PHYS 352

Photon Interactions

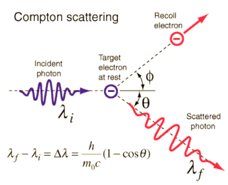
Photon Interactions in Matter

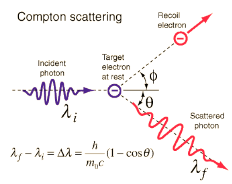
three interactions to consider:

- photoelectric effect
- Compton scattering (including Thomson and Rayleigh scattering)
- pair production
- photodissociation of the nucleus (only at very high γ ray energies)
  - we will ignore

Characteristics of Photon Radiation

- X-rays and γ rays are penetrating because the cross sections for the above processes are much smaller than cross sections for charged particles undergoing inelastic electron collisions
- photons are not degraded in energy as they pass through matter
  - the processes either absorb the photon or scatter them out of the beam
  - thus, the photon that passes straight through has not interacted





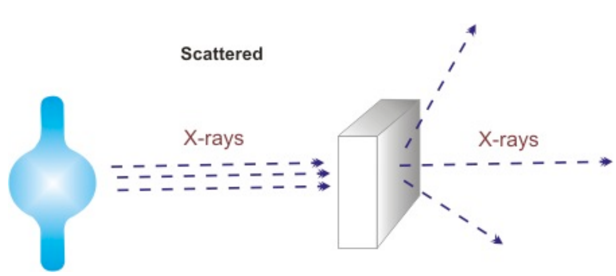
except for: multiple Compton scatters puts photons back into the beam, and they have lower energy

Photon Beam Attenuation

- exponential attenuation of beam intensity:

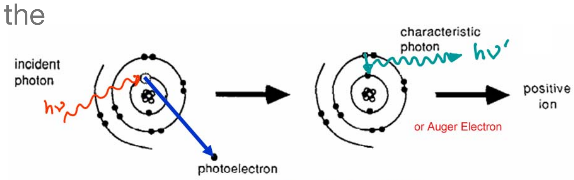
$$I(x) = I_0 \exp(-\mu x); \quad \mu = N\sigma_{tot}$$

- where μ is the absorption coefficient, related to the total cross section that includes the cross section for all the processes (including scattering)

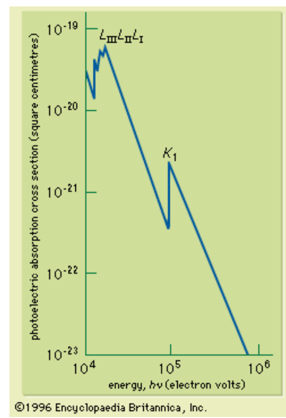


Photoelectric Effect

- the photon is absorbed by the **atom**; the atomic electron is ejected
- the kinetic energy of the electron is:
  - photon energy > binding energy
- requires atomic electrons (does not occur with free electrons) because the atom must take up the momentum
- absorption cross section depends on the material and on which shell electrons are participating
- K-absorption edge, L-absorption edges are clearly visible



KE = hν − BE

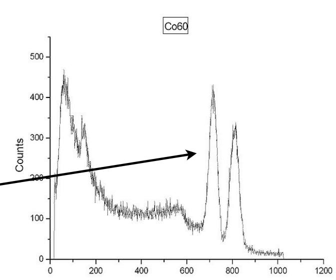


Photoelectric Effect Cross Section

- complicated calculation because it involves wavefunctions for many atomic electrons; some simplifying assumptions can be made
  - for photon energy above the K-absorption edge and $h\nu \ll m_e c^2$

