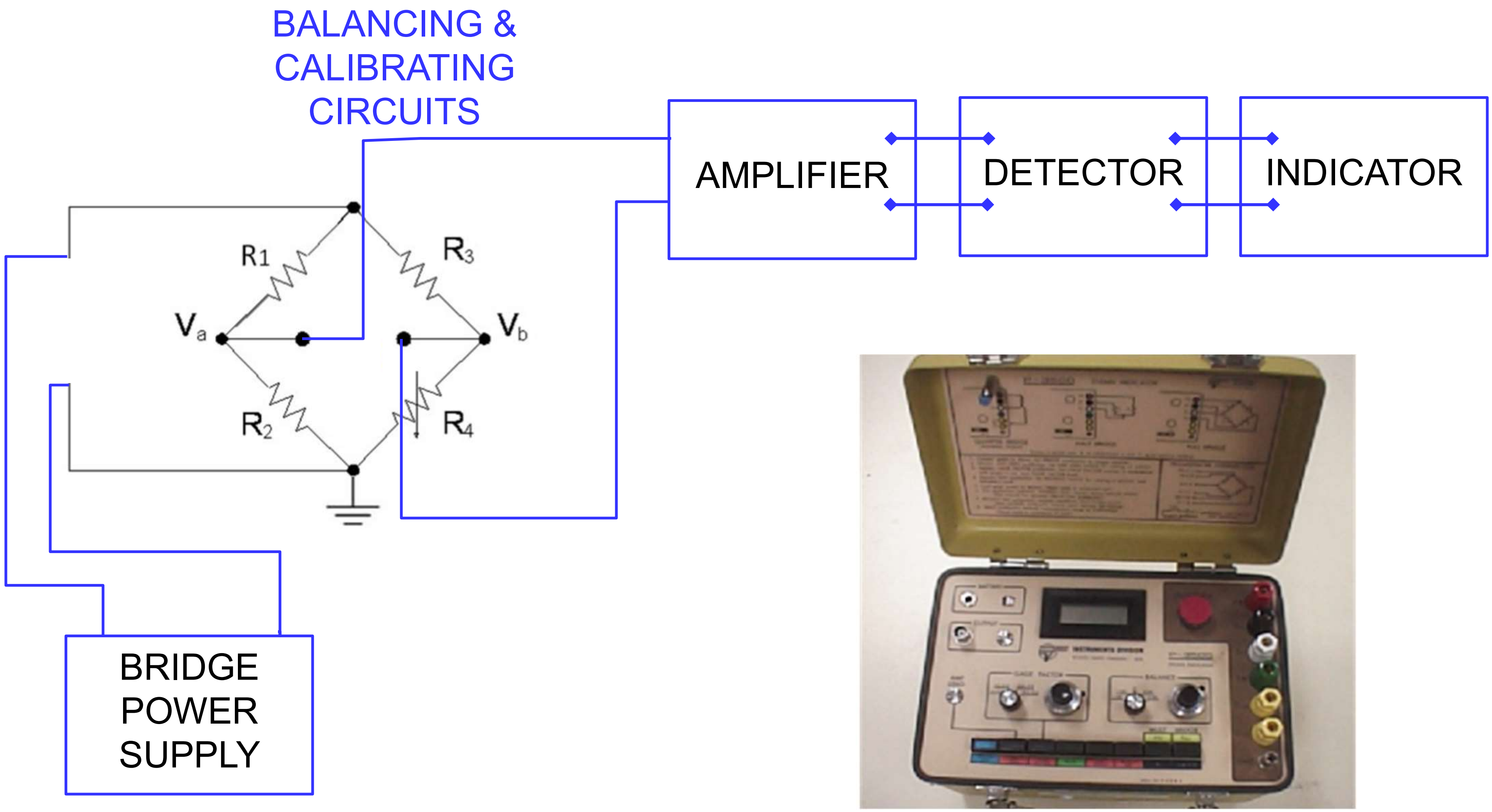
# Strain gauge measurement instrumentation

