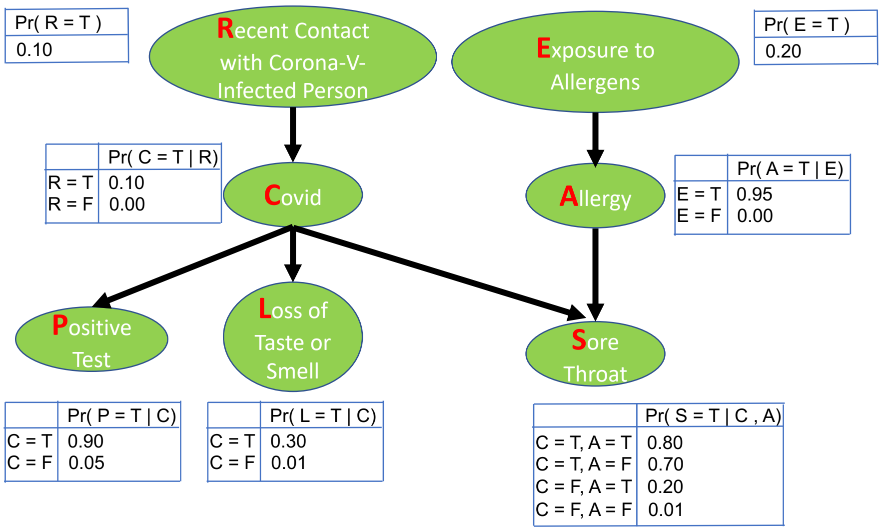
Final Exam

Duration: 120 minutes. **Submit your answers before 5:00 PM**.

Problem 1 (*40 points):*
You are designing a software application to detect Covid, and you came up with the following Bayesian Network. All the variables in this network are Boolean.



	Pr(R = T)
	0.10

	Pr(E = T)
	0.20

	Pr(C = T | R)
R = T	0.10
R = F	0.00

	Pr(A = T | E)
E = T	0.95
E = F	0.00

	Pr(P = T | C)
C = T	0.90
C = F	0.05

	Pr(L = T | C)
C = T	0.30
C = F	0.01

	Pr(S = T | C , A)
C = T, A = T	0.80
C = T, A = F	0.70
C = F, A = T	0.20
C = F, A = F	0.01

1. What is the probability of Covid = true, if:
(Recent Contact with Corona-V-Infected Person) = true,
(Exposure to Allergens) = false,
(Positive Test) = false,
(Loss of Taste or Smell) = true,
and (Sore Throat) = true? In other terms, compute $P(c|r, \neg e, \neg p, l, s)$.

2. Does variable (Covid) depend on variable (Allergy) when you know only the value of variable (Recent Contact with Corona-V-Infected Person) and you don't know the value of any other variable in the network? Why? No computation is needed here, just a very short explanation in English based on what you have learned.

3. Does variable (Recent Contact with Corona-V-Infected Person) depend on variable (Positive Test) when you know only the value of variable (Covid) and you don't know the value of any other variable in the network? Why? Again, no computation is needed here, just a very short explanation in English based on what you have learned.

Problem 2 (*50 points):*

We received a radio signal from a galaxy far, far away, and it seems to be communicated from an extraterrestrial intelligence. The signal is a sequence of 0s and 1s. After analyzing the signal, we realized that it is generated according to some kind of an alien language that uses only two letters. Let's call these two unknown letters A and B.
Letters A and B appear to be used with the same frequency, 50% of time on average for each.
In this language, letter A is 60% of the time followed by letter A, and 40% of the time by letter B. Letter B, on the other hand, is 40% of the time followed by letter A, and 60% of the time by letter B.
Letters A and B are encoded as 0s and $1s$ before being transmitted. So, we cannot observe directly the As and the Bs, only the 0s and $1s$ that we receive. 90% of the time when A is transmitted we observe a "0", but sometimes (10% of the time, due to radio noise) we receive a "1" when A is transmitted. Also, 80% of the time when B is transmitted we observe "1", but sometimes (20% of the time) we receive a "0" when B is transmitted.

1. To formulate the above problem as an HMM (like in homework 3), give the transition model and the evidence (observation) model as conditional probabilities (use any correct notation you like).
2. Filtering: We are receiving a new signal from the same source. We just missed the first bit (no evidence at time $t = 0$), and now (at $t = 1$) we receive a "1". What is the probability that that second letter (at $t = 1$) is a B?
3. Prediction: If the current letter is an A (in other terms, $X_1 = A$), then what is the probability that the next received bit (at $t = 2$) will be a "1" (in other terms, $E_2 = 1$)?
4. Prediction: If the current letter is a B (in other terms, $X_1 = B$), then what is the probability that the next received bit (at $t = 2$) will be a "1" (in other terms, $E_2 = 1$)?
5. Smoothing: We observe that the next bit (at $t = 2$) is indeed also a "1". Use this new information to update your belief that the previous letter (at $t = 1$) was a B.

Bonus question (this question is for an extra 10 points, you don't need to answer it): What is the most likely sequence of letters (at $t = 0$, $t = 1$, and $t = 2$) that were transmitted? Use the Viterbi algorithm.

Problem 3 (*10 points):*

Which ones of the statements below are **true** and which ones are **false**? You do NOT need to provide any explanation.

(a) The Q-learning algorithm does not require a transition model.

(b) The value iteration algorithm requires a transition model.

(c) The value of a given state may vary depending on time if the planning horizon H (number of time-steps ahead) is infinite.

(d) Every MDP has a unique optimal policy.

(e) In a first-order hidden Markov model (temporal model), the next state is independent of past states if the value of the current state is given.

(f) In a second-order hidden Markov model (temporal model), the next state depends on the current state and the previous two states.

(g) A variable in a Bayesian network is independent of its children if its parents are given.

(h) A variable in a Bayesian network is independent of its parents if its children are given.

(i) If the only difference between two MDPs is the discount factor γ then they must have the same optimal policy.

(j) For an MDP with a finite number of states and actions and with a discount factor γ with $0 < \gamma < 1$, policy evaluation (the recursive updates of the value vector V^π using the Bellman equation) is not guaranteed to converge.