Higher Nationals

Internal verification of assessment decisions – BTEC (RQF)

INTERNAL VERIFICATION – ASSESSMENT DECISIONS			
Programme title	BTEC Higher National Diploma in Computing		
Assessor		**Internal Verifier**	
Unit(s)	Unit 11 : Maths for Computing		
Assignment title	Importance of Maths in the Field of Computing		
Student'sname	Ryan Wickramaratne (COL 00081762)		
List which assessment criteria the Assessor has awarded.	**Pass**	**Merit**	**Distinction**

INTERNAL VERIFIER CHECKLIST		
Do the assessment criteria awarded match those shown in the assignment brief?	Y/N	
Is the Pass/Merit/Distinction grade awarded justified by the assessor's comments on the student work?	Y/N	
Has the work been assessed accurately?	Y/N	
Is the feedback to the student: Give details: • Constructive? • Linked to relevant assessment criteria? • Identifying opportunities for improved performance? • Agreeing actions?	Y/N Y/N Y/N Y/N	
Does the assessment decision need amending?	Y/N	

Assessor signature		**Date**	
Internal Verifier signature		**Date**	
Programme Leader signature (if required)		**Date**	

Confirm action completed			
Remedial action taken Give details:			
Assessor signature		**Date**	
Internal Verifier		**Date**	
Programme Leader signature (if		**Date**	

Higher Nationals – Summative Assignment Feedback Form

Student Name/ID	Ryan Wickramaratne (COL 00081762)		
Unit Title	Unit 11 : Maths for Computing		
Assignment Number	1	**Assessor**	
Submission Date	03/08/2022	**Date Received 1st submission**	
Re-submission Date		**Date Received 2nd submission**	

Assessor Feedback:

LO1 Use applied number theory in practical computing scenarios.

Pass, Merit & Distinction Descripts	P1 ☐	P2 ☐	M1 ☐	D1 ☐

LO2 Analyse events using probability theory and probability distributions.

Pass, Merit & Distinction Descripts	P3 ☐	P4 ☐	M2 ☐	D2 ☐

LO3 Determine solutions of graphical examples using geometry and vector methods.

Pass, Merit & Distinction Descripts	P5 ☐	P6 ☐	M3 ☐	D3 ☐

LO4 Evaluate problems concerning differential and integral calculus.

Pass, Merit & Distinction Descripts	P7 ☐	P8 ☐	M4 ☐	D4 ☐

Grade:	**Assessor Signature:**	**Date:**

Resubmission Feedback:

Grade:	**Assessor Signature:**	**Date:**

Internal Verifier's Comments:

Signature & Date:

* Please note that grade decisions are provisional. They are only confirmed once internal and external moderation has taken place and grades decisions have been agreed at the assessment board.

Pearson Higher Nationals in
Computing

Unit 11: Maths for Computing
Assignment 01

General Guidelines

1. A Cover page or title page – You should always attach a title page to your assignment. Use previous page as your cover sheet and make sure all the details are accurately filled.
2. Attach this brief as the first section of your assignment.
3. All the assignments should be prepared using a word processing software.
4. All the assignments should be printed on A4 sized papers. Use single side printing.
5. Allow 1” for top, bottom , right margins and 1.25” for the left margin of each page.

Word Processing Rules

1. The font size should be **12** point, and should be in the style of **Time New Roman**.
2. **Use 1.5 line spacing**. Left justify all paragraphs.
3. Ensure that all the headings are consistent in terms of the font size and font style.
4. Use **footer function in the word processor to insert Your Name, Subject, Assignment No, and Page Number on each pag**e. This is useful if individual sheets become detached for any reason.
5. Use word processing application spell check and grammar check function to help editing your assignment.

Important Points:

1. It is strictly prohibited to use textboxes to add texts in the assignments, except for the compulsory information. eg: Figures, tables of comparison etc. Adding text boxes in the body except for the before mentioned compulsory information will result in rejection of your work.
2. Avoid using page borders in your assignment body.
3. Carefully check the hand in date and the instructions given in the assignment. Late submissions will not be accepted.
4. Ensure that you give yourself enough time to complete the assignment by the due date.
5. Excuses of any nature will not be accepted for failure to hand in the work on time.
6. You must take responsibility for managing your own time effectively.
7. If you are unable to hand in your assignment on time and have valid reasons such as illness, you may apply (in writing) for an extension.
8. Failure to achieve at least PASS criteria will result in a REFERRAL grade .
9. Non-submission of work without valid reasons will lead to an automatic RE FERRAL. You will then be asked to complete an alternative assignment.
10. If you use other people’s work or ideas in your assignment, reference them properly using HARVARD referencing system to avoid plagiarism. You have to provide both in-text citation and a reference list.
11. If you are proven to be guilty of plagiarism or any academic misconduct, your grade could be reduced to A REFERRAL or at worst you could be expelled from the course.

Student Declaration

I hereby, declare that I know what plagiarism entails, namely, to use another's work and to present it as my own without attributing the sources in the correct way. I further understand what it means to copy another's work.

1. I know that plagiarism is a punishable offence because it constitutes theft.
2. I understand the plagiarism and copying policy of the Edexcel UK.
3. I know what the consequences will be if I plagiaries or copy another's work in any of the assignments for this program.
4. I declare therefore that all work presented by me for every aspects of my program, will be my own, and where I have made use of another's work, I will attribute the source in the correct way.
5. I acknowledge that the attachment of this document signed or not, constitutes a binding agreement between myself and Edexcel UK.
6. I understand that my assignment will not be considered as submitted if this document is not attached to the attached.

ryandilthusha@gmail.com

3/08/2022

Student's Signature: **Date:**
(*Provide E-mail ID*) **(*Provide Submission Date*)**

Feedback Form

Formative Feedback : Assessor to Student			
Action Plan			
Summative feedback			
Feedback: Student to Assessor.			
Assessor's Signature		**Date**	
Student's Signature		**Date**	

Assignment Brief

Student Name /ID Number	
Unit Number and Title	Unit 11 : Maths for Computing
Academic Year	2021/2022
Unit Tutor	Ms. Nuwani
Assignment Title	Importance of Maths in the Field of Computing
Issue Date	
Submission Date	3/08/2022
IV Name & Date	

Submission Format:
This assignment should be submitted at the end of your lesson, on the week stated at the front of this brief. The assignment can either be word-processed or completed in legible handwriting. If the tasks are completed over multiple pages, ensure that your name and student number are present on each sheet of paper.

Unit Learning Outcomes:
LO1 Use applied number theory in practical computing scenarios. **LO2** Analyse events using probability theory and probability distributions. **LO3** Determine solutions of graphical examples using geometry and vector methods. **LO4** Evaluate problems concerning differential and integral calculus.

Assignment Brief and Guidance:

Activity 01

Part 1

1. A tailor wants to make square shaped towels. The required squared pieces of cloth will be cut from a ream of cloth which is 20 meters in length and 16 meters in width.
   a) Find the minimum number of squared pieces that can be cut from the ream of cloth without wasting any cloth.
   b) Briefly explain the technique you used to solve (a).

2. On the first day of the month, 4 customers come to a restaurant. Afterwards, those 4 customers come to the same restaurant once in 2,4,6 and 8 days respectively.
   a) On which day of the month, will all the four customers come back to the restaurant together?
   b) Briefly explain the technique you used to solve (a).

Part 2

3. Logs are stacked in a pile with 24 logs on the bottom row and 10 on the top row. There are 15 rows in all with each row having one more log than the one above it.
   a) How many logs are in the stack?
   b) Briefly explain the technique you used to solve (a).

4. A company is offering a job with a salary of Rs. 50,000.00 for the first year and a 4% raise each year after that. If that 4% raise continues every year,
   a) Find the total amount of money an employee would earn in a 10-years career.
   b) Briefly explain the technique you used to solve (a).

Part 3

5. Define the multiplicative inverse in modular arithmetic and identify the multiplicative inverse of 6 mod 13 while explaining the algorithm used.

6. Prime numbers are important to many fields. In the computing field also prime numbers are applied. Provide examples and in detail explain how prime numbers are important in the field of computing.

Activity 02

Part 1

1. Define 'Conditional Probability' with a suitable example.

2. The manager of a supermarket collected the data of 25 customers on a certain date. Out of them 5 purchased Biscuits, 10 purchased Milk, 8 purchased Fruits, 6 purchased both Milk and Fruits.
   Let ***B represents the randomly selected customer purchased Biscuits, M represents the randomly selected customer purchased Milk*** *and* ***F represents the randomly selected customer purchased Fruits.***
   Represent the given information in a Venn diagram. Use that Venn diagram to answer the following questions.
   a) Find the probability that a randomly selected customer either purchased Biscuits or Milk.
   b) Show that the events "The randomly selected customer purchased Milk" and "The randomly selected customer purchased Fruits" are independent.

3. Suppose a voter poll is taken in three states. Of the total population of the three states, 45% live in state A, 20% live in state B, and 35% live in state C. In state A, 40% of voters support the liberal candidate, in state B, 30% of the voters support the liberal candidate, and in state C, 60% of the voters support the liberal candidate.
   Let ***A represents the event that voter is from state A***, ***B represents the event that voter is from state B*** *and* ***C represents the event that voter is from state C****. Let* ***L represents the event that a voter supports the liberal candidate****.*
   a) Find the probability that a randomly selected voter does not support the liberal candidate and lives in state A.
   b) Find the probability that a randomly selected voter supports the liberal candidate.
   c) Given that a randomly selected voter supports the liberal candidate, find the probability that the selected voter is from state B.

4. In a box, there are 4 types [Hearts, Clubs, Diamonds, Scorpions] of cards. There are 6 Hearts cards, 7 Clubs cards, 8 Diamonds cards and 5 Scorpions cards in the box. Two cards are selected randomly <u>without replacement</u>.
   a) Find the probability that the both selected cards are Hearts.
   b) Find the probability that one card is Clubs and the other card is Diamonds.
   c) Find the probability that the both selected cards are from the same type.

Part 2

5. Differentiate between ‘Discrete Random Variable’ and ‘Continuous Random Variable”.

6. Two fair cubes are rolled. The random variable X represents the difference between the values of the two cubes.

   a) Find the mean of this probability distribution. (i.e. Find E[X])
   b) Find the variance and standard deviation of this probability distribution. (i.e. Find V[X] and SD[X])

   The random variables A and B are defined as follows:
   A = X-10 and B = [(1/2)X]-5
   c) Show that E[A] and E[B].
   d) Find V[A] and V[B].
   e) Arnold and Brian play a game using two fair cubes. The cubes are rolled, and Arnold records his score using the random variable A and Brian uses the random variable B. They repeat this for a large number of times and compare their scores. Comment on any likely differences or similarities of their scores.

7. A discrete random variable Y has the following probability distribution.

Y=y	1	2	3	4	5
P(Y=y)	1/3	1/6	1/4	k	1/6

   where k is a constant.
   a) Find the value of k.
   b) Find $P(Y \leq 3)$.
   c) Find $P(Y>2)$.

Part 3

10. The “Titans” cricket team has a winning rate of 75%. The team is planning to play 10 matches in the next season.

   a) Let X be the number of matches that will be won by the team. What are the possible values of X?
   b) What is the probability that the team will win exactly 6 matches?
   c) What is the probability that the team will lose 2 or less matches?
   d) What is the mean number of matches that the team will win?
   e) What are the variance and the standard deviation of the number of matches that the team will win?

11. In a boys’ school, there are 45 students in grade 10. The height of the students was measured. The mean height of the students was 154 cm and the standard deviation was 2 cm. Alex’s height was 163 cm. Would his height be considered an outlier, if the height of the students were normally distributed? Explain your answer.

12. The battery life of a certain battery is normally distributed with a mean of 90 days and a standard deviation of 3 days.

    For each of the following questions, construct a normal distribution curve and provide the answer.

    a) About what percent of the products last between 87 and 93 days?
    b) About what percent of the products last 84 or less days?

    For each of the following questions, use the standard normal table and provide the answer.

    c) About what percent of the products last between 89 and 94 days?
    d) About what percent of the products last 95 or more days?

13. In the computing field, there are many applications of Probability theories. Hashing and Load Balancing are also included to those. Provide an example for an application of Probability in Hashing and an example for an application of Probability in Load Balancing. Then, evaluate in detail how Probability is used for each application while assessing the importance of using Probability to those applications.

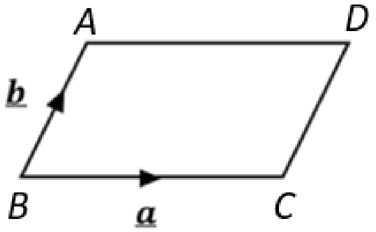
Activity 03

Part 1

1. Find the equation (formula) of a circle with radius r and center C(h,k) and if the Center of a circle is at (3,-1) and a point on the circle is (-2,1) find the formula of the circle.

2. Find the equation (formula) of a sphere with radius r and center C(h, k, l) and show that $x^2 + y^2 + z^2 - 6x + 2y + 8z - 4 = 0$ is an equation of a sphere. Also, find its center and radius.

3. Following figure shows a Parallelogram.



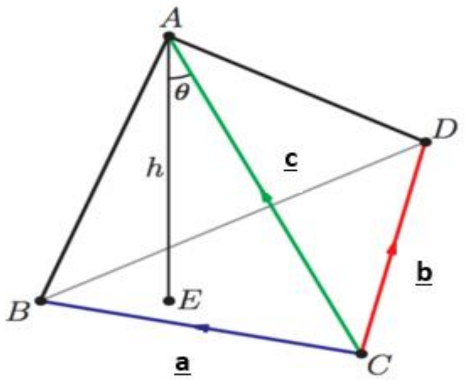
If $\underline{\mathbf{a}}=(\underline{i}+3\underline{j}-\underline{k})$, $\underline{\mathbf{b}}=(7\underline{i}-2\underline{j}+4\underline{k})$, find the area of the Parallelogram.

Part 2

4. If 2x - 4y =3, 5y = (-3)x + 10 are two functions. Evaluate the x, y values using graphical method.

5. Evaluate the surfaces in $\mathbb{R}^3$ that are represented by the following equations.
   i. y = 4
   ii. z = 5

6. Following figure shows a Tetrahedron.



Construct an equation to find the volume of the given Tetrahedron using vector methods and if the vectors of the Tetrahedron are **a**=(i+4j-2k) , **b**=(3i-5j+k) and **c**=(-4i+3j+6k), find the volume of the Tetrahedron using the above constructed equation..

Activity 04

Part 1

1. Determine the slope of the following functions.
   i. $f(x) = 2x - 3x^4 + 5x + 8$
   ii. $f(x) = \cos(2x) + 4x^2 - 3$

2. Let the displacement function of a moving object is $S(t) = 5t^3 - 3t^2 + 6t$. What is the function for the velocity of the object at time t.

Part 2

3. Find the area between the two curves $f(x) = 2x^2 + 1$ and $g(x) = 8 - 2x$ on the interval $(-2) \leq x \leq 1$.

4. It is estimated that *t* years from now the tree plantation of a certain forest will be increasing at the rate of $3t^2 + 5t + 6$ hundred trees per year. Environmentalists have found that the level of Oxygen in the forest increases at the rate of approximately 4 units per 100 trees. By how much will the Oxygen level in the forest increase during the next 3 years?

Part 3

5. Sketch the graph of $f(x) = x^5 - 6x^3 + 3$ by applying differentiation methods for analyzing where the graph is increasing/decreasing, local maximum/minimum points [Using the second derivative test], concave up/down intervals with inflection points.

6. Identify the maximum and minimum points of the function $f(x)= 2x^3 - 4x^4 + 5x^2$ by further differentiation. [i.e Justify your answer using both first derivative test and second derivative test.]

Grading Rubric

Grading Criteria	Achievement (Yes/No)	Feedback
LO1 : Use applied number theory in practical computing scenarios.		
P1 : Calculate the greatest common divisor and least common multiple of a given pair of numbers.		
P2 : Use relevant theory to sum arithmetic and geometric progressions.		
M1 : Identify multiplicative inverses in modular arithmetic.		
D1 : Produce a detailed written explanation of the importance of prime numbers within the field of computing.		
LO2 : Analyse events using probability theory and probability distributions.		
P3 : Deduce the conditional probability of different events occurring within independent trials.		
P4 : Identify the expectation of an event occurring from a discrete, random variable.		
M2 : Calculate probabilities within both binomially distributed and normally distributed random variables.		
D2 : Evaluate probability theory to an example involving hashing and load balancing.		

LO3 : Determine solutions of graphical examples using geometry and vector methods.		
P5 : Identify simple shapes using co-ordinate geometry.		
P6 : Determine shape parameters using appropriate vector methods.		
M3 : Evaluate the coordinate system used in programming a simple output device.		
D3 : Construct the scaling of simple shapes that are described by vector coordinates.		
LO4 : Evaluate problems concerning differential and integral calculus.		
P7 : Determine the rate of change within an algebraic function.		
P8 : Use integral calculus to solve practical problems involving area.		
M4 : Analyse maxima and minima of increasing and decreasing functions using higher order derivatives.		
D4 : Justify, by further differentiation, that a value is a minimum.		

Acknowledgement

I would like to express my special thanks of gratitude to my math lecturer Ms. Nuwani for providing invaluable guidance and giving immense amount of knowledge to work on this assignment perfectly. I specially thanks her because he helped us in doing a lot of research and I came to know about so many new things about the math solving techniques.

Secondly, I would like to thank my parents and friends who helped me a lot in finalizing this project within the limited time frame.

Executive Summery

This entire assignment is based on basics of using applied number theory in practical computing scenarios, analyzing events using probability theory and probability distributions, determining solutions of graphical examples using geometry and vector methods and evaluating problems concerning differential and integral calculus. I've used various of graphing and calculating tools to justify the answers' accuracy and have given small theory parts for each question to understand the question and the answer easily.

List of figures

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Activity 1

Part 1

Question 1

a)

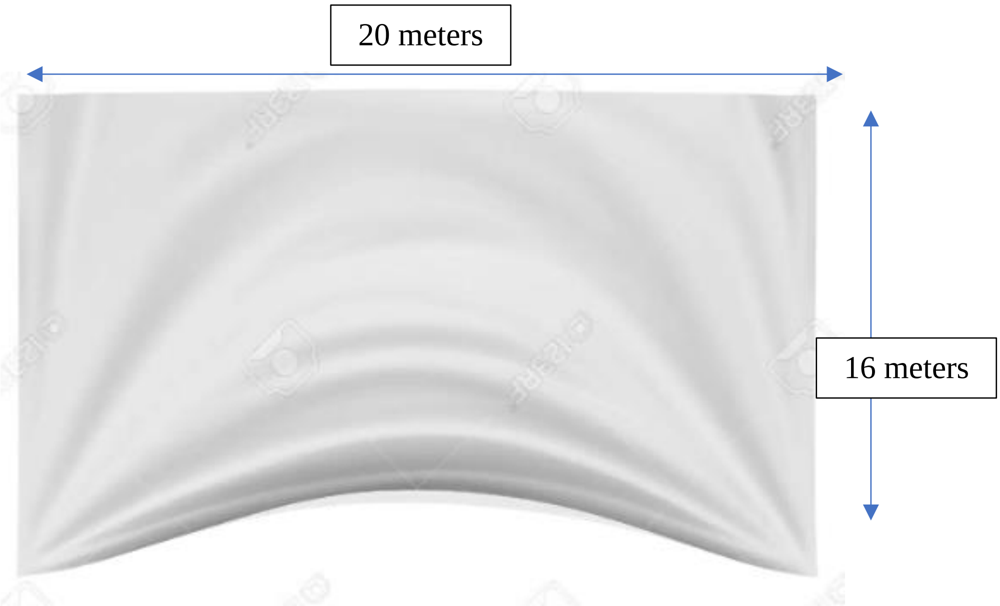


Figure 1. 1 Length and width of the ream of cloth

Prime Factorization of 20 by Upside Down Division Method :-

2	**20**
2	**10**
5	**5**
	1

Factors of 20 → 2 x 2 x 5

Prime Factorization of 16 by Upside Down Division Method :-

2	**16**
2	**8**
2	**4**
2	**2**
	1

Factors of 16 → 2 x 2 x 2 x 2

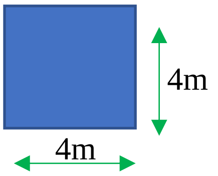
Hence let's find the Greatest Common Divisor (G.C.D.)

Factors of 20 → 2 x 2 x 5

Factors of 16 → 2 x 2 x 2 x 2

G.C.D. = 2 x 2 = 4

Hence when cutting the cloth, each square piece of cloth length and width should be 4m for each side.



To find the minimum number of squared pieces that can be cut from the cloth without wasting any part we should consider area of the cloth →

The total area of the cloth = $20m \times 16m$ = $320m^2$

Each piece of cloth area = $4m \times 4m$ = $16m^2$

Minimum number of squared pieces can be cut from the ream of cloth

= $320m^2 \div 16m^2$ = <u>20 pieces</u>

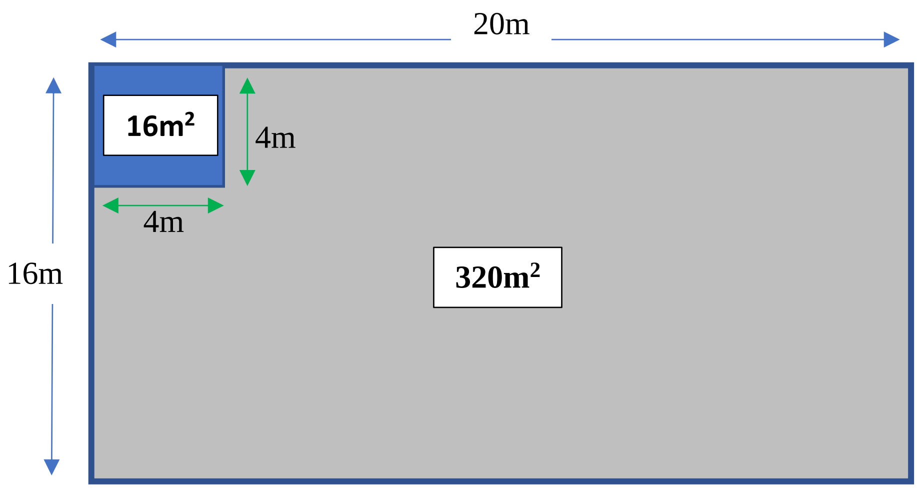


Figure 1. 2 Full ream of cloth area compared to cutting square of pieces area

b)

In the question I've used Greatest Common Divisor (G.C.D.) method. is determined by identifying all common factors between two numbers and choosing the largest. For example, the elements 1, 2 and 4 are shared by 12 and 16. Hence the number 4 is the Greatest Common Divisor.

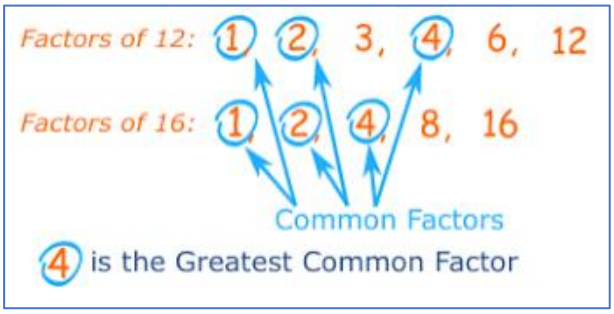


Figure 1. 3 GCD of 12 and 16

The Highest Common Factor (HCF) is another name for it. In number theory, the GCD is utilized for a variety of purposes. Since we can use this method for simple applications like simplifying fractions, I continued to find answer for this question by following this GCD method.

So, to find the factors of length and width of ream of cloth I've used prime factorization of numbers by using upside down division method. We can determine the integers that divide numbers perfectly without leaving a remainder by using this division method.

(Katz, n.d.)

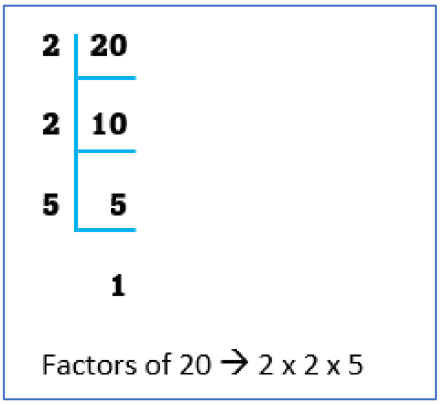


Figure 1. 4 Prime Factorization of 20 by Upside Down Division Method

Question 2

a)

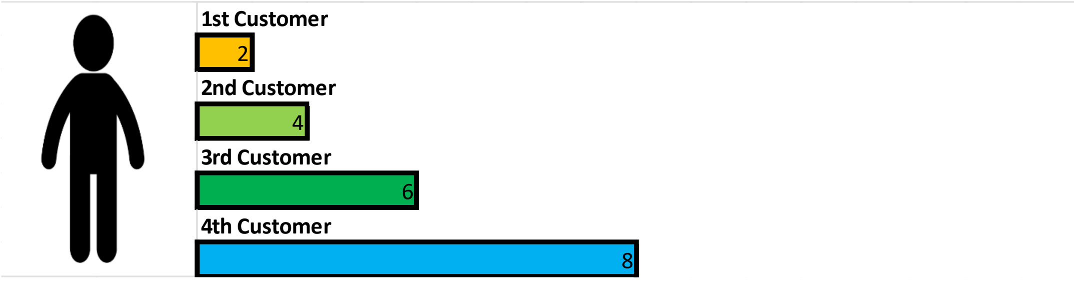


Figure 1. 5 Pattern of the customer arrival to the restaurant

Prime Factorization of 2 by Upside Down Division Method :-

2 | 2

1

Factors of 20 → 2

Prime Factorization of 4 by Upside Down Division Method :-

2 | 4

2 | 2

1

Factors of 20 → 2 x 2

Prime Factorization of 6 by Upside Down Division Method :-

2	6
3	3
	1

Factors of 20 → 2 x 3

Prime Factorization of 8 by Upside Down Division Method :-

2	8
2	4
2	2
	1

Factors of 20 → 2 x 2 x 2

Hence let's find the Least Common Divisor (L.C.D.)

Factors of 2 → 2

Factors of 4 → 2 x 2

Factors of 6 → 2 x 3

Factors of 8 → 2 x 2 x 2

L.C.D. = 2 x 2 x 2 x 3 = <u>24</u>

Since these customers meet after 24 days of the 1st day of month, they meet each other again on 25th Day of the month

b)

The LCM is the smallest number that is a multiple of each of the numbers in a given set of numbers. Least Common Multiple (L.C.M.) is another name for it.

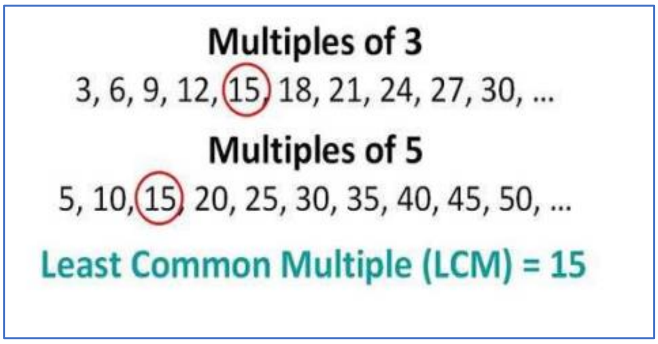


Figure 1. 6 Finding Least Common Multiple

Below figure is representing the pattern of customer arrival to the restaurant. I've marked the multiples of each number. I continued this multiplication until I obtained the same least common multiple for all of the numbers. So, I got the answer 24th day of the month which that all the four customers come back to the restaurant together.

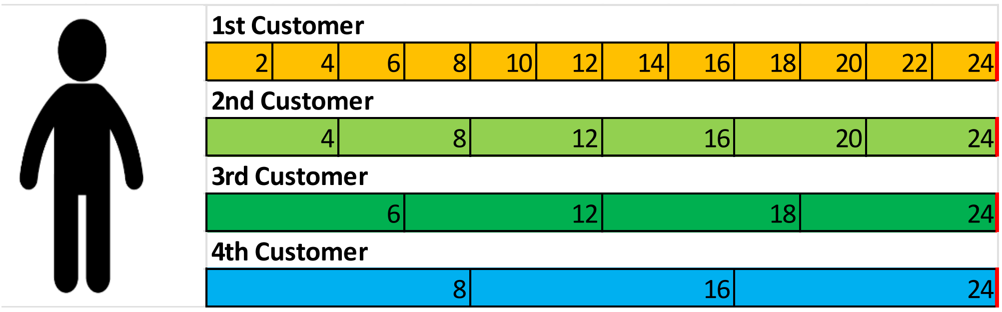


Figure 1. 7 Finding Least Common Multiple by a normal way.

But to solve this problem in the a) part I used Least Common Divisor (L.C.D.) method. Because writing multiples of each number until find least common multiple is difficult. The prime factorization of each number is one of the easiest ways to find the LCM of two numbers, and then the product of the highest powers of the common prime factors is the LCM of those numbers. So, to find the factors of the numbers I’ve used prime factorization of numbers by using upside down division method. We can determine the integers that divide numbers perfectly without leaving a remainder by using this division method.

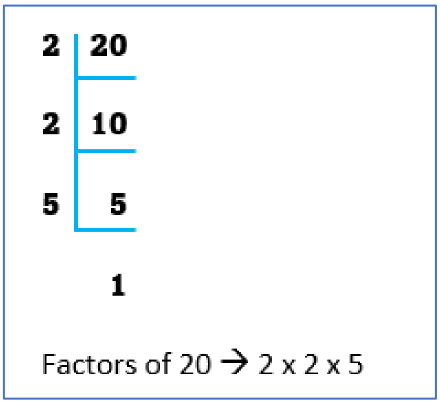


Figure 1. 8 Prime Factorization of 20 by Upside Down Division Method

Part 2

<u>Question 3</u>

a)

1^{st} Row → 10

2^{nd} Row → 11

3^{rd} Row → 11

The Series → 10 , 11 , 12 , 13 , 14 , 15 … 24

$\boxed{a_1}$ $\boxed{a_2}$ $\boxed{a_3}$

Let's check whether this series follows Arithmetic Progression or not.

$$\left.\begin{aligned} a_2 - a_1 &= 11 - 10 = 1 \\ a_3 - a_2 &= 12 - 11 = 1 \end{aligned}\right\}$$

Since 2 successive terms have same different amount this is an Arithmetic Series.

