Reminders

Phys 425
Lecture 11

- MT 1 this Friday
- Problem set 2 due F → skip problems 4 and 5

Last Time

$$\langle E \rangle = (\text{Junk})\, T^4 \int_0^\infty \frac{x^3}{e^x - 1}\, dx = \frac{\pi^4}{15} \Rightarrow \text{Junk}$$

↳ gives $\propto T_3$ behavior @ low T

$$\int_0^\infty \frac{x^n}{e^x - 1}\, dx$$

$$3N = \int_0^{\omega_0} g_{3D}(\omega)\, d\omega$$

$$\Rightarrow \boxed{\omega_0 = \frac{v}{L}\left(6\pi^2 N\right)^{1/3}} \text{ 3D}$$

$$\int dx^2 = \iint r\, dr\, d\theta$$

MT

- powder spectrum →
- geometry BCC, FCC, SC
- conceptual

- The energy is then:

$$\langle E \rangle = \int_0^{\omega_D} g(\omega) \boxed{\left(\frac{1}{e^{\beta\hbar\omega} - 1}\right)} \hbar\omega\, d\omega + \text{const}$$

$$= (\text{Junk})\, T^n \int_0^{T_D/T}$$

requires numerical integration

$$kT_D = \hbar\omega_D$$

We should be able to recover the low T limit.

We're sending T to zero.

$$T \to 0 \Rightarrow T_D/T \to \infty$$

This recovers the Infinite upper limit & gives us $\propto T^3$ behavior

But at high T $(T >> 1)$, we have to look @ $n_B(\beta \hbar \omega)$

$$T >> 1 \Rightarrow \beta \hbar \omega = \frac{\hbar \omega}{k_B T} << 1$$

Bose-Einstein distribution

So:

$$n_{BE}(\beta \hbar \omega) = \frac{1}{e^{\beta \hbar \omega} - 1}$$

$$e^{\beta \hbar \omega} = \underline{1} + \beta \hbar \omega + O(\beta \hbar \omega)$$

expansion that keeps going...

Taylor Expansion

$$\Rightarrow n_{BE}(\beta \hbar \omega) \approx \frac{1}{1 - \beta \hbar \omega - 1} = \frac{1}{\beta \hbar \omega}$$

$$\langle E \rangle = (\text{Junk}) T^4 \int_0^\infty \frac{x^3}{e^x - 1} dx$$

$T << 1$

$$\int_0^{T_D/T} x^3 \left(\frac{1}{x} \right) dx$$

this should recover the Dulong petite limit.

$$C \to 3 N k_B T$$

Shortcomings of Debye model:

1) It's a good model for low T & high T
(for non metals)

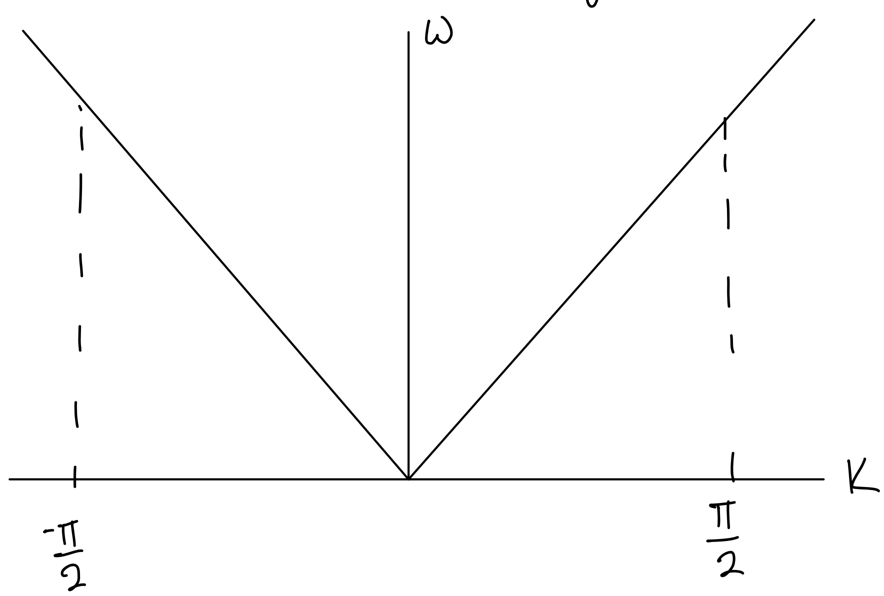
2) $W = v|\vec{k}|$ implies that there are modes (states w/ $K \sim \frac{2\pi}{L}$)
(very large)

but sound waves should have high wavelength.