- We can see the output voltage from Wheatstone bridge is very small
  - Electrical amplification is required to bring the signal to a level where it is useful for reading



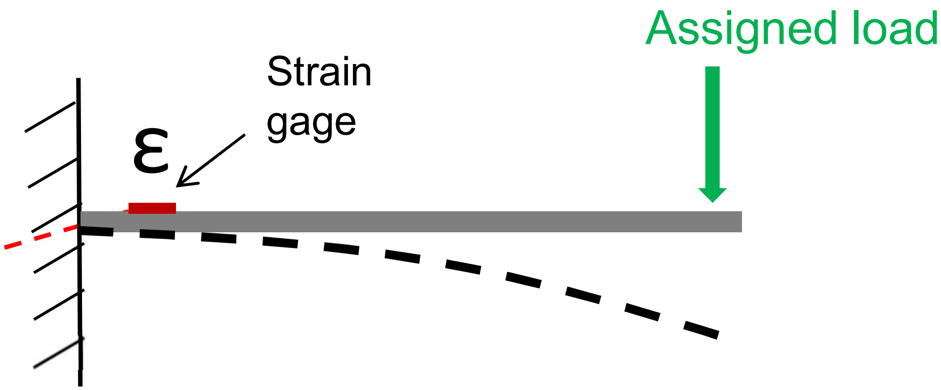
- *In addition*, from power supply (Vin) to recording (Vo) should all be considered as a system (compatibility) to assure satisfactory performance and precision

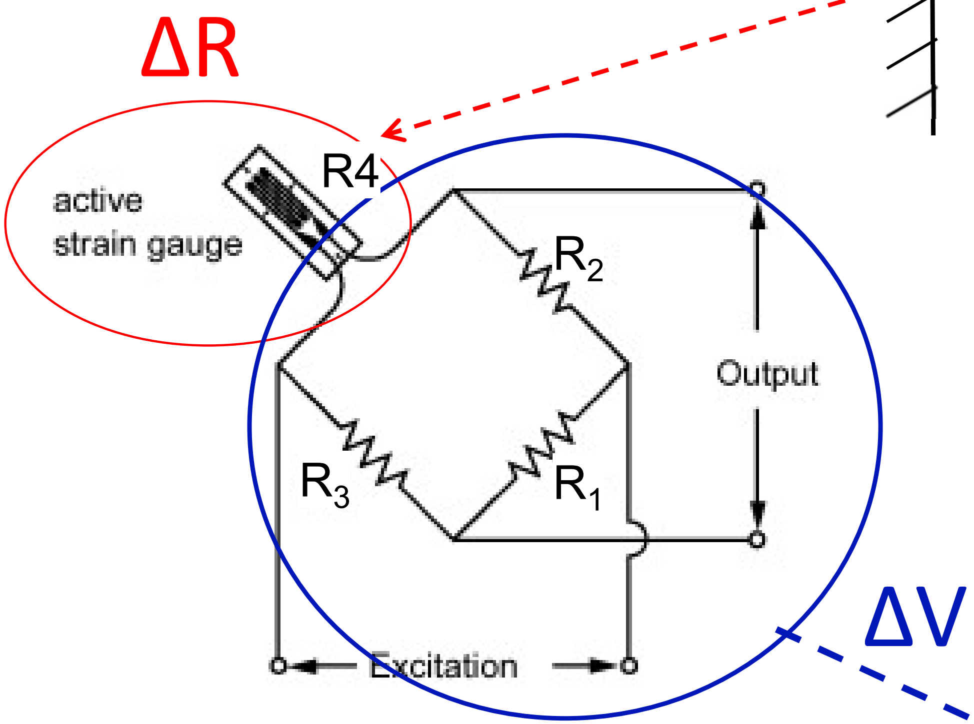
# “Strain gage box”