Formula for the sum of the Arithmetic Series

$$\boxed{S_n = \frac{n}{2}(a_1 + a_n)}$$

S_n = The sum of the Arithmetic Series

n = The total number of terms in this sequence has

a_1 = first term

a_n = The term

$$s_n = \frac{n}{2}\ (a_1 + a_n)$$

n = 15 , a_1 = 10 , a_n = 24

$$s_n = \frac{n}{2}\ (a_1 + a_{15})$$

$$s_n = \frac{15}{2}\ (10 + 24)$$

$$s_n = 7.5\ (34)$$

$$s_n = 255$$

There are 255 logs in the stack

b)

Sequence →

A sequence is a list of numbers in a specific order.

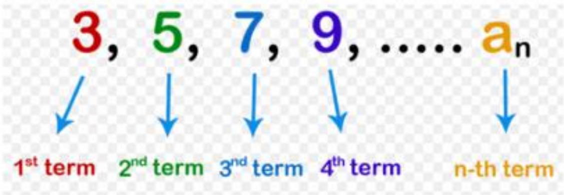
Ex:- 2, 4, 6, 8,10 …

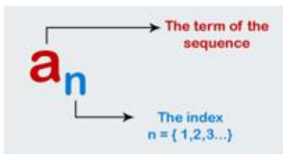
0, -5, -10,-15, -20 …

3, 6, 12, 24, 48, …

The three dots indicate that the pattern should be followed. A term is the name given to each number in the series. The first term in the sequence 1, 3, 5, 7, 9... is 1, the second term is 3, the third term is 5, and so on. The different terms of a series are denoted by the notation a1, a2, a3,... an. The expression an is known as the sequence's nth term.

(Definition and Examples of Sequences, n.d.)





a_i → a_1 a_2 a_3 a_4 a_n

Series →

A series is the summation of a sequence. Hence, a sequence is defined as a set of numbers arranged in a specific order. A series, on the other hand, is defined as the sum of a sequence's elements.

$$\boxed{S_\infty = a_1 + a_2 + a_3 + \ldots}$$

Summation of infinity

There are 2 types of series

1) **Arithmetic Series**
2) **Geometric Series**

Arithmetic Series →

An arithmetic progression is a set of numbers in which any 2 successive terms differ by the same amount, known as the common difference and indicated by the letter d.

So, if any 2 successive terms have same different amount it is an Arithmetic Series.

a_1 , a_2 , a_3 , a_4 , a_5 .. a_n

If we take any 2 successive terms, they should have same different amount.

Ex :- $a_2 - a_1 = d$

$a_3 - a_2 = d$

Ex :- 2, 4, 6, 8,10 , … → d = 2

0, -5, -10,-15, -20 , … → d = (-5)

Geometric Series →

Geometric Progression is a sort of sequence in which each successive term is obtained by multiplying the preceding term by a set number known as the common ratio.

So, if any 2 successive terms have same ratio amount it is a Geometric Series.

a_1 , a_2 , a_3 , a_4 , a_5 .. a_n

If we take any 2 successive terms, they should have same ratio amount.

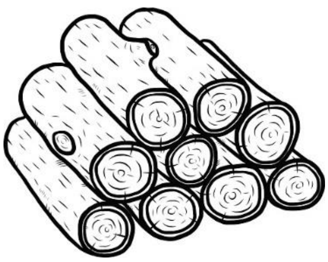
Ex :- $a_2 / a_1 = r$

$a_3 / a_2 = r$

Ex :- 3, 6, 12, 24, 48 , … → r = 2

27, 9, 3 ,1 , 1/3 , … → r = 1/3

So, in this question 1st thing I wanted to find the type of the series. Since the pile of the logs starts 10 from the top and 24 for the bottom, I can consider the 1st term is 10 and last term is 24. And in the question, it tells each row having one more log than the one above it. So, I wrote halfway of the sequence to get an idea as below.



The Series → 10 , 11 , 12 , 13 , 14 , 15 … 24

a_1 a_2 a_3

So, then I checked whether this series follows Arithmetic Progression or not.

$$\left.\begin{aligned} a_2 - a_1 &= 11 - 10 = 1 \\ a_3 - a_2 &= 12 - 11 = 1 \end{aligned}\right\}$$

Since 2 successive terms have same different amount, this is an Arithmetic Series.

Hence, I used the equation for the sum of the Arithmetic Series for this question. By using that I've calculated the total logs in the stack.

Question 4

a)

1st Year → 50,000/=

2nd Year → 50,000 + 50,000 x 4% = 50,000 + 2,000 = 52,000/=

3rd Year → 52,000 + 52,000 x 4% = 52,000 + 2,080 = 54,080/=

4th Year → 54,080 + 54,080 x 4% = 54,080 + 2,163.2 = 56,263.2/=

The Series → 52,000 , 54,080 , 56,263.2 …

$\boxed{a_1}$ $\boxed{a_2}$ $\boxed{a_3}$

Let's check whether this series follows Arithmetic Progression or not.

$a_2 - a_1 = 54{,}080 - 52{,}000 = 2{,}080$

$a_3 - a_2 = 56{,}263.2 - 54{,}080 = 2{,}183.2$

Since 2 successive terms haven't same different amount this is not an Arithmetic Series.

So, let's check whether this series follows Geometric Progression or not.

$a_2 \div a_1 = 54{,}080 \div 52{,}000 = 1.04$

$a_3 \div a_2 = 56{,}263.2 \div 54{,}080 = 1.04$

Since 2 successive terms have same ratio amount this is a Geometric Series.

Formula for the sum of the Geometric Series

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

S_n = The sum of the Geometric Series

a = first term

r = common ratio

n = The total number of terms in this sequence has

$$s_n = \frac{a(1 - r^n)}{1 - r}$$

$$s_n = \frac{50000(1 - 1.04^{10})}{1 - 1.04}$$

$s_n = 600305.36$

Total amount of money an employee would earn in a 10-years career is 600305.36

b)

As in the question number 3 (Stack of logs question), 1st thing I wanted to find the type of the series. I can consider the 1st term as 50,000/= then for each few terms I've calculated the annual salary by using 4% raise as below.

1st Year → 50,000/=

2nd Year → 50,000 + 50,000 x 4% = 50,000 + 2,000 = 52,000/=

3rd Year → 52,000 + 52,000 x 4% = 52,000 + 2,080 = 54,080/=

4th Year → 54,080 + 54,080 x 4% = 54,080 + 2,163.2 = 56,263.2/=

Hence the series is following as below.

The Series → 52,000 , 54,080 , 56,263.2 …

a_1 a_2 a_3

The I checked whether this series follows Arithmetic Progression or not as below.

$a_2 - a_1 = 54{,}080 - 52{,}000 = 2{,}080$

$a_3 - a_2 = 56{,}263.2 - 54{,}080 = 2{,}183.2$

Since 2 successive terms haven't same different amount this is not an Arithmetic Series.

Since this series not an Arithmetic Series, I've checked whether this series follows Geometric Progression or not as below.

$a_2 \div a_1 = 54{,}080 \div 52{,}000 = 1.04$

$a_3 \div a_2 = 56{,}263.2 \div 54{,}080 = 1.04$

Since 2 successive terms have same ratio amount this is a Geometric Series.

As expected, this series was following Geometric progression. Hence, I used the equation for the sum of the Geometric Series. By using that I've calculated the total amount of money an employee would earn in a 10-years career.

Part 3

Question 5

What is Modular Arithmetic?

Modular arithmetic is a type of arithmetic in which only integers are used. Modular arithmetic is sometimes known as clock arithmetic because one of the most common applications of modular arithmetic is in the 12-hour clock, which divides the time period into two equal halves. When two integers are divided, the result is an equation that looks like this:

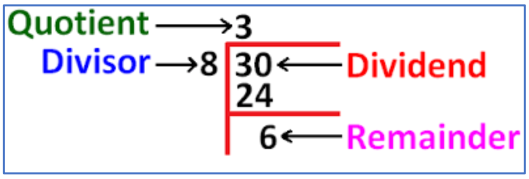
$$\boxed{\frac{A}{B} = Q \text{ remainder } R}$$

A is the dividend

B is the divisor

Q is the quotient

R is the remainder



30 divides by 8

$$\frac{30}{8} = 3 \text{ Remainder } 6$$

A mod B = R

Ex :-

- 10 mod 3 = 1 (1 is the remainder)
- 12 mod 7 = 5

What is multiplicative inverse in Modular Arithmetic?

The term inverse in mathematics refers to the opposite of another operation. We know from basic mathematics that a number multiplied by its inverse equals 1.

Ex:- The inverse of a number A is 1/A since A * 1/A = 1 (e.g. the inverse of 5 is 1/5).

There is no division operation in modular arithmetic. We do, however, have modular inverses. Let's take simple modular equation.

$$A \bmod B = R$$

Suppose we multiply left hand side (A mod B) of this equation with a number X.

If it then results 1, that X should be inverse of left-hand side part (A mod B).

The possible values for X should be less than B.

If → X (A mod B) = 1

Then X is inverse of A mod B.

Identify the multiplicative inverse of 6 mod 13

We should multiple this A mod B (6 mod 13) part with X value.

Possible X values should be range B-1 (Less than 13). → 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12

Then the modular inverse of A mod B is the X value that makes X * A mod 13 = 1.

X (6 mod 13) = R

0 * 6 mod 13 = R → 0 mod 13 = 0

1 * 6 mod 13 = R → 6 mod 13 = 6

2 * 6 mod 13 = R → 12 mod 13 = 12

3 * 6 mod 13 = R → 18 mod 13 = 5

4 * 6 mod 13 = R → 24 mod 13 = 11

5 * 6 mod 13 = R → 30 mod 13 = 4

6 * 6 mod 13 = R → 36 mod 13 = 10

7 * 6 mod 13 = R → 42 mod 13 = 3

8 * 6 mod 13 = R → 48 mod 13 = 9

9 * 6 mod 13 = R → 54 mod 13 = 2

10 * 6 mod 13 = R → 60 mod 13 = 8

11 * 6 mod 13 = R → 66 mod 13 = 1 ← This is the inverse

12 * 6 mod 13 = R → 72 mod 13 = 7

11 is the modular inverse of 6 mod 13 or we can write this as this notation →

$11^{-1} = 6 \bmod 13$

Question 6

A prime number is a whole number greater than one with just one and itself as factors. A factor is a whole number that may be evenly divided into another. For example, 7 cannot be divided into equal-sized units. This is due to the fact that 7 can only be factorized in the following way:

$7 \times 1 = 7$

$1 \times 7 = 7$

This means that the only factors of 7 are 1 and 7. Because it cannot be divided into groups of equal numbers. Hence, 7 is a prime number.

List of Numbers	Prime Numbers
Between 1 and 10	2, 3, 5, 7
Between 11 and 20	11, 13, 17, 19
Between 21 and 30	23, 29
Between 31 and 40	31, 37
Between 41 and 50	41, 43, 47
Between 51 and 100	53, 59, 61, 67, 71, 73, 79, 83, 89, 97

Figure 1. 9 List of numbers with relevant prime numbers

2, 3, 5, 7, 11, 13, 17, 19, 23, and 29 are the first few prime numbers. Composite numbers are those that have more than two factors. For example, 4 can be factorized in a variety of ways. As a result, the factors of four are 1, 2, and 4. There are more than two components involved. Extremely large numbers can be tested to check if they are prime using a computer. However, because there is no limit to how large a natural number may be, testing in this manner will always become too difficult — even for the most powerful supercomputers. For example, in December of 2018, the largest known prime number was 24,862,048 digits.

Table 1. 1 Prime numbers vs Composite numbers

Prime Numbers	**Composite Numbers**
These are numbers that are greater than one and have only two factors: one and the number itself.	These are numbers that are greater than one and have at least three factors.
The smallest and only even prime number is 2.	The smallest composite number is 4.
Prime numbers include 2, 3, 5, 7, 11, 13, and so on.	4, 6, 8, 9, 10, and so on are examples of composite numbers.

Prime numbers are useful for a variety of purposes. For example, Prime Numbers are used in various types of cryptography. In encryption, prime numbers can be particularly useful for generating keys. The prime factors of large numbers are used in most modern computer cryptography. The large number used to encrypt a file can be made public and accessible. Because of the way the encryption works, the only way to decrypt it is to use the prime factors of that large number. Though it is technically only a matter of time to uncover those factors, it would take so much time that we would claim it is impossible.

Quantum computing has received a lot of attention recently, and Google and NASA are working on a project to establish it as a new standard. However, there is no evidence that it can decrypt numbers encrypted with 1024 or 2048 bits. Peter Shor discovered the Quantum Computing Algorithm in 1994. His approach was particularly useful for integer factorization, in which we enter an integer value, and it determines its prime numbers. This technique is quite good at decryption and can factorize smaller prime numbers as well. The RSA scheme, on the other hand, has yet to be cracked. The study is still underway, and once it is completed, all concepts of internet privacy will be shattered.

Prime Numbers can be integrated into our life on several levels. For example, we use prime numbers to communicate our credit card information to online businesses. One of the most important things to remember about prime numbers is that they have unique factorization properties. One of these is that finding greater prime numbers is straightforward. They are, however, more difficult to factorize.

For communications, prime numbers are required, and most computer cryptography relies on them. Not only that, but file encryptions use prime numbers as well. Encryption is used to communicate billing information, log into an account, and even email. The work or prime numbers are the encryption or protection that keeps our personal information from being easily accessible. This feature shows the importance of prime numbers in today's technologically based society. As a result, any limitation in the area of prime numbers is a limitation that we will all experience.

Factorization of more significant prime numbers is used to decrypt the message. However, this factorization is only intended to be used temporarily. Even for a supercomputer, factorization of prime integers is impossible. However, there is hope that quantum computers will be able to solve this challenge in the future.

Prime numbers can also be found on our computer screens. They are used to make the colour of pixels more intense. They're also utilized by manufacturers to get rid of distortions in their products. Furthermore, they provide the foundation of computer security, which is critical in this age of rapid information transmission. Because prime numbers form the foundation of whole numbers, many theorists place a high value on them. They are, however, numbers with unusual mathematical features that have a wide range of uses in today's technology world.

Outside of our beautifully created environment in nature prime numbers play an important function. Cicadas, for example, use prime numbers to lay their eggs and escape their tunnels, according to scientific studies. They leave at 7, 13, or 17 minute intervals. To avoid predators from evolving and preying on them, scientists recommend that they adopt prime numbers. To put it another way, these insects rely on large populations to exist.

Popular culture is full of prime numbers. It has influenced a lot of singers, writers, and artists. Carl Sagan, for example, authored a book on using prime numbers to communicate with aliens. People working in subjects other than science and mathematics are inspired by these positive images and understandings.

Even the most popular game, Candy Crush, is based on scientific theories and mathematical principles, the majority of which would not exist without prime numbers. Without prime numbers, the number theory, for example, would never have seen the light of day. Whatever our thoughts about these unusual numbers, we can't deny that they are an important part of the universe. The continued research will assist us in reaching our full potential as humans. Prime numbers have a lot of potential, and they've been used in a lot of amazing projects. Furthermore, by studying them in depth, we can learn about the world and countless technical achievements.

Activity 2

Part 1

Question 1

What is Probability →

The term "probability" simply refers to the likelihood of something occurring. When we're not sure how an event will turn out, we can talk about the probabilities of several outcomes and how likely they are. Statistics is the study of occurrences determined by probability.

What is Conditional Probability ?

The possibility of an event or consequence occurring based on the occurrence of a preceding event or outcome is known as conditional probability. Conditional probability is derived by multiplying the preceding event's probability by the updated likelihood of the following, or conditional, occurrence. Some conditional probability examples are shown below.

Example 1 :

The first scenario is that a student applying to college is admitted. There is an 80% likelihood that this person will be admitted into college. Event B is that this person will be assigned to a hostel. Only 60% of the approved students will be able to live in hostels.

Hence P (Accepted and hostel accommodation) = P (Hostel accommodation | Accepted) P (Accepted) = (0.60)*(0.80) = 0.48.

Conditional probabilities, as previously stated, are dependent on a past outcome. It makes a number of assumptions as well.

Example 2 :

Let's say we're drawing three marbles from a bag: red, blue, and green. The probability of each marble being drawn is the same. What is the conditional chance of drawing the red marble after the blue one has already been drawn?

First, the likelihood of drawing a blue marble is around 33% because it is one of three possible outcomes. In the event that the first occurrence occurs, there will be two marbles left, each with a 50% chance of being drawn. So the probability of getting a blue marble after drawing a red marble is around 16.5% (33% x 50%).

Example 3 :

Consider the following scenario: a fair die has been rolled, and we are asked to predict the likelihood that it will land on the number five. Your answer is 1/6 because there are six equally likely outcomes.

Consider what might happen if you were given more information before answering, such as the fact that the number rolled was odd. Because there are only three odd numbers that may be rolled, one of which is a five, you'd alter your estimate for the likelihood of a five from 1/6 to 1/3.

The conditional probability of A given B, represented by P(A|B), is a revised likelihood that an event A has occurred, considering the extra information that another event B has certainly occurred on this trial of the experiment.

(BARONE, 2022)

Tree Diagram →

A tree diagram is a great way to visualize what's going on. Let’s take an example and try to visualize in a tree diagram. A bag contains 2 blue and three red marbles. There's a 2/5 probability of getting a Blue marble and a 3/5 chance of getting a Red marble.

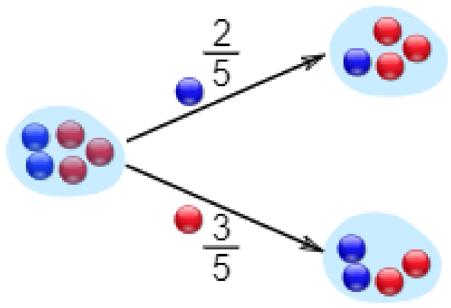


Figure 2. 1 Probability of getting blue and red marbles from the 1st attempt.

If we've already gotten a red marble, there's a 2/4 probability we'll get a blue marble next. If we've already had a blue marble, the chances of getting another one is 1/4. We can take it a step further and observe what happens if we pick up another marble:

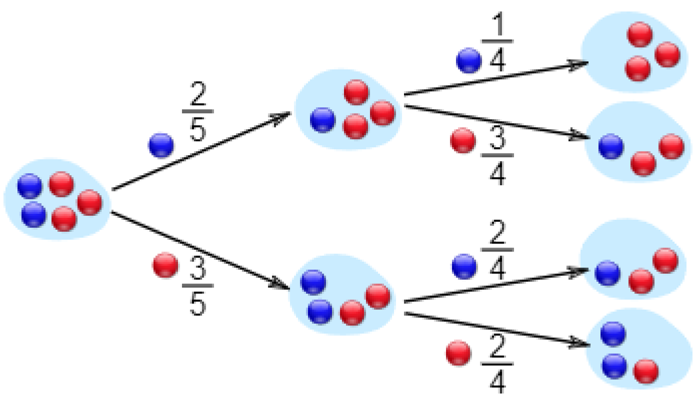


Figure 2. 2 The full tree diagram for the scenario.

We can now answer questions like "How likely can you be to draw two blue marbles?" The probability of drawing two blue marbles is 1/10 (followed by a 2/5 chance and a 1/4 chance).

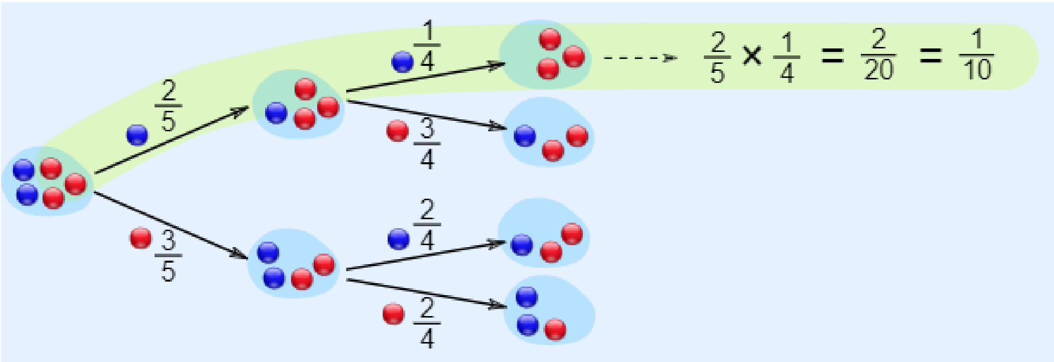


Figure 2. 3 Finding probability of getting 2 marbles using tree diagram.

Notation →

In mathematics, we enjoy notation! That indicates we can interact with the ideas using the power of algebra. Let's visualize and understand the notation for Conditional Probability by using above example. So, the probability of receiving two blue marbles is as follows.

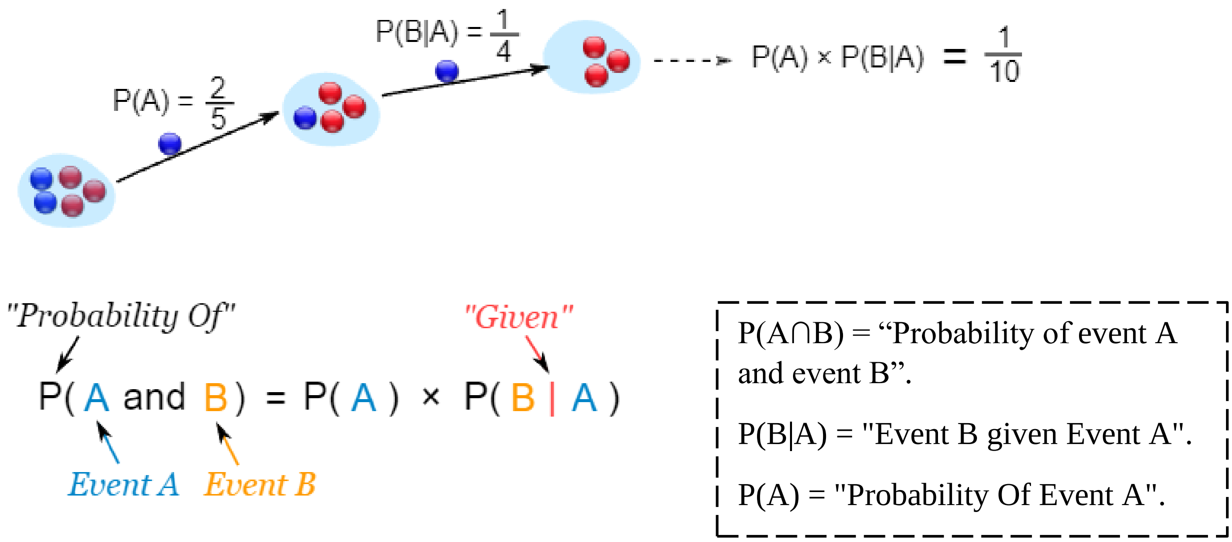


Figure 2. 4 Visualize conditional probability notation using simple example.

(Conditional Probability, n.d.)

Question 2

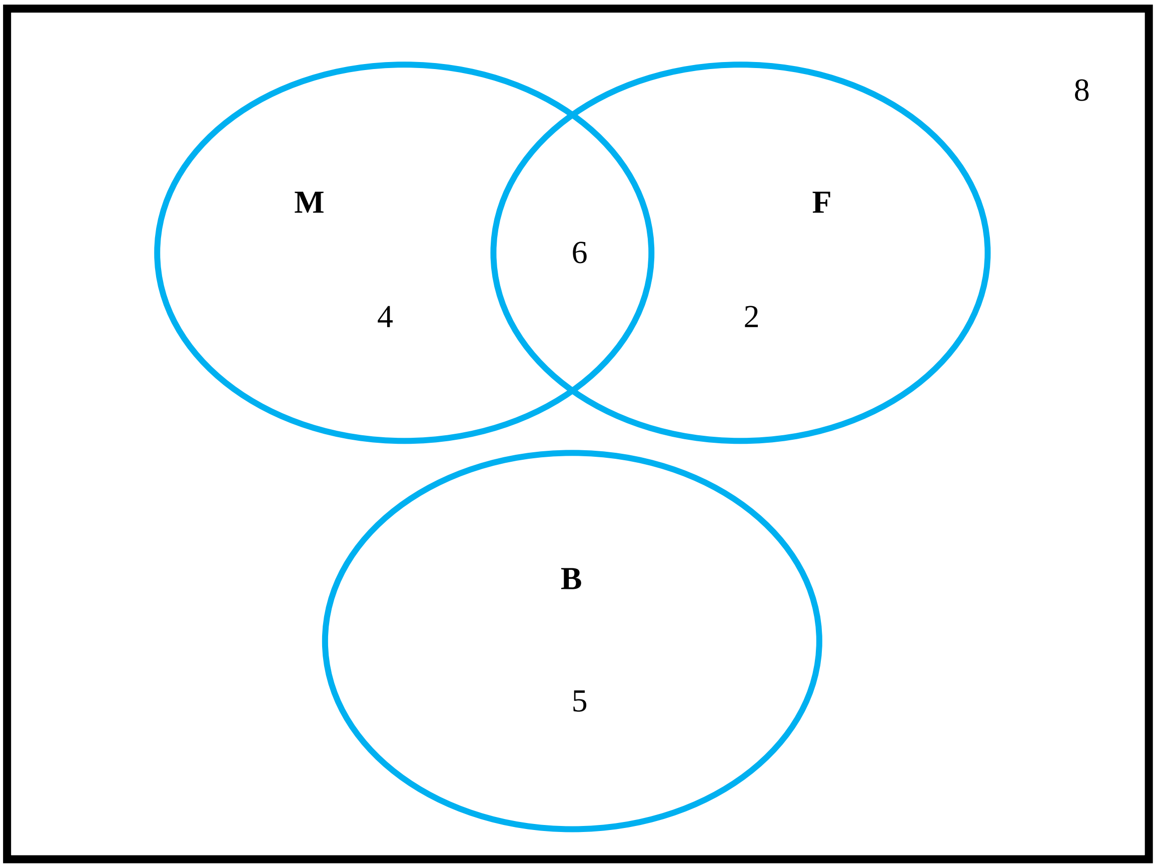


Figure 2. 5 Ven Diagram for the question

a)

n (B) = 5

n (M) = 10

n (F) = 8

n (S) = 25

n (M∩F) = 6

1st method :

Since this is disjoint event to find probability, I can use following equation for this.

$$\boxed{P[E]=\frac{n(E)}{n(S)}=\frac{n(E)}{N}=\frac{\text{no. of outcomes in } E}{\text{total no. of outcomes}}}$$

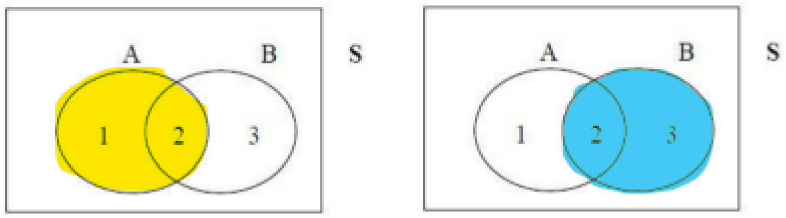
$$\text{P (B or M)} = \text{P (B} \cup \text{M)} = \frac{\text{n (B)} + \text{n (M)}}{\text{n (S)}} = \frac{5+10}{25}$$

$$= 0.6$$

2nd method :

In this question I can use the following theorem of probability also to solve it.

If A and B are any two events, then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$



As we can see the intersection part is adding twice. So, we have to remove 1 additional intersection part.

So, for 2 joined events Probability Union equation is →

$$\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$

Since in our scenario the Biscuit and Milk are 2 separate events since there is no intersection. Hence for our question I can use the following equation.

$$P(B \cup M) = P(B) + P(M) - \cancelto{0}{P(B \cap M)}$$

$$\boxed{P(B \cup M) = P(B) + P(M)}$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{5}{25} = 0.2$$

$$P(M) = \frac{n(M)}{n(S)} = \frac{10}{25} = 0.4$$

$$P(B \text{ or } M) = P(B \cup M) = P(B) + P(M) = 0.2 + 0.4$$

$$\underline{= 0.6}$$

b)

Independent events are events that occurrence is not dependent on any other event.

For example, if we flip a coin in the air and obtain the result Head, we can flip the coin again and get the result Tail. The occurrence of these occurrences is independent of one another in both circumstances.

To check whether M and F independent to each other by using relative formula. If both sides getting same answer it means these events are independent to each other.

$$\boxed{\mathbf{P(\ M \cap F) \quad = \quad P(M)\ \ x\ \ P(F)}}$$

$$P(\ M\ \cap\ F)\ = \frac{n\ (\ M\ \cap\ F)}{n\ (S)} \quad = \frac{6}{25}\ = 0.24$$

$$P\ (M) = \frac{n\ (M)}{n\ (S)} \quad = \frac{10}{25}\ = 0.4$$

$$P\ (F) = \frac{n\ (8)}{n\ (S)} \quad = \frac{8}{25}\ = 0.32$$

Let's replace symbols by found data.

