Homework 3

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Spring semester 2023

1 Part 1: Math

1.1 A

The following steps prove the equality by the properties of logarithms:

$$y_i = e^{\alpha}\delta^{d_i}z_i^{\gamma}e^{\eta_i} \tag{1}$$

$$ln(y_i) = ln(e^{\alpha}\delta^{d_i}z_i^{\gamma}e^{\eta_i}) \tag{2}$$

$$= ln(e^{\alpha}) + ln(\delta^{d_i}) + ln(z_i^{\gamma}) + ln(e^{\eta_i}) \tag{3}$$

$$= \alpha + ln(\delta^{d_i}) + ln(z_i^{\gamma}) + \eta_i \tag{4}$$

$$= \alpha + ln(\delta)d_i + \gamma ln(z_i) + \eta_i \tag{5}$$

1.2 B

The parameter δ is a multiplicative **factor representing the retrofit's effect on energy production**.

1.3 C

We want to show proof of the following equality:

$$\frac{\Delta y_i}{\Delta d_i} = \frac{\delta - 1}{\delta^{d_i}}y_i \tag{6}$$

Since coefficient δ is only "activated" by the binary variable d_i taking a value of 1, we can think of the following expression with potential outcomes notation,

$$y_i(\delta - 1) = \delta y_i - y_i = y_{1i} - y_{0i} = \Delta y_i \tag{7}$$

If the above (8) is the numerator of equation (7), we can use the same notation to rework the denominator too:

$$\delta^{d_i} = d_1 i - d_0 i = \Delta d_i \tag{8}$$

Combining (8) and (9) gives us the original equality,

$$\frac{\Delta y_i}{\Delta d_i} = \frac{\delta - 1}{\delta^{d_i}} y_i \tag{9}$$

This term is the **average marginal effect** of d_i.

1.4 D

We want to show that the following equality holds:

$$\frac{\partial y_i}{\partial z_i} = \gamma \frac{y_i}{z_i} \tag{10}$$

Using properties of exponents and partial derivatives, we manipulate terms.

$$\frac{\partial y_i}{\partial z_i} \left(e^{\alpha} \delta^{d_i} z_i^{\gamma} e^{\eta_i} \right) = \gamma z_i^{\gamma - 1} \left(e^{\alpha} \delta^{d_i} e^{\eta_i} \right) \tag{11}$$

$$= \gamma z_i^{\gamma - 1} \left(\frac{y_i}{z_i^{\gamma}} \right) \tag{12}$$

$$= \gamma \left[\frac{y_i (z_i^{\gamma - 1})}{z_i^{\gamma}} \right] \tag{13}$$

$$= \gamma \left[y_i \left(z_i^{\gamma - 1 - \gamma} \right) \right] \tag{14}$$

$$= \gamma y_i z_i^{-1} \qquad\qquad = \gamma \frac{y_i}{z_i} \tag{15}$$

This term is the **average marginal effect** of z_i.

2 Part 2: Stata

2.1 Estimation

Table 1: Regression results and Marginal Effects

	1	2
Retrofit	-0.101	
	[-0.113 -0.088]	
ln(Square Feet)	0.894	
	[0.880 0.909]	
ln(Temperature (F))	0.281	
	[0.040 0.523]	
Intercept	-0.769	
	[-1.834 0.296]	
Marginal Effects, Retrofit		-114.050
		[-114.521 -113.578]
Marginal Effects, Square Feet		0.629
		[0.628 0.629]
Marginal Effects, Temperature (F)		4.077
		[3.969 4.185]
Number of observations	1000	1000

2.2 Graph