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Quantum time scales in alpha tunneling

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Abstract – The theoretical treatment of alpha-decay by Gamow is revisited by investigating the quantum time scales in tunneling. The time spent by an alpha-particle in front of the barrier and traversing it before escape is evaluated using microscopic alpha-nucleus potentials. The half-life of a nucleus is shown to correspond to the time spent by the alpha knocking in front of the barrier. Calculations for medium and super heavy nuclei show that from a multitude of available tunneling time definitions, the transmission dwell time gives the bulk of the lifetime of the decaying state, in most cases.

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The concept of quantum tunneling has been with us for decades and has been successfully applied in many branches of physics. It essentially expresses the fact that in quantum mechanics there is always a non-zero finite probability for a particle to go from one region to another even if the regions are separated by a large but finite potential barrier and the particle carries a kinetic energy less than the height of the barrier. Some of the very first applications were by Gamow [1] and by Gurney and Condon [2] to study the alpha-decay of radioactive nuclei. It was however soon noticed that the tunneling phenomenon was not restricted to nuclear physics, but was rather a general result of quantum mechanics which is now commonly used to study the physics of semiconductors and superconductors, in constructing electron tunneling microscopes, to find the lifetimes of the newly discovered super heavy elements [3] and sometimes even to understand the early cosmology of the universe [4]. Its many ramifications reached the realm of atomic physics [5] with ultracold atoms [6], as well as chemistry [7,8] and biology [7,9]. In spite of the fact that tunneling is now widely applied in many fields, the question of how much time does the particle require to tunnel and what connection does it have with the physically measured lifetimes of spontaneously decaying objects still remains unanswered in the cases where we encounter a bound state before tunneling.

In the present work we attempt to answer the above question. To be specific, within the framework of semiclassical approximations, we derive expressions

for the times spent by a tunneling particle in front of the barrier as well as within the barrier. Examining the known expressions for the lifetimes of decaying (metastable) states (or resonances), a connection is found between the so-called dwell times in tunneling and the lifetimes of metastable states. The dwell time is considered to be a measure of the average time spent by a particle in a given region of space. The concept was first introduced by Smith [10] in the context of quantum collisions and to derive a lifetime matrix for multichannel resonances. In the one-dimensional case, it was first introduced by Büttiker [11]. Indeed, earlier in 1966, Baz had proposed [12] the use of Larmor precession as a clock to measure the duration of quantum-mechanical collision events and Rybachenko [13] had applied the method to the simpler case of particles in one dimension. The work of Büttiker was an extension of these works. Büttiker and Landauer also defined [14] a traversal or interaction time of transmitted particles in tunneling, which, as we shall see later also finds a new physical significance in the tunneling process. More recently, the dwell time formalism for the transition from a quasilevel to a continuum of states was discussed in the context of electron and alpha-particle tunneling by Price [15]. Some other recent applications of dwell time can be found in [16].

In what follows we shall present the formalism connecting the dwell times with the decay widths (and hence the half-lives) and conclude with the help of realistic examples that the dwell times of transmitted alpha-particles in the region before entering the barrier correspond to the half-lives of radioactive nuclei decaying by alpha-decay.

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Apart from the dwell time, there exist other tunneling time concepts in the literature and the subject in principle is replete with controversies [17]. The controversy stems from definitions of phase times or the so-called “group delay times” which can also predict superluminal tunneling velocities. The Hartman effect [17] for example, states that the tunneling time becomes independent of the thickness of the barrier length for thick enough barriers and thus results in unbounded tunneling velocities. Such controversies, however, are not relevant for the calculations done in the present work. We do not evaluate the phase time but rather the dwell time and the barrier region itself makes a small contribution to the total dwell time (with the bulk coming from the region in front of the barrier). Hence we do not expect saturation effects such as the Hartman effect resulting in interpretations of superluminal alpha propagation.