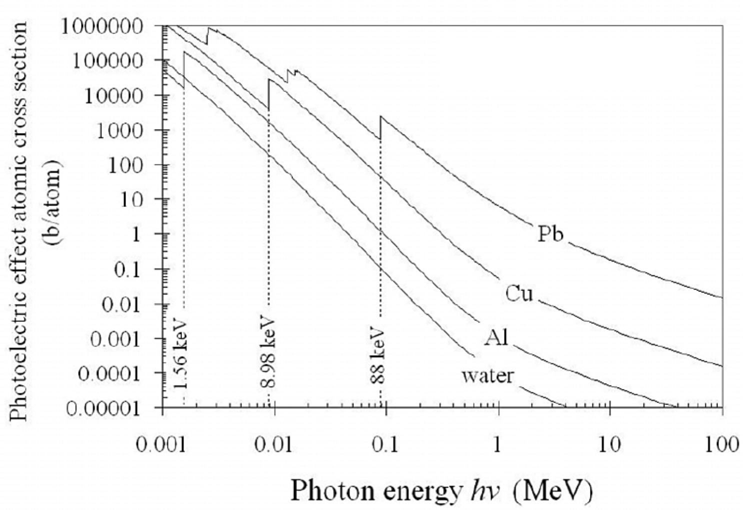
$$\sigma_{photo} = 4\alpha^4 \sqrt{2} Z^5 \frac{8\pi r_e^2}{3} \left(\frac{m_e c^2}{h\nu}\right)^{7/2}$$

  - where $\alpha \cong 1/137$ is the fine structure constant, r_e is the classical electron radius = 2.817×10^{-13} cm
- cross section falls rapidly with energy (other processes become more important around one to a few MeV
- scales approximately as Z^5 or Z^4
  - higher Z desirable in detectors (and shielding)
    - more photoelectric effect interactions
    - larger "photopeak"



Photoelectric Cross Section Over Broad Range

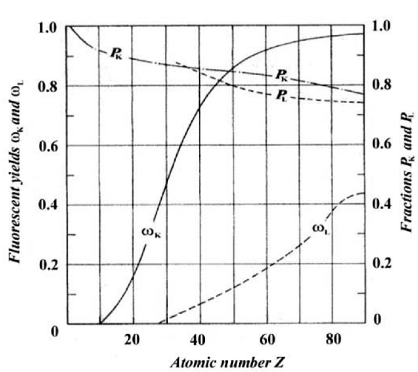
- at high E, cross section goes as $1/(h\nu)$; at lower E, σ goes as $1/(h\nu)^{3.5}$
- the dependence on Z varies somewhat with energy too (that's why Z^5 or Z^4)



X-ray Fluorescence Yield

- as already described in this course, the atom with the K-shell vacancy is excited and can give off characteristic X-rays

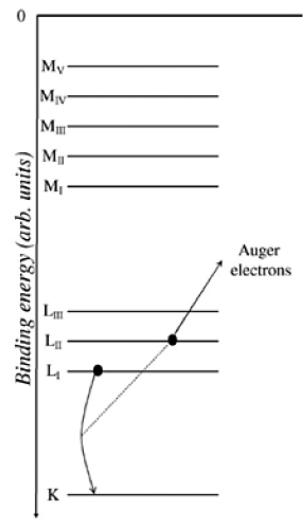
$$\textit{fluorescent yield} \equiv w_K \equiv \frac{\textit{number of atoms that emit K radiation}}{\textit{number of atoms with K shell vacancy}}$$



Note: Figure above shows the dependence of fluorescent yield with *Z* and also the functions P_K and P_L which are the fractions of all photoelectric effect interactions that occur in the *K* and *L* shells if the energy is above the shell binding energies.

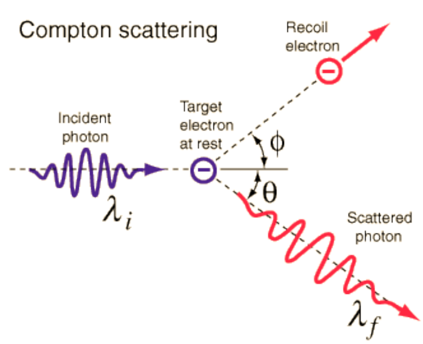
Auger Electrons

- emission of Auger electrons competes with X-rays
- "important" consideration for detectors
  - in thin detectors, the X-rays after photoelectric effect might escape
  - Auger electrons will certainly add to the detected signal in all detectors



Compton Scattering

- the photon scatters off of quasi-free electrons
  - if the energy of the photon is large compared to the electron's binding energy, the binding energy can be ignored and the electron can be considered "free"
- the energy of the photon is degraded
- the photon is scattered out of the beam
  - incoming photon energy $h\nu_i$
  - outgoing photon energy $h\nu_f$
  - recoil electron kinetic energy T_e
  - recoil electron momentum $\mathbf{p}_e$