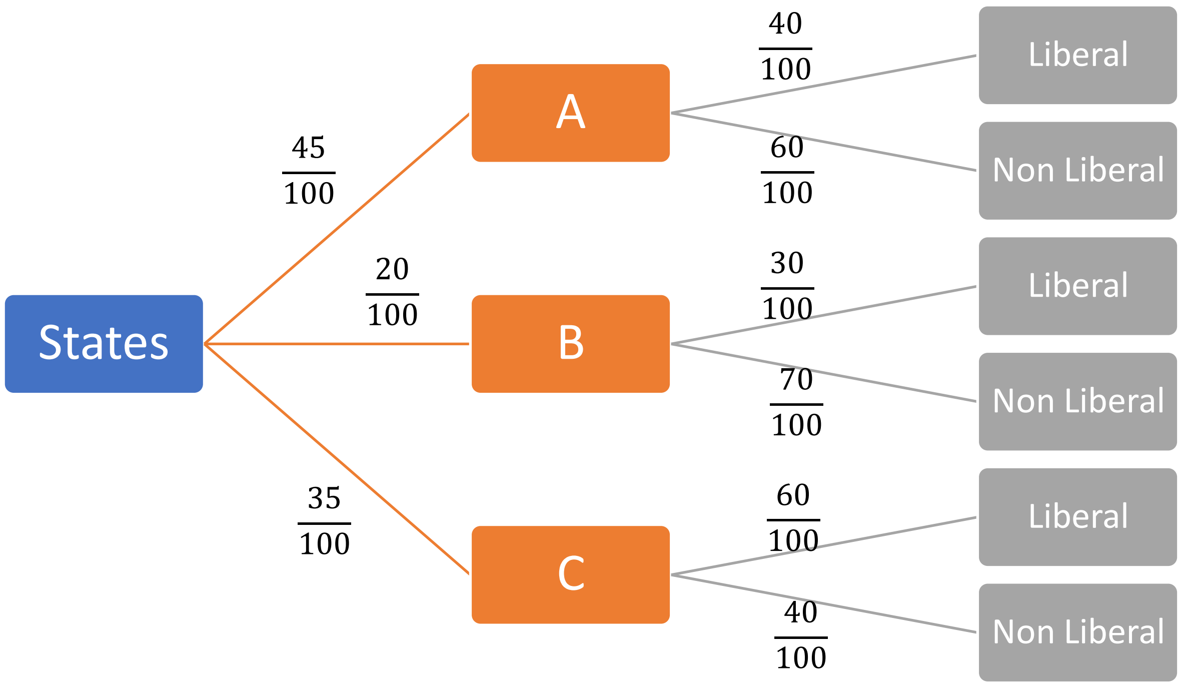
$$P(\ M \cap F) \quad = \quad P(M)\ \ x\ \ P(F)$$

$0.24 \quad = \quad 0.4\ \ x\ \ 0.32$

$0.24 \quad \neq \quad 0.128$

<u>Hence P(M ∩ F) ≠ P(F) x P(F) → M and F is not independent events</u>

Question 3



a)

P (A) = 0.45

P (B) = 0.20

P (C) = 0.35

P (L | A) = 0.4

P (L′ | A) = 0.6

P (L | B) = 0.3

P (L′ | B) = 0.7

P (L | C) = 0.6

P (L′ | C) = 0.4

For this question we can use Conditional Probability notation for find the answer. The relevant Conditional Probability notation for this question is as following:

"Probability Of" *"Given"*

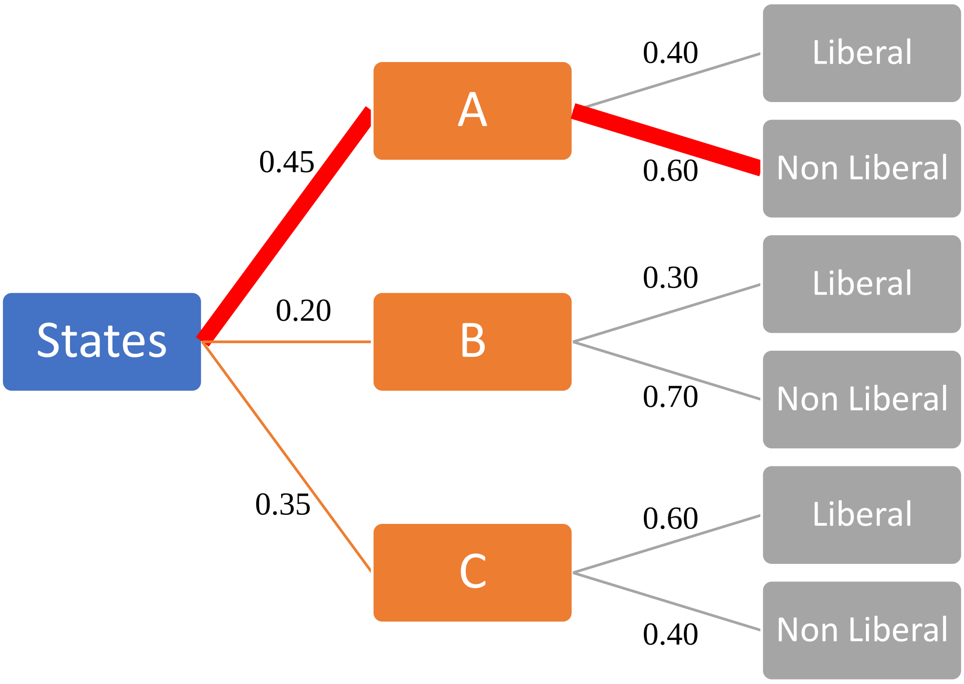
P(A and B) = P(A) × P(B | A)

Event A *Event B*

P(A∩B) = "Probability of event A and event B".

P(B|A) = "Event B given Event A".

P(A) = "Probability Of Event A".

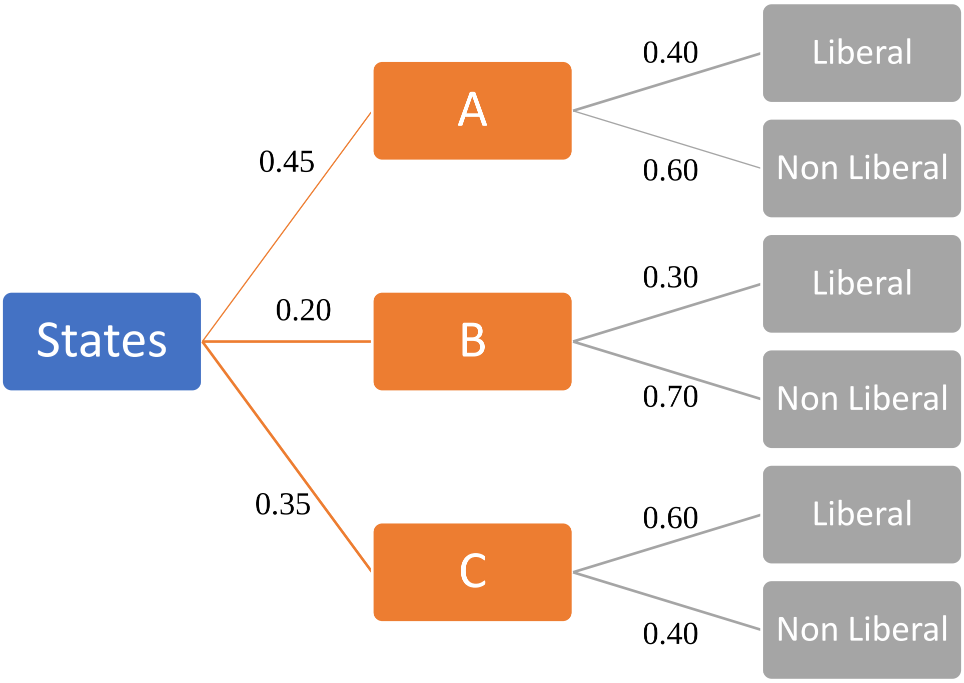


$$P(A \cap L') = P(A) \times P(L' \mid A)$$

$$= 0.45 \times 0.6$$

$$= \underline{\underline{0.27}}$$

b)



$$P(A \cap L) = P(A) \times P(L \mid A)$$
$$= 0.45 \times 0.40$$
$$= \underline{0.18}$$

$$P(B \cap L) = P(B) \times P(L \mid B)$$
$$= 0.20 \times 0.30$$
$$= \underline{0.06}$$

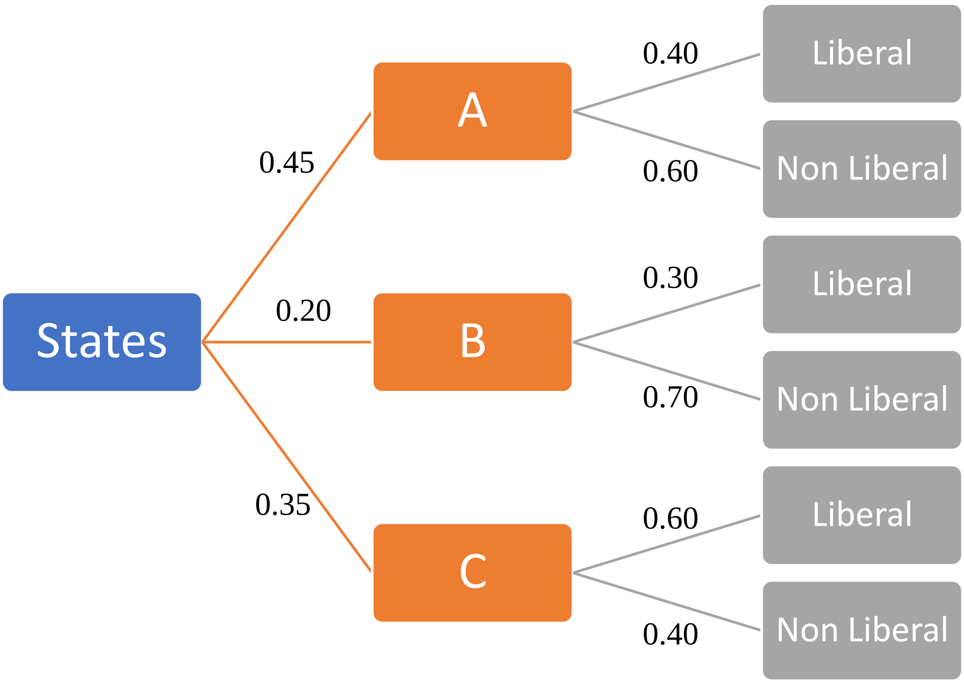
$$P(C \cap L) = P(C) \times P(L \mid C)$$
$$= 0.35 \times 0.60$$
$$= \underline{0.21}$$

$$P(L) = P(A \cap L) + P(B \cap L) + P(C \cap L)$$
$$= 0.18 + 0.06 + 0.21$$
$$= \underline{0.45}$$

c)

For this question we can use Conditional Probability notation for find the answer. The relevant Conditional Probability notation for this question is as following:

$$\boxed{P(A|\mathbf{B}) = \frac{P(A \cap B)}{P(B)}}$$



$$P(L \mid B) = \frac{P(B \cap L)}{P(B)}$$

$P(B \cap L) = P(\mathrm{B}) \quad \mathrm{x} \quad P(\mathrm{L} \mid B)$

$= 0.20 \quad \mathrm{x} \quad 0.30$

$= \underline{0.06}$

$$P(B|L) = \frac{P(B \cap L)}{P(L)} \rightarrow \frac{0.06}{0.45}$$

$$= 0.133$$

(Solutions, 2015)

Question 4

Hearts (H) : 6

Clubs (C) : 7

Diamonds (D) : 8

Scorpions (S) : 5

Total n(s) → 26

1st Attempt :

$$P\ (H)\ = \frac{n\ (H)}{n\ (s)} = \frac{6}{26}$$

$$P\ (C)\ = \frac{n\ (C)}{n\ (s)} = \frac{7}{26}$$

$$P\ (D)\ = \frac{n\ (D)}{n\ (s)} = \frac{8}{26}$$

$$P\ (S)\ = \frac{n\ (S)}{n\ (s)} = \frac{5}{26}$$

2nd Attempt for Hearts given :

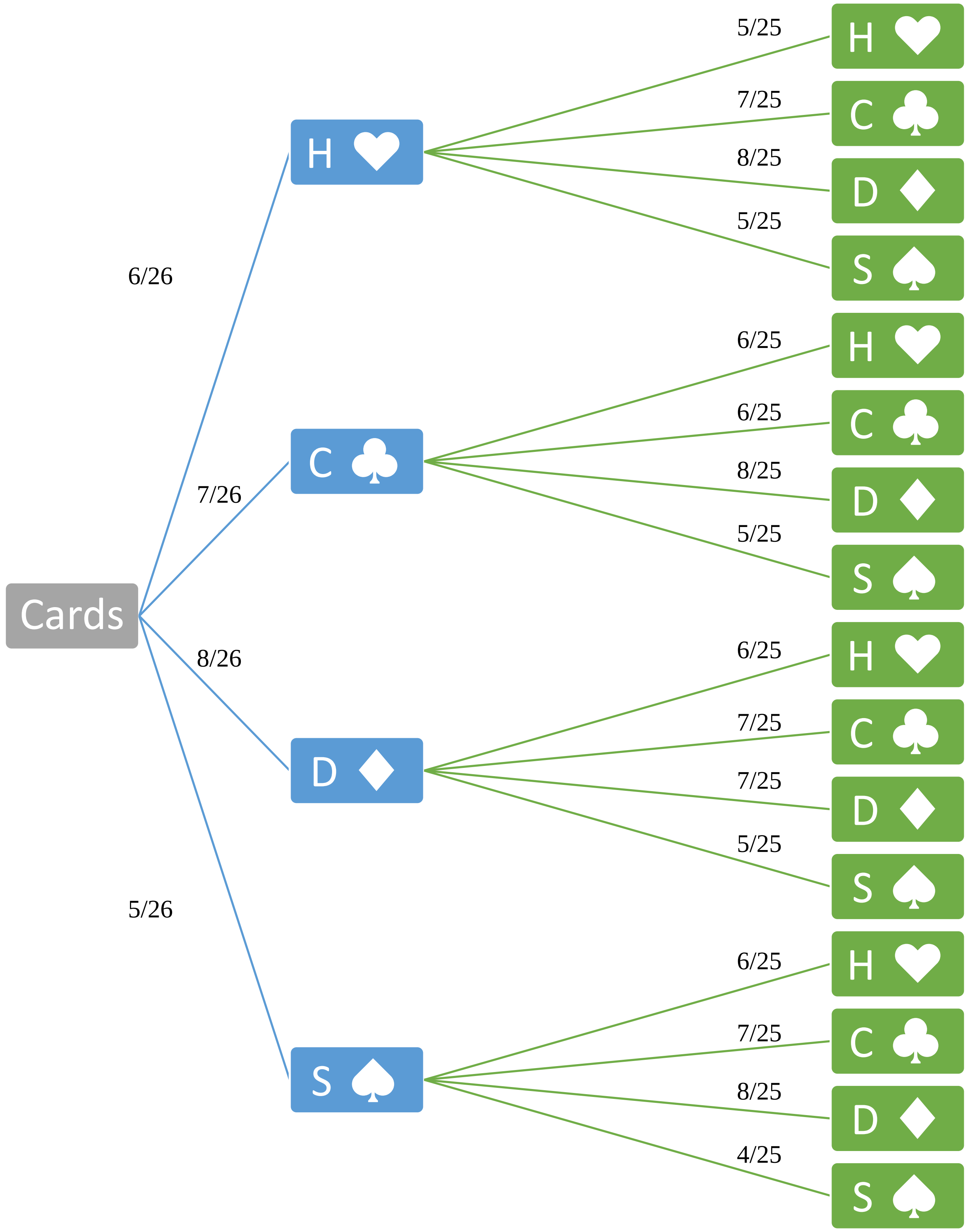
$$P\ (H')\ = \frac{n\ (H')}{n\ (s)} = \frac{5}{25}$$

$$P\ (C')\ = \frac{n\ (C')}{n\ (s)} = \frac{7}{25}$$

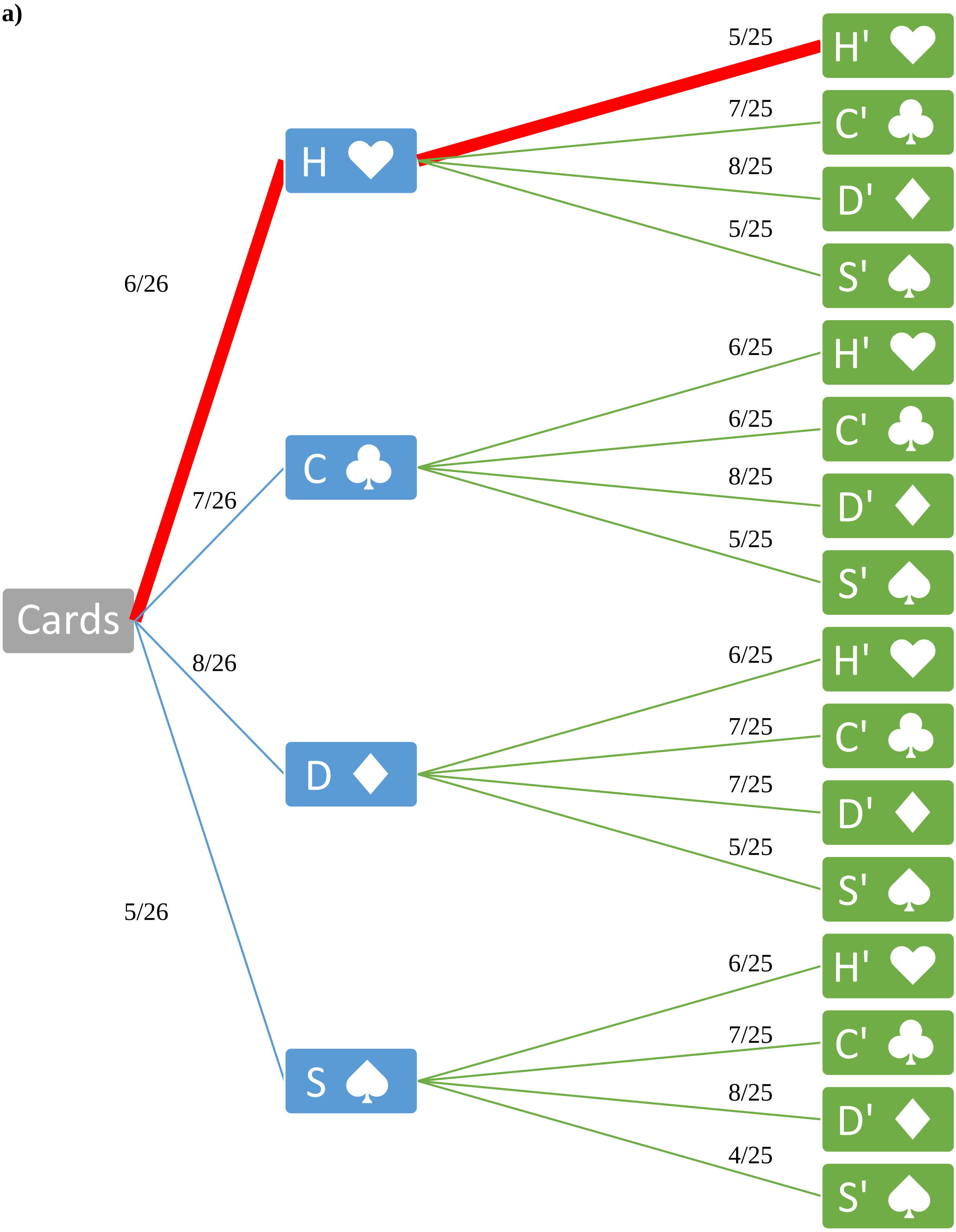
$$P\ (D')\ = \frac{n\ (D')}{n\ (s)} = \frac{8}{25}$$

$$P\ (S')\ = \frac{n\ (S')}{n\ (s)} = \frac{5}{25}$$

Likewise for each 1st Attempt 2nd row's cards probability change accordingly like below tree diagram.


Cards
6/26
7/26
8/26
5/26
H
C
D
S
5/25
7/25
8/25
5/25
6/25
6/25
8/25
5/25
6/25
7/25
7/25
5/25
6/25
7/25
8/25
4/25

a)


Cards
6/26
7/26
8/26
5/26
H
C
D
S
5/25
7/25
8/25
5/25
6/25
6/25
8/25
5/25
6/25
7/25
7/25
5/25
6/25
7/25
8/25
4/25
H'
C'
D'
S'
H'
C'
D'
S'
H'
C'
D'
S'
H'
C'
D'
S'

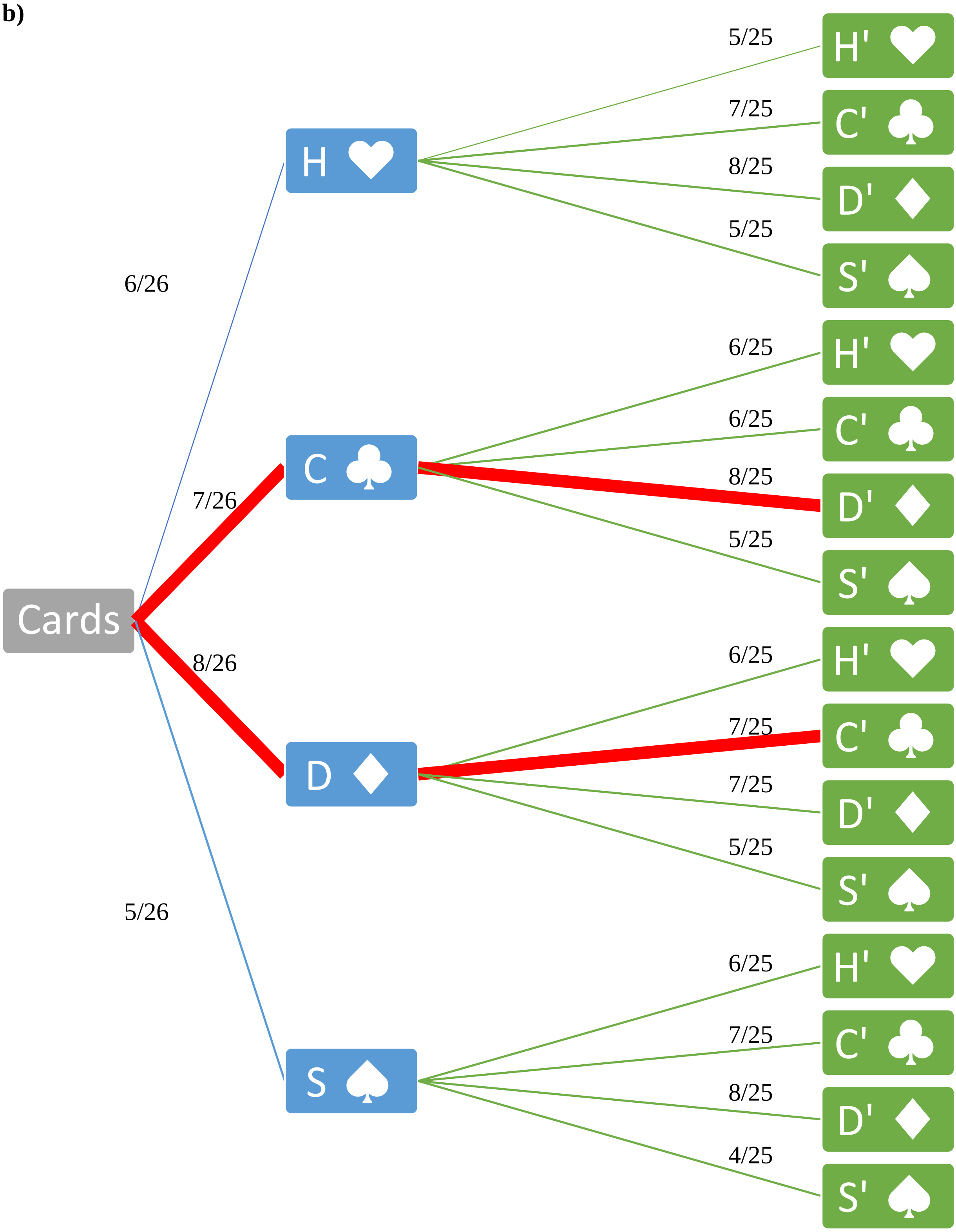
$$P(H \cap H') = P(H) \times P(H' \mid H)$$

$$= \frac{6}{26} \times \frac{5}{25}$$

$$= \frac{30}{650}$$

$$= \frac{3}{65}$$

b)


Cards
6/26
7/26
8/26
5/26
H
C
D
S
5/25
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5/25
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6/25
8/25
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H'
C'
D'
S'
H'
C'
D'
S'
H'
C'
D'
S'
H'
C'
D'
S'

$$P(C \cap D') = P(C) \ x \ P(D' \mid C)$$

$$= \frac{7}{26} \ x \ \frac{8}{25}$$

$$= \frac{56}{650}$$

$$= \frac{28}{325}$$

$$P(D \cap C') = P(D) \ x \ P(C' \mid D)$$

$$= \frac{8}{26} \ x \ \frac{7}{25}$$

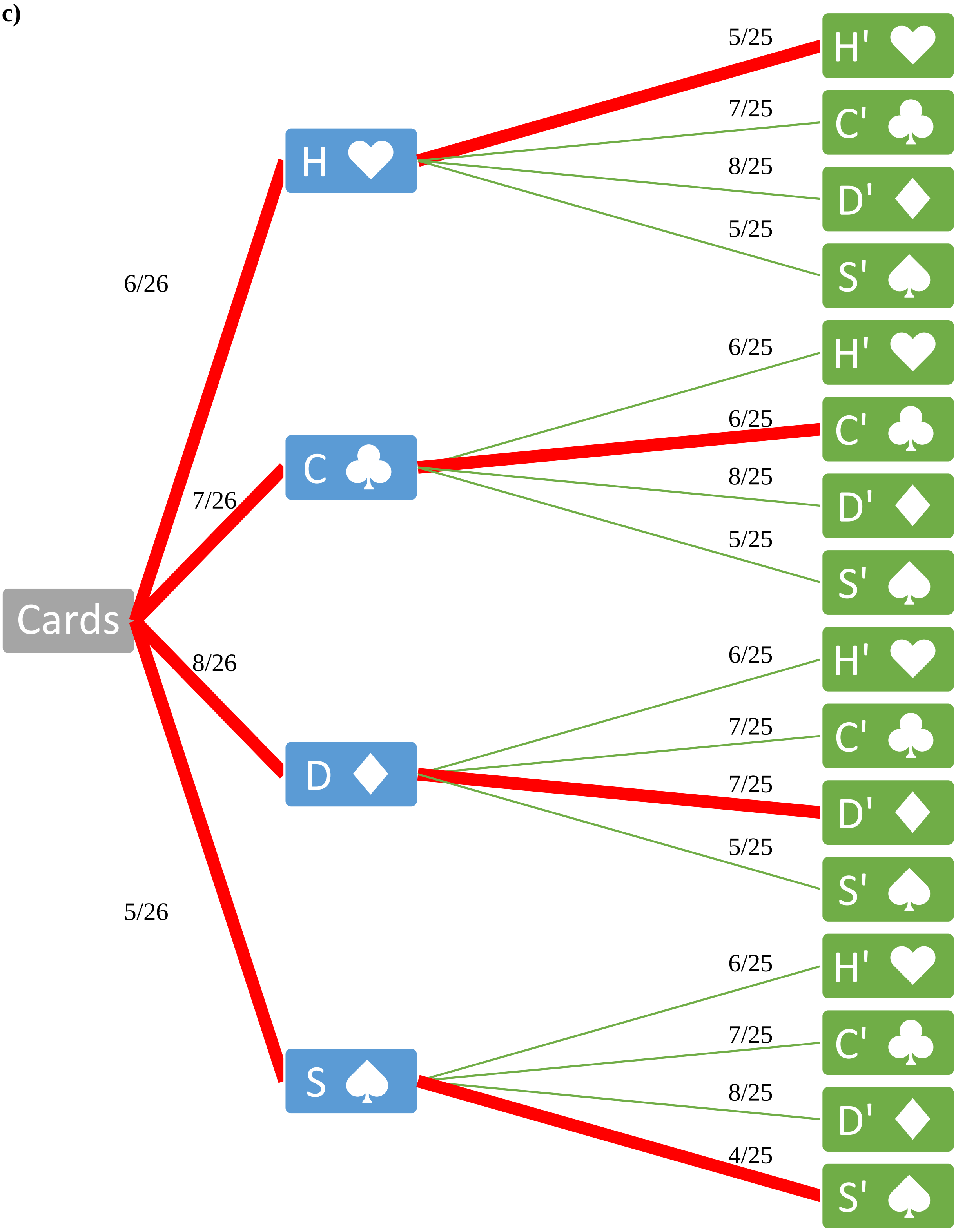
$$= \frac{56}{650}$$

$$= \frac{28}{325}$$

$$P[\ (C \cap D') \cup (D \cap C')\ ] = \frac{28}{325} + \frac{28}{325}$$

$$= \frac{56}{325}$$

c)


Cards
H
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H'
C'
D'
S'
H'
C'
D'
S'
H'
C'
D'
S'
H'
C'
D'
S'

$$P(H \cap H') = P(H) \times P(H' \mid H)$$

$$= \frac{6}{26} \times \frac{5}{25}$$

$$= \frac{30}{650}$$

$$P(C \cap C') = P(C) \times P(C' \mid C)$$

$$= \frac{7}{26} \times \frac{6}{25}$$

$$= \frac{42}{650}$$

$$P(D \cap D') = P(D) \times P(D' \mid D)$$

$$= \frac{8}{26} \times \frac{7}{25}$$

$$= \frac{56}{650}$$

$$P(S \cap S') = P(S) \times P(S' \mid S)$$

$$= \frac{5}{26} \times \frac{4}{25}$$

$$= \frac{20}{650}$$

$$P[\ (H \cap H') \cup (C \cap C') \cup (D \cap D') \cup (S \cap S')\ ] = \frac{30}{650} + \frac{42}{650} + \frac{56}{650} + \frac{20}{650}$$

$$= \frac{148}{650}$$

$$= \frac{72}{325}$$

Part 2

Question 5

Random Variable →

A random variable is a variable used to calculate the outcome of a random experiment. As there are two forms of data, discrete and continuous, there are two types of random variables. A discrete random variable can have an exact value, whereas a continuous random variable's value will lie within a certain range. Probability distributions are diagrams that illustrate how probabilities are spread throughout the values of a random variable.