Though the nuclear potentials used in this work are not the most modern ones, these potentials and the semiclassical approach [18] used suffice to illustrate the main findings of this work. There exist potentials for which the semiclassical WKB approach may not be appropriate. However, it is known to work reasonably well for the alpha tunneling problem. This approach is indeed commonly used in the literature for the evaluation of the standard Gamow factor and prediction of half-lives of heavy and super heavy nuclei [19]. We shall see below that the dwell time (which one would generally not expect to be an experimentally measurable quantity) in the region in front of the barrier corresponds to the commonly used definition of the measured half-life. Since the WKB approximation is widely used for the calculation of half-lives, we too use the WKB wave function for the evaluation of dwell times in the present work.

For an arbitrary barrier $V(x)$ in one-dimension (a framework which is also suitable for spherically symmetric problems), confined to an interval (x_1, x_2) , the dwell time is given by the number of particles in the region divided by the incident flux j :

$$\tau_D = \frac{\int_{x_1}^{x_2} |\Psi(x)|^2 dx}{j}. \quad (1)$$

Here $\Psi(x)$ is the time-independent solution of the Schrödinger equation in the given region. Though we shall restrict to using a semiclassical approximation for the wave function in the present work, one can always define a dwell time as above in a given region of space, be it with an exact or an approximate wave function. The standard definition of dwell time is the time spent in the region (x_1, x_2) regardless of how the particle escaped (by reflection or transmission) and $j = \hbar k_0 / \mu$ (see footnote ¹)

¹We do not explicitly mention the normalization factor of the wave function and its corresponding appearance in the current density, since we are only interested in evaluating dwell times, where the normalization cancels in the ratio.

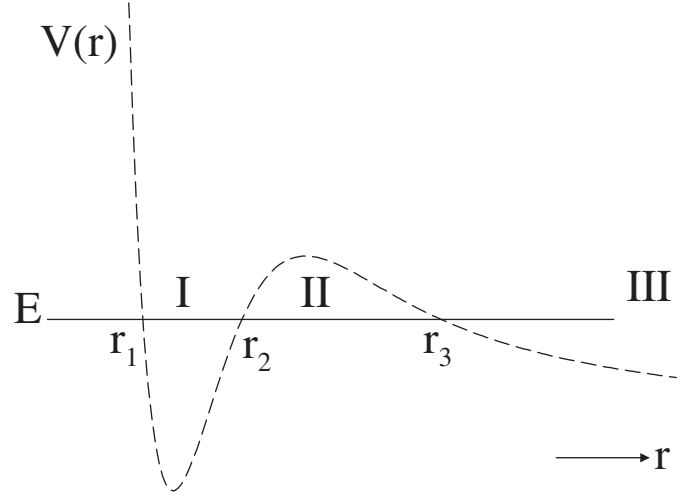


Fig. 1: Typical potential $V(r)$ in an alpha-nucleus tunneling problem. r_1 , r_2 and r_3 are the classical turning points for a given kinetic energy E of the tunneling particle.

(with $k_0 = \sqrt{2\mu E}/\hbar$) for a free particle. However, one can also define transmission and reflection dwell times for the particular cases when the particle is bound in a region and later either got transmitted or reflected. The flux j in these cases would get replaced by the transmitted or reflected fluxes, $j_T = \hbar k_0 |T|^2 / \mu$ and $j_R = \hbar k_0 |R|^2 / \mu$ [20], respectively. The current j_T is the particle's flux in region III (see fig. 1). One would then obtain [20],

$$\frac{1}{\tau_D} = \frac{|T|^2}{\tau_D} + \frac{|R|^2}{\tau_D} = \frac{1}{\tau_{D,T}} + \frac{1}{\tau_{D,R}}, \quad (2)$$

where $|T|^2$ and $|R|^2$ are the transmission and reflection coefficients (with $|T|^2 + |R|^2 = 1$ due to conservation of probability) and $\tau_{D,T} = \int |\Psi|^2 dx / j_T$ and $\tau_{D,R} = \int |\Psi|^2 dx / j_R$, define the transmission and reflection dwell times, respectively. The traversal time defined by Büttiker is somewhat different and is given as,

$$\tau_{\text{trav}}(E) = \int_{x_1}^{x_2} \frac{\mu}{\hbar k(x)} dx, \quad (3)$$

where, $k(x) = \sqrt{2\mu(|V(x) - E|)}/\hbar$ with E being the kinetic energy of the tunneling particle and μ the reduced mass. Having defined the tunneling times relevant to the present work, we shall now apply them to the study of the alpha-decay of nuclei.

