Lecture 02: Conduction

Fourier's Law

Form A: $q'' = -k \cdot \nabla T$ ← temp. gradient, $\frac{dT}{dx}$, $\left[\frac{K}{m}, \frac{°C}{m}\right]$

$\frac{dT}{dx}$ ↑ distance of conduction

q'' ↑ heat flux $\left[\frac{W}{m^2}\right]$

$\frac{q}{A}$

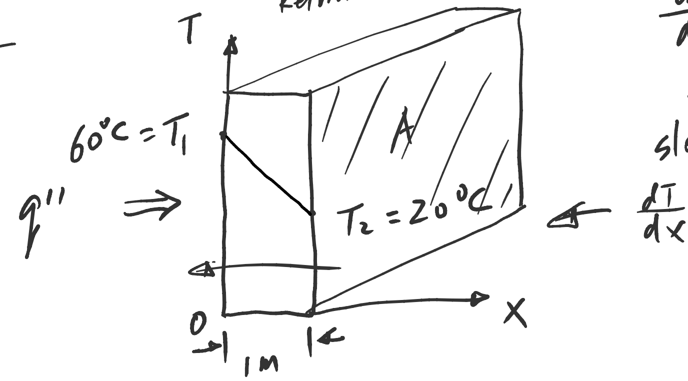
k ↑ thermal conductivity $\left[\frac{W}{m \cdot K}\right]$ or $\frac{W}{m \cdot °C}$

K ↑ Kelvin

$$\frac{dT}{dx} = \frac{T_2 - T_1}{1 - 0} = \frac{20 - 60}{1} = -40 \ \frac{°C}{m}$$

$\frac{dT}{dx}$ ↓ slope

"–" means



q'': heat flux.

CPU → ~ 40 ~ 120 $\frac{W}{cm^2}$

air cooling < 10 $\frac{W}{cm^2}$

heat spreader
Cu, Al, diamond film,
graphite sheet

k: thermal conductivity.

$k_{Cu} \sim 380 \ \frac{W}{m \cdot K}$

$k_{air} \sim 0.06 \ \frac{W}{m \cdot K}$

$k_{Al} \sim 170 \ \frac{W}{m \cdot K}$

$k_{diamond} \sim 1000 \ \frac{W}{m \cdot K}$

graphite/graphene $\sim 500 \ \frac{W}{m \cdot K}$ ~ $2000 \ \frac{W}{m \cdot K}$

$5000 \ \frac{W}{m \cdot K}$

$k_{water} \sim 0.6 \ \frac{W}{m \cdot K}$

$K_{ss} \sim 55 \ \frac{W}{m \cdot K}$

∇T : temp. gradient

$$\underline{\underline{\nabla T}} = \frac{dT}{dX} \quad \frac{^\circ C}{m} \equiv \frac{K}{m}$$

dT → temp. diff.

dX → distance

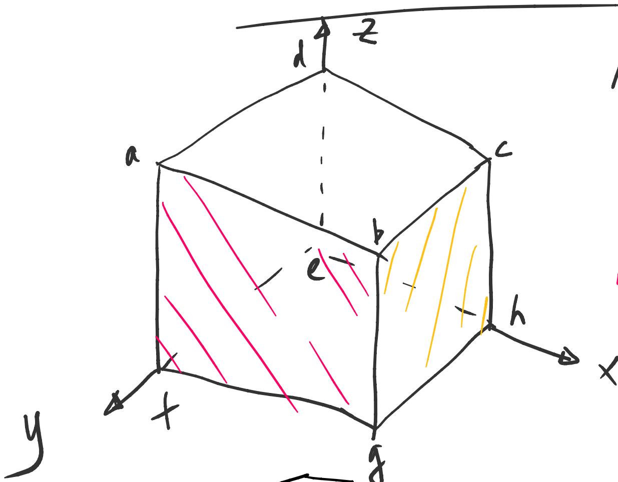
temp. diff. $T_1 - T_2 = 10^\circ C$

$$(T_1 + 273.15) - (T_2 + 273.15) = T_1 - T_2 = 10 \ K$$

Form B: $q = -K \cdot A \cdot \nabla T$

A ↑ heat transfer area for conduction.

