

WOLVERINE



The Wolverine Worldwide (a shoe company in Rockford) improperly disposed of chemicals (PFAS), which have leaked into the ground water. The state of Michigan says that if more than 7% of households in a city exceed the safe limit of chemicals, the city needs to switch to bottled water. A concerned citizen takes a random sample of 100 households and finds that 12 have unsafe water. Do the data provide convincing evidence that Rockford should switch to bottled water?

1. State appropriate hypotheses for performing a significance test. Use $\alpha = 0.05$.

p = proportion of households
with unsafe amt. of
chemicals

$$H_0 : p = .07$$

$$H_a : p > .07$$

2. (a) Conduct a significance test (aka a hypothesis test), find the P-value, and interpret this value. You may assume that conditions and assumptions hold, and do not need to check them.

$$\begin{aligned} x &= 12 & \hat{p} &= .12 \\ n &= 100 & SD(\hat{p}) &= \sqrt{\frac{.07 \cdot .93}{100}} \\ p &= .07 & \alpha &= .05 \end{aligned}$$

$$1 - \text{pnorm}(\hat{p}, p, SD(\hat{p})) = .025$$

= p-value

- (b) State a conclusion. Should Rockford keep the current water or switch to bottled water? Explain.

- (c) Let's suppose this decision is wrong. What would be a consequence of this error?

- (d) If the water is actually safe, what is the probability that this error will occur?

3. (a) Now suppose the p-value you got for the test in question 2 was 0.217. Would you reject H_0 or fail to reject H_0 ?

(b) Let's suppose this decision is wrong. What would be a consequence of this error?

4. Are the consequences in question #2 or question #3 more serious?

Type I and Type II Errors

		Truth about the population	
		H_0 true	H_a true
Conclusion based on sample	Reject H_0		
	Fail to reject H_0		

Type I Error:

Type II Error:

Power (of a test):

2. The flu vaccine for a given year is found to be 55% effective. A new flu vaccine is developed and it is time to do a clinical trial on its effectiveness, as compared to the existing vaccine. The null and alternative hypotheses are as follows, and they use an alpha of 0.01.

$$H_0: p = 0.55$$

$$H_a: p > 0.55$$

- a. Describe a Type I and Type II error in this scenario, and their consequences. Which is the worse type of error here?
- b. What is the probability of a Type I error occurring?
- c. Which clinical trial has higher **power**: a trial involving 100 participants, or a trial involving 1000 participants?
- d. How could the researchers adjust alpha to reduce the probability of a Type II error, and what is the downside to that?
- e. Discuss how the actual (unknown) efficacy of the new vaccine factors into the power of this test.

3. Mr. Wilcox purchased a trick coin that is supposed to land heads up 75% of the time. One of his students volunteer to test this claim. The student flips the coin 50 times and finds that the coin lands heads up 35 times. The student then performs a test of the following hypotheses at the $\alpha = 0.10$ significance level:

$$H_0: p = 0.75$$

$$H_a: p < 0.75$$

where p = the true proportion of tosses of this coin that would land heads-up.

- Describe a Type I error and a Type II error in this setting.
- Which type of error may result in Mr. Wilcox returning the coin and writing a negative review of the product?
- If the student were to use $\alpha = 0.05$ instead of $\alpha = 0.10$, how would this affect the probability of getting each type of error?
- If the student instead flips the coin 100 times, will the power of this test increase, decrease, or remain the same?

4. People wonder if the Nike Vaporfly 4% running shoe is worth the money. Nike claims that it improves running economy by 4%, thus resulting in the runner being “4% faster”.
 - a. Suppose an outside firm is conducting testing on the shoe to help buyers decide if they should purchase the shoe. What would their null and alternative hypotheses be? You should write them in words, don't try to use any notation.
 - b. Which type of error is “worse” in this scenario?
 - c. Should the outside firm use an alpha of .05 or .01? Why?
 - d. If you worked for Nike and were leading the testing for the shoe with the hopes of publishing the results as part of an ad campaign, which alpha would you choose to use?