Numerical solution of the geostrophic adjustment problem

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- When the atmosphere or ocean are perturbed away from geostrophy, how do they return to equilibrium?

- non-dimensional linear SW equations
  - f-plane dynamics
  - constant depth
  - small aspect ratio

- Poincaré wave excitation (dispersive)
  - $\omega^2/f^2 = 1+k^2\lambda_R^2$

- Scale dependence of transient and steady response
  - L relative to λ_R

- Gill (7.2,7.3), Holton (7.6), Pedlosky (Lecture 12)

$$\frac{\partial u}{\partial t} - v = -\frac{\partial \eta}{\partial x}$$

$$\frac{\partial v}{\partial t} + u = 0$$

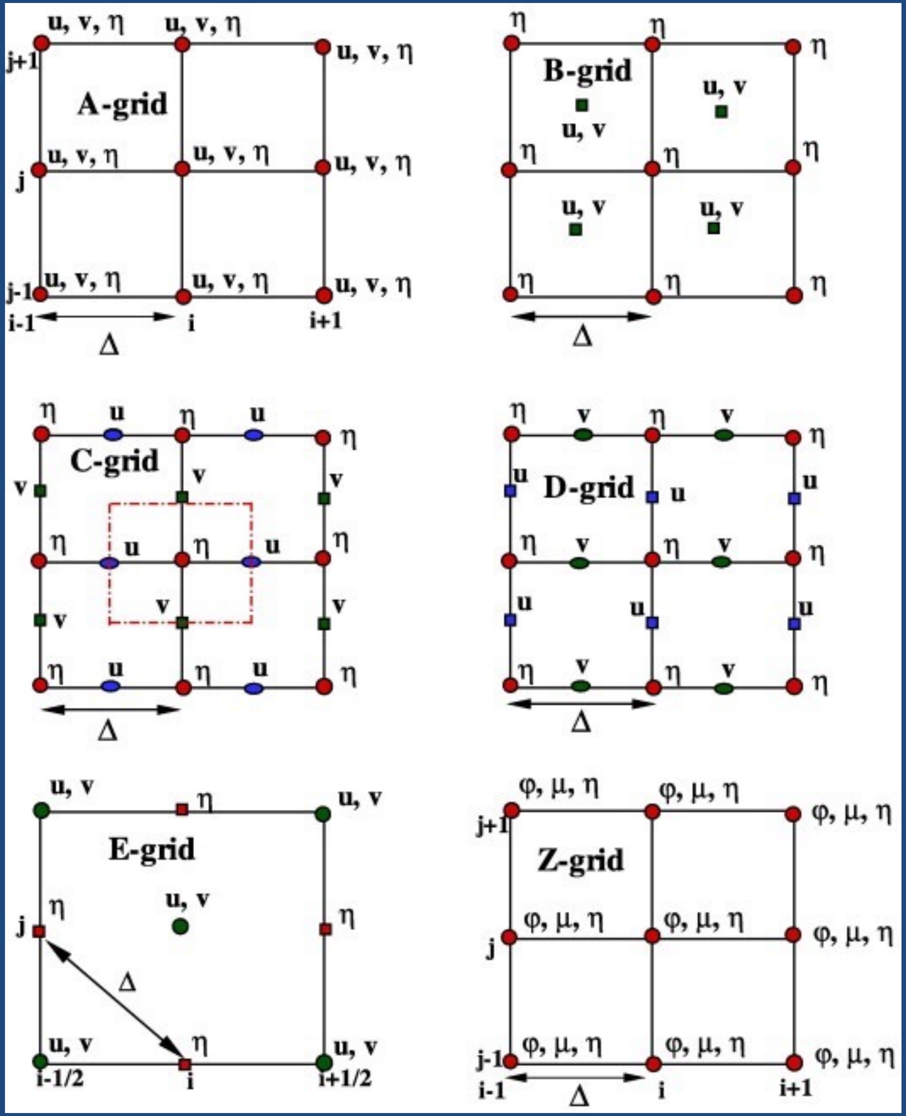
$$\frac{\partial \eta}{\partial t} + \left(\frac{\lambda_R}{L}\right)^2 \frac{\partial u}{\partial x} = 0$$

$$\lambda_R^2 = \frac{gH}{f^2}$$

- Staggering of the spatial grid
  - "Arakawa" grids
    - (A),(B),…,(E)
    - frequently employed in models of the ocean and atmosphere (why?)