We study the alpha-decay of nuclei as a tunneling of the α through the potential barrier of the alpha-daughter nucleus system using a semiclassical approach. Typically, one considers the tunneling of the α through a spherically symmetric r -space potential (see fig. 1) of the form, $V(r) = V_n(r) + V_c(r) + \hbar^2(l+1/2)^2/2\mu r^2$, where $V_n(r)$ and $V_c(r)$ are the attractive nuclear and repulsive Coulomb parts of the α -(daughter) nucleus potential, r the distance between the centres of mass of the daughter nucleus and alpha and μ their reduced mass. The last term represents the

Table 1: Comparison of the calculated half-lives (from the transmission dwell times) with experiment, for medium and super heavy alpha emitters.

Radioactive nucleus	Q value (MeV)	$\ln 2 \tau_{D,T}^I$ (s)	$\ln 2 \tau_{D,T}^{II}$ (s)	$\ln 2 (\tau_{D,T}^I + \tau_{D,T}^{II})$ (s)	$\tau_{1/2}$ (expt) (s)	$P_\alpha^{\text{full} (a)}$	Number of assaults
$^{108}_{52}\text{Te}$	3.445	1.506	0.261	1.767	4.286	0.41	7×10^{21}
$^{169}_{77}\text{Ir}$	6.151	0.466	0.069	0.535	1.28	0.42	2×10^{21}
$^{173}_{79}\text{Au}$	6.836	9.29×10^{-3}	1.38×10^{-3}	10.67×10^{-3}	26.6×10^{-3}	0.40	4×10^{19}
$^{180}_{74}\text{W}$	2.508	2.69×10^{25}	3.74×10^{24}	3.06×10^{25}	5.68×10^{25}	0.54	1×10^{47}
$^{273}_{110}\text{Ds}$	11.368	2.038×10^{-5}	2.76×10^{-6}	2.32×10^{-5}	17×10^{-5}	0.14	9×10^{16}
$^{277}112$	11.3	1.16×10^{-4}	1.52×10^{-5}	1.31×10^{-4}	6.90×10^{-4}	0.19	5×10^{17}

^(a) $\ln 2 (\tau_{D,T}^I + \tau_{D,T}^{II}) / \tau_{1/2}(\text{expt}) = P_\alpha^{\text{full}}$.

Langer modified centrifugal barrier [21]. Writing the wave functions in region I and region II (up to a normalization factor) using the semiclassical Wentzel-Kramers-Brillouin (WKB) approximation [22],

$$\begin{aligned}\Psi_I(r) &= \frac{2\sqrt{k_0}}{\sqrt{k(r)}} \cos \left[\int_r^{r_2} dr' k(r') - \frac{\pi}{4} \right], \\ \Psi_{II}(r) &= \frac{\sqrt{k_0}}{\sqrt{\kappa(r)}} \exp \left[- \int_{r_2}^r dr' \kappa(r') \right],\end{aligned}\quad (4)$$

where $k(r) = \sqrt{2\mu(E - V(r))}/\hbar$ and $\kappa(r) = \sqrt{2\mu(V(r) - E)}/\hbar$, the dwell times τ_D^I and τ_D^{II} in regions I and II, respectively, are given as follows:

$$\begin{aligned}\tau_D^I(E) &= \frac{4\mu}{\hbar} \int_{r_1}^{r_2} \frac{dr}{k(r)} \cos^2 \left[\int_r^{r_2} dr' k(r') - \frac{\pi}{4} \right] \\ &\simeq \frac{2\mu}{\hbar} \int_{r_1}^{r_2} \frac{dr}{k(r)} = 2\tau_{\text{trav}}^I(E),\end{aligned}\quad (5)$$

where the second line follows from replacing the squared cosine term by $(1/2)$. The dwell time in region II is

$$\tau_D^{II}(E) = \frac{\mu}{\hbar} \int_{r_2}^{r_3} \frac{dr}{\kappa(r)} \exp \left[-2 \int_{r_2}^r dr' \kappa(r') \right]. \quad (6)$$