Derivation of Compton Scattering Kinematics

- using 4-momentum, the quantity P^2 is Lorentz invariant

$$P \equiv (E, \boldsymbol{p}c); \quad P^2 \equiv E^2 - \boldsymbol{p}c \bullet \boldsymbol{p}c = E^2 - (pc)^2 = (mc^2)^2$$

$$P_i + P_e = P_r + P_f; \quad P_i + P_e - P_f = P_r$$

$$(E_i + m_e c^2 - E_f)^2 - (\boldsymbol{p}_i c - \boldsymbol{p}_f c)^2 = (P_r)^2 = (m_e c^2)^2$$

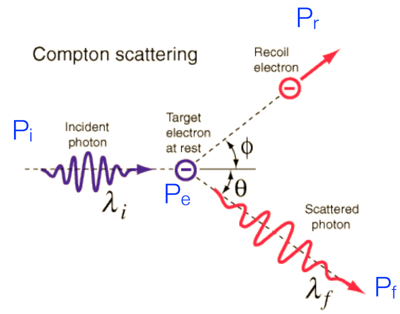
$$E_i^2 + (m_e c^2)^2 + E_f^2 + 2E_i m_e c^2 - 2E_f m_e c^2 - 2E_i E_f - p_i^2 c^2 - p_f^2 c^2 + 2\boldsymbol{p}_i \bullet \boldsymbol{p}_f c^2 = (m_e c^2)^2$$

$$2E_i m_e c^2 - 2E_f m_e c^2 - 2E_i E_f + 2 p_i p_f c^2 \cos\theta = 0$$

$$m_e c^2 (h\nu_i - h\nu_f) = h\nu_i h\nu_f (1 - \cos\theta)$$

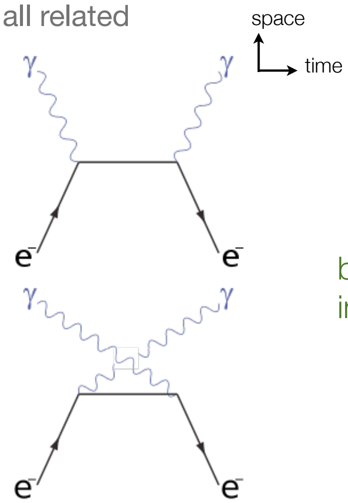
$$h\nu_f = \frac{h\nu_i \, m_e c^2}{m_e c^2 + h\nu_i (1 - \cos\theta)} = \frac{h\nu_i}{1 + \frac{h\nu_i}{m_e c^2}(1 - \cos\theta)}$$

$$T_e = h\nu_i - h\nu_f$$

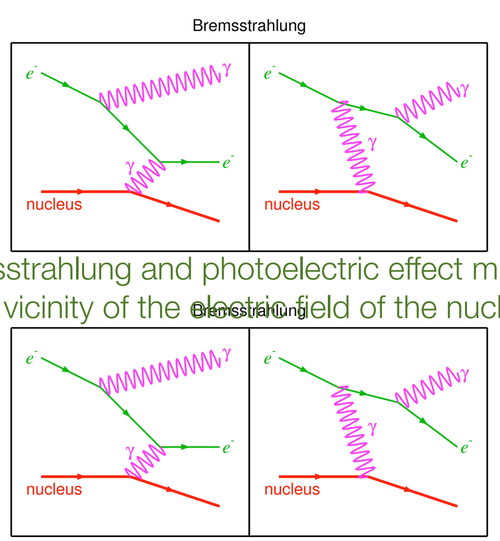


Aside: Feynman Diagrams

- Compton scattering, photoelectric effect, Bremsstrahlung cross sections are all related



Compton scattering



photoelectric effect

Compton Scattering Cross Section

- Klein-Nishina formula (calculated to 1st order in quantum electrodynamics)

$$\frac{d\sigma}{d\Omega} = F_{KN} \frac{r_e^2}{2}(1+\cos^2\theta)$$

$$F_{KN} = \left(\frac{1}{1+A(1-\cos\theta)}\right)^2 \left\{1 + \frac{A^2(1-\cos\theta)^2}{[1+A(1-\cos\theta)](1+\cos^2\theta)}\right\}$$