- Analytical solutions to the transient problem known only for select initial conditions
  - special functions (Bessel) or integral (Fourier) transform methods

- Physical insight
  - examine many possible initial states



Rajpoot et al., 2012, *J. Comp. Phys.*

Numerical approach

Regular spatial grid, leapfrog time stepping

$$u_j^{n+1} = u_j^{n-1} + 2\Delta t\left[v_j^n - \left(\frac{h_{j+1}^n - h_{j-1}^n}{2\Delta x}\right)\right]$$

$$v_j^{n+1} = v_j^{n-1} + 2\Delta t u_j^n$$

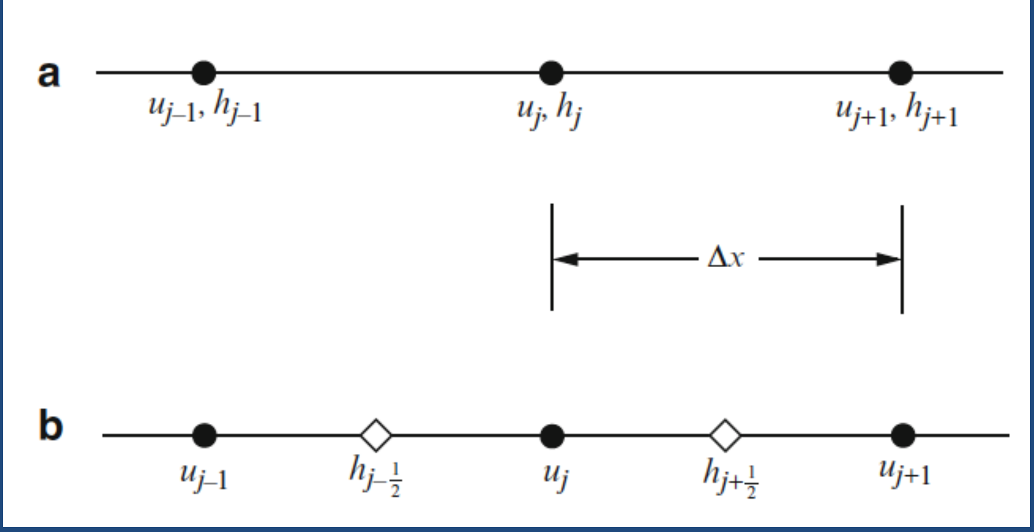
$$h_j^{n+1} = h_j^{n-1} + \frac{\Delta t}{\Delta x}\left[u_{j+1}^n - u_{j-1}^n\right]$$

Staggered spatial grid, explicit forward-backward in time

$$u_j^{n+1} = u_j^n + \Delta t\left[v_j^n - \left(\frac{h_{j+1/2}^n - h_{j-1/2}^n}{\Delta x}\right)\right]$$

