

STAT 153. Homework # 1

DUE by 9/2/2016 before LAB

Linear Algebra (3 points total)

1. (1 pt) Express $(5, 9, 5)$ as linear combination of $u = (2, 1, 4)$, $v = (1, -1, 3)$ and $w = (3, 2, 5)$.
2. (1 pt) Consider the basis $B = \{u_1, u_2\}$ and $B' = \{v_1, v_2\}$ for R^2 , where

$$u_1 = (1, 0)', u_2 = (0, 1)', v_1 = (2, 1)', v_2 = (-3, 4)'$$

Compute the coordinate matrix $[w]_B$, where $w = (3, -5)'$ and use the transition matrix from B to B' to compute $[w]_{B'}$.

3. (1 pt) Find the real values of λ that make the following vectors linearly dependent set in R^3 .

$$v_1 = (\lambda, -0.5, 5), v_2 = (-0.5, \lambda, -0.5), v_3 = (-0.5, 5, \lambda)$$

Mathematical Statistics (3 points total)

4. (1 pt) Let X_1 and X_2 have the joint pdf $f(x_1, x_2) = x_1 + x_2$, $0 < x_1 < 1$, $0 < x_2 < 1$, zero elsewhere. Find the conditional mean and variance of X_2 , given $X_1 = x_1$, $0 < x_1 < 1$.
5. (1 pt) Let X_1, X_2, \dots, X_{25} and Y_1, Y_2, \dots, Y_{25} be two independent random samples from two normal distributions $N(0, 16)$ and $N(1, 9)$ respectively. Let \bar{X} and \bar{Y} be the corresponding sample means. Compute $\Pr(\bar{X} > \bar{Y})$.
6. (1 pt) Let X and Y be random variables such that $Z = X - 2Y$ has variance equal to 28. If $\sigma_X^2 = 4$ and $\rho_{XY} = 0.5$, find the variance σ_Y^2 of Y .

Regression (4 points total)

The data below show, for a consumer finance company operating in six cities, the number of competing loan companies operating in the city (X) and the number per thousand of the company's loans made in that city that are currently delinquent (Y).

i	1	2	3	4	5	6
X_i	4	1	2	3	3	4
Y_i	16	5	10	15	13	22

Assume that the regression model $Y_i = \beta_0 + \beta_1 X_i + \epsilon_i$ is applicable.

7. (1 pt) Find the matrix representation of X and Y from this regression model, $Y = X\beta + \epsilon$.
8. (1 pt) Find $Y'Y$, $X'X$, $X'Y$ and $(X'X)^{-1}$
9. (1 pt) Find the vector of estimated regression coefficients, vector of residuals
10. (1 pt) Find SSE & SSR