COMP 157 Assignment 2

Exercises 2.1:

4. a. *Glove selection* There are 22 gloves in a drawer: 5 pairs of red gloves, 4 pairs of yellow, and 2 pairs of green. You select the gloves in the dark and can check them only after a selection has been made. What is the smallest number of gloves you need to select to have at least one matching pair in the best case? in the worst case? (after [Mos01], #18)

Exercises 2.2:

3. For each of the following functions, indicate the class $\Theta(g(n))$ the function belongs to. (Use the simplest $g(n)$ possible in your answers.) Prove your assertions.

   a. $(n^2+1)^{10}$

   b. $\sqrt{10n^2+7n+3}$

   c. $2n \lg(n+2)^2 + (n+2)^2 \lg \frac{n}{2}$

   d. $2^{n+1} + 3^{n-1}$

   e. $\lfloor \log_2 n \rfloor$

9. We mentioned in this section that one can check whether all elements of an array are distinct by a two-part algorithm based on the array's presorting.

   a. If the presorting is done by an algorithm with the time efficiency in $\Theta(n \log n)$, what will be the time efficiency class of the entire algorithm?

   b. If the sorting algorithm used for presorting needs an extra array of size n, what will be the space efficiency class of the entire algorithm?

Exercises 2.3:

1. Compute the following sums.

   a. $1 + 3 + 5 + 7 + ... + 999$

   b. $2 + 4 + 8 + 16 + ... + 1024$

   c. $\sum_{i=3}^{n+1} 1$ d. $\sum_{i=3}^{n+1} i$ e. $\sum_{i=0}^{n-1} i(i+1)$

   f. $\sum_{j=1}^{n} 3^{j+1}$ g. $\sum_{i=1}^{n} \sum_{j=1}^{n} ij$ h. $\sum_{i=0}^{n-1} 1/i(i+1)$

2. Find the order of growth of the following sums.

   a. $\sum_{i=0}^{n-1} (i^2+1)^2$ b. $\sum_{i=2}^{n-1} \lg i^2$

   c. $\sum_{i=1}^{n} (i+1)2^{i-1}$ d. $\sum_{i=0}^{n-1} \sum_{j=0}^{i-1} (i+j)$

   Use the $\Theta(g(n))$ notation with the simplest function $g(n)$ possible.

5. Consider the following algorithm.

   Algorithm *Secret*$(A[0..n-1])$
   //Input: An array $A[0..n-1]$ of n real numbers
   $minval \leftarrow A[0]$; $maxval \leftarrow A[0]$
   for $i \leftarrow 1$ **to** $n-1$ **do**
   if $A[i] < minval$
   $minval \leftarrow A[i]$
   if $A[i] > maxval$
   $maxval \leftarrow A[i]$
   return $maxval - minval$

   Answer questions a–e of Problem 4 about this algorithm.

a. What does this algorithm compute?

b. What is its basic operation?

c. How many times is the basic operation executed?

d. What is the efficiency class of this algorithm?

e. Suggest an improvement, or a better algorithm altogether, and indicate its efficiency class. If you cannot do it, try to prove that, in fact, it cannot be done.

Exercises 2.4:

1. Solve the following recurrence relations.

   a. $x(n) = x(n-1) + 5 \quad \text{for } n > 1, \quad x(1) = 0$

   b. $x(n) = 3x(n-1) \quad \text{for } n > 1, \quad x(1) = 4$

   c. $x(n) = x(n-1) + n \quad \text{for } n > 0, \quad x(0) = 0$

   d. $x(n) = x(n/2) + n \quad \text{for } n > 1, \quad x(1) = 1 \quad (\text{solve for } n = 2^k)$

   e. $x(n) = x(n/3) + 1 \quad \text{for } n > 1, \quad x(1) = 1 \quad (\text{solve for } n = 3^k)$

8. Consider the following recursive algorithm.

   Algorithm *Min1*$(A[0..n-1])$
   //Input: An array $A[0..n-1]$ of real numbers
   if $n = 1$ **return** $A[0]$
   else $temp \leftarrow$ *Min1*$(A[0..n-2])$
   if $temp \leq A[n-1]$ **return** $temp$
   else return $A[n-1]$

   a. What does this algorithm compute?

   b. Set up a recurrence relation for the algorithm's basic operation count and solve it.

9. Consider another algorithm for solving the same problem as the one in Problem 8 which recursively divides an array into two halves:

call $Min2(A[0..n-1])$ where

Algorithm $Min2(A[l..r])$
if $l = r$ **return** $A[l]$
else $temp1 \leftarrow Min2(A[l..\lfloor (l+r)/2 \rfloor])$
$temp2 \leftarrow Min2(A[\lfloor (l+r)/2 \rfloor + 1..r])$
if $temp1 \leq temp2$ **return** $temp1$
else return $temp2$

a. Set up a recurrence relation for the algorithm's basic operation and solve it.

b. Which of the algorithms *Min1* or *Min2* is faster? Can you suggest an algorithm for the problem they solve that would be more efficient than either of them?

Submission Requirements:

- Submit your answers via Canvas.
- All submissions must be typeset. No handwritten work will be accepted.
- Word or PDF formats are preferred. If submitting documents in another format, include a separate text note indicating tools needed to read the document.