$$v_j^{n+1} = v_j^n + \Delta t u_j^{n+1}$$

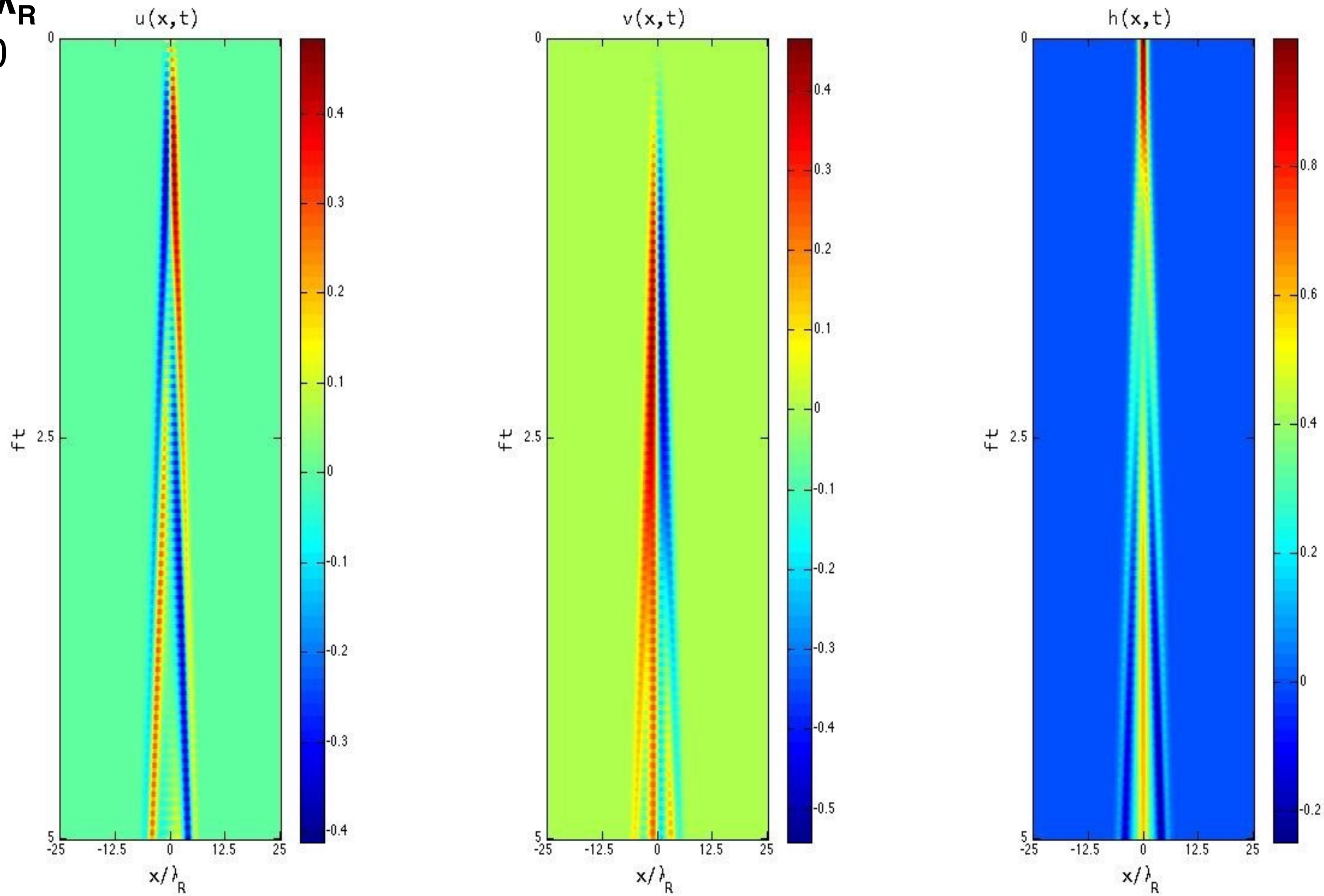
$$h_{j+1/2}^{n+1} = h_{j-1/2}^n + \frac{\Delta t}{\Delta x}\left[u_{j+1}^{n+1} - u_j^{n+1}\right]$$