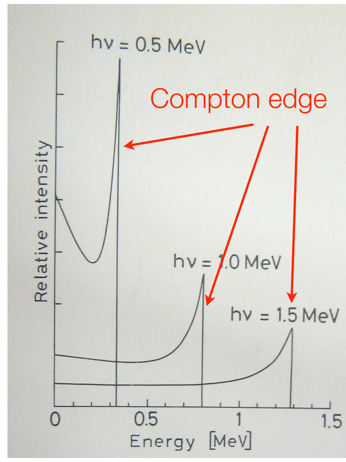
$$A = \frac{h\nu_i}{m_e c^2}$$

ratio of $h\nu_f / h\nu_i$

- integrate over θ to get total Compton scattering cross section
- expressed as differential cross section versus recoil electron kinetic energy
  - gives the spectrum of Compton recoil electrons seen in a detector

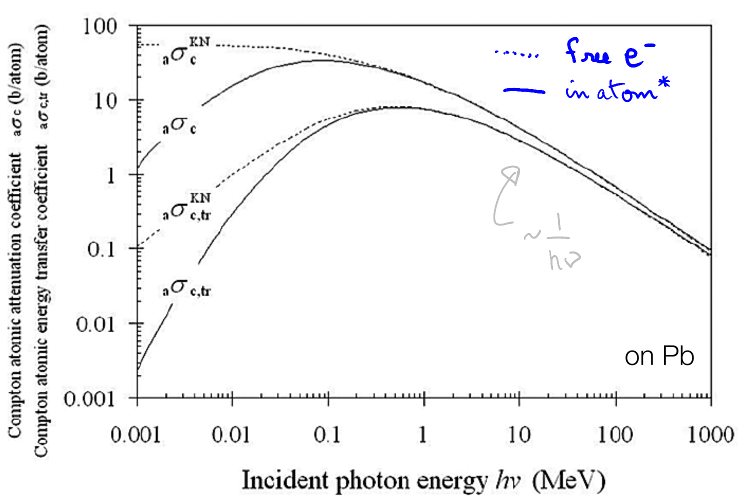
$$\frac{d\sigma}{dT_e} = \frac{\pi r_e^2}{A^2 m_e c^2}\left[2 + \frac{s^2}{A^2(1-s)^2} + \frac{s}{1-s}\left(s - \frac{2}{A}\right)\right]$$

$$s = \frac{T_e}{h\nu_i}$$



Total Compton Cross Section

- correction for binding of electrons appears at low energies
- here, photoelectric effect dominates, so it almost doesn't matter if the free electron calculation overestimates



Features of Compton Scattering

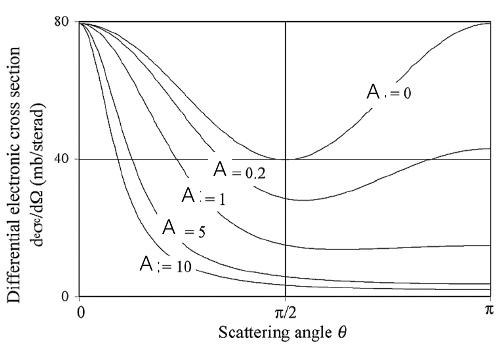
- the atomic cross section (versus the electronic one): multiply by Z electrons in the atom
  - # of atoms per gram = N_A/A
  - # of electrons per gram = Z N_A/A
    - but Z/A ~ 1/2 for most materials
- thus, the Compton cross section *per gram* is roughly independent of the material
- the probability for Compton scattering goes as the number of grams in the path or equivalently proportional to density
- energy dependence goes approximately as 1/hv (at higher energies)