3) It feels very forced (ad-hoc)
↳ had to manually cut T^3 dependence

4) Metals have $C \propto \alpha T^3 + \gamma T$ at low T

(electron-phonon interaction)
↳ BCS theory



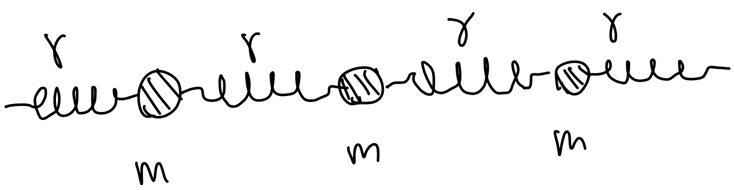
We started with a very classical theory: Dulong-Petit → Einstein → Debye
↗ Quantum
↑ phonon (extremes)

☆ EXAM I ends here ☆

Normal Modes

1D Toy Model (1D Solid)

(Monoatomic model) → atoms of one specie



• in equilibrium, they want to be at a distance away (a = atomic spacing)

In equilibrium

displacement

$$U_n = X_n - X_n^{(eq)}$$

$$= X_n - na$$

The interatomic potential (harmonic) → up to the 2 term

$$\phi_j = \frac{\gamma}{2}\left(X_{j+1} - X_j - a\right)^2$$

regular ole spring potential ($U_{sp} = \frac{1}{2}k(\Delta x)^2$)

We can now calculate the force

$$F = \frac{-d}{dx_j}\phi_j$$

and we can show that the equation of motion for atom j is

$$m\underbrace{\frac{d^2}{dt_j^2}U_j}_{a_j} = \underbrace{\gamma(U_{j+1} + U_{j-1} - 2U_j)}_{F_j}$$

$$m \quad a_j = F_j$$

This has wave solutions

i: imaginary number

$$u_n = Ae^{-i(kna - \omega t)}$$

Plug in to ☆

$$-m\omega^2 e^{-i(kna - \omega t)}$$

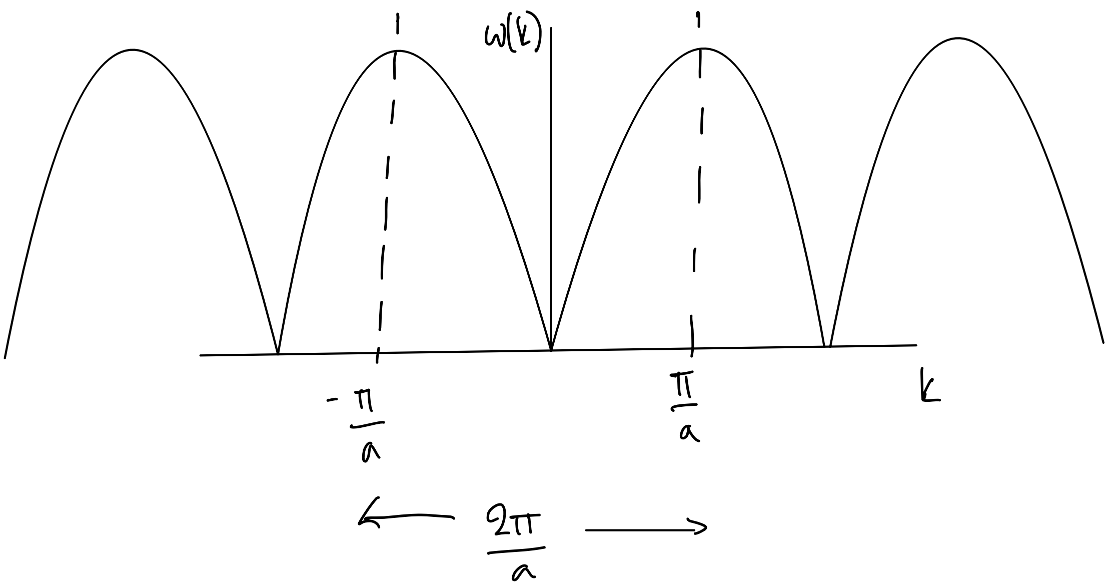
$$= \gamma A e^{i\omega t}\left[e^{-ika(n+1)} + e^{-ika(n-1)} - 2e^{ikan}\right]$$

$$\hookrightarrow \boxed{\omega(k) = 2\sqrt{\frac{\gamma}{m}}\left|\sin\left(\frac{ka}{2}\right)\right|}$$

The dispersion relation for our 1D solid is periodic

w/ period $\frac{2\pi}{a}$ (the distance of a reciprocal lattice vector)

$\hookrightarrow$ periodicity of the lattice presents itself in the dispersion relation.



(Whole reciprocal lattice unit cell (Brillouin zone))