Durran, Fig. 4.1

Unstaggered grid, leapgfrog
unit Gaussian IC

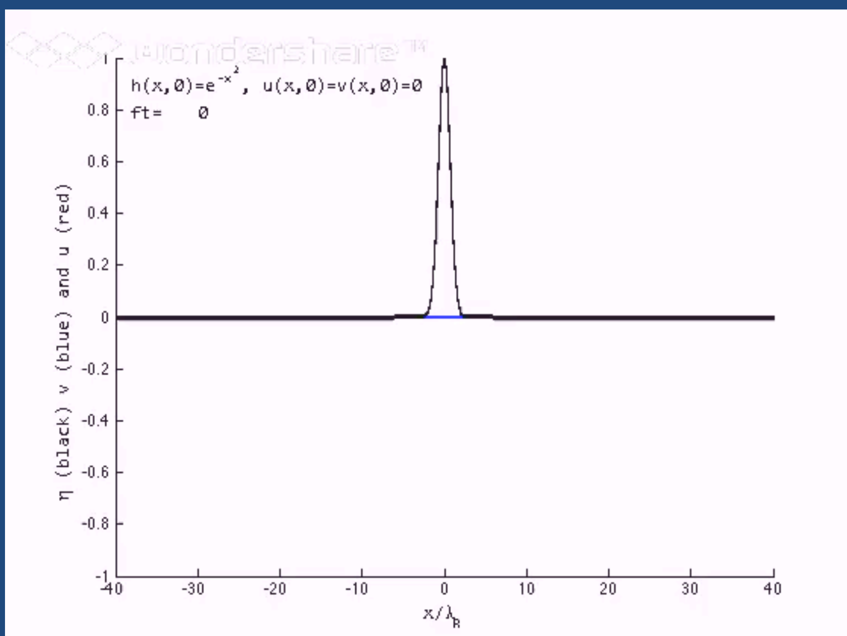
$L = \lambda_R$
$f > 0$

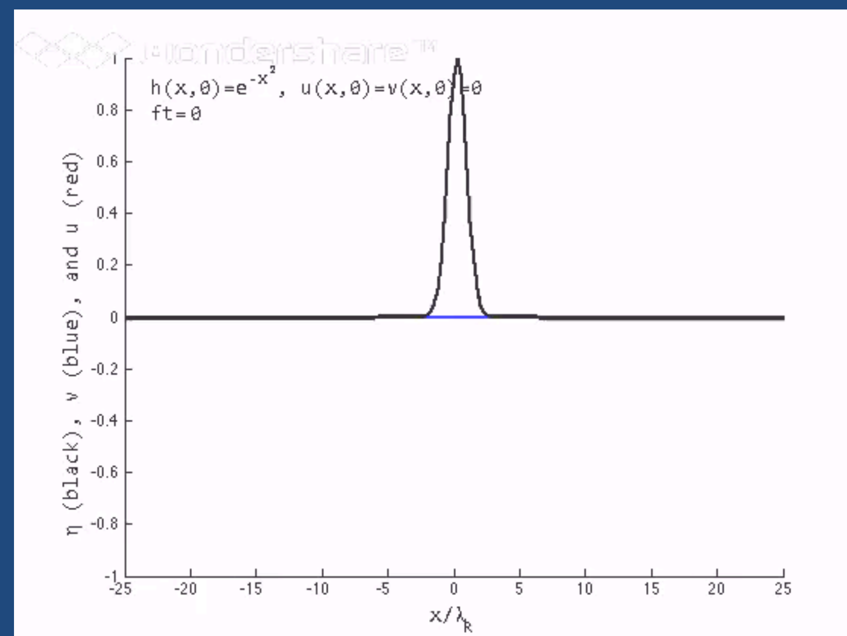