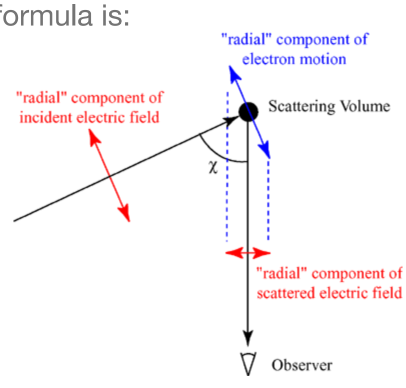
Classical Limit – Thomson Scattering

- J.J. Thomson calculated the classical scattering cross section for photons by free electrons; classical limit of Klein-Nishina formula is:

$$h\nu_i \ll m_e c^2; \quad A \to 0; \quad F_{KN} \to 1$$

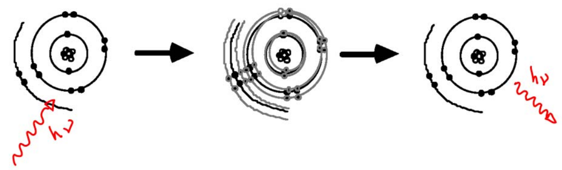
$$\frac{d\sigma}{d\Omega} = \frac{r_e^2}{2}(1+\cos^2\theta); \quad \sigma = \frac{8\pi}{3} r_e^2 = 0.665 \text{ barns}$$





Rayleigh Scattering

- is classical Thomson that takes place coherently for all electrons in the atom

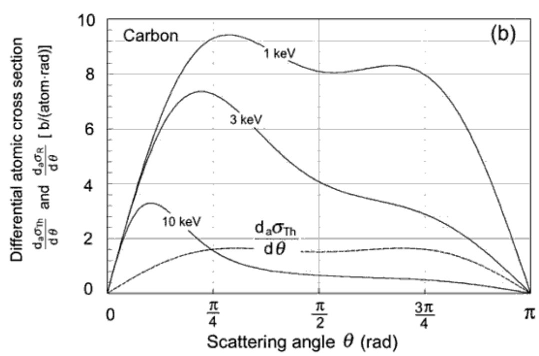


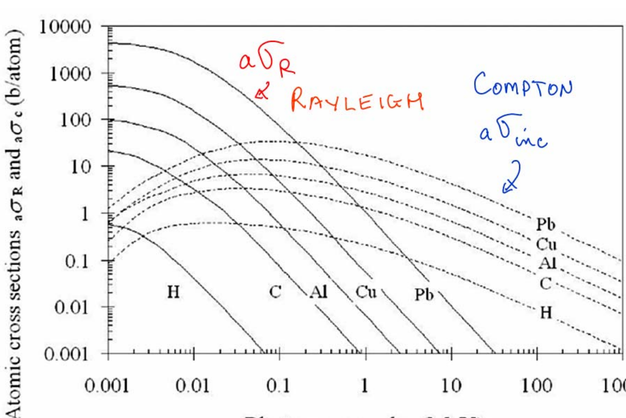
- atom is polarized by the electric field; oscillating polarization emits EM radiation
- Z electrons participate; the effect is coherent so goes as Z^2 (amplitude of the fields add and not just incoherent sum of the power)
- Rayleigh has coherence (and potentially resonance) on top of Thomson; and materials have atoms and not plasma (free electrons)
  - hence the low energy behaviour of photons in matter becomes Rayleigh scattering and not Thomson

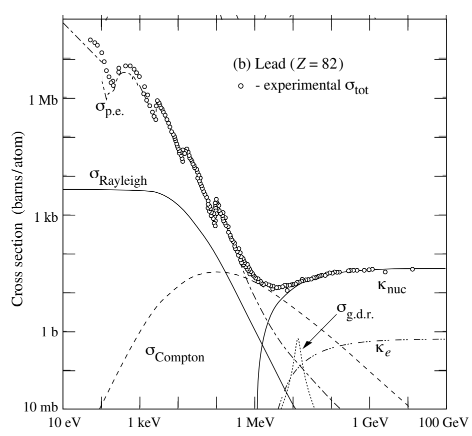
Rayleigh Cross Section in Comparison

- Rayleigh form factor multiplies Thomson differential cross section

$$\frac{d\sigma}{d\Omega} = \frac{r_e^2}{2}(1+\cos^2\theta)\{F(\theta,h\nu_i,Z)\}^2$$