A random variable is a variable that has a wide range of possible values. This is due to the fact that a random event might have numerous outcomes. As a result, a random variable and an algebraic variable should not be confused. A random variable, on the other hand, can have a collection of values that could be the result of a random experiment.

For an example, assume two dice are rolled and the sum of the numbers is represented by the random variable X. The smallest value of X will therefore be 2 (1 + 1), with the largest value being 12 (6 + 6). As a result, X may be any number between 2 and 12. (inclusive). After assigning probabilities to each outcome, the probability distribution of X can be calculated.

A random variable, commonly denoted by the letter X, is a variable whose possible values are numerical results of a random event.

For example, the random values we can get when rolling a die are X = 1,2,3,4,5,6.

The random values we can obtain while tossing a coin are X = H and T.

Depending on the type of data provided, random variables can be categorized into two types.

1. Discrete random variable
2. Continuous random variable

A discrete random variable is described by a probability mass function, while a continuous random variable is described by a probability density function

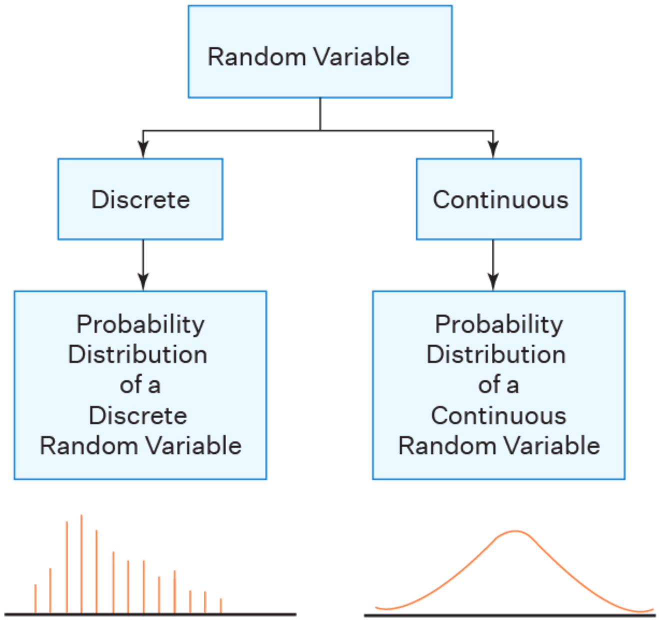


Figure 2. 6 2 Types of Random Variables

1) Discrete Random Variable →

A discrete random variable can take any whole number value as the result of a random experiment. The discrete random variable has a countable number of alternative outcomes, such as 0, 1, 2, 3, 4, and so on. The values of discrete random variables are represented using probability distributions. A stochastic variable is another name for a discrete random variable. A binomial random variable and a Poisson random variable are examples of discrete random variables. In general, there are two sorts of data: discrete and continuous, so we've looked at a discrete random variable here. An algebraic variable should not be confused with a discrete random variable. An algebraic variable can only have one value, whereas a discrete random variable can have many.

For an example, assume two dice are rolled and the sum of the numbers is represented by the random variable X. The smallest value of X will then be 2, arising from the outcomes 1 + 1 = 2, while the largest value will be 12, resulting from the outcomes 6 + 6 = 12. As a result, X may be any number between 2 and 12. (inclusive). Here the random variable "X" takes only 11 values. Since "X" takes only finite values it is called Discrete Random Vaiable. The probability distribution of X may now be calculated by assigning probabilities to each outcome.

Hence Discrete Random Variable simply means it is a variable that can take only countable or finite set of outcomes.

Ex :-

- Number of family members in a family   X = 2,3,4,5 likewise
- Number of vehicles in a park   Y = 20,30,45 likewise
- Number of ants born in tomorrow (Even if there are billions, we can count them)

2) Continues Random Variable →

Random variables are divided into two types: continuous random variables and discrete random variables. A random variable is a variable whose value is determined by all of an experiment's possible outcomes. A continuous random variable has a range of values, whereas a discrete random variable has a fixed value.

As explained before, continuous random variable is variable that can take on a wide range of values. In other words, if a random variable has a value that falls between two intervals, it is said to be continuous. Measurements such as height, weight, time, and so on are represented by continuous random variables. A continuous random variable is represented by the area under a density curve. Hence, Continuous Random Variable is a variable with an infinite number of possible values. As a result, there is no chance that a continuous random variable will take on an exact value.

For an example, the human life span interval is [0 yrs, 100 years]. What are the possible values for "X" if it is a random variable that represents a human's life span?

- X = 3 years 2 months 3 weeks 5 days 3 hours and 5 minutes.
- X = 43 years 1 months 2 weeks 4 days 3 hours and 15 minutes.
- X = 89 years 3 months 1 weeks 6 days 5 hours and 2 minutes.

Can we say all the potential values for "X" in the interval [0 yrs, 100 yrs] as we have described above? "No," is the answer. In the span [0 yrs, 100 yrs], we have an endless number of values for "X." Since "X" can take any value in the interval [0 yrs, 100 yrs], it is referred to be a Continuous Random Variable.

Hence Continous Random Variable simply means it is a variable that can contain infinite set of outcomes (But they may be reported "discretely").

Ex :-

- Height of a student X = 50.01cm,50.11cm,50.20cm,50.21cm… likewise
- Temperature in a city Y = 20.01^{o}C,20.02 ^{o}C,20.03 ^{o}C … likewise
- Exact winning time 100m in 2016 Olympics Z = 9.56s, 9.561s, 956012s likewise could have infinite number of values)

Random Variables are denoted by upper case letters (X,Y etc.).

Individual outcomes for an RV are denoted by lower case letters (x , y etc.)

Ex:-

X = Odd number

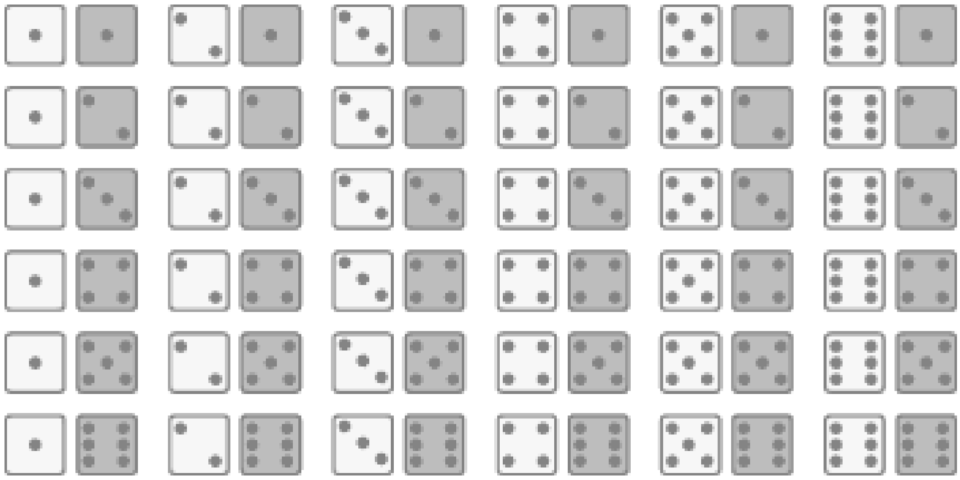
X = {x1, x2, x3… x_n}

1 3 5

Table 2. 1 Discrete Random Variable vs Continuous Random Variable

Discrete Random Variable	Continuous Random Variable
A discrete random variable is one that takes only a finite or countable number of values.	Continuous random variables are those that take all possible values between specific set limitations.
In tossing two coins three times, the number of heads.	The number of hours that a particular bulb will last.
When 10 cards are picked from a 52-card pack that has been well-shuffled, the number of aces is the number of aces.	The "ph" value of a chemical compound randomly chosen.
The number of doctors among students in their final years of school.	Wages of factory workers.
When five dice are thrown once, we get fours.	University students' heights.

Question 6



1st Draw

2nd Draw

Number	**1**	**2**	**3**	**4**	**5**	**6**
1	0	1	2	3	4	5
2	1	0	1	2	3	4
3	2	1	0	1	2	3
4	3	2	1	0	1	2
5	4	3	2	1	0	1
6	5	4	3	2	1	0

X = Difference bretween the values of the 2 cubes ← Random Variable

X = 0 , 1, 2, 3, 4, 5

x1 x2 x3 x4 x5 x6 ← Possible outcomes

x_i	**x1 = 0**	**x2 = 1**	**x3 = 2**	**x4 = 3**	**x5 = 4**	**x6 = 5**
P (X = x_i)	6 / 36	10 / 36	8 / 36	6 / 36	4 / 36	2 / 36

a) The mean of the above probability distribution is as follows:

$$E(X) = \sum x \,.\, P\,(X = x)$$

$$E(X) = \sum_{i=1}^{6} x_i P\,(X = x_i)$$

$$E(X) = \mathrm{x1.P(X = x1) + } x2\mathrm{.P(X = x2) + x3.P(X = x3) + x4.P(X = x4) + x5.P(X = x5) + x6.P(X = x6)}$$

$$E(X) = \mathrm{0.P(X = x1) + 1.P(X = x2) + 2.P(X = x3) + 3.P(X = x4) + 4.P(X = x5) + 5.P(X = x6)}$$

$$E(X) = 0\left(\frac{6}{36}\right) + 1\left(\frac{10}{36}\right) + 2\left(\frac{8}{36}\right) + 3\left(\frac{6}{36}\right) + 4\left(\frac{4}{36}\right) + 5\left(\frac{2}{36}\right)$$

$$E(X) = 0 + \frac{10}{36} + \frac{16}{36} + \frac{18}{36} + \frac{16}{36} + \frac{10}{36}$$

$$E(X) = \frac{10 + 16 + 18 + 16 + 10}{36}$$

$$E(X) = \frac{70}{36} = \frac{35}{18} = 1.94$$

b)

$$V(X) = \sum x^2 \,.\; P\,(X = x) - [E(X)]^2$$

$$V(X) = \sum_{i=1}^{6} x_i P\,(X = x_i)\; - [E(X)]^2$$

$$V(X) = x_1^2.P(X = x1) + x_2^2.P(X = x2) + x_3^2.P(X = x3) + x_4^2.P(X = x4) + x_5^2.P(X = x5) + x_6^2.P(X = x6) - [E(X)]^2$$

$$V(X) = 0^2.P(X = x1) + 1^2.P(X = x2) + 2^2.P(X = x3) + 3^2.P(X = x4) + 4^2.P(X = x5) + 5^2.P(X = x6) - [E(X)]^2$$

$$V(X) = 0\left(\frac{6}{36}\right) + 1\left(\frac{10}{36}\right) + 4\left(\frac{8}{36}\right) + 9\left(\frac{6}{36}\right) + 16\left(\frac{4}{36}\right) + 25\left(\frac{2}{36}\right) - [E(X)]^2$$

$$V(X) = 0 + \frac{10}{36} + \frac{32}{36} + \frac{54}{36} + \frac{64}{36} + \frac{50}{36} - [E(X)]^2$$

$$V(X) = \frac{10 + 32 + 54 + 64 + 50}{36} - [E(X)]^2$$

$$V(X) = \frac{210}{36} = \frac{105}{18} = 5.83 - [E(X)]^2$$

$$V(X) = 5.83 - [1.94]^2$$

V(X) = 2.05

$$SD(X) = \sqrt{var(X)}$$

$$SD(X) = \sqrt{2.06}$$

SD (X) = 1.43

c)

$$E(A) = E(X - 10)$$

We can use these Expected Functions sperately.

$$= E(X) - E(10)$$

Expected value of contant term is equal to corresponding contant term.

$$= 1.94 - 10$$

$$\underline{E(A) = -8.05}$$

$$E(B) = E\left(\frac{1}{2}X - 5\right)$$

We can use these Expected Functions sperately.

$$= E\left(\frac{1}{2}X\right) - E(5)$$

Expected value of contant term is equal to corresponding contant term.

$$= \frac{1}{2}E(X) - 5$$

$$= \frac{1}{2}(1.94) - 5$$

$$\underline{E(B) = -4.02}$$

d)

$$V(A) = V(X - 10)$$

We can use these Variance Functions speratly.

$$= V(X) - V(10)$$

A constant's variance is zero.

$$= 2.05 + 0$$

$$\underline{V(A) = 2.05}$$

Variance never reduces. It is always added.

$$V(B) = V\left(\frac{1}{2}X - 5\right)$$

We can use these Variance Functions speratly.

$$= V\left(\frac{1}{2}X\right) - V(5)$$

A constant's variance is zero.

$$= \left(\frac{1}{2}\right)^2 \; V(X) + 0$$

$$= \frac{1}{4}(2.05) - 0$$

Variance never reduces. It is always added.

$$\underline{V(B) = 0.51}$$

e)

Small Theory Part →

We discuss Binomial Distribution under Discrete Continuous Random Variable.

And we discuss Normal Distribution under Continuous Random Variable.

- The binomial distribution has two possible outcomes (the prefix “bi” means 2 or twice). → The experiment should be a series of independent trials.
- A coin flip, for example, has only two possible outcomes: heads or tails, but taking a test has two possible outcomes: pass or fail.
- A Binomial Distribution either shows (S)uccess or (F)ailure.

Let p be the probability of success, so that q = 1 − p is the probability of failure.

X →	The number of successes out of n trials
p →	Probability of success
q = 1 – p →	Probability of failure
n →	Number of trials

We use this notation to show X follows Binomial Distribution.

$$x \sim Bin$$

Binomial

Follows

Binomial Distribution has 2 parameters. 1 is “n” and other 1 is “p”

$$x \sim Bin\ (n, p)$$

Number of trials

Probability of success

Answer to the Question →

When Arnold gets his score using random variable A and Brian gets his score using random variable B.

I can use this following notation to show how Arnold follows Binomial Distribution.

A → The number of successes out of n trials Arnold gets

p → Probability of success

n → Number of trials Arnold rolls the pair of fair cubes

$$A \sim Bin\ (n\ ,p)$$

I can use this following notation to show how Brian follows Binomial Distribution.

B → The number of successes out of n trials Brian gets

p → Probability of success

n → Number of trials Brian rolls the pair of fair cubes

$$B \sim Bin\ (n\ ,p)$$

The probability distribution for rolling fair cubes will therefore have the same Binomial distribution with parameters "n" and "p" for both.

Question 7

Small theory part for this question →

A person is rolling a die

X = Having odd number ← My Random Variable

X = 1 , 3, 5

x1 x2 x3 ← Possible outcomes

Find Probability when my Random variable X is equal to x1 → P(X = x1)

→ P(X =1)

Should use like these notations.

Find Probability when my Random variable X is equal to x3 → P(X = x3)

→ P(X =5)

When taking probability summation of each probability value, it should give 1.

Ex:- $\sum P(X = x_i) = P(X = x1) + P(X = x2) + P(X = x3) = 1$

a)

	y1	y2	y3	y4	y5
Y = y	**1**	**2**	**3**	**4**	**5**
P (Y = y)	1/3	1/6	1/4	k	1/6

P (Y = y1)

P (Y = 1)

Probability when Y equals to 1

P (Y = y5)

P (Y = 5)

Probability when Y equals to 12

a)

$$\sum_{i=x_1}^{x_5} P\ (Y = y_i) = 1$$

$$\sum P(Y = y_i) = \mathrm{P(Y\ =\ y1) + P(Y\ =\ y2) + P(Y\ =\ y3) + P(Y\ =\ y4)\ + P(Y\ =\ y5)}\ \ = 1$$

$$= \frac{1}{3} + \frac{1}{6} + \frac{1}{4} + k + \frac{1}{6} = 1$$

$$k = 1 - \left(\frac{1}{3} + \frac{1}{6} + \frac{1}{4} + \frac{1}{6} \right)$$

$$k = 1 - \left(\frac{22}{24} \right)$$

$$k = \left(\frac{2}{24} \right) = \left(\frac{1}{12} \right) = 0.083$$

b)

$$\sum P(\mathrm{Y} \leq 3) = \mathrm{P}(\mathrm{Y} = 1) + \mathrm{P}(\mathrm{Y} = 2) + \mathrm{P}(\mathrm{Y} = 3)$$

$$= \frac{1}{3} + \frac{1}{6} + \frac{1}{4}$$

$$= \frac{8+4+6}{24}$$

$$= \frac{18}{24}$$

$$= \underline{0.75}$$

c)

$$\sum P(\mathrm{X} > 2) = \mathrm{P}(\mathrm{X} = 3) + \mathrm{P}(\mathrm{X} = 4) + \mathrm{P}(\mathrm{X} = 5)$$

$$= \frac{1}{4} + \frac{2}{24} + \frac{1}{6}$$

$$= \frac{6+2+4}{24}$$

$$= \frac{12}{24}$$

$$= \underline{0.50}$$

Part 3

Question 10

a)

X = number of matches will be won by the team

{0,1,2,3,4,5,6,7,8,9,10}

b)

$$x \sim Bin\ (n, p)$$

X → The number of successes

p → Probability of success = 0.75

q = 1 – p → Probability of failure = 1 – 0.75 = 0.25

n → Number of trials = 10

$$\boxed{P(x) = {}^{n}C_{x} p^{x} q^{n-x} = \frac{n!}{(n-x)!x!} p^{x} q^{n-x}}$$

$$P\ (X = 6) = \binom{10}{6} (0.75)^{6}\ (0.25)^{10-6}$$

$$\binom{10}{6} = \frac{n!}{(n-x)!\ x!} = \frac{10!}{(10-6)!\ 6!} = \frac{10!}{(4)!\ \ 6!} = \frac{10 \times\ 9 \times\ 8 \times\ 7 \times \cancel{6} \times \cancel{5} \times \cancel{4!}}{\cancel{(4)!}\ \cancel{6} \times \cancel{5} \times\ 4 \times\ 3 \times\ 2 \times\ 1}$$

$$\binom{10}{6} = \frac{5040}{24} = 210$$

Hence,

$$P\ (X = \ 6) = \binom{10}{6}\,(0.75)^6\ (0.25)^{10-6}$$

$$P\ (X = \ 6) = 210\ (0.75)^6\ (0.25)^4$$

$$\underline{P\ (X = 6\ ) = 0.145}$$

c)

$$P\ (X \geq \ 8) = P\ (X = \ 8) + \ P\ (X = \ 9) + \ P\ (X = \ 10)$$

To find P (X = 8) →

$$P\ (X = \ 8) = \binom{10}{8}\,(0.75)^8\ (0.25)^{10-8}$$

$$\binom{10}{8} = \frac{n!}{(n-x)!\ x!} = \frac{10!}{(10-8)!\ 8!} = \frac{10!}{(2)!\ \ 8!} = \frac{10 \times 9 \cancel{\times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2\,!}}{\cancel{(2)}!\ \cancel{8 \times 7 \times 6 \times\ 5 \times\ 4 \times\ 3} \times\ 2 \times\ 1}$$

$$\binom{10}{8} = \frac{90}{2} = 45$$

Hence,

$$P\ (X = \ 8) = \binom{10}{8}\,(0.75)^8\ (0.25)^{10-8}$$

$$P\ (X = \ 8) = 45\ (0.75)^8\ (0.25)^2$$

$$\underline{P\ (X = 8\ ) = 0.281}$$

To find P (X = 9) →

$$P\ (X =\ 9) = \binom{10}{9}\,(0.75)^{9}\ (0.25)^{10-9}$$

$$\binom{10}{9} = \frac{n!}{(n-x)!\ x!} = \frac{10!}{(10-9)!\ 9!} = \frac{10!}{(1)!\ \ 9!} = \frac{10 \times \cancel{9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2} \times \cancel{1!}}{\cancel{(1)}!\ \ \cancel{9 \times 8 \times 7 \times 6 \times\ 5 \times\ 4 \times\ 3 \times\ 2} \times\ 1}$$

$$\binom{10}{9} = \frac{10}{1} = 10$$

Hence,

$$P\ (X =\ 9) = \binom{10}{9}\,(0.75)^{9}\ (0.25)^{10-9}$$

$$P\ (X =\ 9) = 10\ (0.75)^{9}\ (0.25)^{1}$$

P (X = 9) = 0.187

To find P (X = 10) →

$$P\ (X =\ 10) = \binom{10}{10}\,(0.75)^{10}\ (0.25)^{10-10}$$

$$\binom{10}{10} = \frac{n!}{(n-x)!\ x!} = \frac{10!}{(10-10)!\ 10!} = \frac{10!}{(0)!\ \ 9!} = \frac{10\ \cancel{\times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}}{\cancel{9 \times 8 \times 7 \times 6 \times\ 5 \times\ 4 \times\ 3 \times\ 2 \times\ 1}}$$

$$\binom{10}{10} = 1$$

Hence,

$$P\ (X =\ 10) = \binom{10}{10}\,(0.75)^{10}\ (0.25)^{10-10}$$

$$P\ (X =\ 9) = 1\ (0.75)^{10}\ (0.25)^{0}$$

P (X = 10) = 0.056

Hence the s the probability that the team will lose 2 or less matches is →

$$P\ (X \geq\ 8) = P\ (X =\ 8) +\ P\ (X =\ 9) +\ P\ (X =\ 10)$$

$$P\ (X \geq 8\ )\ =\ 0.281 + 0.187 + 0.056$$

$$\underline{P\ (X \geq 8\ )\ =\ 0.524}$$

d)

mean = E (X) = n × p = 10 (0.75) = 7.5

e)

variance = Var (X) = σ^2 = n × p × q = 10 (0.75) (0.25) = 1.875

standard deviation = σ = $\sqrt{\sigma^2}$ = $\sqrt{1.875}$ = 1.369

Question 11

Small Theory Part →

- The most significant of all probability distributions is the continuous normal distribution.
- It has a bell-shaped graph. Almost all fields use this bell-shaped curve.
- The entire area under the curve is just one because it is a continuous distribution.

Empirical Rule →

According to this rule, 68% of the data is within 1 standard deviation of the mean.

95% of the data is within 2 standard deviations of the mean.

And 99.7% of the data within 3 standard deviations of the mean.

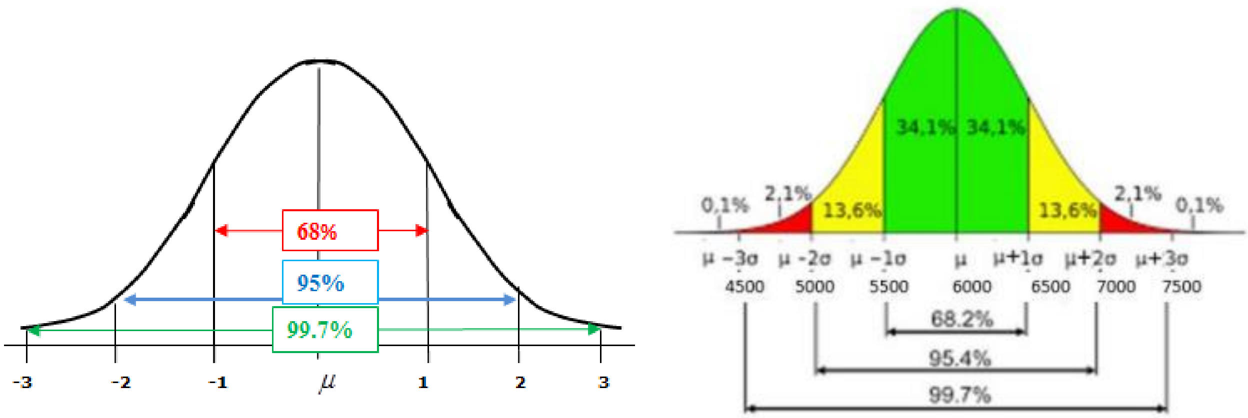


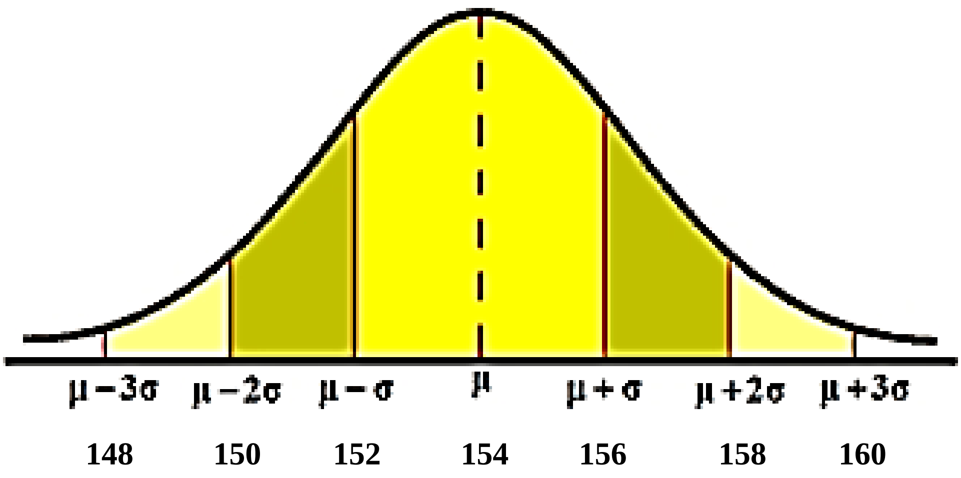
Figure 2. 7 Bell curve with Empirical Rule

Within these sigma intervals, 99.7% of the data is aligned. The data outside of it is referred to as an "outlier."

Answer to the Question →

Mean Height (μ) = 154cm

SD (σ) = 2cm



Alex's Height = 163cm

Alex's Z- score value $= \frac{value - mean}{S.D.} = \frac{x-\mu}{\sigma} = \frac{163-154}{2}$ = 4.5

Z-scores of +/-3 or greater from zero are commonly used to identify outliers.

Alex's height is an outlier since Alex's height's Z-score value 4.5 is above than 3.

Question 12

a)

We use this notation to show X follows Normal Distribution. Has 2 parameters.

$$x \sim Normal\ (\mu, \sigma^2)$$

Follows | Normal | Population Mean | Population Variance

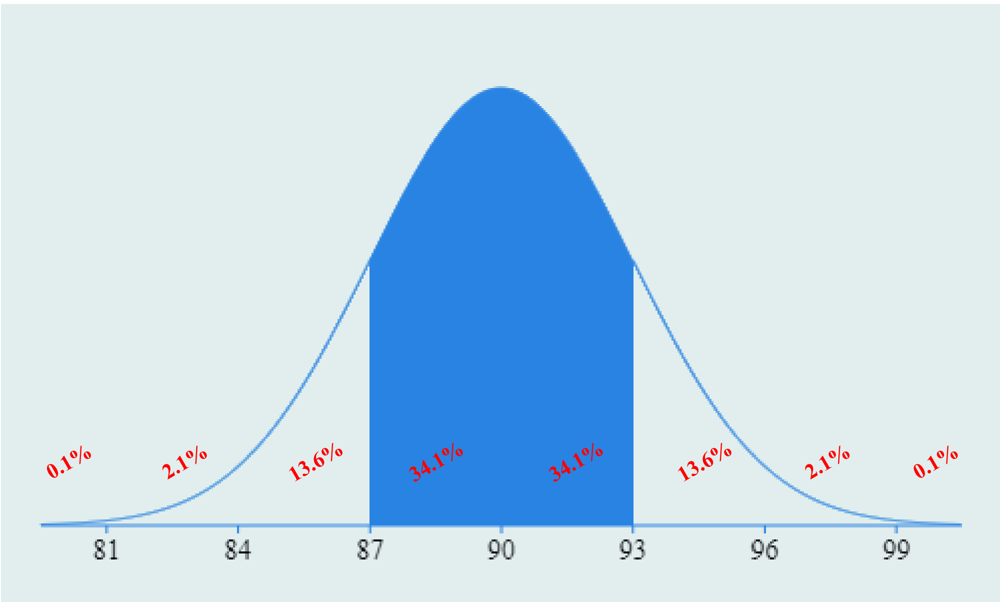


Figure 2. 8 Bell Curve for the question

Since the data is within 1 standard deviation of the mean →

P (87 < X < 93) = 34.1% + 34.1% = 68.2% or (0.68)

b)

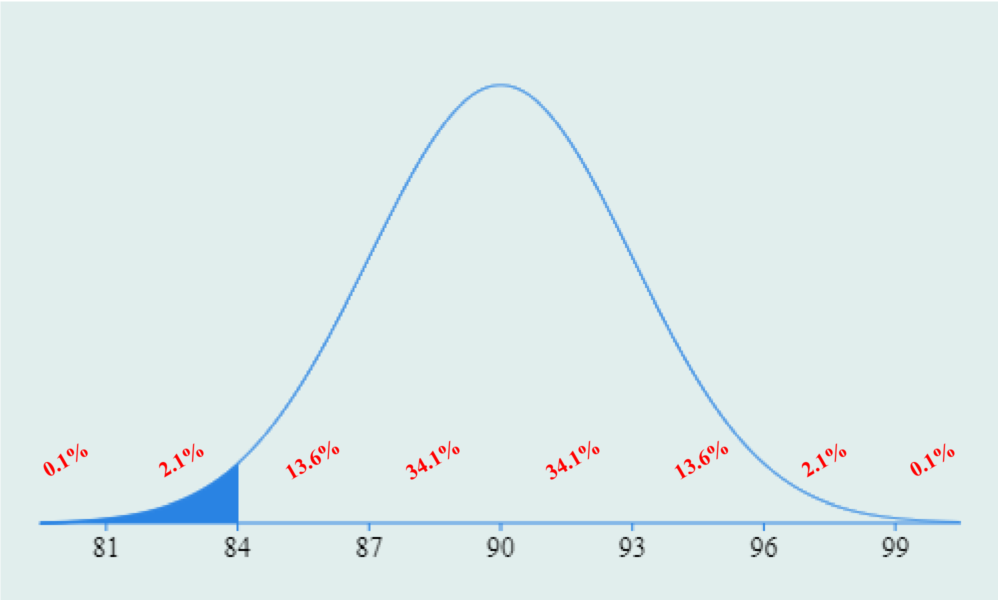


Figure 2. 9 Bell Curve for the question

P (X ≤ 84) = 2.1% + 0.1% = 2.2% or (0.02)

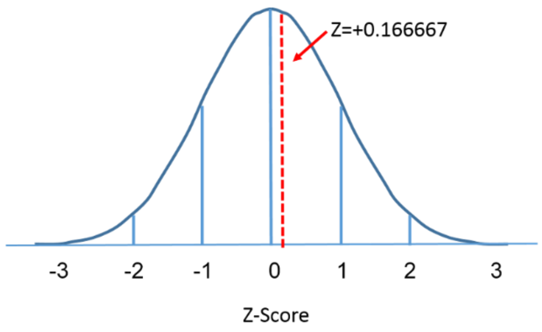
c)

Small Theory Part →

- We can't utilize Empirical Rule to determine the probability of this one.
- To find these probabilities, we use Probability Distribution Tables.
- Before we can understand the Probability Distribution Table, we must first understand the Standard Normal Distribution.