Evolution of Gaussian free surface deviation: comparison of numerical methods

Grid: unstaggered
T-step: Leapfrog

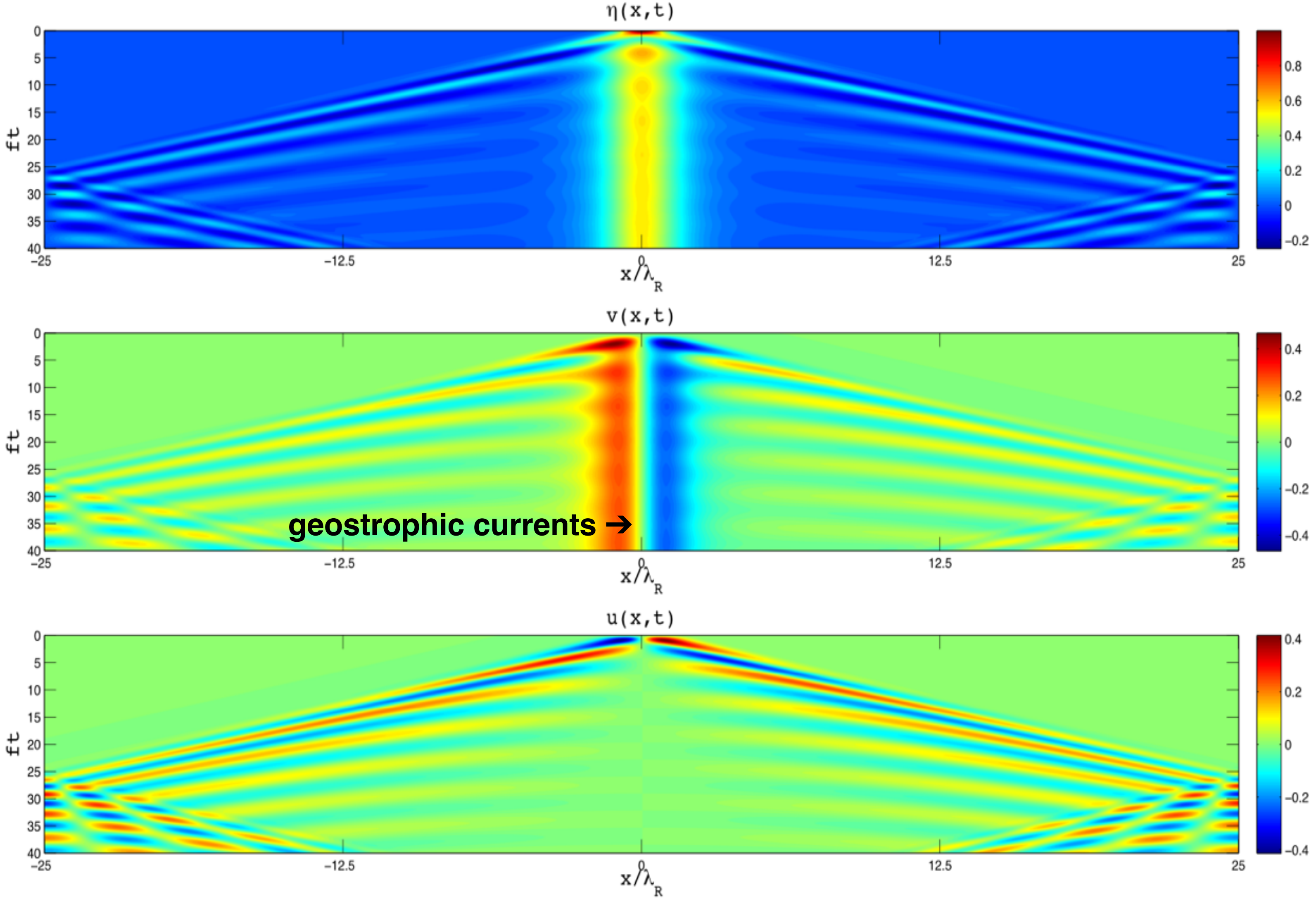
Grid: staggered
T-step: Forward-backward