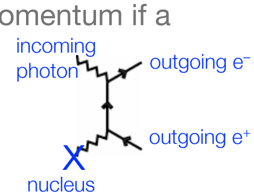


Rayleigh and Thomson are Elastic Scattering

- photon does not lose energy; atom is not ionized or excited
  - nucleus/atom absorbs momentum recoil and takes a negligible amount of energy from the photon
- hence, the processes are not relevant for dose calculations (e.g. in medical physics)
- on the other hand, they do scatter photons out of the beam; add to the overall attenuation
  - ~5% or less of the X-rays (i.e. X-ray energies, in a typical diagnostic beam) suffer Rayleigh scattering
- photoelectric effect dominates at low energies also (see previous slide)

Pair Production

- photon must have energy greater than 2 $m_e c^2$ or 1.022 MeV then a photon can transform into an electron and positron
- must occur in the vicinity of the nuclear Coulomb field also
  - otherwise cannot simultaneously conserve energy and momentum if a photon turned into an electron and positron in free space



- the nucleus is able to absorb the momentum of the photon
- can follow same approach as for bremsstrahlung
  - swap (outgoing e⁺ and incoming γ) for (incoming e⁻ and outgoing γ)
  - screening of the nucleus can be important
- e.g. total cross section for pair production (at high energies)

$$\sigma_{pair} = 4Z^2 \alpha r_e^2 \{\tfrac{7}{9}[\ln(183 Z^{1/3}) - f(Z)] - 1/54\}$$

Pair Production Cross Section

- scales as Z^2
- energy dependence: yes

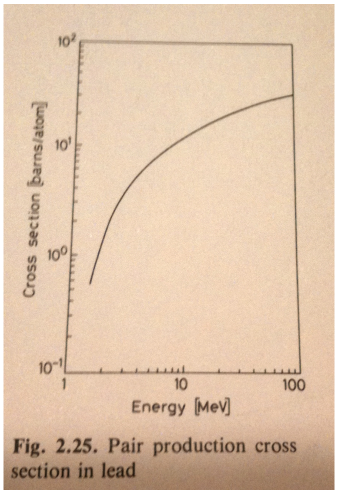
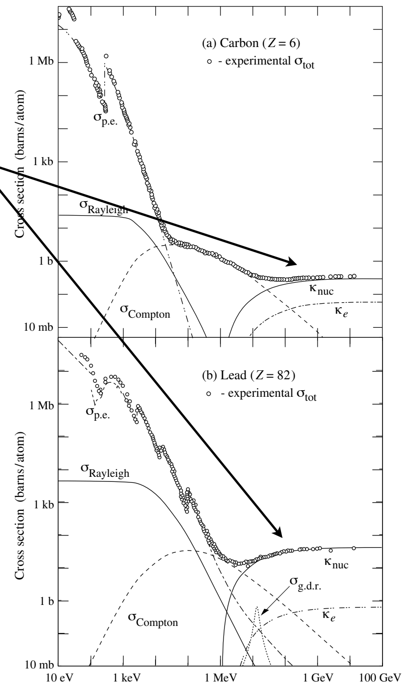


Fig. 2.25. Pair production cross section in lead

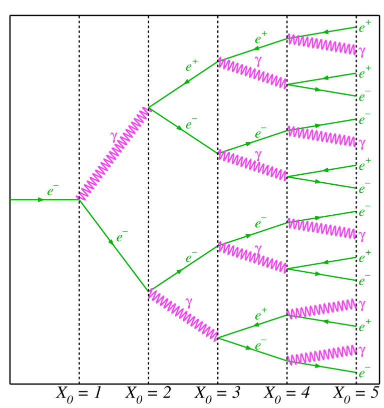


Electron-Photon Showers

- at very high energies, pair production dominates and we can define a mean free path length for pair production to occur L_{pair}:
- and it turns out to be about equal to L_{rad}
  - L_{rad} is the mean free path length for bremsstrahlung
- *not surprising since bremsstrahlung and pair production are Feynman diagrams are closely related to each other*
- in high energy particle physics, electrons (and positrons) and gammas produce electron-photon showers and the radiation length L_{rad} characterizes the distance over which the showers develop

$$\mu = N\sigma_{pair} = \frac{1}{L_{pair}}$$

$$L_{pair} \simeq \frac{9}{7} L_{rad}$$



$$X_0 \equiv L_{rad}$$

Simulation 5 GeV e^- on Lead-Glass

- from S. Menke's web page
  MPI Munich