Though the semiclassical WKB approach which is still often used for the evaluation of decay widths [23] is sufficient for the objectives of the present work, better methods such as the Gamow-state formalism [24] do exist. In fact, earlier, starting in the seventies, the general formula for the lifetime of a nucleus decaying by α -decay was obtained on the basis of a Gamow-state formalism in [25]. If we now consider the standard definition of the WKB decay width [26]

$$\Gamma(E) = P_\alpha \frac{\hbar^2}{2\mu} \left[\int_{r_1}^{r_2} \frac{dr}{k(r)} \right]^{-1} e^{-2 \int_{r_2}^{r_3} \kappa(r) dr}, \quad (7)$$

(P_α is the pre-formation probability of the alpha cluster inside the radioactive nucleus decaying later by alpha-particle emission) and compare it with eq. (5), taken

along with the fact that the transmission coefficient $|T|^2 = e^{-2 \int_{r_2}^{r_3} \kappa(r) dr}$, we arrive at the first main result of this work:

$$\Gamma(E) = P_\alpha \hbar |T|^2 [\tau_D^I(E)]^{-1} = P_\alpha \hbar [\tau_{D,T}^I(E)]^{-1}. \quad (8)$$

The decay width is given by the inverse of the *transmission dwell time* in the region in front of the barrier. This implies that the half-life (which is evaluated at the energy $E = Q$, where Q is the amount of energy released in the decay),

$$\tau_{1/2} = \frac{\hbar \ln 2}{\Gamma} = \frac{\ln 2}{P_\alpha} \tau_{D,T}^I, \quad (9)$$

is essentially given by the *transmission dwell time* in region I. It is of further interest to note that the frequency of assaults at the barrier, ν , can be written as the inverse of the time required to traverse the distance back and forth between the turning points r_1 and r_2 as [27]

$$\nu = \frac{\hbar}{2\mu} \left[\int_{r_1}^{r_2} \frac{dr}{k(r)} \right]^{-1}. \quad (10)$$

However, from eq. (5), it follows that $\nu = [\tau_D^I]^{-1}$ and taken along with eq. (8) with $P_\alpha = 1$, we can see that the number of assaults that the particle makes before tunneling is $N_a = \nu \tau_{D,T}^I = (|T|^2)^{-1}$.

In table 1, we list the transmission dwell times for four medium heavy nuclei with $l=0$ and two recently studied super heavy nuclei [28]. In the absence of much information on the l values of the super heavy nuclei studied, we assume the angular momentum $l=0$. The case of the light nucleus ^8Be (with $l=2$) is presented separately in table 2. The alpha-alpha potential for ^8Be is described analytically and is taken from [29]. The alpha-nucleus potential for the medium and super heavy nuclei is constructed using a double folding model with realistic nucleon-nucleon (NN) interactions as given in [29] and also used in some recent works [23,30]. An accurate estimate would however demand the latest available NN potentials [31] and considerations of the four-particle

Table 2: Same as table 1 but for the light nucleus ^8Be .

Radioactive nucleus	Q value (MeV)	$\ln 2 \tau_{D,T}^I$ (s)	$\ln 2 \tau_{D,T}^{II}$ (s)	$\ln 2 (\tau_{D,T}^I + \tau_{D,T}^{II})$ (s)	$\tau_{1/2}$ (expt) (s)	P_α^{full} ^(a)	Number of assaults
$^8\text{Be} (l=2)$	3.1218	3.7×10^{-22}	2.56×10^{-22}	6.26×10^{-22}	3.02×10^{-22}	2.1	1
		$\ln 2 \tau_D^I$ (s)	$\ln 2 \tau_D^{II}$ (s)	$\ln 2 (\tau_D^I + \tau_D^{II})$ (s)			
		1.59×10^{-22}	2.19×10^{-22}	3.78×10^{-22}	3.02×10^{-22}	1.25	1

$$^{(a)} \ln 2 (\tau_{D,T}^I + \tau_{D,T}^{II}) / \tau_{1/2}(\text{expt}) = P_\alpha^{\text{full}}.$$

correlations in nuclei [32]. The Coulomb potential is also obtained via a double-folding procedure where the matter densities of nuclei are replaced by their charge densities. The details of the potentials used in the present work can be found in [33].

