PHYS7721 Homework 7

Ryan Carelli

Due Thursday May 4, 2017

Exercise 1. Exercise 7.5.

Solution.

(a) Start with the equations for these relations,

$$\kappa_T = \frac{1}{n}\Big(\frac{\partial n}{\partial P}\Big)_T$$

$$\kappa_S = \frac{1}{n}\Big(\frac{\partial n}{\partial P}\Big)_z$$

From 7.1.8 we borrow the result that when $N_0 << N$, $n = aT^{3/2}g_{3/2}(z)$, which implies

$$dn = aT^{3/2}\Big(\frac{1}{z}g_{1/2}(z)\Big)dz + \frac{3}{2}aT^{1/2}g_{3/2}(z)dT$$

and using that $P = cT^{5/2}g_{5/2}(z)$,

$$dP = cT^{5/2}\Big(\frac{1}{z}g_{3/2}(z)\Big)dz + \frac{5}{2}cT^{3/2}g_{5/2}(z)dT$$

which on evaluation of the fraction and letting $c = ak$ gives the result. As $z \to 1$, κ_T diverges.

(b) Note

$$P = \frac{2U}{3V}$$

$$\Big(\frac{\partial P}{\partial T}\Big)_V = \frac{2C_V}{3V}$$

$$C_P - C_V = TV\frac{1}{nkT}\frac{g_{1/2}(z)}{g_{3/2}(z)}\frac{4C_V^2}{9V^2} = \frac{4C_V^2 g_{1/2}(z)}{9Nkg_{3/2}(z)}$$

which gives the relation desired.

□

Exercise 2. Exercise 7.11.

Solution.

$$N_e = (N_e)_0 + (N_e)_1 = \frac{V}{\lambda^3}g_{3/2}\Big(e^{\frac{\mu}{kT}}\Big) + \frac{V}{\lambda^3}g_{3/2}\Big(e^{\frac{\mu-\epsilon_1}{kT}}\Big)$$

Which gives the constraint for the critical temperature,

$$\frac{V}{\lambda^3}g_{3/2}(1) + \frac{V}{\lambda^3}g_{3/2}(e^{-\epsilon_1/kT_c}) = N$$

Taking $x << 1$ we get,

$$\lambda_c \approx \frac{V}{N}(\zeta(3/2) + x)$$

$$\frac{T_c}{T_0} \approx 1 - \frac{2/3}{\zeta(3/2)}x$$

on approximation and expansion with $x \leq 1$.

$$\lambda_c^3 \approx \frac{2V}{N}(\zeta(3/2) - \sqrt{\pi}\sqrt{(\epsilon_1/kT_c)} \implies \frac{T_c}{T_c^0} \approx 2^{-2/3}\Big(1 + \frac{2}{3}\frac{\pi^{1/2}}{\zeta(3/2)}2^{1/3}\frac{\epsilon_1^{1/2}}{(kT_c^0)^{1/2}}\Big)$$

□

Exercise 3. Exercise 7.14

Solution. We want to start with the density of states,

$$a(\epsilon)d\epsilon = \frac{V}{h^n}\frac{2\pi^{n/2}}{\delta(n-2)}p^{n-1}dp = \frac{V}{h^n}\frac{2\pi^{n/2}}{sA^{n/s}\delta(n/2)}\epsilon^{(n/s)-1}d\epsilon$$

Which gives upon the integration given in the book,

$$\frac{V}{h^n}\frac{2\pi^{n/2}\delta(n/s)}{sA^{n/s}\delta(n/2)}(kT)^{n/s}g_{n/s}(z)$$

$$P = \frac{1}{h^n}\frac{2\pi^{n/2}\delta(n/s)}{sA^{n/s}}(kT)^{n/s+1}g_{n+s+1}(z)$$

$$U = kT^2\Big(\frac{\partial}{\partial T}\frac{PV}{kT}\Big) = \frac{n}{s}PV \implies P = \frac{sU}{nV}$$

and we get Bose-Einstein condensation when $z \to 1$ at finite temperature. So $n > s$ in which case

$$N = \frac{V}{h^n}\frac{2\pi^{n/2}\partial(n/s)}{sA^{n/s}\delta(n/2)}(kT_c)^{n/s}\zeta(n/s)$$

Now for specific heat,

$$\frac{1}{2}\Big(\frac{\partial z}{\partial T}\Big)_v = -\frac{n}{s}\frac{1}{T}\frac{g_{n/s}(z)}{g_{n/s-1}(s)}$$

$$\frac{C_V}{Nk} = \frac{n}{s}(\frac{n}{s}+1)\frac{g_{n/s+1}(z)}{g_{n/s}(z)} - \frac{n^2}{s^2}\frac{g_{n/s}(z)}{g_{n/s-1}(s)}$$

$$\frac{C_P}{Nk} = \Big(\frac{n}{s}+1\Big)^2\frac{(g_{n/s+1}(z)^2 g_{n/s-1}(z)}{g_{n/s}(z)^3} - \frac{n}{s}(\frac{n}{s}+1)\frac{g_{n/s+1}(z)}{g_{n/s}(z)}$$

The desired limiting case for C_V and C_P just comes from noting the limiting case of g as $T \to \infty$ which cancels everything but the n/s and $n/s+1$ then we multiply by Nk. □