The standard normal distribution, commonly known as the z-distribution, is a type of normal distribution with a mean of zero and a standard deviation of one. Any normal distribution's values can be normalized by converting them to z-scores. Z-scores indicate how many standard deviations each result is from the mean.

This is an example.



Answer to the Question →

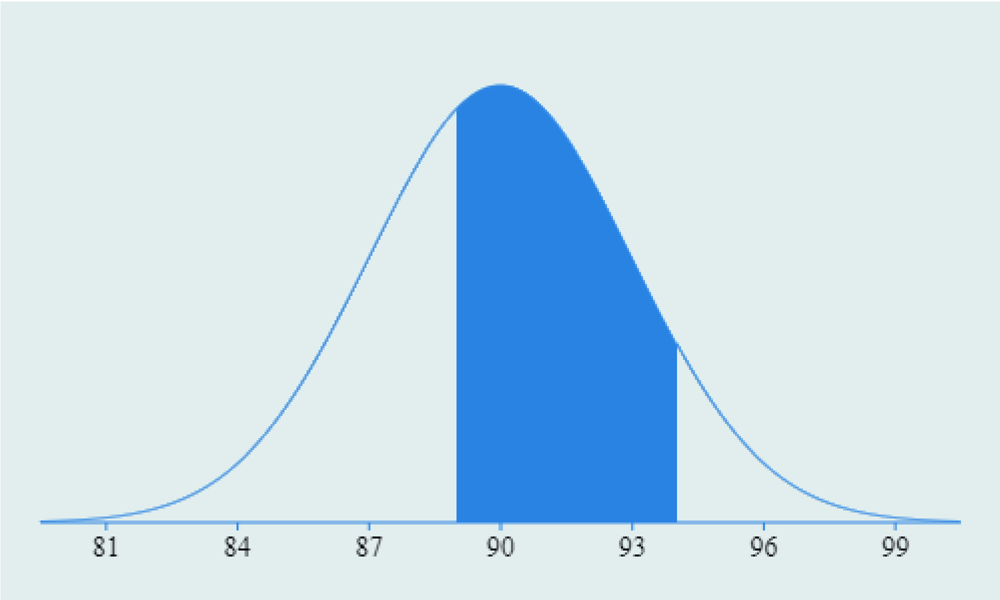


Figure 2. 10 Bell Curve for the question

$$x \sim Normal\ (\mu, \sigma^2)$$

$$x \sim Normal\ (90{,}9)$$

Should convert this to standard normal distribution using this equation → $Z = \frac{X-\mu}{\sigma}$

If I do change a left-hand side, I have to do the same change to right hand side also.

$\mu \rightarrow 90$

$\sigma \rightarrow 3$

$$P(89 < X < 94)$$

$$P\left(\frac{89-\mu}{\sigma} < \frac{X-\mu}{\sigma} < \frac{94-\mu}{\sigma}\right)$$

$$P\left(\frac{89-90}{3} < \frac{X-90}{3} < \frac{94-90}{3}\right)$$

$$P\left(\frac{-1}{3} < Z < \frac{4}{3}\right)$$

$$P(-0.333 < Z < 1.333)$$

We need to find this area's Probability.

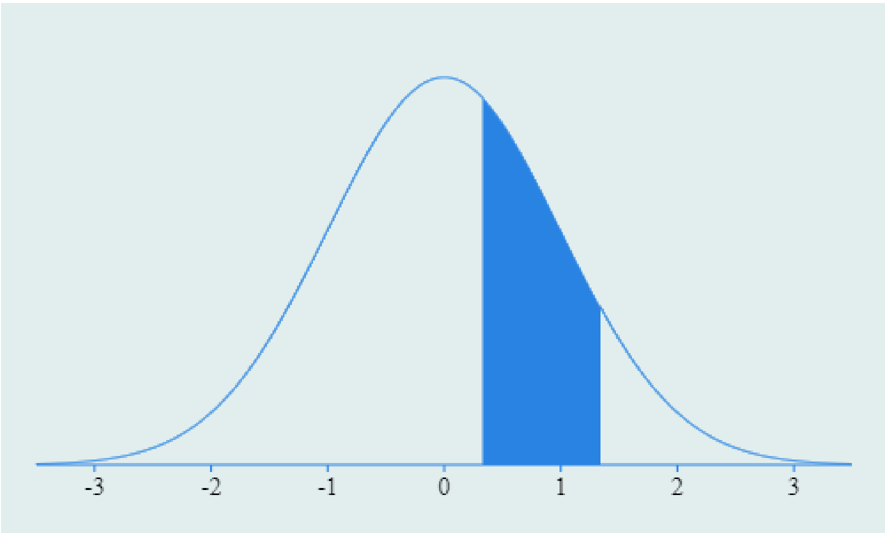
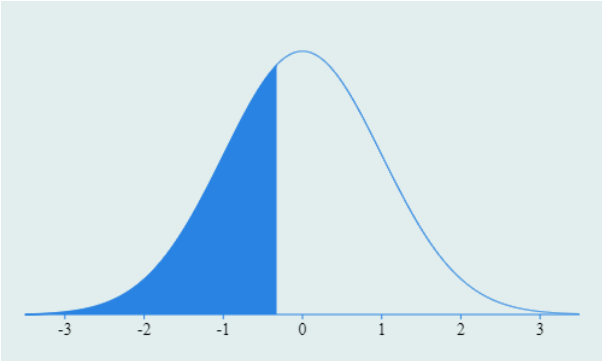


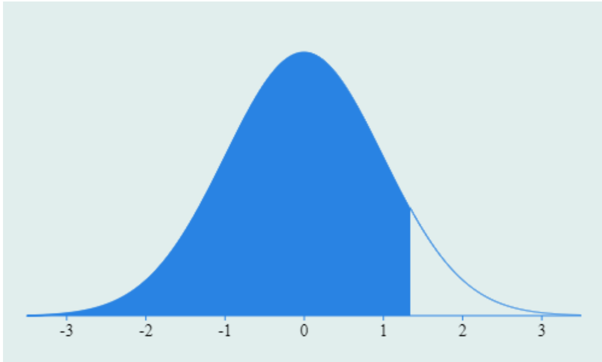
Figure 2. 11 Z distribution for the question

But, when we are using Standard Normal Distribution Chart to find above probability, we would find these areas probabilities.

Area 1 → $P(Z < -0.333)$



Area 2 → $P(Z < 1.333)$



To find exact probability we need to use simple area math.

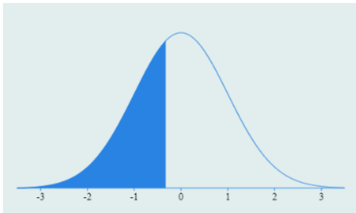
To do that, we reduce the Area 1 from Area 2 to get the answer.

Solution

=

Area 2

-



Area 1

Area 2 Probability According to S.N.D. Chart is →

$P(Z < 1.333) = 0.90824$

Area 1 Probability According to S.N.D. Chart is →

$P(Z < -0.333) = 0.37070$

Answer is → $P(Z < 1.333) - P(Z < -0.333) = 0.90824 - 0.37070 =$

0.53754 or 53.75%

d)

We need to find this area's Probability.

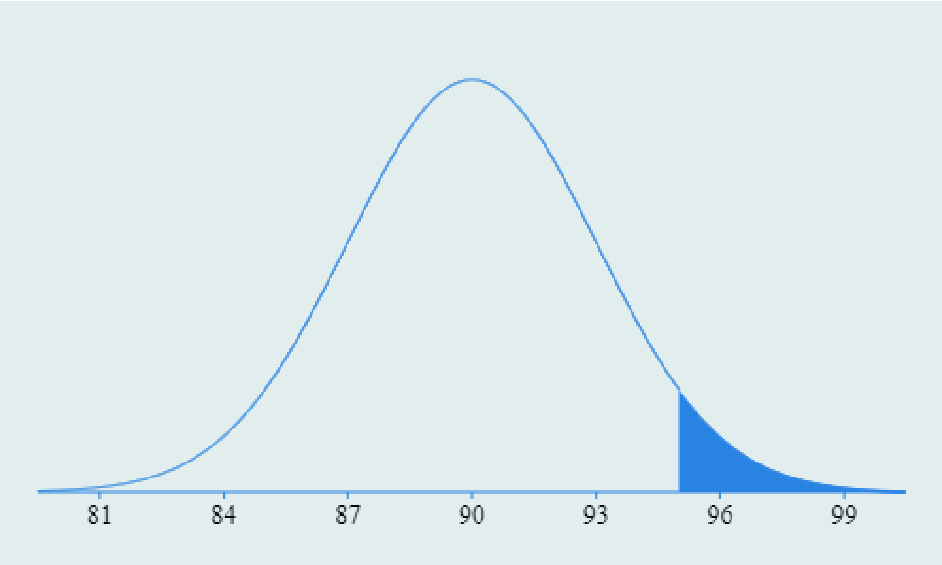


Figure 2. 12 Z distribution for the question

Should convert this to standard normal distribution using this equation → $Z = \frac{X-\mu}{\sigma}$

If I do change a left-hand side, I have to do the same change to right hand side also.

μ → 90

σ → 3

$$P(95 < X)$$

$$P\left(\frac{95-\mu}{\sigma} < \frac{X-\mu}{\sigma}\right)$$

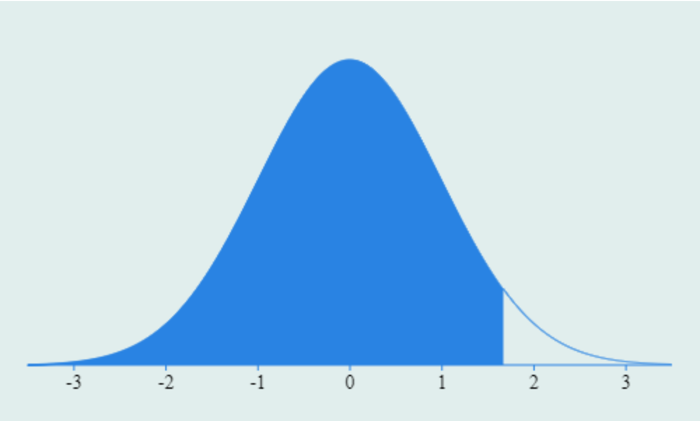
$$P\left(\frac{95-90}{3} < \frac{X-90}{3}\right)$$

$$P\left(\frac{5}{3} < Z\right)$$

$$P(1.666 < Z)$$

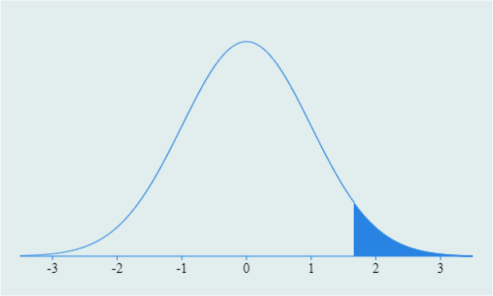
But, when we are using Standard Normal Distribution Chart to find above probability, we would find this area probability.

Area 1 → $P(Z < 1.666)$



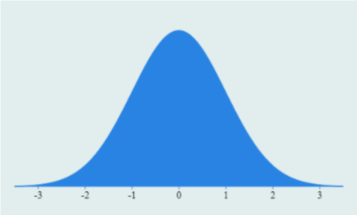
To find exact probability we need to use simple area math.

To do that, we reduce the Area 1 from Total area to get the answer.



Solution

=



Total Area

-

Area 1

Total Area Probability is → 1

Area 1 Probability According to S.N.D. Chart is →

$P(Z < 1.666) = 0.95154$

Answer is → $1 - P(Z < 1.666)$ = 1 - 0.95154 =

0.04846 or 4.8%

Question 13

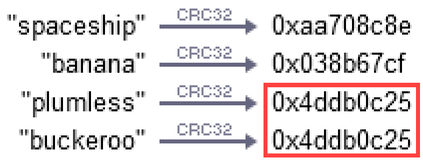
Application of Probability in Hashing

Hashing is the process of converting one value into another based on a specified key or string of characters. This is frequently represented by a shorter, fixed-length value or key that represents the original string and makes it easier to find or use it. The most common application for hashing is the creation of hash tables. A hash table holds key-value pairs in a list that can be accessed via its index. Since this number of key-value pairs is infinite, the hash function will map the keys to the table size. A hash value is then used to index a specific element. A hash value, also known as a hash, is created by a hash function and is based on a mathematical hashing method. This algorithm takes data and hashes it to create a uniform message. A good hash always utilizes a one-way hashing technique to avoid the conversion of the hash back into the original key.

A hash function generates an integer hash value within a certain range from an item of a specific type. Strings, built shader programs, files, and even directories can be used as input. The same input always produces the same hash value, whereas a good hash function produces distinct hash values when given different inputs. A hash function is unaware of "other" elements in the input set. It simply does do some arithmetic and/or bit-magic techniques on the input object. As a result, there is always the probability that two different inputs will produce the same hash value.

Take, for example, the well-known hash function CRC32. If we pass this function the strings "plumless" and "buckeroo," it will return the same result. This is known as a hash collision. What is the probability of a collision between hashes? The birthday problem from mathematics is really a generalized version of this problem. Because the answer is not always obvious, it can be difficult to predict correctly.

Hash collision example



Due to probability, an event is sometimes more likely to occur than we assume it is. In this scenario, if we survey a random sample of only 23 people, there is a 50-50 chance that two of them will have a birthday. The birthday paradox is the name given to this situation. We can put it to the test and observe how mathematical probability works.

There are numerous hash function options, and research is constantly ongoing to develop a solid hash function. Some hash functions are quick, while others are slow. Some spread hash values evenly across the allowed range, whereas others do not. Let's pretend the hash function is quite good for our purposes — it distributes hash values uniformly across the available range.

Here's a graph with $N=2^{32}$. This graph represents the probability of a collision while utilizing 32-bit hash values. It's worth mentioning that when the number of hashes reaches 77163, there's a 50% risk of a collision. Also, regardless of N, the graph has the same S-curve shape.

Graph with N=232 which represents the probability of a collision while utilizing 32-bit hash values.

Avoiding collisions is important in some applications, such as when employing hash values as IDs. Because of this, the small probability is the most intriguing. The following table offers a range of small probability if our hash values are 32-bit, 64-bit, or 160-bit. We can simply find the nearest matching row if we know the number of hash values. I've included a few real-world probabilities that I pulled from the internet to assist put the statistics into context, such the chances of winning the lottery.

Number of **32-bit** hash values	Number of **64-bit** hash values	Number of **160-bit** hash values	**Odds of a hash collision**
77163	5.06 billion	1.42×10^{24}	1 in 2
30084	1.97 billion	5.55×10^{23}	1 in 10
9292	609 million	1.71×10^{23}	1 in 100
2932	192 million	5.41×10^{22}	1 in 1000
927	60.7 million	1.71×10^{22}	1 in 10000
294	19.2 million	5.41×10^{21}	1 in 100000
93	6.07 million	1.71×10^{21}	1 in a million
30	1.92 million	5.41×10^{20}	1 in 10 million
10	607401	1.71×10^{20}	1 in 100 million
	192077	5.41×10^{19}	1 in a billion
	60740	1.71×10^{19}	1 in 10 billion
	19208	5.41×10^{18}	1 in 100 billion
	6074	1.71×10^{18}	1 in a trillion
	1921	5.41×10^{17}	1 in 10 trillion
	608	1.71×10^{17}	1 in 100 trillion
	193	5.41×10^{16}	1 in 10^{15}
	61	1.71×10^{16}	1 in 10^{16}
	20	5.41×10^{15}	1 in 10^{17}
	7	1.71×10^{15}	1 in 10^{18}

Odds of a full house in poker
1 in 693

Odds of four-of-a-kind in poker
1 in 4164

Odds of being struck by lightning
1 in 576000

Odds of winning a 6/49 lottery
1 in 13.9 million

Odds of dying in a shark attack
1 in 300 million

Odds of a meteor landing on your house
1 in 182 trillion

Figure 2. 13 A few real-world probabilities taken from the internet, such as lottery odds

By using following link's calculator, we can measure the Hash Table Collision Probability.

[https://opendsa-server.cs.vt.edu/Calculate the probability of a collision.](https://opendsa-server.cs.vt.edu/)

Application of Probability in Load Balancing

In order to overcome the problems of increasing network delay and lower handling capacity induced by network congestion in 4G networks, a probability-based load balancing method is developed. The nodes use a probabilistic approach for routing admission and historical load data to map the network load state and balance loads. The historical state based load mapping may effectively remove the difficulties in determining the load status in distributed independent operation, etc. Hence the probabilistic approach can effectively overcome the fuzzy judgment upon threshold admission, while simultaneously reducing the number of broadcast packets to save channel resources.

H&P DSR (History and Probability Based Dynamic Source Routing) was developed on the foundation of combining the traditional on-demand routing protocol to considerably improve network throughput and reduce time delay without increasing channel occupancy. The majority of load balancing route admission methods used today are based on threshold criteria. In other words, a threshold value is established to determine routing admission, namely admitting (or prohibiting) routes with metrics less than (or more than) the threshold value. In contrast to the threshold-based routing admission mechanism, the probability-based algorithm does not directly determine whether the route is admitted. However, it provides an admission probability via information summary in order for the node to implement routing admission based on the probability.

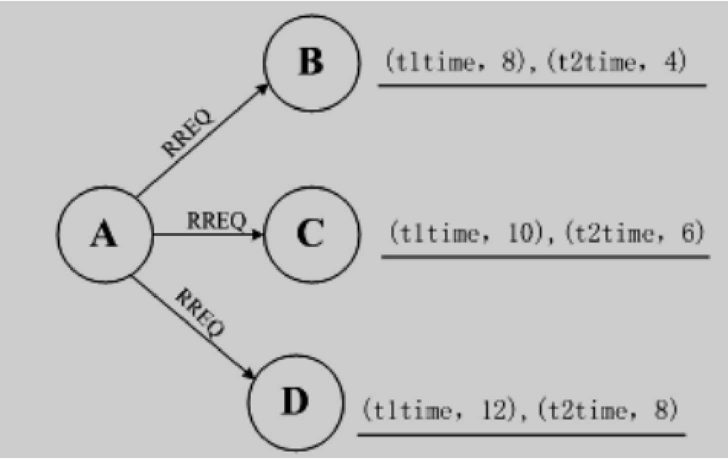


Figure 2. 14 Load description for nodes at different moments

As indicated in the image above, nodes B, C, and D received the routing request from node A, and the load descriptions for nodes B, C, and D at t1 are 8, 10, and 12, respectively. If the threshold value is 7, such a parameter cannot be utilized to differentiate the loads of nodes B, C, and D. Because the loads on all three nodes are greater than the threshold value [7-8].

Similarly, at t2, the load descriptions for nodes B, C, and D are 4, 6, and 8, respectively. The three nodes can still not be distinguished if the threshold value is 10, and there is a significant load variation between the three nodes.

The goal of the probabilistic algorithm is to determine various routing admission probabilities based on various load description variables. For instance, the probabilistic algorithm returns 80%, 60%, and 30% as the admission probability and the routing admission results for the three nodes B, C, and D are definitely distinct from one another if the load description values are respectively 8, 10, and 12. Node D definitely transmits more RREQs than the other two nodes.

The probability-based algorithm can correctly differentiate between the loads of various nodes and then apply various techniques to the various loads. For a fixed load, an appropriate admission probability must be given. If the preset load is considered an independent variable and the corresponding admission probability is considered a function value, then the following equation can be used to derive the relevant function connection between the load and the admission probability.

$$\boxed{P = F(L)}$$

P is the admission probability, L is the node load, and F is the probability function. If a specific load L is specified, the above method can be used to calculate the routing admission probability P. Meanwhile, the probability function F can be fitted using a variety of curves.

Activity 3

Part 1

Question 1

What is equation of a Circle →

A circle is a closed curve formed from a fixed point known as the center, with all points on the curve having the same distance from the center. The radius is defined as the distance from the center.

(Equation of a circle, n.d.)

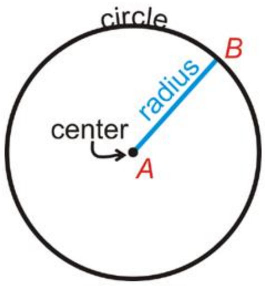


Figure 3. 1 A circle with a radius line and a central point.

The circle can be labeled to represent the center and a point on the circle. The circle's center is (h, k), and a point on the circle is (x, y).

The radius length is the distance between (h, k) and (x, y).

In the below picture, the following points have been marked.

The center point coordinate of a circle is given by **(h , k).**

The radius of a circle is given by **r.**

A general point on the circle is given by **(x , y).**

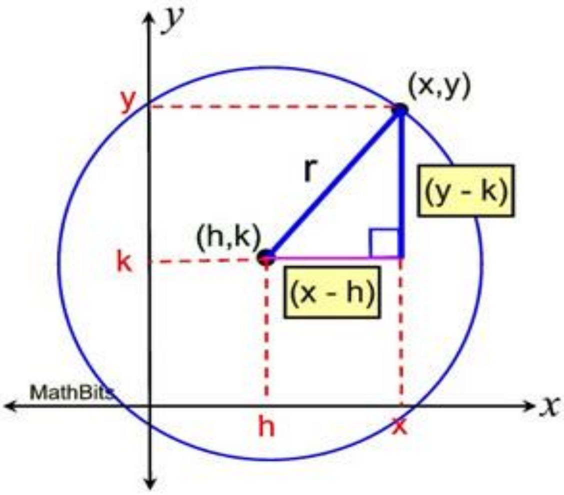


Figure 3. 2 Typical circle with basic coordinates

We can get the equation by using the distance formula, which can be used to calculate the distance between two points.

$$r^2 = a^2 + b^2$$

$$\boxed{r^2 = (y-k)^2 + (x-h)^2}$$

Finding equation of circle for the assignment question →

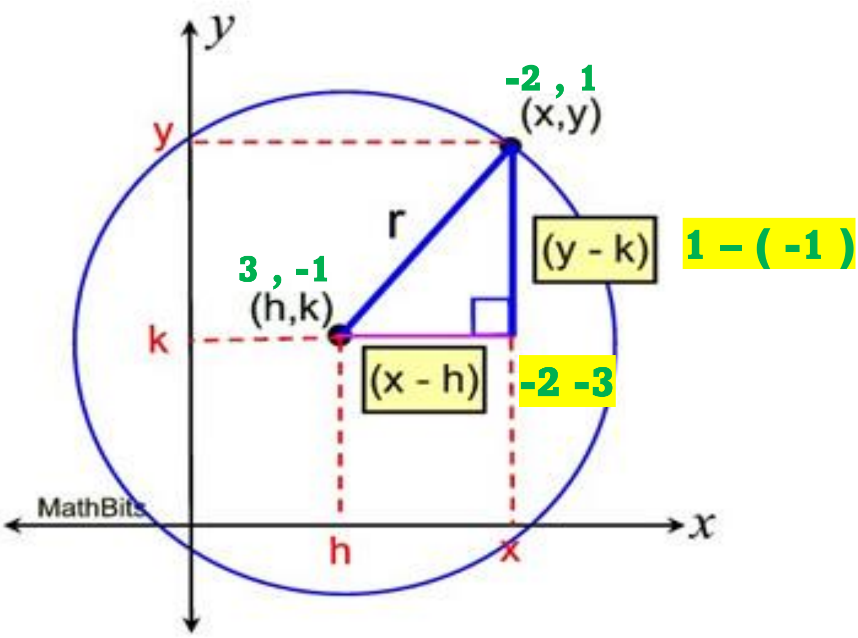


Figure 3. 3 Finding equation of circle with basic coordinates

$$r^2 = (y-k)^2 + (x-h)^2$$

Let's find "r" value (radius of the circle) first.

$$r^2 = [1-(-1)]^2 + [-2-3]^2$$

$$r^2 = [1+1]^2 + [-2-3]^2$$

$$r^2 = [+2]^2 + [-5]^2$$

$$r^2 = 4 + 25$$

$$r^2 = 29$$

$$\underline{r = \sqrt{29}}$$

Now we can find the equation by substituting "r" value to distance formula.

$$r^2 = (y-k)^2 + (x-h)^2$$

$$r^2 = (y-(-1))^2 + (x-h)^2$$

$$\underline{29 = (y+1)^2 + (x-3)^2}$$

We can graph the equation of the circle as follows.

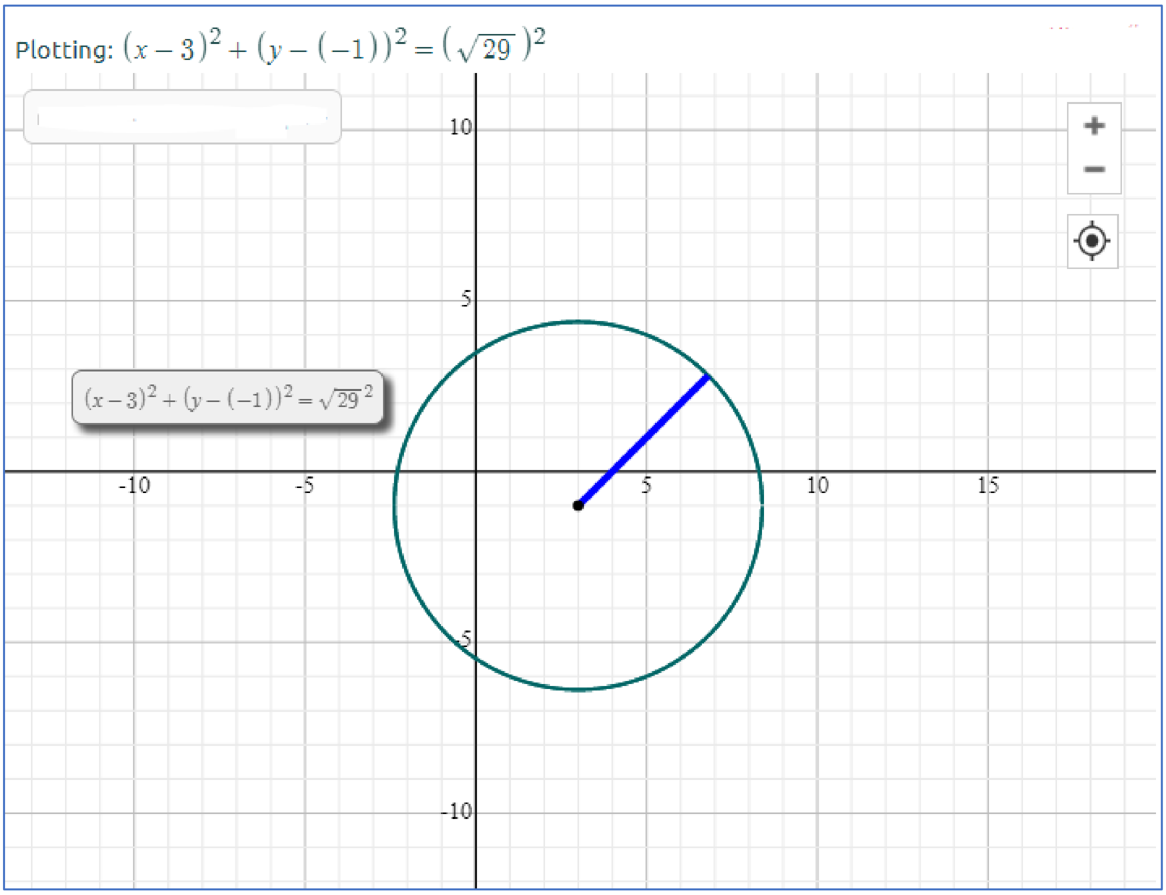


Figure 3. 4 Graph of the Equation of the Circle

Question 2

What is equation of a Sphere →

A sphere is a three-dimensional object with a circular shape. The sphere is split into 3 axes: the x-axis, the y-axis, and the z-axis. This is the key distinction between a circle and a sphere. A sphere, unlike other 3D shapes, has no edges or vertices. The points on the sphere's surface are evenly spaced from the center. As a result, the distance between the sphere's center and surface is equal at any point. The radius of the sphere is the length of this distance. A ball, a globe, the planets, and other spheres are examples.

(Sphere, n.d.)