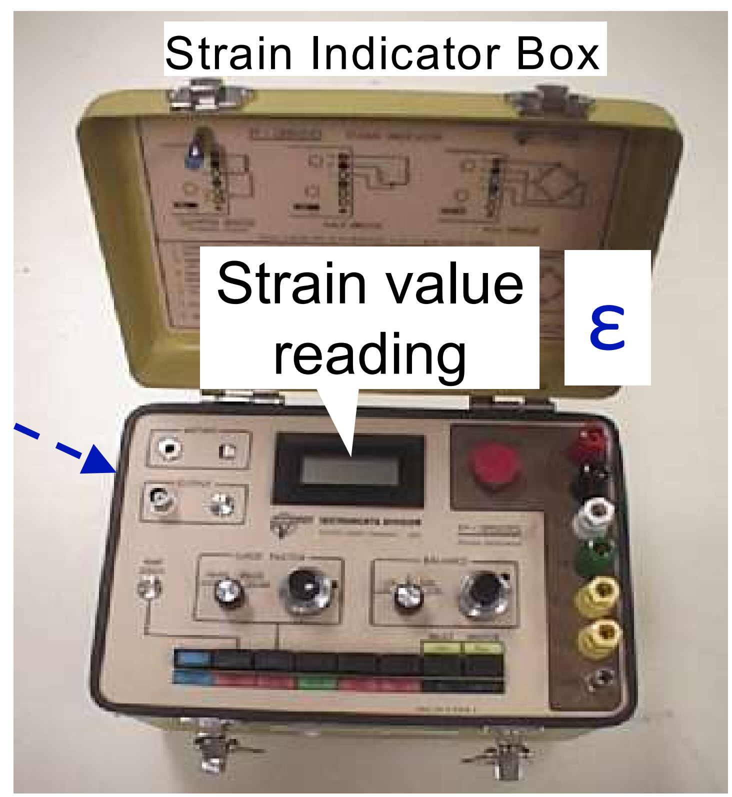


# Lab 5: strain Measurement

## Cantilever beam

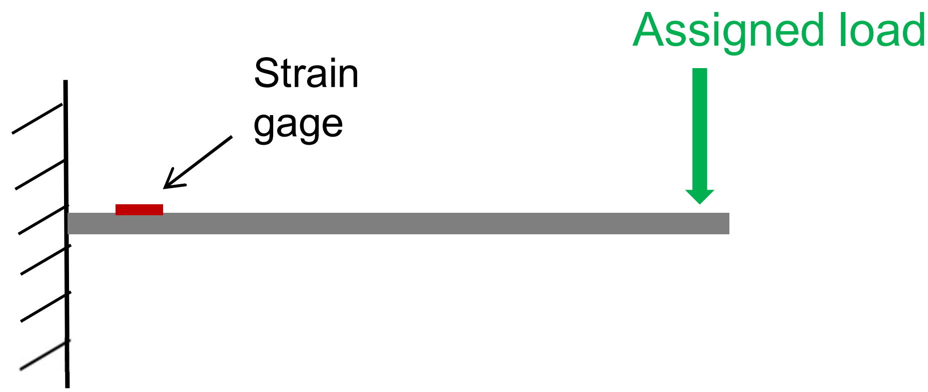


# Lab 3 contents

Cantilever beam




1. A load is applied near the free end by dead weight or displacement or what applicable in the lab
2. Resulted (bending) stress at the location of the strain gage is calculated as ____
3. While the strain at the location of the strain gage is measured as ____

