Treating m as fixed, a as parameter. support is $[0,\infty)$ so condit 1 ✓

① ✓ $f(y) = \frac{1}{a} m y^{m-1} e^{-y^m/a}$

$$\frac{d}{da} = my^{m-1}\left(\frac{-e^{-y^m/a}}{a^2} + \frac{e^{-y^m/a}\, y^m}{a^3}\right)$$

② ✓ so condit 2 holds because ↑ exists

③ $$\int_0^\infty my^{m-1}\left(\frac{-e^{-y^m/a}}{a^2} + \frac{e^{-y^m/a}\, y^m}{a^3}\right)dy \stackrel{?}{=} 0$$

$$\boxed{\begin{array}{l} \text{Let } w = \frac{y^m}{a},\ dw = my^{m-1}dy \quad dy = \frac{dw}{my^{m-1}} \\ y = (aw)^{1/m} \end{array}}$$

$$= \int_0^\infty my^{m-1}\left(\frac{-e^{-y^m/a}}{a^2}\right)dy + \int_0^\infty my^{m-1}y^m \frac{e^{-y^m/a}}{a^3}\,dy$$

$$= \int_0^\infty \cancel{my^{m-1}}\left(\frac{-e^{-w}}{a^2}\right)\frac{dw}{\cancel{my^{m-1}}}$$

$$= \frac{-1}{a^2}\int_0^\infty e^{-w}\,dw + \int_0^\infty \cancel{my^{m-1}}\,y^m \frac{e^{-w}}{a^3}\frac{dw}{\cancel{my^{m-1}}}$$

$$= \frac{-1}{a^2}\int_0^\infty e^{-w}\,dw + \frac{1}{a^3}\int_0^\infty (aw)^{m/m} e^{-w}\,dw$$

$$= -\frac{1}{a^2}\int_0^\infty e^{-w}\,dw + \frac{a}{a^3}\int_0^\infty w e^{-w}\,dw$$

$$\int_0^\infty \frac{\partial^2}{\partial a^2} \cdots\, dy =$$

$$\frac{1}{a^5}\left[\int_0^\infty m y^{m-1}\left(-4a e^{-y^m/a} y^m\right) dy + \int_0^\infty m y^{m-1} e^{-y^m/a} y^{2m}\, dy + \int_0^\infty m y^{m-1}\left(2a^2 e^{-y^m/a}\right) dy\right.$$

$$\boxed{w = y^m/a\,,\quad dw = \frac{m y^{m-1}}{a}\, dy \Rightarrow dy = \frac{a\, dw}{m y^{m-1}} \qquad y = (aw)^{1/m}}$$

$$\frac{1}{a^5}\left[\int_0^\infty \cancel{m y^{m-1}}\left(-4a^2 e^{-w} y^m\right) \frac{dw}{\cancel{m y^{m-1}}} + \int_0^\infty \cancel{m y^{m-1}} e^{-w} y^{2m} \frac{a\, dw}{\cancel{m y^{m-1}}} + \int_0^\infty \cancel{m y^{m-1}}\left(2a^3 e^{-w}\right) \frac{dw}{\cancel{m y^{m-1}}}\right.$$

$$\frac{1}{a^5}\left[-4a^3 \int_0^\infty w e^{-w}\, dw + a^3 \int_0^\infty w^2 e^{-w}\, dw + 2a^3 \int_0^\infty e^{-w}\, dw\right]$$

$$\uparrow$$

$$\Gamma(3) = 2$$

$$2 + 2 + -4 = 0$$

condit 4 holds ✓

$$I(a) = -E\left[\frac{\partial^2}{\partial a^2} \ln(f_X(x \mid \theta))\right]$$

$$= -E\left[\frac{\partial^2}{\partial a^2}\left(-\ln(a) + \ln(m) + (m-1)\ln(y) - \frac{y^m}{a}\right)\right]$$

$$= -E\left[\frac{\partial}{\partial a}\left(\frac{-1}{a} + 0 + 0 + \frac{y^m}{a^2}\right)\right]$$

$$= -E\left(\frac{1}{a^2} + \frac{-2y^m}{a^3}\right]$$

$$= -\left[\frac{1}{a^2} + \frac{-2}{a^3} E[y^m]\right]$$

$$= \frac{1}{a^2} + \frac{2}{a^3}(m^{\text{th}}\ \text{moment})$$

- $CRLB = \dfrac{1}{n\left(\frac{-1}{a^2} + \frac{2}{a^3}(m^{th} \text{ moment of weibull})\right)}$

- $L(a) = \prod_{i=1}^{n} \frac{1}{a} m y^{m-1} e^{-y^m/a}$

$$= \frac{1}{a^n} \cdot m^n \cdot \prod_{i=1}^{n} y_i^{m-1} \cdot e^{\frac{-1}{a}\sum_{i=1}^{n} y_i^m}$$

$\frac{-1}{a}\sum_{i=1}^{n} y_i^m$ is suff stat for a

- log likelihood:

$$\ln(1) - n\ln(a) + n\ln(m) + \ln\left(\prod_{i=1}^{n} y_i^{m-1}\right) + \frac{-1}{a}\sum_{i=1}^{n} y_i^m$$

- deriv wrt a:

$$0 - \frac{n}{a} + 0 + 0 + \frac{1}{a^2}\sum_{i=1}^{n} y_i^m$$

- ~~Now we want to maximize this, so want $\left(\frac{-n}{a}\right)$ to be as close to zero as possible. Otherwise it'll a~~ [illegible] ~~larger negative.~~
- ~~Then want large a~~
- ~~Notice $\forall y$ must be > 0~~
- ~~$a \uparrow$ implies $\min(y_i)$ must be > 0~~
- ~~Then $\hat{a}_{MLE} = \min(y_1, \ldots, y_n)$~~

$$\Rightarrow \frac{n}{a} = \frac{\sum y_i^m}{a^2} \Rightarrow na = \sum y_i^m$$

$$\Rightarrow \hat{a}_{MLE} = \frac{\sum y_i^m}{n}$$