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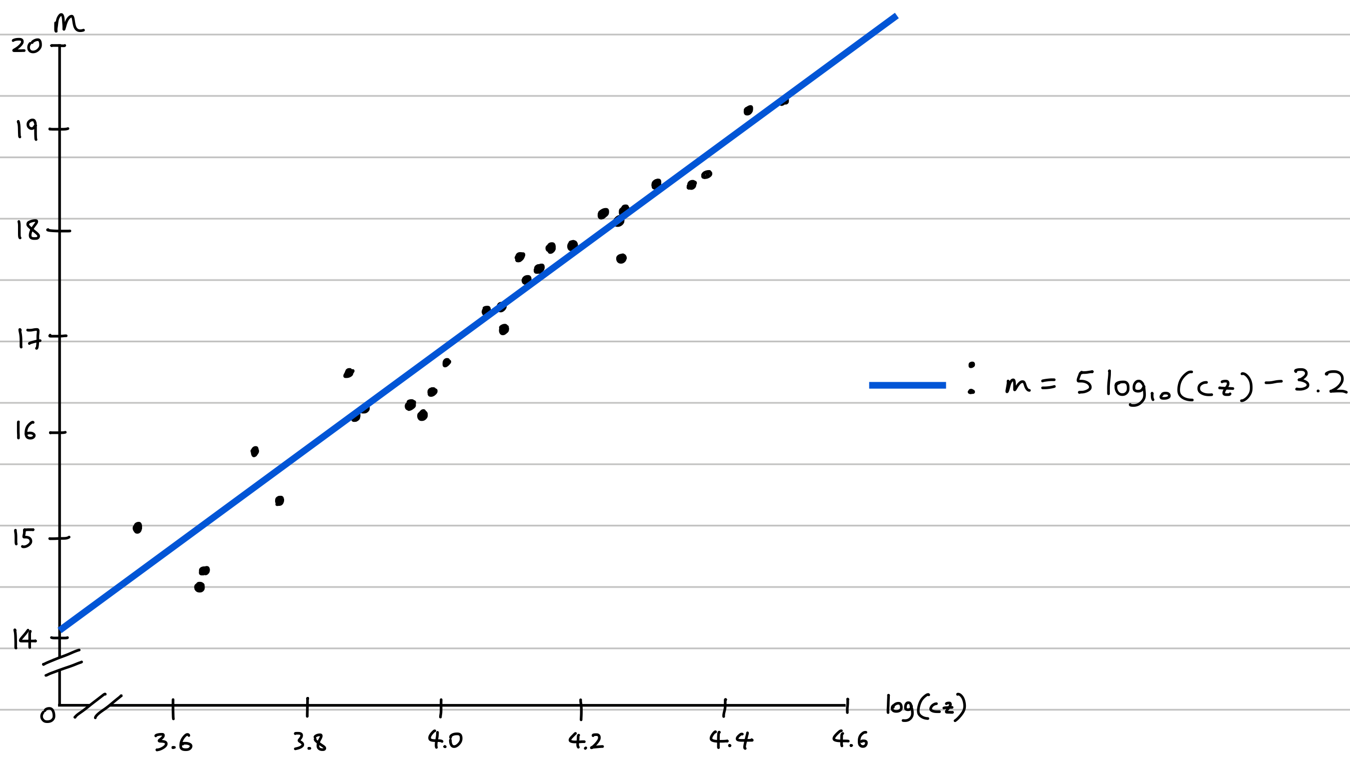
STA302 Assignment

1.



2. Let m be the apparent magnitude and $\log_{10}(cz)$ be the base-10 logarithm of the recessional velocity ($v_r = cz$) of a type Ia supernovae. The linear model I am estimating is $m = \beta_1 \log_{10}(cz) + \beta_0$, where β_0 (the m-intercept) and β_1 (the slope) are the parameters.

3. Set n = 29 (the number of data points). Denote x_i as the i-th $\log_{10}(cz)$ data value and y_i as the i-th m data value, for $i \in \{1, \dots, n\}$.

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{1}{29} 117.877 = 4.064724, \quad \bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i = \frac{1}{29} 496.88 = 17.13379.$$

Using the method of least squares,

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n}(x_i - \bar{x})^2} = \frac{8.87091}{1.748158} = 5.074433 \approx 5$$

which is the same as the coefficient of $\log_{10}(cz)$ in the redshift-magnitude relation

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x} = 17.13379 - (5)(4.064724) = -3.18983 \approx -3.2$$

Thus, the estimation for the model based on the n = 29 data points is:

$$m = \hat{\beta}_1 \log_{10}(cz) + \hat{\beta}_0 = 5 \log_{10}(cz) - 3.2$$

4. See 1.

5. Equating the redshift-magnitude relation and my model,

$$m = M_{ra} + 5\log_{10}(cz) - 5\log_{10}(H_0) + 25 \qquad \text{the redshift-magnitude relation}$$
$$= -19 + 5\log_{10}(cz) - 5\log_{10}(H_0) + 25$$
$$= 6 + 5\log_{10}(cz) - 5\log_{10}(H_0)$$
$$= 5\log_{10}(cz) - 3.2 \qquad \text{my model}$$
$$\Rightarrow 6 - 5\log_{10}(H_0) = -3.2$$
$$\Rightarrow 9.2 = 5\log_{10}(H_0)$$
$$\Rightarrow H_0 \approx 10^{9.2/5} = 10^{1.84} = 69.18309709 \approx 69$$

so $H_0 \approx 69 \frac{\text{km}}{\text{s}} \frac{1}{\text{Mpc}}$.