<u>What we do further?</u>

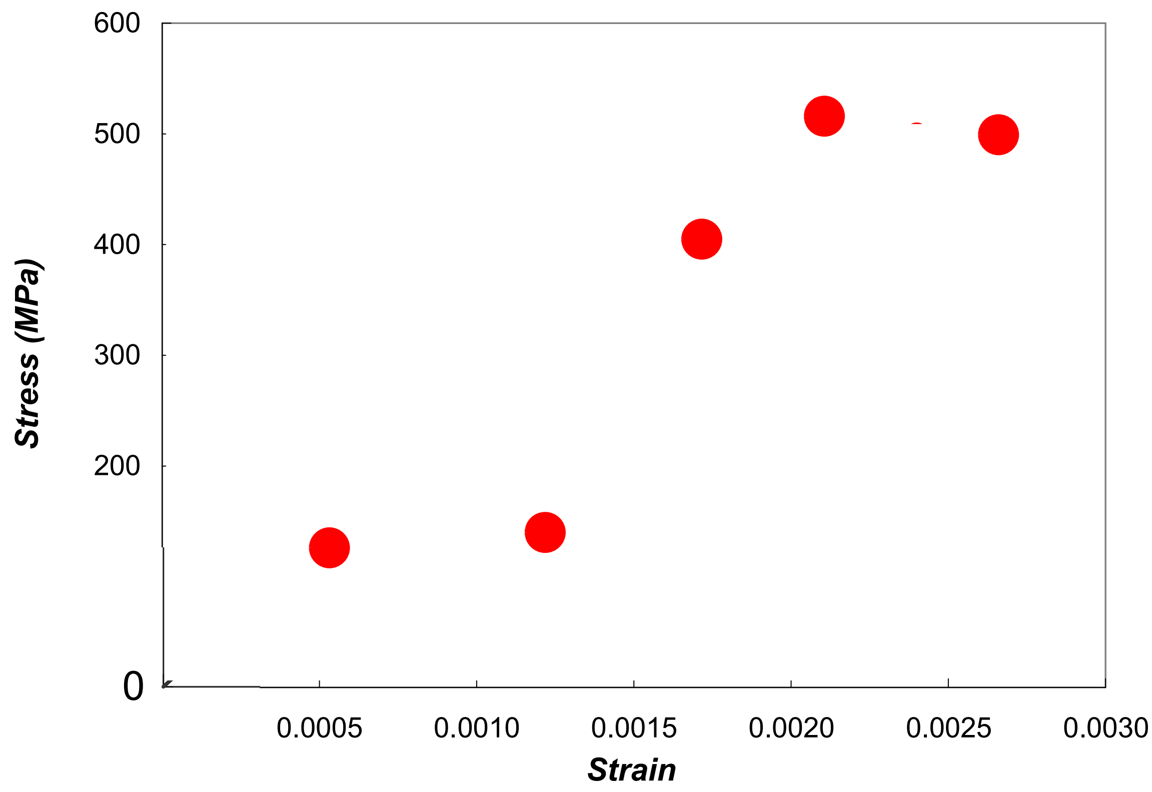
Now: determine beam material's

# Young's modulus

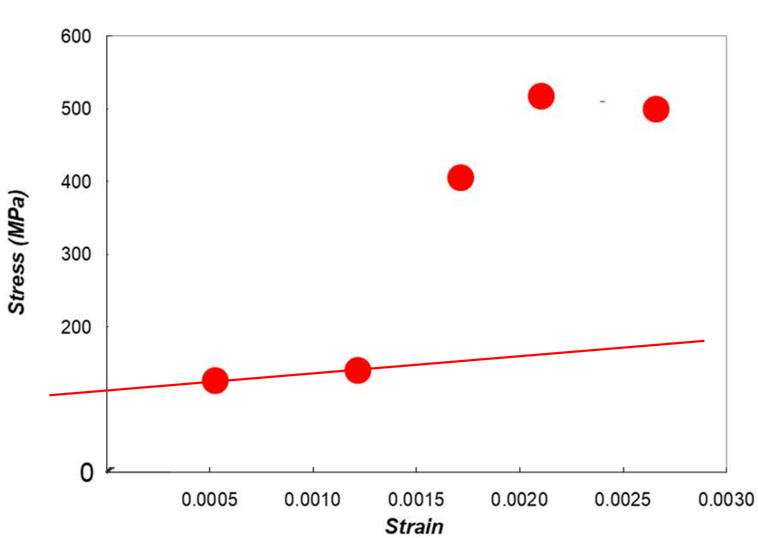
$$\sigma = E\varepsilon$$

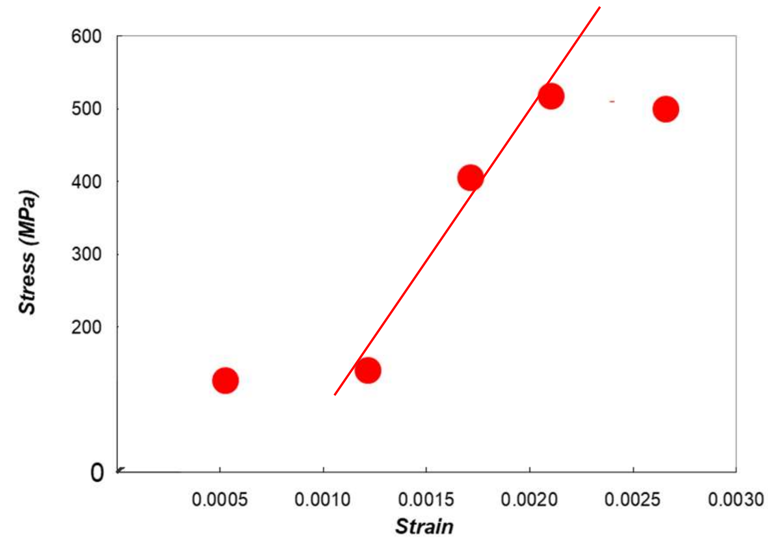
What methods do we have?

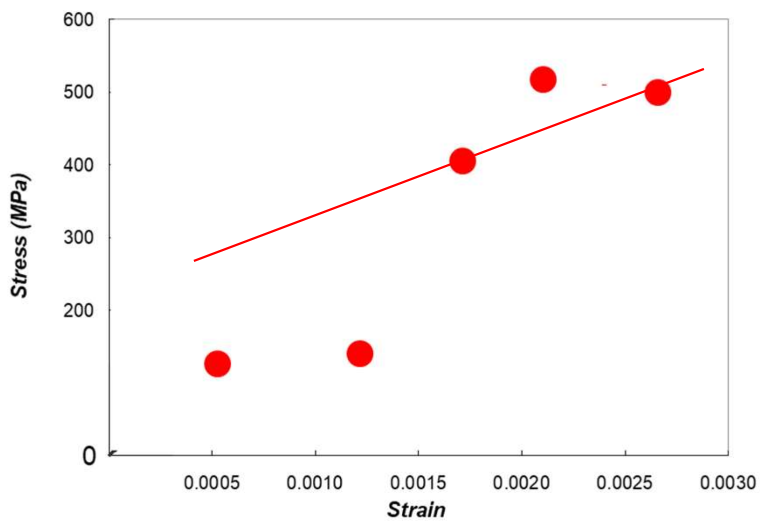