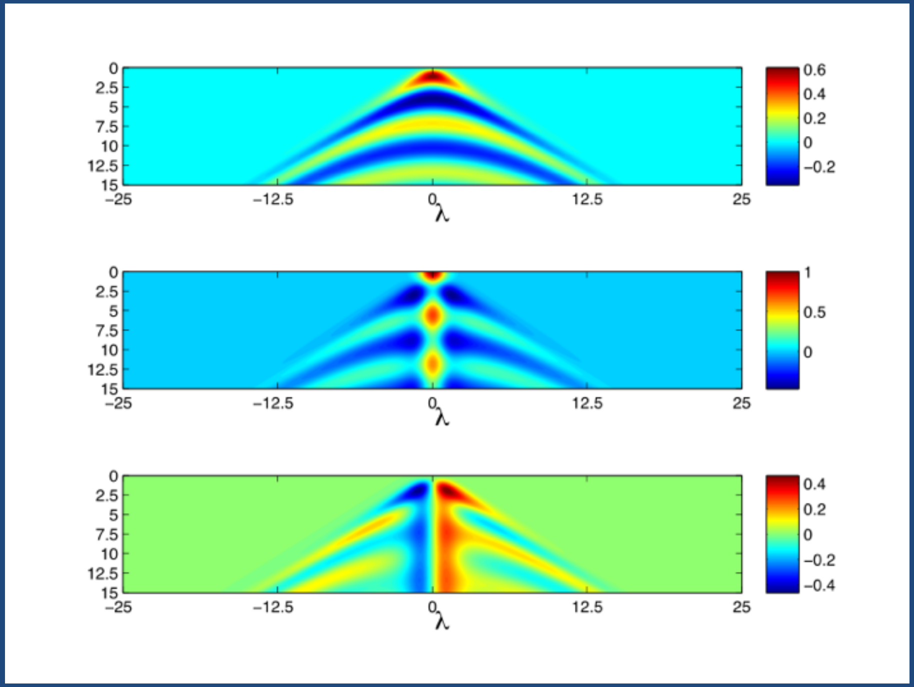
Staggered grid, forward-backward
unit Gaussian IC

$L = \lambda_R$
$f > 0$

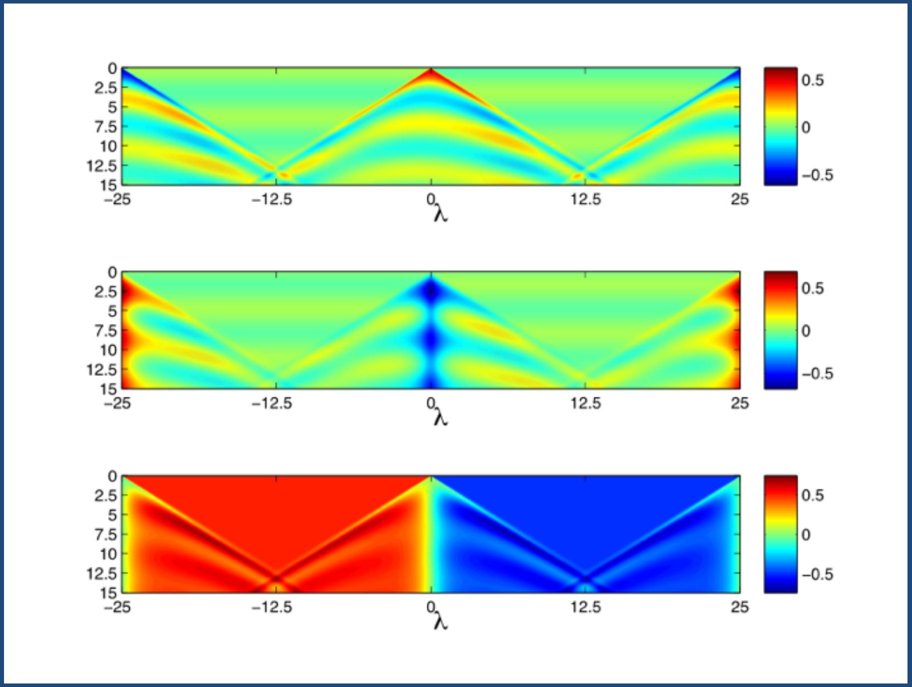


Examples of other initial states:

(A) Northward, laterally sheared jet
Free surface flat, zero zonal flow



(B) Step function displacement at origin,
motionless *(u = v = 0)*



Gill version (see notes)