We also list the transmission dwell times in region II, in the tables. To evaluate the total time spent in regions I and II, one would start with the definition of the dwell time as

$$\tau_D^{\text{full}}(E) = \int_{r_1}^{r_3} \frac{|\Psi(r)|^2 dr}{j}. \quad (11)$$

The lifetime of the decaying nucleus should in principle be related to the total transmission dwell time spent by the α in the two regions and not just in region I as given by (9). Defining such a time as, say,

$$\tau_{D,T}^{\text{full}}(E) = \tau_{D,T}^I(E) + \tau_{D,T}^{II}(E), \quad (12)$$

the half-life expression in (9) would rather take the form,

$$\tau_{1/2}^{\text{full}} = \frac{\ln 2}{P_\alpha^{\text{full}}} \tau_{D,T}^{\text{full}}, \quad (13)$$

with P_α^{full} being the preformation factor in this case. For the medium heavy nuclei, agreement with experimental half-lives is obtained with $P_\alpha^{\text{full}} \sim 0.4\text{--}0.55$. These values are close to those obtained in the literature [19] with similar potentials. The P_α^{full} values for the super heavy ones are around 0.2, which could change with possible higher angular momenta for these nuclei. In principle, the values of P_α^{full} depend on the details of the nuclear potentials which in turn affect the tunneling probabilities. The values of P_α^{full} found here should hence not be interpreted as a constraint on four-particle correlations [32] in microscopic nuclear-structure models. A detailed study of the preformation factors for super heavy alpha emitters can be found in [34]. The half-lives are evaluated at the experimental Q values, *i.e.*, at $E = Q$ in eq. (9).

We find that the major bulk of the half-life of a medium or super heavy radioactive nucleus is spent in region I, in front of the barrier before tunneling. Though the time spent inside the barrier is much smaller, it is not negligible and should be added to that in region I to obtain the total time spent in tunneling. The number of knocks

(assaults) made at the barrier is inversely proportional to the transmission coefficient which for the heavy nuclei is extremely small leading to a huge number of assaults by the alpha at the barrier. With $|R|^2$ being almost unity in these cases, the reflection dwell times, $\tau_{D,R}$ as well as the average dwell times τ_D are orders of magnitude smaller as compared to $\tau_{D,T}$ as well as the half-lives. The times spent in region II (the barrier) are always an order of magnitude smaller. The case of ^8Be (with $l=2$ studied here) is however, very different. With the transmission coefficient being reasonably big ($|T|^2 = 0.86$ at the Q value of 3.1218 MeV), the times, $\tau_{D,R}$, $\tau_{D,T}$ and τ_D are all comparable and of the same order of magnitude in both regions. It makes more sense then, to compare the standard average dwell time definition with the experimental half-life. An interesting implication of the large transmission coefficient here is that the alpha in ^8Be escapes after one knock.

In conclusion, we can say that the present work provides a new look at the physics of the quantum tunneling problem in general and the alpha-decay problem in particular. In principle, one does not really know how alpha-particles or clusters of heavier nuclei are formed inside the nucleus. In fact, there are arguments that they can be formed in virtual states [35] different from the states of those objects in free space. Such objects could undergo some restructuring in the nucleus [36] which can then not be described by a simple potential consideration. The present work does not attempt to address these issues. The conclusions drawn in this work are hence limited to the picture of a preformed cluster (say the alpha) inside the nucleus, which tries to tunnel through a potential barrier. Further, we note that in contrast to the lifetime definition for the decay of an elementary particle, say muon decay, the definition of lifetime in the context of a tunneling problem is somewhat different. Here the tunneling object has to traverse a region of space, thus giving rise to the question of how long does it require for a particle to tunnel (for a general discussion, see [37]). The dwell time concept answers this question. In the alpha tunneling problem studied in the present work, it is found that the dwell time of the α -particle in front of the barrier as well as within the barrier is important. This can be relevant not only to the alpha radioactive nuclei of medium mass and the recent discoveries of super heavy

nuclei, but also to other branches of science and physics where the quantum concepts of time in conjunction with bound states and tunneling might prove fruitful to have a measure of the speed of a reaction.

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