| Strain | Stress (MPa) |
|---|---|
| 0 | 0 |
| 0.00049 | 100 |
| 0.0011 | 200 |
| 0.00155 | 300 |
| 0.0019 | 400 |
| 0.0024 | 500 |

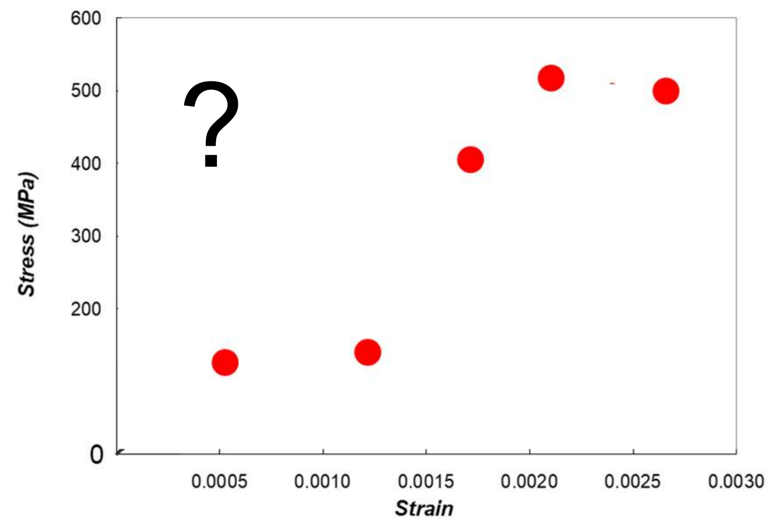



Example


600
500
400
300
200
0
Stress (MPa)
0.0005
0.0010
0.0015
0.0020
0.0025
0.0030
Strain