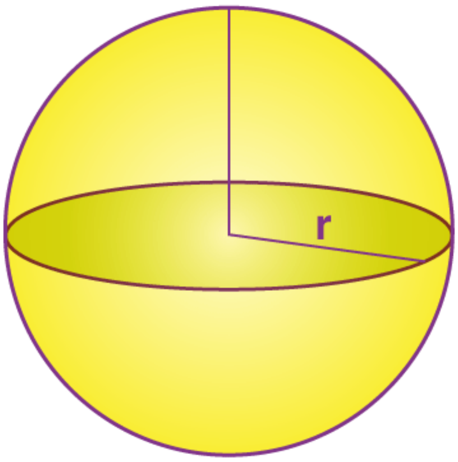
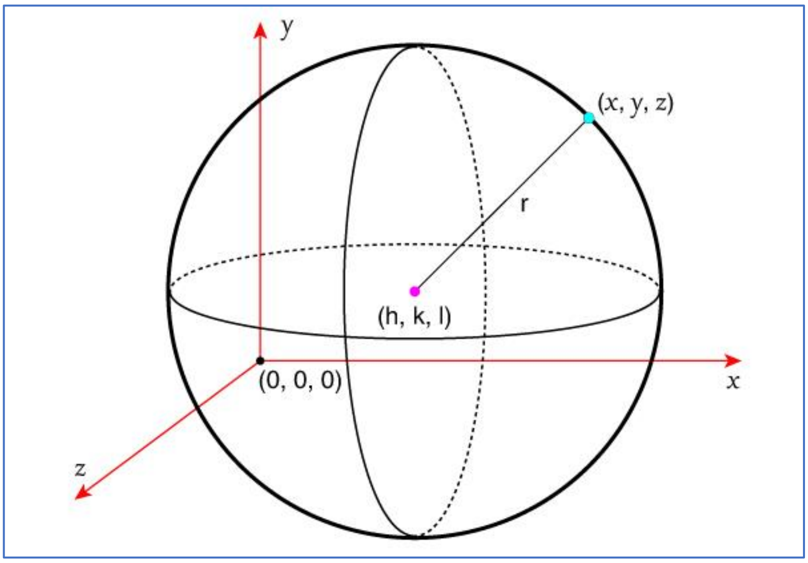


Figure 3. 5 A sphere with a radius

Spheres are three-dimensional representations of circles. A sphere's equation is similar to that of a circle, but with an additional variable for the extra dimension.

$$\boxed{r^2 = (y-k)^2 + (x-h)^2 + (z-l)^2}$$

Equation of circle has 2 coordinates in each point. But in equation of sphere, it has 3 coordinates in each point.



At here → r = radius

(h , k , l) = center

(x , y , z) = any point on the surface

Finding equation of circle for the assignment question →

We need to arrange this equation as our general equation as below. A sphere's standard form equation is:

$$r^2 = (y-k)^2 + (x-h)^2 + (z-l)^2$$

To arrange the equation in this manner, we must first group like terms variables together.

$$0 = x^2 + y^2 + z^2 - 6x + 2y + 8z - 4$$

$$4 = x^2 - 6x + y^2 + 2y + z^2 + 8z$$

$$4 = (x^2 - 6x) + (y^2 + 2y) + (z^2 + 8z)$$

Next, we'll finish the square for each grouping. When completing the square, we want to halve the coefficient in front of the variable, square it, and add it to both ends.

As for the given question, in order to complete the square for the "x" variable, I took the -6 and divided it, $\frac{-6}{2} = -3$, squared it, $(-3)^2 = 9$, and added it to both sides.

$$4 + \left(\frac{-6}{2}\right)^2 = (x^2 - 6x + \left(\frac{-6}{2}\right)^2) + (y^2 + 2y) + (z^2 + 8z)$$

$$4 + 9 = (x^2 - 6x + 9) + (y^2 + 2y) + (z^2 + 8z)$$

$$4 + 9 + \left(\frac{+2}{2}\right)^2 = (x^2 - 6x + 9) + (y^2 + 2y + \left(\frac{+2}{2}\right)^2) + (z^2 + 8z)$$

$$4 + 9 + 1 = (x^2 - 6x + 9) + (y^2 + 2y + 1) + (z^2 + 8z)$$

$$4 + 9 + 1 + \left(\frac{+8}{2}\right)^2 = (x^2 - 6x + 9) + (y^2 + 2y + 1) + (z^2 + 8z + \left(\frac{+8}{2}\right)^2)$$

$$4 + 9 + 1 + 16 = (x^2 - 6x + 9) + (y^2 + 2y + 1) + (z^2 + 8z + 16)$$

$$30 = (x^2 - 6x + 9) + (y^2 + 2y + 1) + (z^2 + 8z + 16)$$

We can factor each grouping once we've completed the square.

$$(x^2 \; - \; 6x + 9\,) + (\,y^2 + \; 2y + 1) \quad + (z^2 \; + \; 8z + 16) = 30$$

$$(x^2 \; - \; 6x + 9\,) + (\,y^2 + \; 2y + 1) \quad + (z^2 \; + \; 8z + 16) = 30$$

$$(x - 3)^2 + (y + 1)^2 \; + (z + 4)^2 = 30$$

We now have our equation in an understandable form. The number being subtracted is at the sphere's center.

$$(x - \underset{h}{3})^2 + (y - \underset{k}{(-1)})^2 \; + (z - \underset{l}{(-4)})^2 = 30$$

The center of spear represents by (h , k , l).

Hence the center of the sphere is at (3 , -1 , -4)

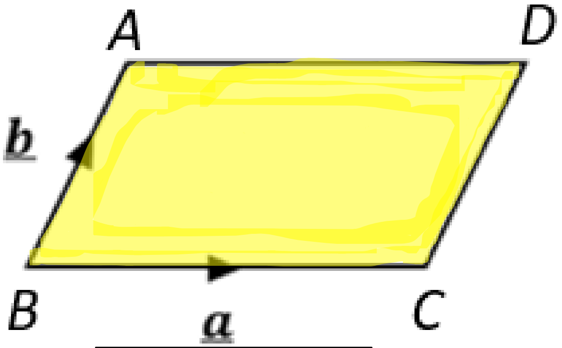
The radius is equal to the square root of the final number.

$$r = \sqrt{30} = 5.47$$

Hence this sphere has a radius of 5.47

Question 3

The magnitude of the cross-product is the area of the parallelogram spanned by two vectors.



b=(7i-2j+4k)

a=(i+3j-k)

$$\text{Area} = |\vec{a} \times \vec{b}|$$

$$|\vec{a} \times \vec{b}|$$

a = (i + 3j -k)

b = (7i - 2j + 4k)

We can write these i , j , k components using a matrix as below.

1st row should contain i , j , k unit vectors. Since we need to find a x b, the 2nd row should contain vector a's corresponding components of unit vectors. And the 3rd row should contain vector b's corresponding components of unit vectors. We need to consider the order. The cross product of a x b versus cross product of b x a gives different result.

$$\underline{a} \times \underline{b} = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} i & j & k \\ 1 & 3 & -1 \\ 7 & -2 & 4 \end{vmatrix}$$

The determinant can be expanded along the first row. And dividing each two by two as follows.

$$\underline{a} \times \underline{b} = \begin{vmatrix} 3 & -1 \\ -2 & 4 \end{vmatrix} i - \begin{vmatrix} 1 & -1 \\ 7 & 4 \end{vmatrix} j + \begin{vmatrix} 1 & 3 \\ 7 & -2 \end{vmatrix} k$$

Then we need to take cross product as follows.

$\underline{a}$ x $\underline{b}$ = [((3 x 4) – (-1 x -2)) i - ((1 x 4) – (-1 x 7)) j + ((1 x -2) – (3 x 7)) k]

= [(12 – 2) i - (4 – -7) j + (-2 – 21) k]

= [10 i - 11j + (-23) k]

= <u>[10 i - 11j - 23 k]</u>

This is the magnitude of the cross product of the vectors for which we are looking for the area. So we're looking for the vector magnitude with components 10, -11, and -23.

The magnitude of the vector is simply the square root of the sum of the components' squares.

Thus, area should get as the following.

$$|\vec{a} \times \vec{b}| = \sqrt{10^2 + (-11^2) + (-23^2)} = \sqrt{100 + 121 + 529} = \sqrt{750} = 5\sqrt{30}$$

$$\underline{Area = |\vec{a} \times \vec{b}| = 5\sqrt{30} = 5.92}$$

Part 2

<u>Question 4</u>

What is solving systems of linear equations by graphing→

Two or more equations make up a system of linear equations, such as y=0.5x+2 and y=x-2. We graph both equations in the same coordinate system to solve a system of linear equations graphically. The system's solution will be found at the intersection of the two lines. The two lines intersect in (-3, -4) in the graph below, which is the solution to this system of equations. The graphical method is the process of solving a system of simultaneous linear equations into variables by drawing a graph.

(Graphing and Functions, n.d.)

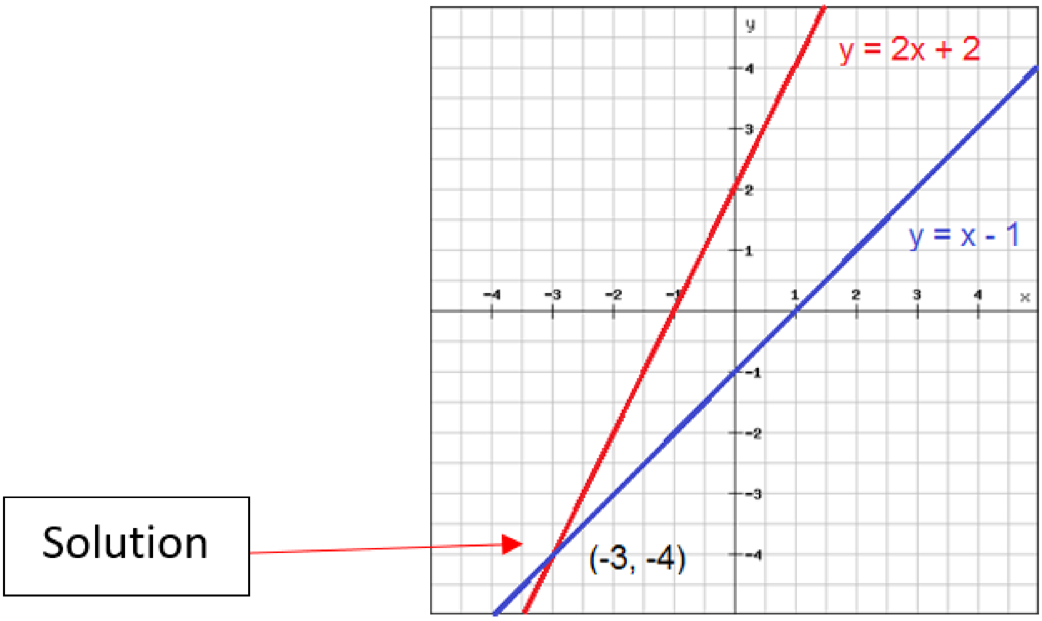


Figure 3. 6 Graph 2 sample equations in a coordinate plane

Finding solution for systems of linear equations by graphing of the assignment question →

Answer (Using Method 1) →

First, we need to graph each equation.

Let's take several coordinate points for 1st equation.

2x – 4y = 3

2x – 4y = 3			
X point	**Equation**	**Y point**	**Coordinates**
If x = -2	2(-2) – 4y = 3	y = -1.75	(-2 , -1.75)
If x = -1	2(-1) – 4y = 3	y = -1.25	(-1 , -1.25)
If x = 0	2(0) – 4y = 3	y = - 0.75	(0 , - 0.75)
If x = 1	2(+1) – 4y = 3	y = -0.25	(+1 , -0.25)
If x = 2	2(+2) – 4y = 3	y = 0.25	(+2 , +0.25)

Let's take several coordinate points for 2nd equation.

5y = -3x + 10

5y = -3x + 10			
X point	**Equation**	**Y point**	**Coordinates**
If x = -2	5y = -3(-2) + 10	y = 3.2	(-2 , +3.2)
If x = -1	5y = -3(-1) + 10	y = 2.6	(-1 , +2.6)
If x = 0	5y = -3(0) + 10	y = 2	(0 , +2)
If x = 1	5y = -3(+1) + 10	y = 1.4	(+1 , +1.4)
If x = 2	5y = -3(+2) + 10	y = 0.8	(+2 , +0.8)

Then we need to determine whether the lines are intersected or not. If they intersect, the point of intersection is the solution.

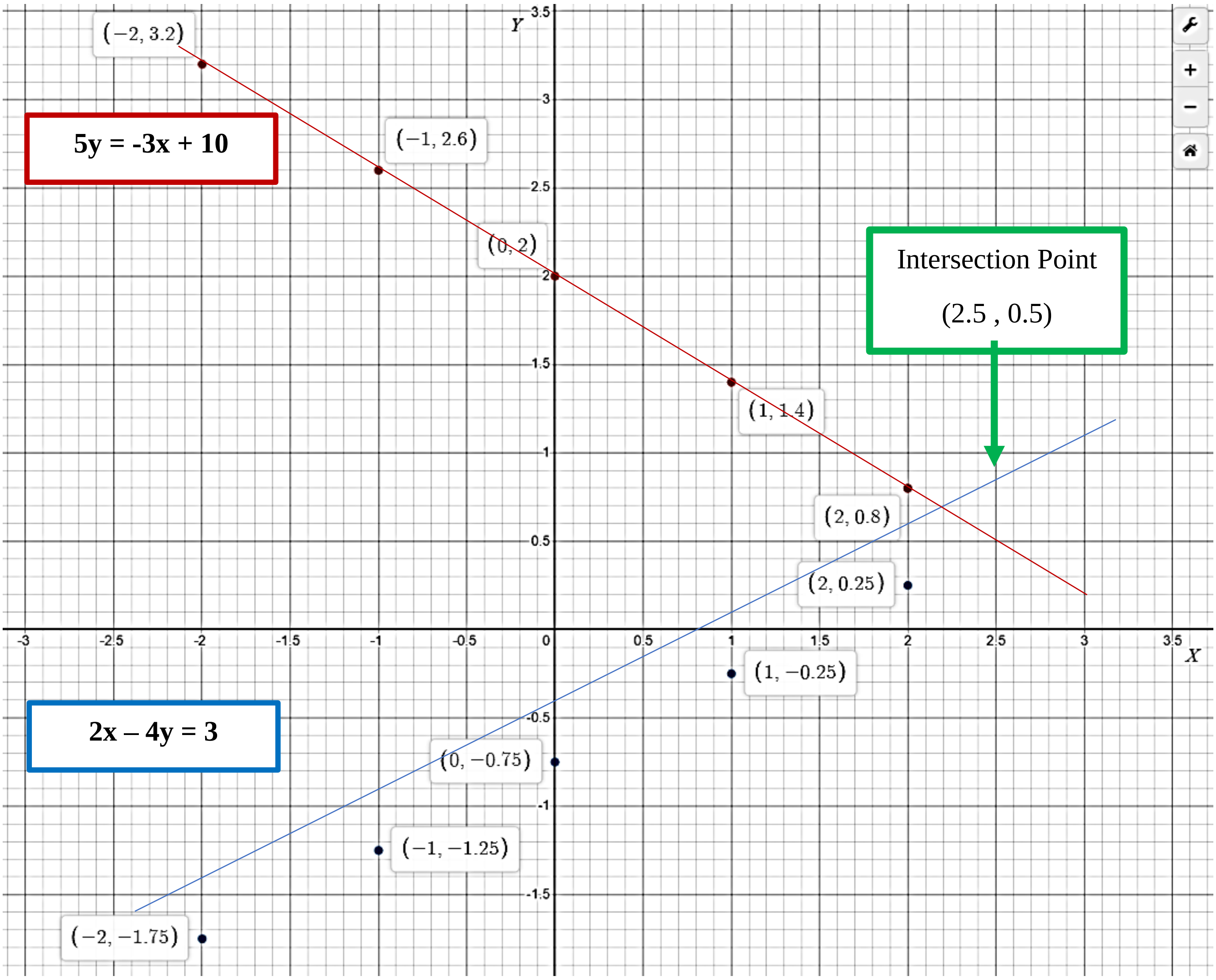


Figure 3. 7 Solution assignment question systems of linear equation by graph

Intersection point is (2.5 , 0.5)

Hence the answer is (2.5 , 0.5)

Answer (Using Method 2) →

First, we need to graph each equation as in the method 1.

In this method I'm gonna use 2 simple points to draw each equation in the graph.

Let's take 2 easy coordinate points for 1st equation.

2x – 4y = 3

2x – 4y = 3			
Y axis intercept point X = 0			
X point	**Equation**	**Y point**	**Coordinates**
If x = 0	2(0) – 4y = 3	y = - 0.75	(0 , - 0.75)
X axis intercept point Y = 0			
If y = 0	2x – 4(0) = 3	x = 1.5	(+1.5 , 0)

Then let's take 2 easy coordinate points for the 2nd equation also.

5y = -3x + 10

5y = -3x + 10			
Y axis intercept point X = 0			
If x = 0	5y = -3(0) + 10	y = 2	(0 , +2)
X axis intercept point Y = 0			
If y = 0	5(0) = -3x + 10	x = 3.33	(+3.33 , 0)

As in the method 1, the intersecting point is the solution.

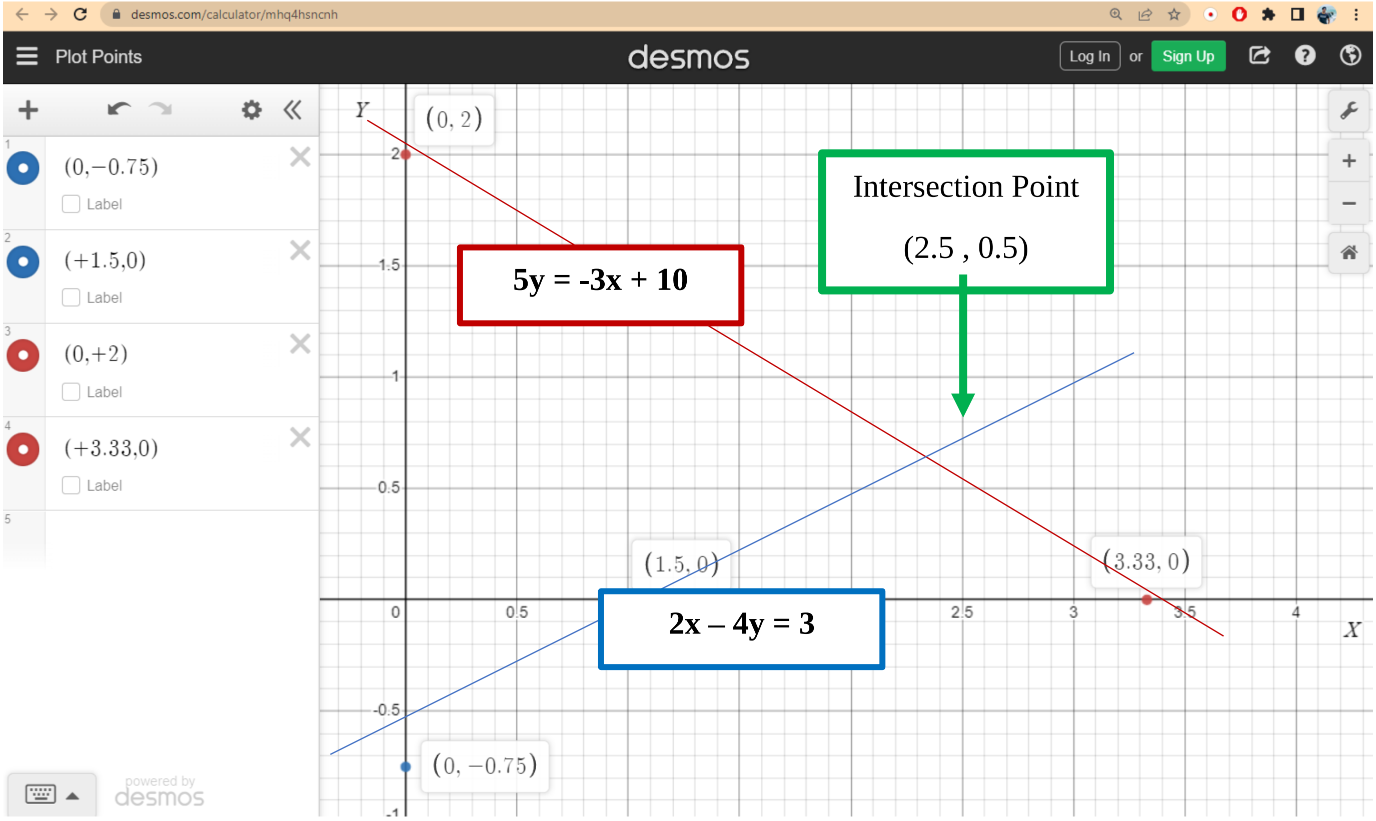


Figure 3. 8 Solution assignment question systems of linear equation by graph (Using desmos.com website)

Intersection point is (2.5 , 0.5)

Hence the answer is (2.5 , 0.5)

Question 5

What is Plane in 3-Dimentional co-ordinate system→

To name a point in 3-dimensional space, we use three coordinates in a 3-dimensional coordinate system. This system includes a z-axis in addition to the x- and y-axes. With the exception of the third axis, which is perpendicular to the other two axes, it appears to be a two-dimensional coordinate system.

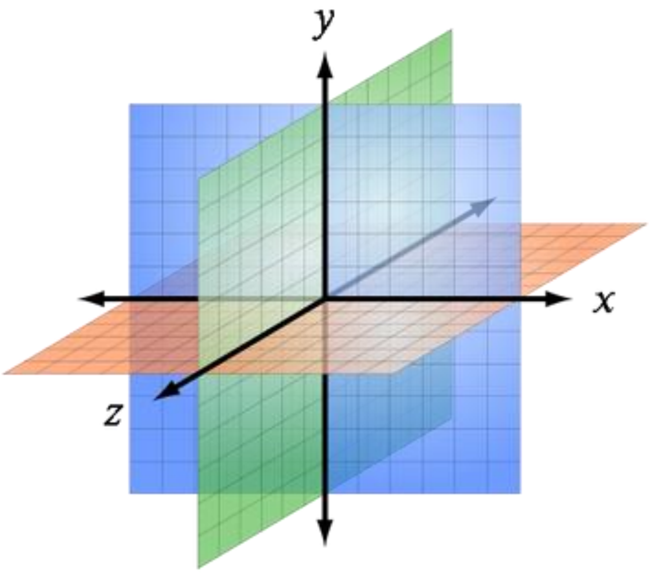


Figure 3. 9 3D coordinate system planes

- If I consider Orange color plane, I can represent X and Y coordinates, but Z coordinates are 0.
- If I consider Blue color plane, I can represent Y and Z coordinates, but X coordinates are 0.
- If I consider Green color plane, I can represent X and Y coordinates, but Z coordinates are 0.

The xy-plane is the plane that contains the x- and y-axes. The xz- plane is the plane that contains the x- and z-axes., while the yz- plane is the plane that contains the y- and z-axes.

i) Evaluate the surfaces in $\mathbb{R}^3$ that are represented by y = 4 in the assignment question →

(x , z) plane shifted 5 units for positive side of y axis.

The set {(x, y, z) | y = 4} represented by the equation z = 3, which is the set of all points in whose y-coordinate is 4 (x and z can have any value).

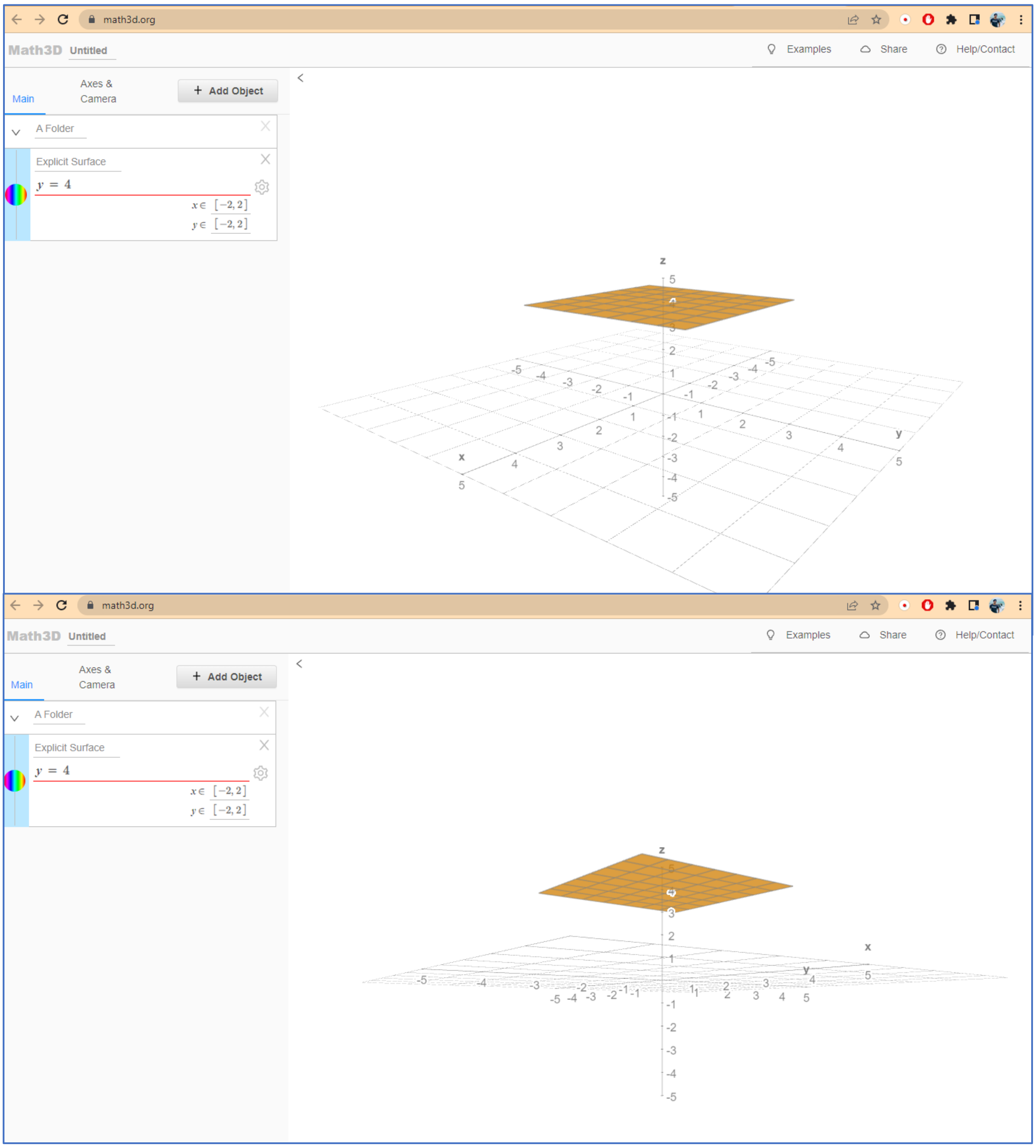


Figure 3. 10 Solution for assignment question by 3-D coordinate system tool (Using https://www.math3d.org/ website)

ii) Evaluate the surfaces in $\mathbb{R}^3$ that are represented by z = 5 in the assignment question →

(x , y) plane shifted 5 units for positive side of z axis.

The set {(x, y, z) | z = 5} represented by the equation z = 5, which is the set of all points in whose z-coordinate is 5 (x and y can have any value).

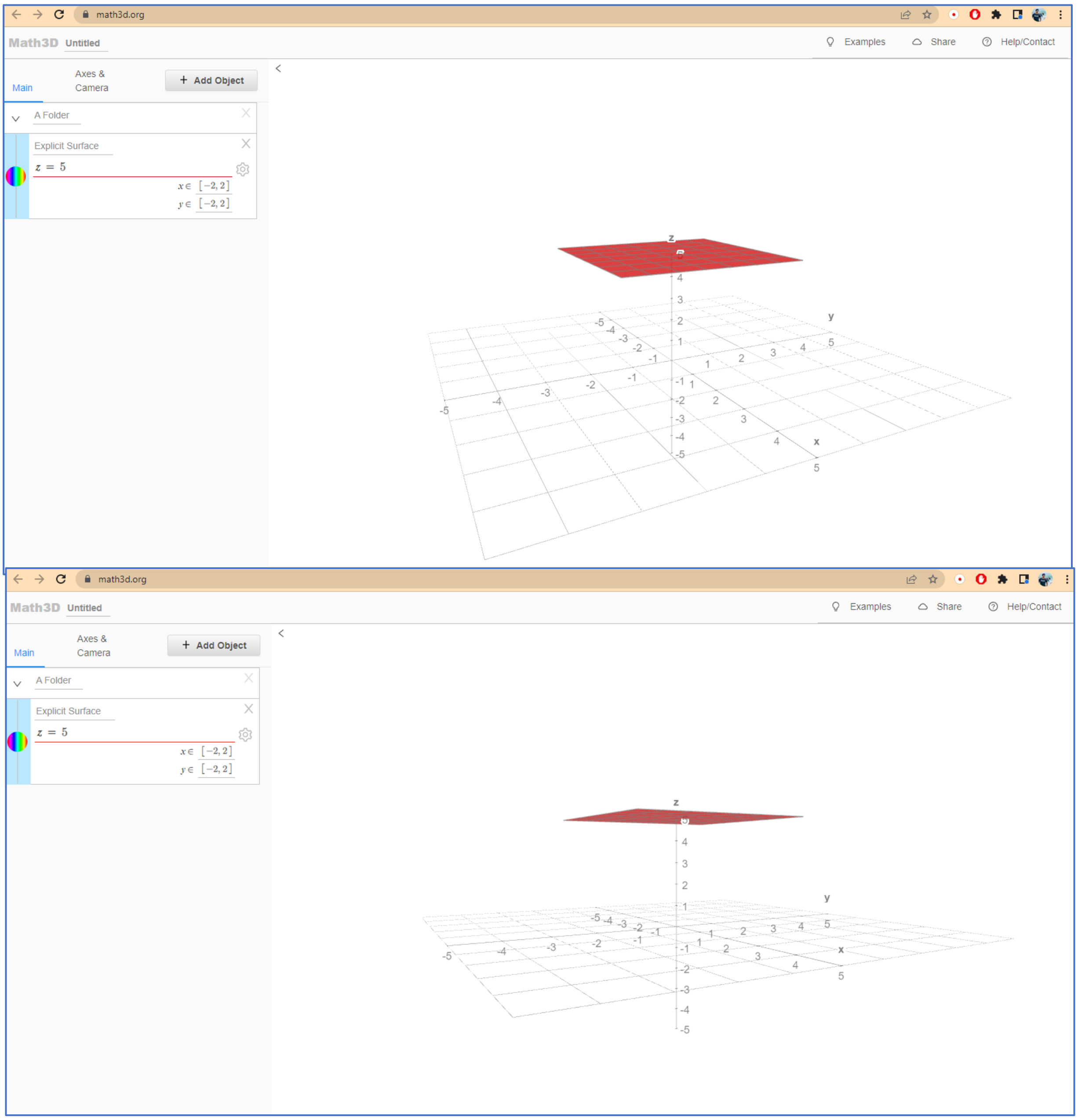
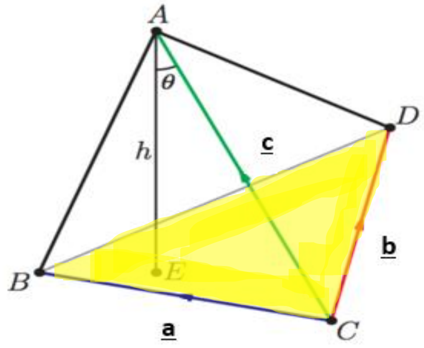


Figure 3. 11 Solution for assignment question by 3-D coordinate system tool (Using https://www.math3d.org/ website)

Question 6

The dot product of a vector with the cross product of two other vectors is known as the scalar triple product. If a, b, and c are three vectors, then their scalar triple product is a .(b × c).

