Bonus question:

- The closed-form of $\hat{\beta}_1$, the estimate of the slope parameter, is $\hat{\beta}_1 = \dfrac{\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})}{\sum_{i=1}^{n}(x_i-\bar{x})^2}$
  where $\bar{x}$ is the sample mean for variable x, $\bar{y}$ the sample mean for variable y.

- The definition of r_{xy}, the sample correlation coefficient for variables x and y, is
  $r_{xy} = \dfrac{\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})}{(n-1)s_x s_y}$, where $\bar{x}, \bar{y}$ are as before, $s_x = \sqrt{s_x^2} \geq 0$ is the sample standard deviation of x, $s_y = \sqrt{s_y^2} \geq 0$ is the sample standard deviation of y.

- Notice $\dfrac{\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})}{\sum_{i=1}^{n}(x_i-\bar{x})^2} = \dfrac{\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})}{(n-1)\,s_x^2} = \dfrac{\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})}{(n-1)s_x s_y}\,\dfrac{s_y}{s_x} = r_{xy}\dfrac{s_y}{s_x}$

  where $s_x^2 = \dfrac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{(n-1)}$ is the sample variance for x,

- Thus, $\hat{\beta}_1 = r_{xy}\dfrac{s_y}{s_x}$ is the relation between $\hat{\beta}_1$ and r_{xy}.

- Overall, as the standard deviations of the dependent and independent variables become more similar, the rate of change in the dependent variable for a one-unit change in the independent variable becomes closer to the measure of how well they correlate.

  If the standard deviation of two variables are equal, the estimate of the slope parameter is equal to the sample correlation coefficient.

  If the standard deviation of the dependent variable is greater than that of the independent variable, the estimate for the slope parameter is larger than the sample correlation coefficient.

  If the standard deviation of the dependent variable is smaller than that of the independent variable, the estimate for the slope parameter is smaller than the sample correlation coefficient.