600
500
400
300
200
0
Stress (MPa)
0.0005
0.0010
0.0015
0.0020
0.0025
0.0030
Strain


600
500
400
300
200
0
Stress (MPa)
0.0005
0.0010
0.0015
0.0020
0.0025
0.0030
Strain

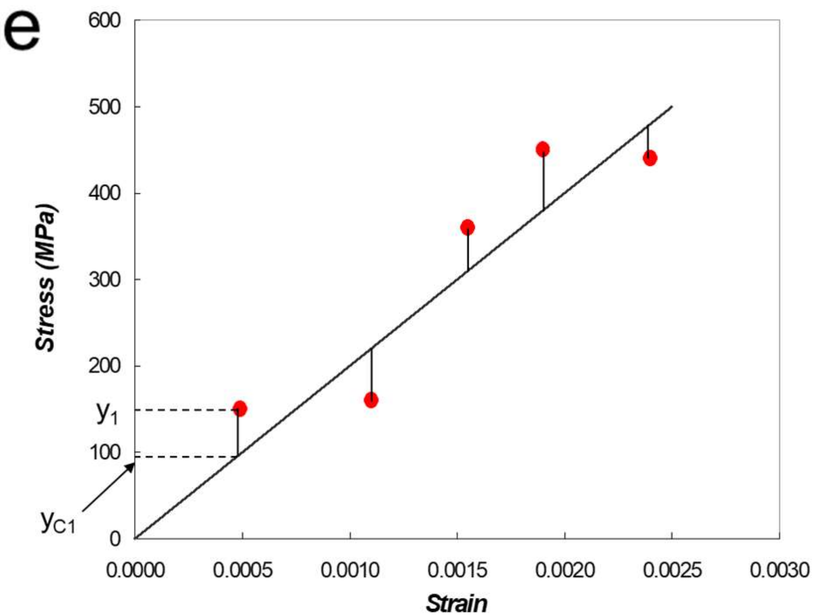

600
500
?
400
300
200
0
Stress (MPa)
0.0005
0.0010
0.0015
0.0020
0.0025
0.0030
Strain

# Least square curve fitting using Excel

- Linear least square method is the most common method of curve fitting
- The goal is trying to fit all the points with a linear line

$$y = ax + b$$

- and choose the line function parameters $(a, b)$ so as to minimize the fitting error in terms of the difference between the data value $y_i$ and the y-values (mathematically correlated results) on the fitted curve.




Minimize $S = \sum_{i=1}^{N} (y_i - y_{Ci})^2$

$$= (y_1 - y_{C1})^2$$
$$+ (y_2 - y_{C2})^2$$
$$+ (y_3 - y_{C3})^2 + \ldots\ldots.$$

# The fitting method

$$S = \sum_{i=1}^{n} \boxed{[y_i - \boxed{(ax_i + b)}]^2}$$

The y value obtained using the assumed relation

$(x_i, y_i)$ your measurements

To minimize a function of multiple variables, we compute the partial derivatives w.r.t. each of the variables and set them equal to zero.