It is also referred to as the triple scalar product, the box product, and the mixed product. The volume of the tetrahedron is equal to (1/6) times the scalar triple product of the vectors on which it is based. For this question I've considered following highlighted base vector.



$$\text{Volume} = \frac{1}{6}|c.(a \times b)|$$

$$\frac{1}{6}|c.(a \times b)|$$

a = (i + 4j -2k)

b = (3i - 5j + k)

c = (-4i + 3j + 6k)

The determinant of the components of the three vectors then gives their scalar triple product. We can write these a , b , c components using a matrix as below.

1st row should contain components of "c". 2nd row should contain components of "a". And the 3rd row should contain components of "b".

$$c.(a \times b) = \begin{vmatrix} c_1 & c_2 & c_3 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} -4 & 3 & 6 \\ 1 & 4 & -2 \\ 3 & -5 & 1 \end{vmatrix}$$

The determinant can be expanded along the first row. And dividing each two by two as follows.

$$c.(a \times b) = \begin{vmatrix} 4 & -2 \\ -5 & 1 \end{vmatrix} - 4 - \begin{vmatrix} 1 & -2 \\ 3 & 1 \end{vmatrix} 3 + \begin{vmatrix} 1 & 4 \\ 3 & -5 \end{vmatrix} 6$$

Then we need to take cross product as follows.

c.(a x b)= [((4 x 1) – (-2 x -5)) -4 - ((1 x 1) – (-2 x 3))3 + ((1 x -5) – (4 x 3))6]

= [(4 – +10) -4 - (1 – -6) 3 + (-5 – 12) 6]

= [(-6)-4 - (7)3 + (-17)6]

= [24 - 21 - 102]

= -99

Thus, volume should get as the following.

$$\text{Volume} = \frac{1}{6} . |c.(a \times b)|$$

$$= \frac{1}{6} . |c.(a \times b)|$$

$$= \frac{1}{6} . |-99|$$

$$= \frac{99}{6}$$

Volume = 16.5

Activity 4

Part 1

<u>Question 1</u>

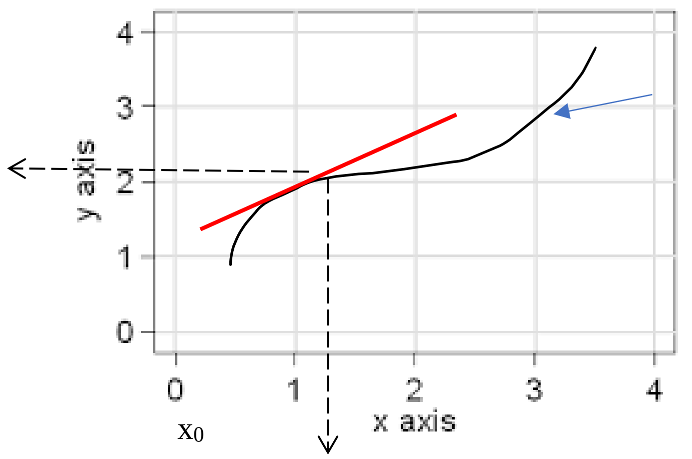
Small Theory Part →

We can use Derivatives for graph sketching purpose because the derivative values give slope of tangent line.

The below graph shows the function of x.

The slope of the line (tangent) is representing the derivative of the function when $x = x_0$.

m (slope) = $\frac{dy}{dx}$



Assignment Question →

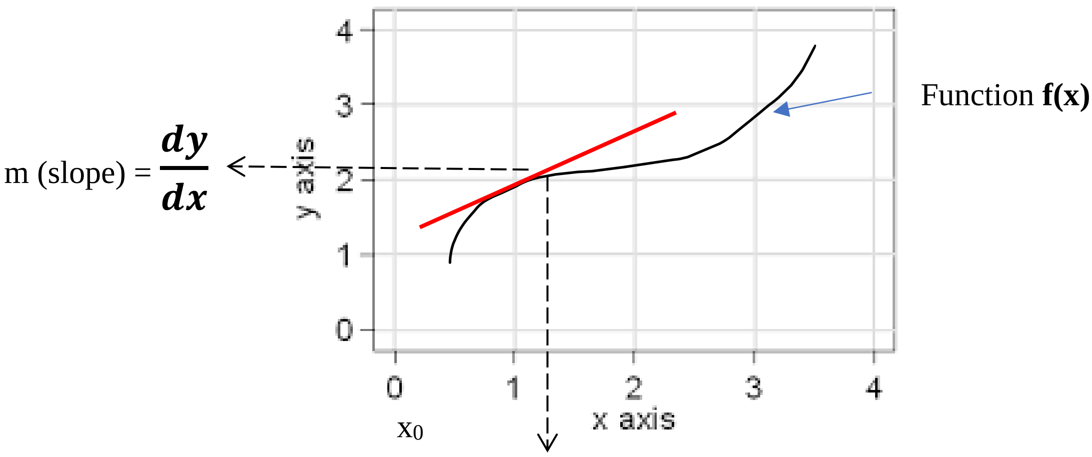
i) Determine the slope of f(x) = 2x – 3x⁴ + 5x + 8

The given function is

$$f(x) = 2x - 3x^4 + 5x + 8$$

If I differentiate this function, it gives the slope of tangent line of any X point.

$$\frac{d\,y}{dx} = f'(x) = 2x - 3x^4 + 5x + 8$$



$$\frac{d\,y}{dx} = f'(x) = 2x - 3x^4 + 5x + 8$$

$$\frac{d\,y}{dx} = f'(x) = (2 \times 1x^{1-1}) - (3 \times 4x^{4-1}) + (5 \times 1x^{1-1}) + 0$$

$$= 2x^0 - 12x^3 + 5x^0 + 0$$

$$= 2 - 12x^3 + 5$$

$$m\ (slope) = f'(x) = 7 - 12x^3$$

The slope (m) is $7 - 12x^3$

Assignment Question →

ii) Determine the slope of f(x) = cos(2x) + 4x² - 3

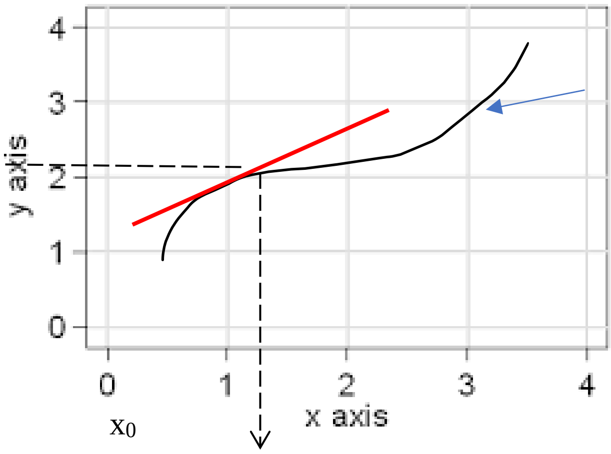
The given function is

$$f(x) = \cos(2x) + 4x^2 - 3$$

If I differentiate this function, it gives the slope of tangent line of any X point.

$$\frac{d\,y}{dx} = f'(x) = \cos(2x) + 4x^2 - 3$$

m (slope) = $\frac{\boldsymbol{dy}}{\boldsymbol{dx}}$



Function **f(x)**

$$\frac{d\,y}{dx} = f'(x) = \cos(2x) + 4x^2 - 3$$

$$\frac{d\,y}{dx} = f'(x) = -2(sin2x) + \left(4 \times 2x^{2-1}\right) - 0$$

$$= -2(sin2x) + 8x^1 - 0$$

$$= -2(sin2x) + 8x$$

$$m\ (slope) = f'(x) = -2(sin2x) + 8x$$

The slope (m) is -2(sin2x) + 8x

Question 2

Velocity is instantaneous change of displacement.

The function for the velocity of the object at time t can get by differentiate the given function.

$$S(t) = 5t^3 - 3t^2 + 6t$$

$$V = \frac{d\,y}{dx} = \frac{d\,s(t)}{dx} = \frac{d\,(5t^3 - 3t^2 + 6t)}{dx}$$

$$V = \frac{d\,s(t)}{dx} = 5 \times 3t^{3-1} - \ 3 \times 2t^{2-1} + \ 6 \times 1t^{1-1}$$

$$= 15t^2 - \ 6t^1 + \ 6t^0$$

$$V = \frac{d\,s(t)}{dx} = 15t^2 - \ 6t \ + \ 6$$

At here I've used 2 rules in Derivatives.

- The Constant Rule
- The Power Rule

Part 2

Question 3

I’ve sketched the graph using GeoGebra website Online Graphing tool. And I’ve realized the higher function for both x axis negative area and positive area is the g(x) = 8 – 2x function.

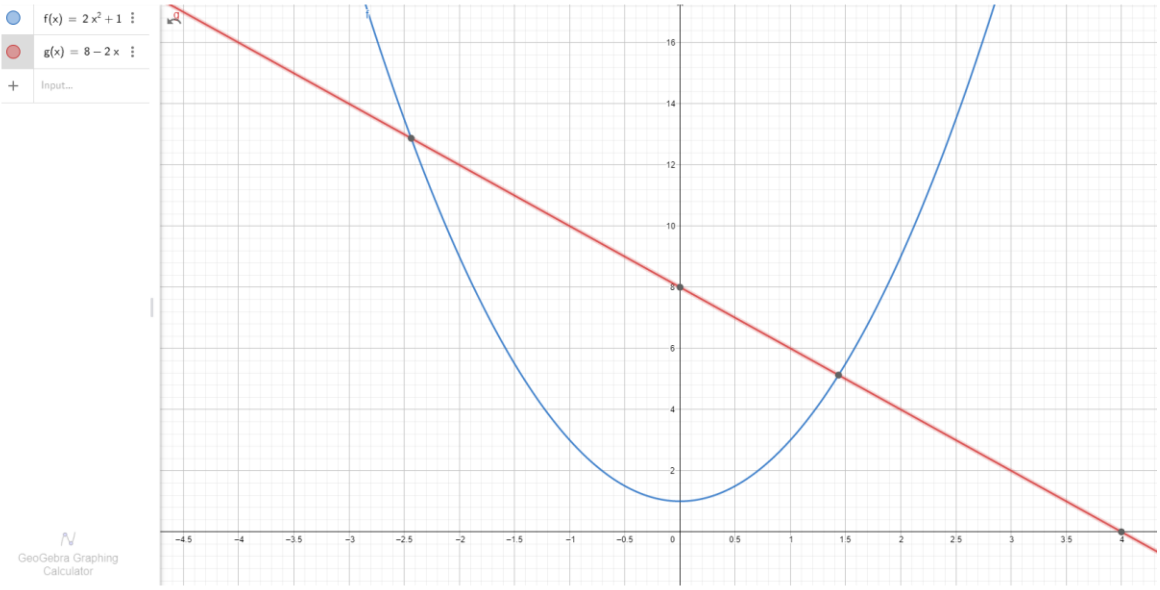


Figure 4. 1 Sketched the graph using GeoGebra website

In this case, we want to find the area between the functions y = f(x) and y = g(x) on the interval -2 to +1.

We need to find area by directly using integration function for this question.

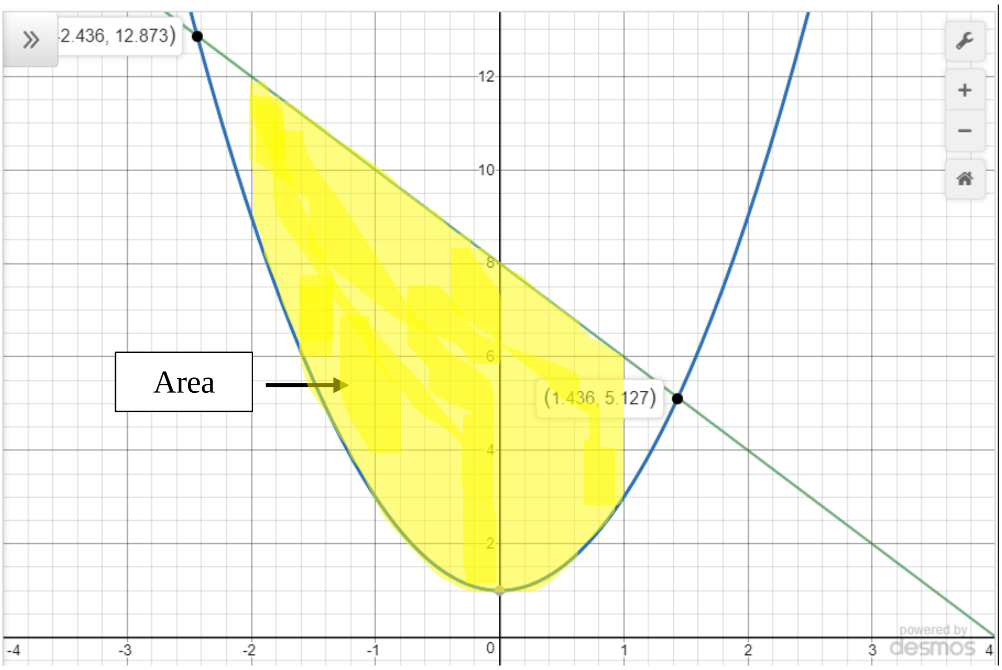


Figure 4. 2 Area in a graph

Finding Area →

According to the graph the g(x) function is higher than f(x) function in both right and left

$$Area\ 1 = \int_{-2}^{1} g(x) - f(x)$$

$$= \int_{-2}^{1} (8 - 2x) - (2x^2 + 1)\, dx$$

$$= \int_{-2}^{1} -2x^2 - 2x + 7\, dx$$

Applying the sum rule

$$= -\int_{-2}^{1} 2x^2\, dx - \int_{-2}^{1} 2x\, dx + \int_{-2}^{1} 7\, dx$$

Taking constants out and applying power rule

$$= -2\left[\frac{x^3}{3}\right]_{-2}^{1} - 2\left[\frac{x^2}{2}\right]_{-2}^{1} + \left[\frac{7x}{1}\right]_{-2}^{1}$$

Substituting upper limit and lower limit

$$= -\frac{2}{3}[1^3 - (-2^3)] - \frac{2}{2}[1^2 - (-2^2)] + \frac{7}{1}[1 - (-2)]$$

$$= -(6.00) - (-3.00) + 21$$

$$\underline{\text{Area} = 18.00}$$

Question 4

We can integrate the given rate (add up all trees) to give us the number of trees in the forest.

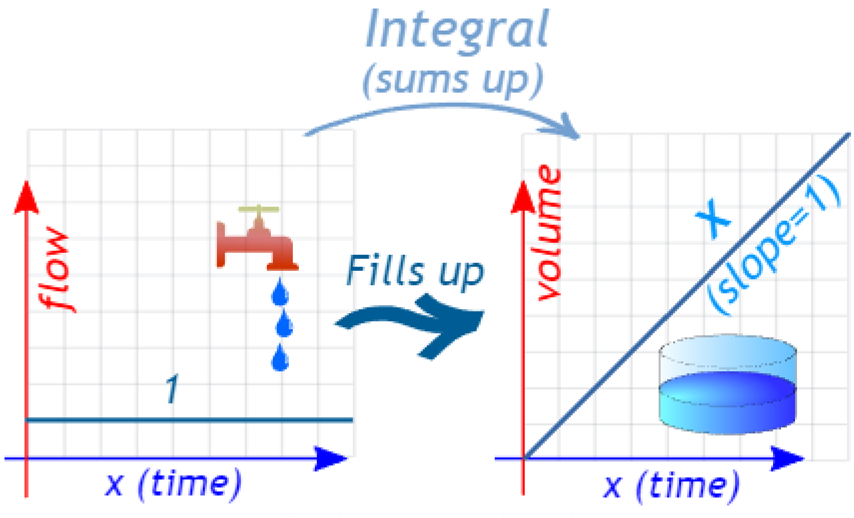


Figure 4. 3 Use of Integration

With a grow rate of $3t^2 + 5t + 6$ hundred trees per year, the trees increase by $3t^2 + 5t + 6$ hundred every year, so would increase by 3t 2 + 5t + 6 three hundred after 3 years.

So, 1st I considered the rate of growth change of tree with respect to time as following.

I assumed the rate of tree growing as $G(t)$

$$\frac{dG(t)}{dt} = 3x^2 + 5t + 6 \; hundred\ trees\ per\ year$$

For the next 3 years we can get the number of trees by the rate change of definite integral.

$$= \int_{0}^{3} 3t^2 + 5t + 6 \ dx$$

$$= \int_{0}^{3} 3t^2 + 5t + 6 \ dx$$

$$= \int_{0}^{3} 3t^2 \, dx + \int_{0}^{3} 5t \, dx + \int_{0}^{3} 6 \, dx$$

Taking constants out and applying power rule

$$= 3\left[\frac{t^3}{3}\right]_0^3 + 5\left[\frac{x^2}{2}\right]_0^3 + \left[\frac{6t}{1}\right]_0^3$$

Substituting upper limit and lower limit

$$= \frac{3}{3}[3^3 - (0^3)] + \ \frac{5}{2}[3^2 - (0^2)] \ + \frac{6}{1}[3 - (0)]$$

$$= \ (27) + (22.5) + 18$$

<u>Number of trees grow in 3 years → 67.5 hundred trees (6750 trees)</u>

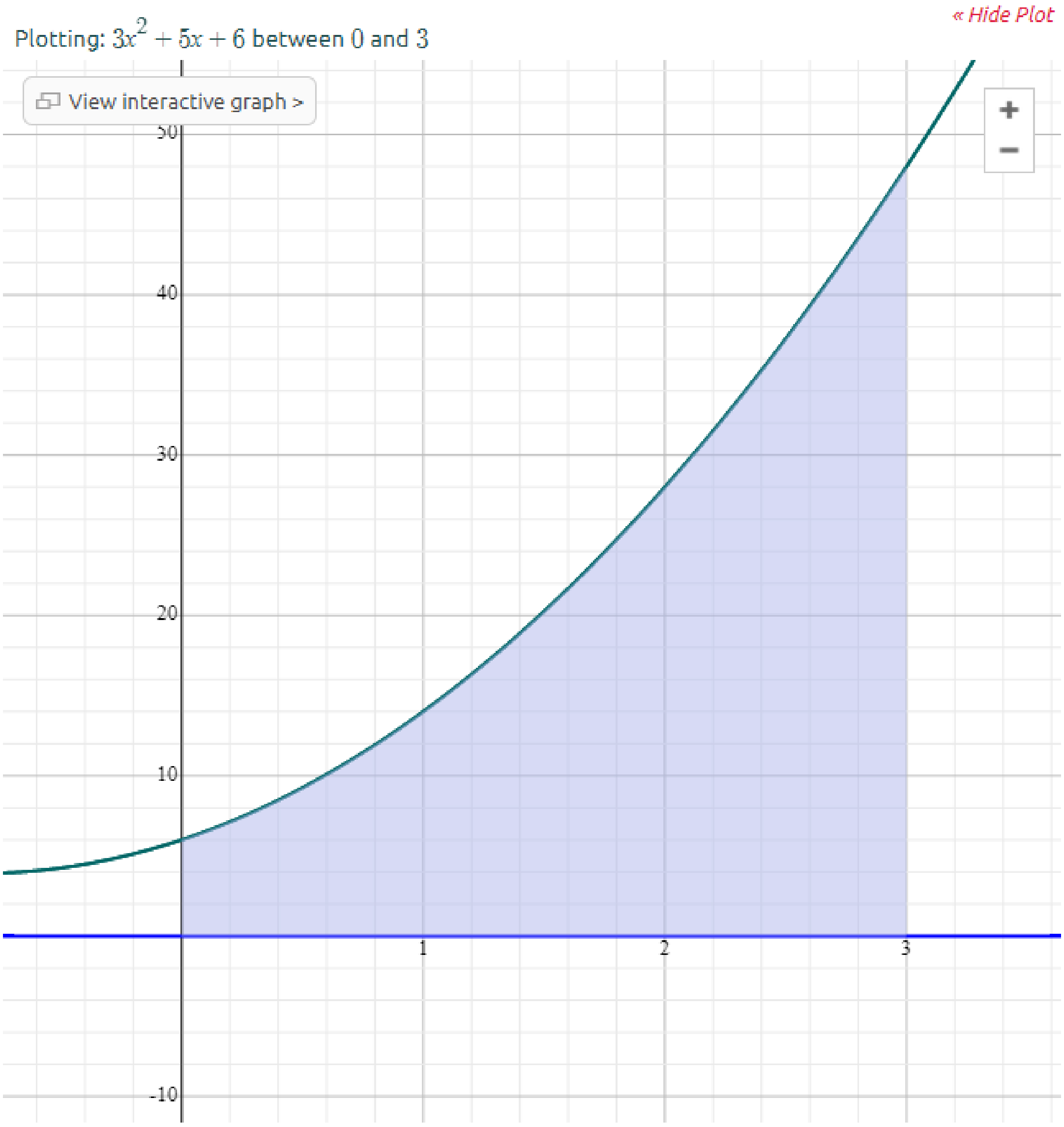


Figure 4. 4 Graphical view of the area

Since oxygen in forest increase at the rate of 4 units per 100 trees, we can calculate the increased Oxygen level in the forest units for 6750 trees as following.

$$\text{For 100 trees} \rightarrow 4 \text{ units}$$

$$\text{For 6750 trees} \rightarrow = \frac{4 \times 6750}{100} = \underline{270 \text{ units}}$$

Part 3

Question 5

First, we need to do is finding critical points (turning points).

To find the critical points of a single-variable function y = f(x), we set the function's derivative to zero and solve.

- Step 1: Find the derivative f'(x).
- Step 2: Set f '(x) = 0 and solve it to find all the values of x that satisfy it (if any).
- Step 3: Find all the x values (if any) for which f '(x) is equal to zero.
- Step 4: All of the x values from Steps 2 and 3 are the x-coordinates of the critical points.

1st thing I did was find the derivative $f'(x)$.

$$f(x) = x^5 - 6x^3 + 3$$

$$f(x) = \frac{d\,f(x)}{dx} = f'(x) = x^5 - 6x^3 + 3$$

$$f'(x) = 1 \times 5x^{5-1} - 6 \times 3x^{3-1} + 0$$

$$\underline{\underline{f'(x) = 5x^4 - 18x^2}}$$

Then I set $f'(x)$ = 0 and solve it to find all the values of x that satisfy it (if any).

$$f'(x) = 5x^4 - \ 18x^2$$

To find critical point set $f'(x) = 0$

$$5x^4 - \ 18x^2 = 0$$

$$x^2(5x^2 - 18) = 0$$

OR

$$x^2 = 0$$

$$x = \sqrt{0}$$

$$x = 0$$

$$5x^2 - 18 = 0$$

$$x = \pm\sqrt{\frac{18}{5}}$$

OR

$$x = +\sqrt{\frac{18}{5}}$$

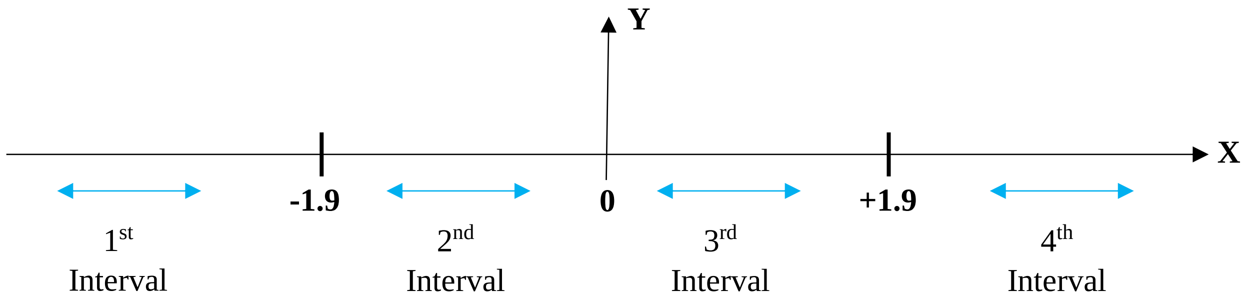
$$x = 1.9$$

$$x = -\sqrt{\frac{18}{5}}$$

$$x = -1.9$$

Second step is, we need to do is finding increasing and decreasing intervals →

According to above results I know there are 3 critical points and which containing 4 intervals. And I've marked those critical points along with the relevant intervals below number line.



Finding Increasing or Decreasing intervals →

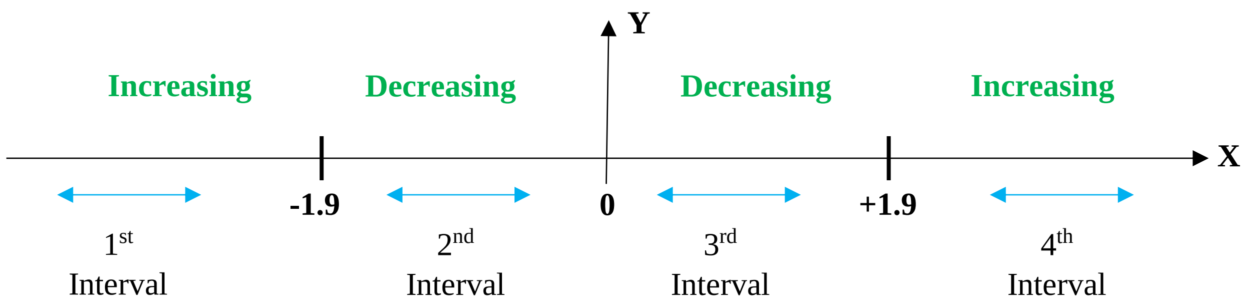
I can find whether function is Increasing or Decreasing by using below table.

Table 4. 1 Finding whether function is Increasing or Decreasing

Interval	Test Value	Result	Function Behaviour
x < -1.9	x = - 2	+8 (+)	Increasing
-1.9 < x < 0	x =- 1	-13 (-)	Decreasing
0 < x < 1.9	x = 1	-13 (-)	Decreasing
1.9 < x	x = 2	+8 (+)	Increasing

The intervals of increasing are → (-∞ , -1.9) and (1.9 , ∞)

The intervals of decreasing are → (-1.9 , 0) and (0 , 1.9)



To get the results for corresponding test values I used first derivative function. Below I've given the results I've obtained for corresponding test values for each interval.

For test value -2 →

$$f'(x) = 5x^4 - \ 18x^2$$

$$f'(-2) = 5(-2)^4 - \ 18(-2)^2$$

$$f'(-2) = \ +8$$

For test value -1 →

$$f'(x) = 5x^4 - \ 18x^2$$

$$f'(-1) = 5(-1)^4 - \ 18(-1)^2$$

$$f'(-1) = \ -13$$

For test value 1 →

$$f'(x) = 5x^4 - \ 18x^2$$

$$f'(1) = 5(1)^4 - \ 18(1)^2$$

$$f'(1) = \ -13$$

For test value 2 →

$$f'(x) = 5x^4 - \ 18x^2$$

$$f'(2) = 5(2)^4 - \ 18(2)^2$$

$$f'(2) = \ +8$$

Finding Local Maximum and Minimum points using First Derivative Test →

First Derivative Test is the process of locating these Maximum and Minimum points using increasing and decreasing intervals.

- If function goes decreasing to increasing, it is a Minimum Point.
- If function goes increasing to decreasing, it is a Maximum Point.
- If function goes has neither Maximum nor Minimum, it is a Inflection Point (Concavity Changing Point).

At x = -1.9 point

At x = -1.9 point the f ′(x) function goes from increasing (+) to decreasing (-).

So, there is Local Maximum Point at x = -1.9.

The corresponding "y" axis value can get by substituting corresponding x value to the original function.

$$f(x) = x^5 - 6x^3 + 3$$

$$f(-1.9) = (-1.9)^5 - 6(-1.9)^3 + 3$$

$$f(-1.9) = 19.39$$

The corresponding "y" axis value at x = -1.9 point is 19.39

At x = 0 point

At x = 0 point the f ’(x) function goes from decreasing (-) to decreasing (-).

So, there is neither maximum nor minimum point. It can be an Inflection Point (Concavity Changing Point) at x = 0.

