# Electrodynamics Final Exam Study Guide

Exam Date: May 5, 2016

## 1. General Information

Write down the different geometrical curls. Write down the different geometrical divergences. Write down the different volume elements.

## 2. Electrostatics

Coulomb's Law:

$$\bar{F}_{01} = \frac{1}{4\pi\epsilon_0}\frac{q_0 q_1}{r_{01}^2}\hat{r_{01}}$$

Electric Field Definition:

$$\bar{E} = \lim_{q_0 \to 0} \frac{F_0}{q_0}$$

The units of an electric field are $\frac{N}{C}\frac{V}{m}$.
We have three possible charge elements:

$$dq = \lambda dl = \sigma da = \rho dV$$

Gauss' Law:

$$\phi_E = \frac{q_{in}}{\epsilon_0} \implies \int_S \bar{E} da = \frac{q_{enc}}{\epsilon_0}$$

Vector Potential:

$$V(\bar{r}) = -\int_{\bar{r}_1}^{\bar{r}_2} \bar{E} \cdot dl \implies E = -\nabla V$$

Induced Surface Charge Density:

$$\sigma(\theta) = -\epsilon_0 \frac{\partial V}{\partial r}$$

## 3. Laplace Equation

Getting rid of e:

$$e^{kx} + e^{-kx} = 2cosh(kx)$$

2D Cartesian General Solution:

$$V(x,y) = (Ae^{kx} + Be^{-kx})(Csin(ky) + Dcos(ky))$$

2D Spherical General Solution (Azimuthal Symmetry):

$$V(r,\theta) = \sum_{l=0}^{\infty} \left( A_l r^l + \frac{B_l}{r^{l+1}} P_l(cos\theta) \right)$$

where the $P_l$'s are the Legendre Polynomials defined by

$$P_l(x) = \frac{1}{2^l l!}\left(\frac{d}{dx}\right)^l (x^2-1)^l$$

## 4. Multipole Expansion

Generalized Multipole:

$$V(\bar{r}) = \frac{1}{4\pi\epsilon_0}\sum_{n=0}^{\infty}\frac{1}{r^{n+1}}\int (r')^n P_n(cos\alpha)\rho(r')d\tau$$

Monopole Term:

$$V_{mon}(\bar{r}) = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}$$

Dipole Term:

$$V_{dip}(\bar{r}) = \frac{1}{4\pi\epsilon_0}\frac{\bar{p}\cdot\hat{r}}{r^2}$$

## 5. Polarization

Atomic Polarizability:

$$\bar{p} = \alpha\bar{E}$$

Torque Dipole:

$$\bar{N} = \bar{p}\times\bar{E}$$

Force Dipole

$$\bar{F} = (\bar{p}\cdot\nabla)\bar{E}$$

Surface Bound Charge:

$$\sigma_b = \bar{P}\cdot\hat{n}$$

Volume Bound Charge:

$$\rho_b = -\nabla\cdot\bar{P}$$

Electric Displacement:

$$\nabla\cdot(\epsilon_0\bar{E}+\bar{P}) = \rho_f \implies \bar{D} = \epsilon_0\bar{E}+\bar{P} \implies \int \bar{D}\cdot da = Q_{f_{enc}}$$

## 6. Linear Dielectrics

Polarization:

$$\bar{P} = \epsilon_0\chi_e E$$

D Field:

$$\bar{D} = \epsilon\bar{E} = \epsilon_0(1+\chi_e)\bar{E}$$