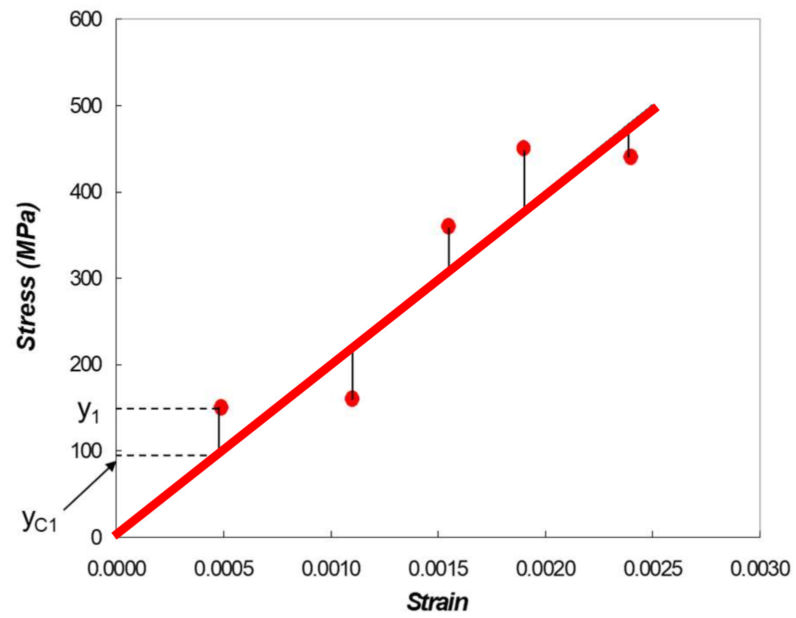
$$\frac{\partial S}{\partial a} = 0 \quad , \quad \frac{\partial S}{\partial b} = 0$$

# The fitting method (cont'd)

Solved:

$$a = \frac{n\sum x_i y_i - \left(\sum x_i\right)\left(\sum y_i\right)}{n\sum x_i^2 - \left(\sum x_i\right)^2}$$

$$b = \frac{\left(\sum y_i\right)\left(\sum x_i^2\right) - \left(\sum x_i y_i\right)\left(\sum x_i\right)}{n\sum x_i^2 - \left(\sum x_i\right)^2}$$



# How good is the fit?

- Visual examination of graph, or
- By "regression analysis" using a correlation coefficient, *R*:

$$\boldsymbol{R} = \left(1 - \frac{\sigma_{y,x}^2}{\sigma_y^2}\right)^{\frac{1}{2}}$$

$$\sigma_y = \left[\frac{\sum_{i=1}^{n}(y_i - y_m)^2}{n-1}\right]^{1/2}$$

$$\sigma_{y,x} = \left[\frac{\sum_{i=1}^{n}(y_i - y_{ic})^2}{n-2}\right]^{1/2}$$

yi – actual data
yic – calculated from the relation

The coefficient R tells how the data set match with a straight line:

