MAT237Y1 - Interview Activity

1. *Claim:* Let $S \subseteq \mathbb{R}^n$ be a finite set. Prove that S has zero Jordan measure.

Solution: Fix $\varepsilon > 0$. Set $N = |S|$, the cardinality of the set. Label the elements of S as $x_1, x_2, ..., x_N$. For $i \in \{1, ..., N\}$, define the rectangles

$$R_i = \left[(x_i)_1 - \sqrt[n]{\frac{\varepsilon}{2^{n+1}N}},\ (x_i)_1 + \sqrt[n]{\frac{\varepsilon}{2^{n+1}N}}\ \right] \times ... \times \left[(x_i)_n - \sqrt[n]{\frac{\varepsilon}{2^{n+1}N}},\ (x_i)_n + \sqrt[n]{\frac{\varepsilon}{2^{n+1}N}}\ \right] .$$

Notice that $x \in R_i$ by the construction of R_i and

$$\mathrm{vol}(R_i) = (2\sqrt[n]{\frac{\varepsilon}{2^{n+1}N}})^n = (\sqrt[n]{\frac{\varepsilon}{2N}})^n = \frac{\varepsilon}{2N} .$$

Therefore, $S \subseteq \bigcup_{i=0}^{N} R_i$ and

$$\sum_{i=1}^{N} \mathrm{vol}(R_i) = N\frac{\varepsilon}{2N} = \frac{\varepsilon}{2} < \varepsilon$$

as needed.

The skills needed to produce this solution include:

- The ability to recall the definition of a set having zero Jordan measure, which in turn involves the definitions of rectangles, the volume of a rectangle, and the finite union and sum operations.
- The ability to match the structure of the solution with the format of the definition. The steps must be taken in this order for correctness: fix an arbitrary ε, set an appropriate N, construct specific rectangles depending on ε and N, then ensure the conditions of the definition are satisfied.
- The ability to recognize what a finite set is and how to construct a simple rectangle that uses the properties of such a set to meet the conditions of the definition.
- The ability to recognize and avoid potential sources of confusion. Above $(x_i)_1$ was used to avoid confusion with notation such as $x_{i1}, ..., x_{in}$, where it would be difficult to distinguish between the different subscripts.