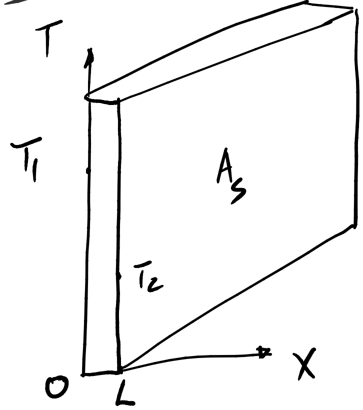
A is normal to ∇T



A_x = $Area_{bchg}$ or $Area_{adef}$

A_y = $Area_{abgf}$

e.g. Case I:



K

$T_1 > T_2$

$$q = -KA\frac{dT}{dx}$$

$$= -K \cdot A_s \cdot \frac{T_2 - T_1}{L - 0}$$

$$= K \cdot A_s \cdot \frac{T_1 - T_2}{L} \quad > 0$$

-distance

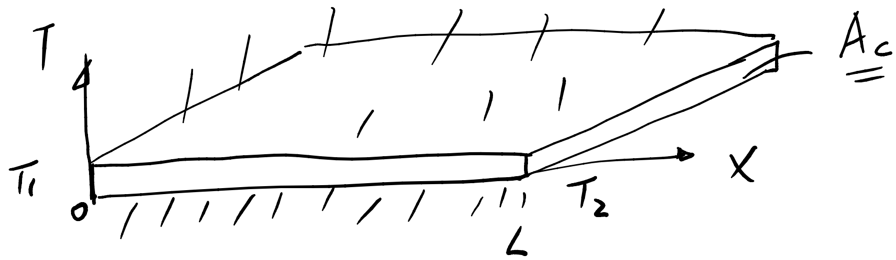
1-D.

⇒ 1D. const K, w/o q''' internal heat generation

$$q = K \cdot A \cdot \frac{\Delta T}{L}$$

$q = k \cdot A \cdot \frac{\quad}{L}$

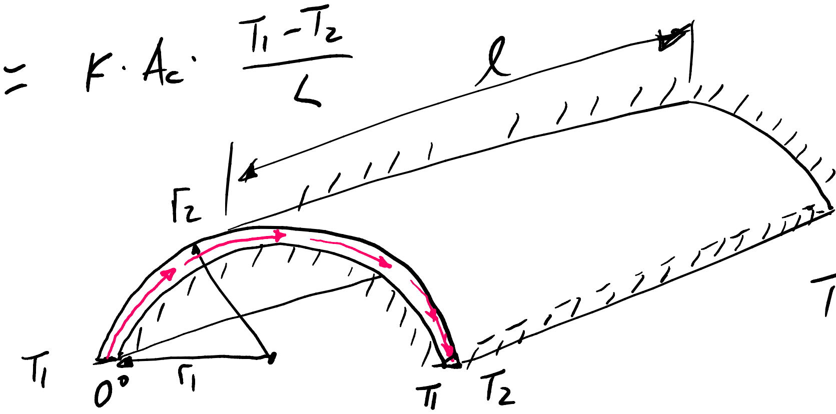
Case II:



$T_1 > T_2$

$$q = K \cdot A_c \cdot \frac{T_1 - T_2}{L}$$

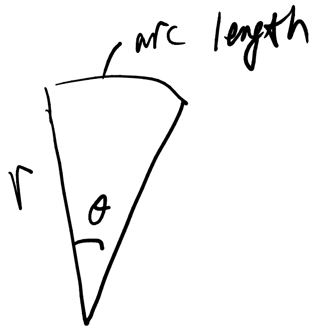
Case III



$T_1 > T_2$

K

$$q = -K \cdot A \cdot \frac{dT}{dS} = -K \cdot \underbrace{(r_2 - r_1)}\cdot \ell \cdot \frac{T_2 - T_1}{\underbrace{\left[r_1 + \frac{r_1 + r_2}{2}\right] \cdot \pi}_{\text{arc length}}}$$



$r \cdot \theta$

if $r_2 - r_1$ is small

r_1 or r_2

T: linear Temp distribution, Temp as a function of location.

$T(x) = ax + b$

if. $T(x) = ax^2 + bx + c$ [°C]

i> a, b, c units

$\left[\frac{°C}{m^2}\right]$ $\left[\frac{°C}{m}\right]$ [°C]

ii> K is given

ii) K is given

$q''(x)$ heat flux distribution.

$$q'' = -K \cdot \frac{dT}{dx} \quad \checkmark$$

$$\frac{dT(x)}{dx} = \frac{d}{dx}\left(ax^2 + bx + c\right) = 2ax + b$$

$$\Rightarrow q''(x) = -K(2ax + b)$$

$$°C = \frac{5}{9} \cdot °F - 32$$