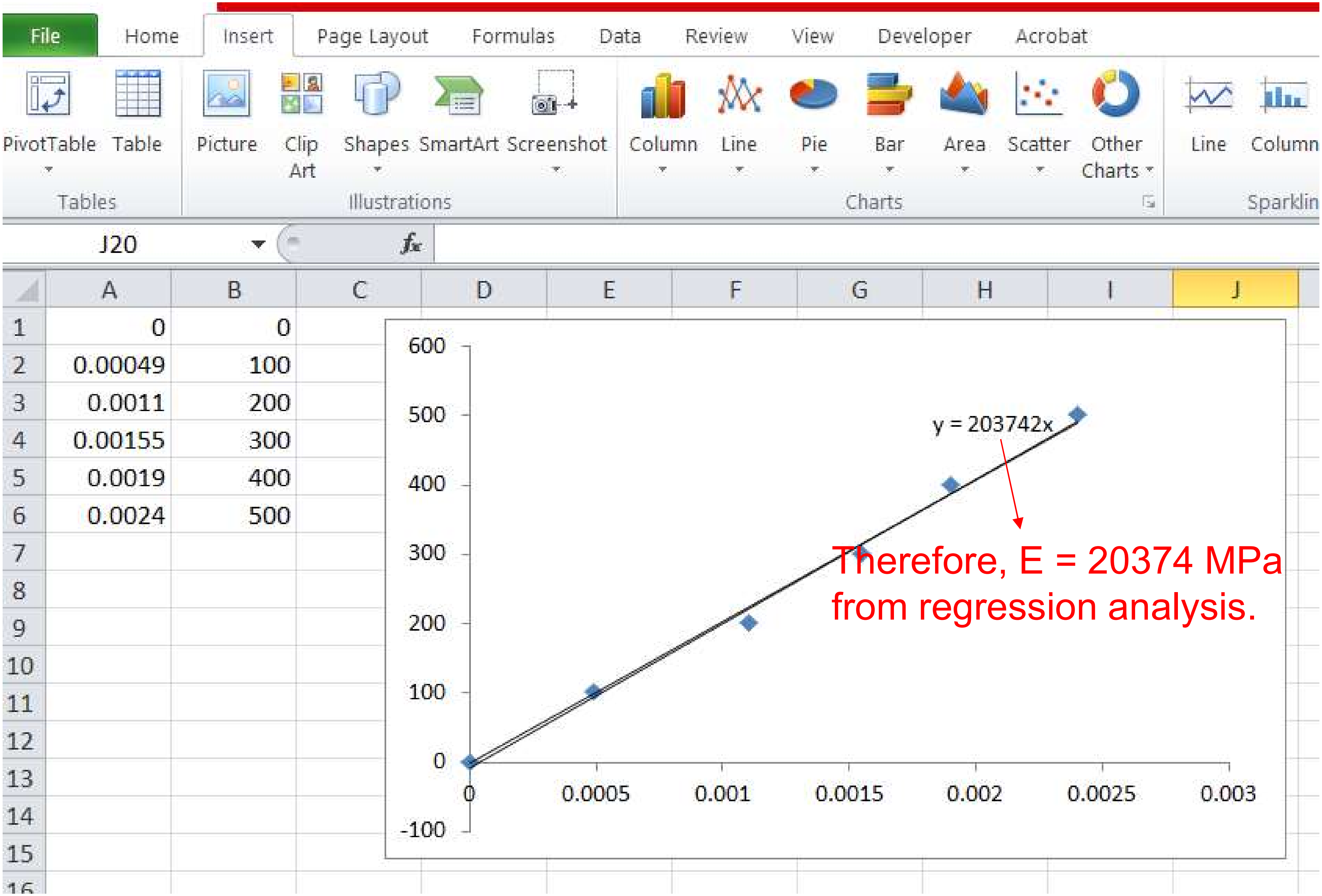
- 0<R<1 in general
- R =1, perfect fit, all variability in data fit into a straight line and hence fully explained by the model
- R = 0, the data are all over the place and hence there is no linear relation between the data
- Good fit: R as close to unity as possible

# Linear curve fitting with Excel

<u>EXCEL procedure</u>

- Enter the data columns in Excel and choose “Scatter with Only Markers” plotting function in Charts from the toolbar
- The plot will show up. [Select the grid and legend and hit “Delete” key, respectively if they show up in the plot.]
- Click data point on chart to select them
- Right click the mouse and select “Add Trendline”’ from the popup menu
- In “Trendline Options”, set “Trend/Regression Type” to be Linear and check Set Intercept and set the value to be 0. And make sure “Display Equation on Chart” is checked. All other use default settings
- Click Close and the linear regression line shows up

*Note: contents in [ ] are optional for appearance only.*




Therefore, E = 20374 MPa from regression analysis.

# Method 2 for deriving E with the experimental data in lab 3

**Alternatively**, Young's modulus can be calculated directly from beam theory

| *Strain* | *Stress (MPa)* | *E (MPa)* |
|---:|---:|---:|
| 0 | 0 | |
| 0.00049 | 100 | 204081.6327 |
| 0.0011 | 200 | 181818.1818 |
| 0.00155 | 300 | 193548.3871 |
| 0.0019 | 400 | 210526.3158 |
| 0.0024 | 500 | 208333.3333 |

- Find mean value, standard deviation
- Write E= ??? ± ??? for a pre-assigned confidence level

# Lab 3 report due time

- Because of Spring Break, lab 3 report is set to be due on 5 PM Wednesday, March 16.

**END of Lecture #10**