The corresponding “y” axis value can get by substituting corresponding x value to the original function.

$$f(x) = x^5 - 6x^3 + 3$$

$$f(0) = (0)^5 - 6(0)^3 + 3$$

$$f(0) = 3.00$$

The corresponding “y” axis value at x = 0 point is 3.00

At x = +1.9 point

At x = 1.9 point the f ’(x) function goes from decreasing (-) to increasing (+).

So, there is Local Minimum Point at x = 1.9.

The corresponding “y” axis value can get by substituting corresponding x value to the original function.

$$f(x) = x^5 - 6x^3 + 3$$

$$f(1.9) = (1.9)^5 - 6(1.9)^3 + 3$$

$$f(1.9) = -13.39$$

The corresponding “y” axis value at x = 1.9 point is -13.39

Finding Local Maximum and Minimum points using Second Derivative Test →

Second Derivative Test is the process of locating these Maximum and Minimum points substituting critical points to second derivative function.

- If Second Derivative value for a critical point is greater than 0 (+ value) it is Minimum Point.
- If Second Derivative value for a critical point is lesser than 0 (- value) it is Maximum Point.
- If Second Derivative value is equal to 0 for a critical point, it can be Inflection Point (Concavity Changing Point).

$$f''(c) < 0 \rightarrow \text{Maximum Point}$$

$$f''(c) = 0 \rightarrow \text{Concavity Changing Point}$$

$$f''(c) > 0 \rightarrow \text{Minimum Point}$$

$$f(x) = x^5 - 6x^3 + 3 \leftarrow \boxed{\text{Original Function}}$$

$$\frac{d\,f(x)}{dx} = f'(x) = 5x^4 - 18x^2 \leftarrow \boxed{\text{First Derivative}}$$

$$\frac{d\,f'(x)}{dx} = f''^{(x)} = 5 \times 4x^{4-1} - 18 \times 2x^{2-1}$$

$$\underline{\underline{f''(x) = 20x^3 - 36x}} \leftarrow \boxed{\text{Second Derivative}}$$

At x = -1.9 point

At x = -1.9 point to the f ′′(x) function substitute "x" value.

$$f''(x) = 20x^3 - \ 36x$$

$$f''(-1.9) = 20(-1.9)^3 - \ 36(-1.9)$$

$$f''(-1.9) = -68.78$$

Since the second derivative value to corresponding x = -1.9 is lesser than 0 (- value) it is Maximum Point.

At x = 0 point

At x = 0 point to the f ′′(x) function substitute "x" value.

$$f''(x) = 20x^3 - \ 36x$$

$$f''(0) = 20(0)^3 - \ 36(0)$$

$$f''(0) = 0$$

Since the second derivative value to corresponding x = 0 is equal to 0, it can be Inflection Point (Concavity Changing Point).

At x = +1.9 point

At x = +1.9 point to the f ″(x) function substitute “x” value.

$$f''(x) = 20x^3 - \ 36x$$

$$f''(1.9) = 20(1.9)^3 - \ 36(1.9)$$

$$f''(1.9) = 68.78$$

Since the second derivative value to corresponding x = 1.9 is greater than 0 (+value) it is Minimum Point.

Finding Concavity changing intervals →

To find concavity changing intervals for function y = f(x), we set the function's second derivative to zero and solve.

So, I've set $f''(x)$ = 0 and solve it to find all the values which are the concavity changing points.

$$f''(x) = 20x^3 - \ 36x$$

To find concavity changing intervals set $f''(x) = 0$

$$20x^3 - \ 36x = 0$$

$$4x\ (5x^2 - 9) = 0$$

OR

$$4x = 0$$

$$\underline{\underline{x = 0}}$$

$$5x^2 - 9 = 0$$

$$x = \pm\sqrt{\frac{9}{5}}$$

OR

$$x = +\sqrt{\frac{9}{5}}$$

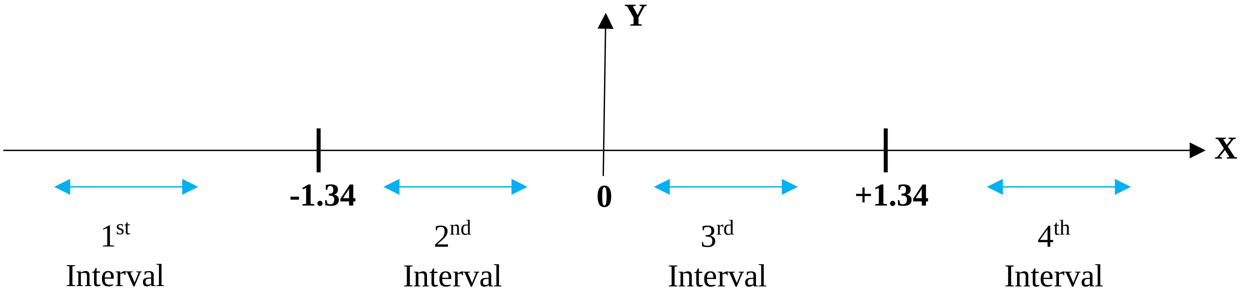
$$\underline{\underline{x = 1.34}}$$

$$x = -\sqrt{\frac{9}{5}}$$

$$\underline{\underline{x = -1.34}}$$

Then we need to do is finding increasing and decreasing intervals →

According to above results I know there are 3 concavity changing points and which containing 4 intervals. And I've marked those concavity changing points along with the relevant intervals below number line.



I can find concavity changing intervals by using below table.

Table 4. 2 Finding concavity changing intervals

Interval	**Test Value**	**Result**	**Concavity Behaviour**
x < -1.34	x = - 2	-88 (-)	Concave Down
-1.34 < x < 0	x =- 1	16 (+)	Concave Up
0 < x < 1.34	x = 1	-16 (-)	Concave Down
1.34 < x	x = 2	+88 (+)	Concave Up

The intervals of Concave Down are → (-∞ < x < -1.34) and (0 < x < 1.34)

The intervals of Concave Up are → (-1.34 < x < 0) and (1.34 < x < ∞)

To get the results for corresponding test values I used second derivative function. Below I've given the results I've obtained for corresponding test values for each interval.

For test value -2 →

$$f''(x) = 20x^3 - \ 36x$$

$$f''(-2) = 20(-2)^3 - \ 36(-2)$$

$$f''^{(-2)} = \ -88$$

For test value -1 →

$$f''(x) = 20x^3 - \ 36x$$

$$f''(-1) = 20(-1)^3 - \ 36(-1)$$

$$f''(-1) = \ 16$$

For test value 1 →

$$f''(x) = 20x^3 - \ 36x$$

$$f''(1) = 20(1)^3 - \ 36(1)$$

$$f''^{(1)} = \ -16$$

For test value 2 →

$$f''(x) = 20x^3 - \ 36x$$

$$f''(2) = 20(2)^3 - \ 36(2)$$

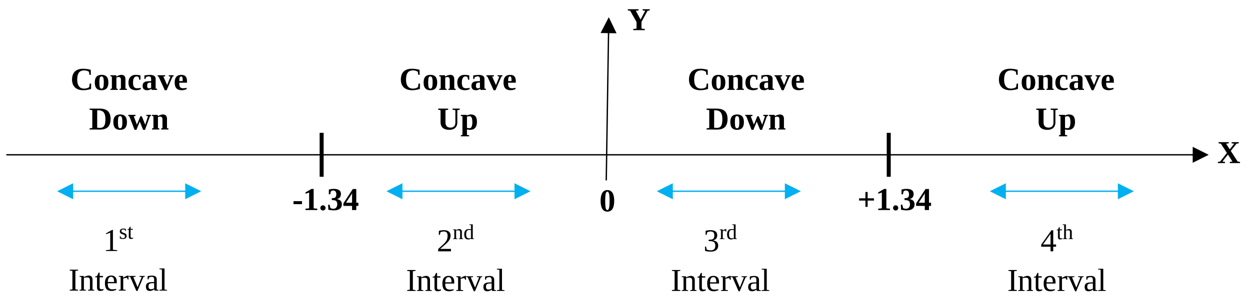
$$f''(2) = \ +88$$

Finding Inflection Points→

Table 4. 3 Finding Inflection Points

Interval	Test Value	Result	Concavity Behaviour
x < -1.34	x = - 2	-88 (-)	Concave Down
-1.34 < x < 0	x =- 1	16 (+)	Concave Up
0 < x < 1.34	x = 1	-16 (-)	Concave Down
1.34 < x	x = 2	+88 (+)	Concave Up

According to above results which got from Second Derivative Test, I've marked those concavity changing points along with the relevant intervals below number line.



At x = -1.34 the concavity changes suddenly from Down to Up.

At x = 0 the concavity changes suddenly from Up to Down.

At x = +1.34 the concavity changes suddenly from Down to Up.

According to above number line at x = -1.34 , 0 and x = +1.34, the concavity changes suddenly.

Hence, the x = -1.34 , 0 and x = +1.34 are the Inflection Points (Concavity Changing Points).

Graphing the Function →

To graph this function, I've used below Online Graphing Tool and then gave the original function as the input.

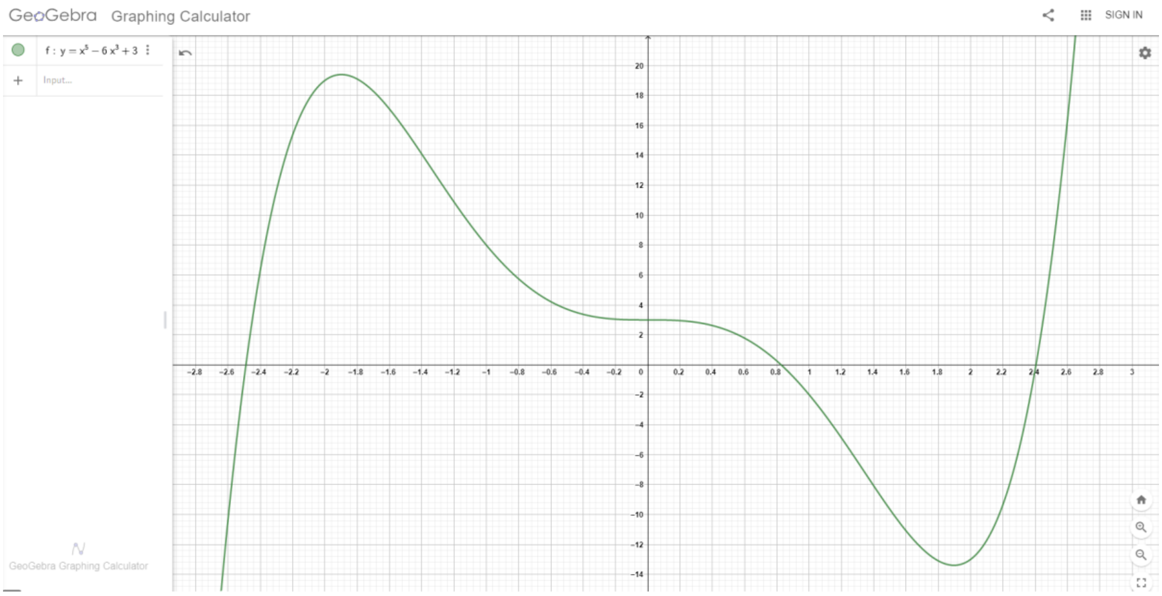


Figure 4. 5 Graphing the Function

https://www.geogebra.org/graphing?lang=en

Finding “x” axis intersect points →

To find “x” axis intersect points, I equalled “y” value to 0

So, I’ve set original $f(x)$ function’s “y” value to 0 and solve it to find all the values which are intersecting “x” axis.

$$f(x) = x^5 - 6x^3 + 3$$

To find “y” axis intersect points set $f(x) = 0$

$$0 = x^5 - 6x^3 + 3$$

$x = 0.83$ or $x = -2.49$ or $x = 2.41$

To finding corresponding x values I’ve used this online algebra calculator : https://www.symbolab.com/geometry

Finding “y” axis intersect points →

To find “y” axis intersect points, I equalled x value to 0

So, I’ve set original $f(x)$ function’s “x” value to 0 and solve it to find all the values which are intersecting “y” axis.

$$f(x) = x^5 - 6x^3 + 3$$

To find “x” axis intersect points set $x = 0$

$$f(x) = (0)^5 - 6(0)^3 + 3$$

$y = 3.0$

To finding important point values of the graph I've used this Online Graphing Tool:

https://www.transum.org/Maths/Activity/Graph/Desmos.asp

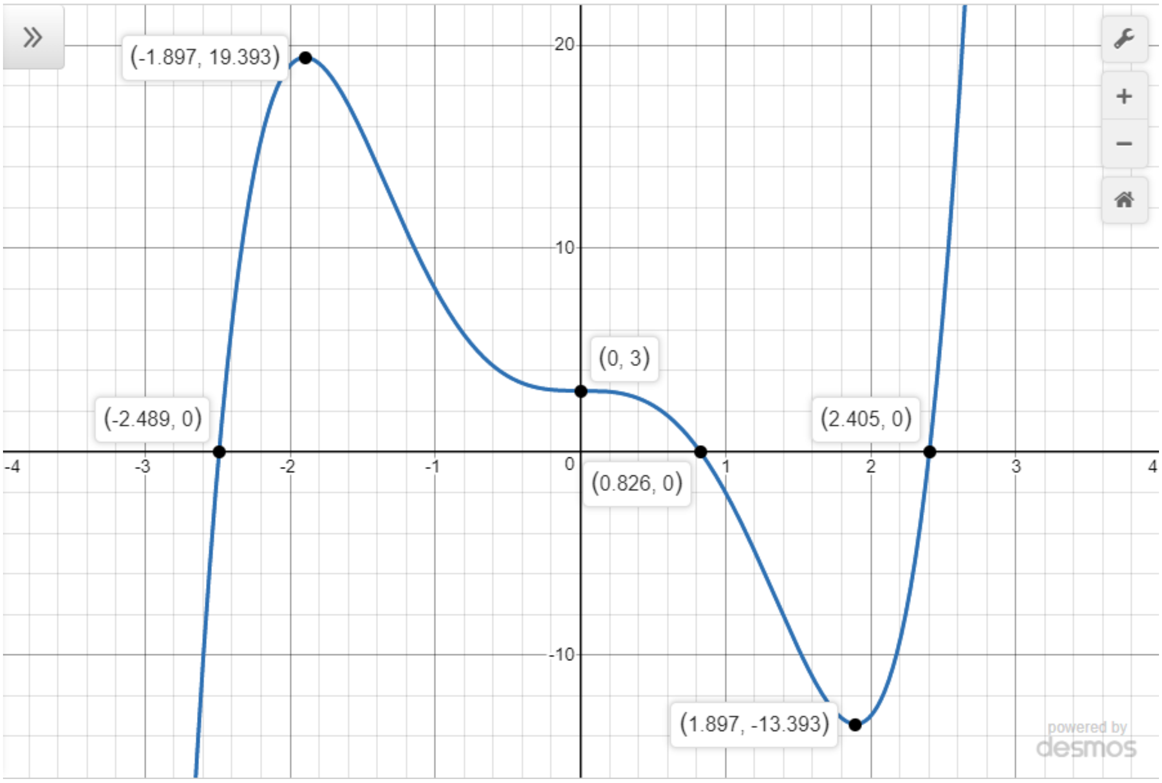


Figure 4. 6 Finding important point values of the graph I've used this Online Graphing Tool

Question 6

First, we need to do is finding critical points (turning points).

To find the critical points of a single-variable function y = f(x), we set the function's derivative to zero and solve.

- Step 1: Find the derivative f'(x).
- Step 2: Set f '(x) = 0 and solve it to find all the values of x that satisfy it (if any).
- Step 3: Find all the x values (if any) for which f '(x) is equal to zero.
- Step 4: All of the x values from Steps 2 and 3 are the x-coordinates of the critical points.

1st thing I did was find the derivative $f'(x)$.

$$f(x) = 2x^3 - 4x^4 + 5x^2$$

$$f(x) = \frac{d\,f(x)}{dx} = f'(x) = 2x^3 - 4x^4 + 5x^2$$

$$f'(x) = 2 \times 3x^{3-1} - 4 \times 4x^{4-1} + 5 \times 2x^{2-1}$$

$$\underline{\underline{f'^{(x)} = 6x^2 - 16x^3 + 10x}}$$

Then I set $f'(x)$ = 0 and solve it to find all the values of x that satisfy it (if any).

$$f'(x) = 6x^2 - \ 16x^3 + \ 10x$$

To find critical point set $f'(x) = 0$

$$6x^2 - \ 16x^3 + \ 10x = 0$$

$$2x(3x - 8x^2 + 5) = 0$$

OR

$$2x = 0$$

$$\underline{\underline{x = 0}}$$

$$3x - 8x^2 + 5 = 0$$

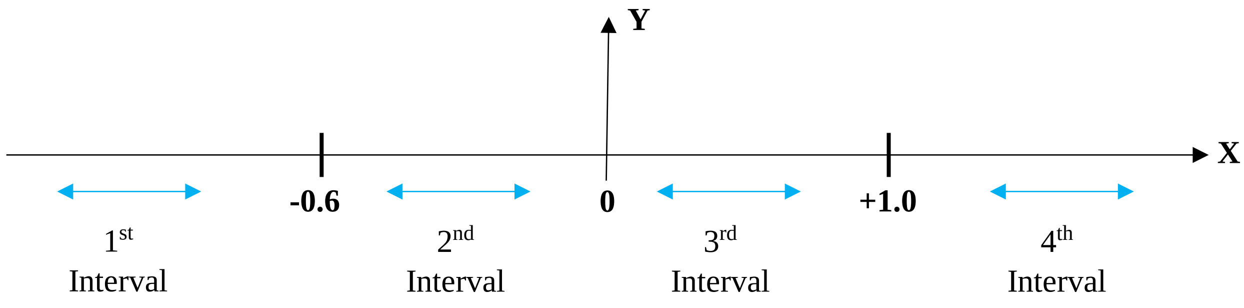
$$x = -\frac{5}{8} \qquad x = 1$$

OR

$$\underline{\underline{x = -0.6}} \qquad \underline{\underline{x = 1.0}}$$

Second step is, we need to do is finding increasing and decreasing intervals →

According to above results I know there are 3 critical points and which containing 4 intervals. And I've marked those critical points along with the relevant intervals below number line.



Finding Increasing or Decreasing intervals →

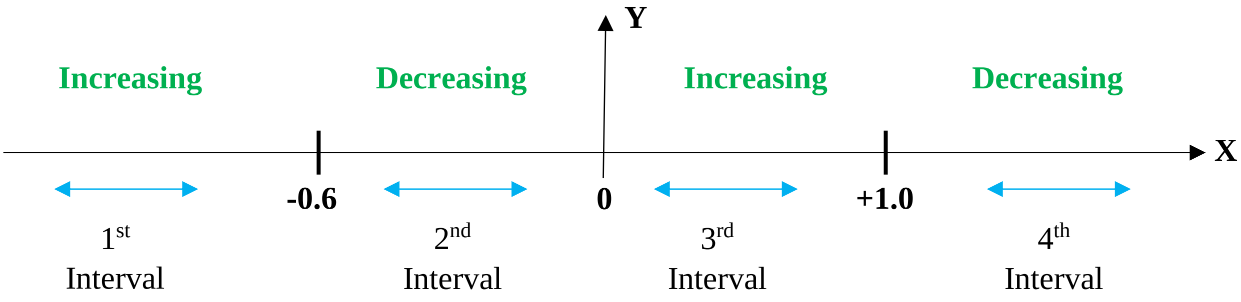
I can find whether function is Increasing or Decreasing by using below table.

Table 4. 4 Finding Increasing or Decreasing intervals

Interval	Test Value	Result	Function Behaviour
x < -0.6	x = - 1	+12 (+)	Increasing
-0.6 < x < 0	x = - 0.5	-1.5 (-)	Decreasing
0 < x < 1.0	x = 0.5	+45 (-)	Increasing
1.0 < x	x = 2	-84 (+)	Decreasing

The intervals of increasing are → (-∞ , -0.6) and (1.9 , ∞)

The intervals of decreasing are → (-1.9 , 0) and (0 , 1.9)



To get the results for corresponding test values I used first derivative function. Below I've given the results I've obtained for corresponding test values for each interval.

For test value -1 →

$$f'(x) = 6x^2 - \ 16x^3 + \ 10x$$

$$f'(-1) = 6(-1)^2 - \ 16(-1)^3 + \ 10(-1)$$

$$f'(-1) = \ +12$$

For test value -0.5 →

$$f'(x) = 5x^4 - \ 18x^2$$

$$f'(-0.5) = 6(-0.5)^2 - \ 16(-0.5)^3 + \ 10(-0.5)$$

$$f'(-0.5) = \ -1.5$$

For test value 0.5 →

$$f'(x) = 5x^4 - \ 18x^2$$

$$f'(0.5) = 6(0.5)^2 - \ 16(0.5)^3 + \ 10(0.5)$$

$$f'(0.5) = \ 4.5$$

For test value 2 →

$$f'(x) = 5x^4 - \ 18x^2$$

$$f'(2) = 6(2)^2 - \ 16(2)^3 + \ 10(2)$$

$$f'(2) = \ -84$$

Finding Local Maximum and Minimum points using First Derivative Test →

First Derivative Test is the process of locating these Maximum and Minimum points using increasing and decreasing intervals.

- If function goes decreasing to increasing, it is a Minimum Point.
- If function goes increasing to decreasing, it is a Maximum Point.
- If function goes has neither Maximum nor Minimum, it is an Inflection Point (Concavity Changing Point).

At x = -0.6 point

At x = -0.6 point the f ′(x) function goes from increasing (+) to decreasing (-).

So, there is Local Maximum Point at x = -0.6.

The corresponding "y" axis value can get by substituting corresponding x value to the original function.

$$f(x) = 2x^3 - 4x^4 + 5x^2$$

$$f(-0.6) = 2(-0.6)^3 - 4(-0.6)^4 + 5(-0.6)^2$$

$$\underline{\underline{f(-0.6) = 0.85}}$$

The corresponding "y" axis value at x = -0.6 point is 0.85

At x = 0 point

At x = 0 point the f ’(x) function goes from decreasing (-) to increasing (+).

So, there is Local Minimum Point at x = 0.

The corresponding “y” axis value can get by substituting corresponding x value to the original function.

$$f(x) = 2x^3 - 4x^4 + 5x^2$$

$$f(0) = 2(0)^3 - 4(0)^4 + 5(0)^2$$

$$f(0) = 0$$

The corresponding “y” axis value at x = 0 point is 0

At x = +1.0 point

At x = 1.0 point the f ’(x) function goes from increasing (-) to decreasing (+).

So, there is Local Maximum Point at x = 1.0

The corresponding “y” axis value can get by substituting corresponding x value to the original function.

$$f(x) = 2x^3 - 4x^4 + 5x^2$$

$$f(1.0) = 2(1.0)^3 - 4(1.0)^4 + 5(1.0)^2$$

$$f(1.0) = 3.0$$

The corresponding “y” axis value at x = 1.0 point is 3.00

Finding Local Maximum and Minimum points using Second Derivative Test →

Second Derivative Test is the process of locating these Maximum and Minimum points substituting critical points to second derivative function.

- If Second Derivative value for a critical point is greater than 0 (+ value) it is Minimum Point.
- If Second Derivative value for a critical point is lesser than 0 (- value) it is Maximum Point.
- If Second Derivative value is equal to 0 for a critical point, it can be Inflection Point (Concavity Changing Point).

$f''(c) < 0$ → Maximum Point

$f''(c) = 0$ → Concavity Changing Point

$f''(c) > 0$ → Minimum Point

$$f(x) = 2x^3 - 4x^4 + 5x^2$$ ← Original Function

$$\frac{d\,f(x)}{dx} = f'(x) = 6x^2 - 16x^3 + 10x$$ ← First Derivative

$$\frac{d\,f'(x)}{dx} = f''(x) = 6 \times 2x^{2-1} - 16 \times 3x^{3-1} + 10 \times 1x^{1-1}$$

$$\underline{\underline{f''(x) = 12x - 48x^2 + 10}}$$ ← Second Derivative

At x = -0.6 point

At x = -0.6 point to the f ″(x) function substitute "x" value.

$$f''(x) = 12x - \ 48x^2 + 10$$

$$f''(-0.6) = 12(-0.6) - \ 48(-0.6)^2 + 10$$

$$\underline{f''(-0.6) = -14.48}$$

Since the second derivative value to corresponding x = -0.6 is lesser than 0 (- value) it is Maximum Point.

At x = 0 point

At x = 0 point to the f ″(x) function substitute "x" value.

$$f''(x) = 12x - \ 48x^2 + 10$$

$$f''(0) = 12(0) - \ 48(0)^2 + 10$$

$$\underline{f''(0) = 10}$$

Since the second derivative value to corresponding x = 0 is greater than 0 (+ value) it is Minimum Point.

At x = +1.0 point

At x = +1.0 point to the f ″(x) function substitute "x" value.

$$f''(x) = 12x - 48x^2 + 10$$

$$f''(1.0) = 12(1.0) - 48(1.0)^2 + 10$$

$$\underline{f''(1.0) = -26.00}$$

Since the second derivative value to corresponding x = 1.0 is lesser than 0 (- value) it is Maximum Point.

Justify answer retrieved from both first derivative test and second derivative test →

At x = -0.6 in both methods (first derivative test and second derivative) I received Maximum Point.

At x = 0 in both methods (first derivative test and second derivative) I received Minimum Point.

At x = +1.0 in both methods (first derivative test and second derivative) I received Maximum Point.

Finding Concavity changing intervals →

To find concavity changing intervals for function y = f(x), we set the function's second derivative to zero and solve.

So, I've set $f''(x)$ = 0 and solve it to find all the values which are the concavity changing points.

$$f''(x) = 12x - \ 48x^2 + 10$$

To find concavity changing intervals set $f''(x) = 0$

$$12x - \ 48x^2 + 10 = 0$$

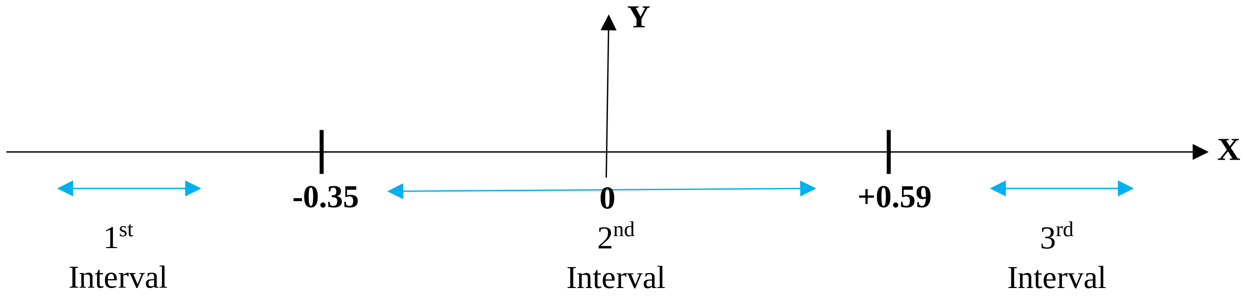
$$6x - 24x^2 + 5 = 0$$

OR

$$x = -0.35 \qquad\qquad x = 0.59$$

Then we need to do is finding increasing and decreasing intervals →

According to above results I know there are 2 concavity changing points and which containing 3 intervals. And I've marked those concavity changing points along with the relevant intervals below number line.



I can find concavity changing intervals by using below table.

Table 4. 5 Finding Concavity changing intervals

Interval	Test Value	Result	Concavity Behaviour
x < -0.35	x = - 1	-40 (-)	Concave Down
- 0.35 < x < 0.59	x = 0	10 (+)	Concave Up
0.59 < x	x = 1	-36 (-)	Concave Down

The intervals of Concave Down are → (-∞ < x < -0.35) and (0.59 < x < ∞)

The intervals of Concave Up are → (-0.35 < x < 0.59)

To get the results for corresponding test values I used second derivative function. Below I've given the results I've obtained for corresponding test values for each interval.

For test value -1 →

$$f''(x) = 22x - \ \ 48x^2 + 10$$

$$f''(x-1) = 12(-1) - 48(-1)^2 + 10$$

$$f''(-1) = \ -40.0$$

For test value 0 →

$$f''(x) = 2x - \ 48x^2 + 10$$

$$f''(0) = 12(0) - 48(0)^2 + 10$$

$$f''(0) = \ 10$$

For test value 1 →

$$f''(x) = 2x - \ 48x^2 + 10$$

$$f''(1) = 12(1) - 48(1)^2 + 10)$$

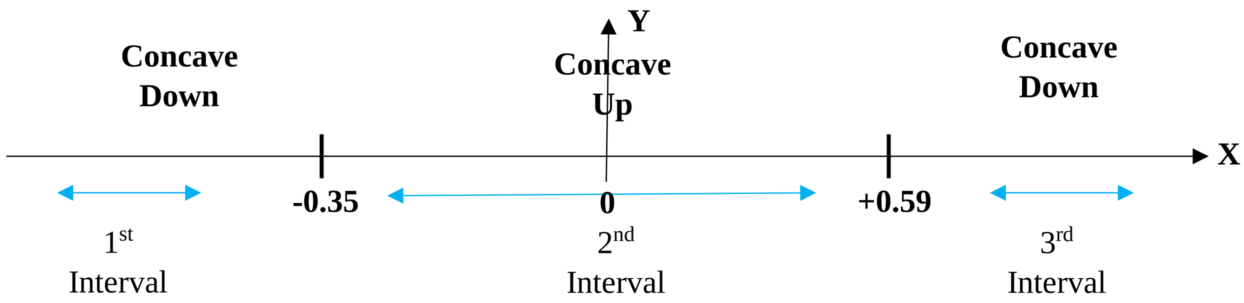
$$f''^{(2)} = \ -26$$

Finding Inflection Points→

Table 4. 6 Finding Inflection Points

Interval	Test Value	Result	Concavity Behaviour
$x < -0.35$	$x = -1$	-50 (-)	Concave Down
$-0.35 < x < 0.59$	$x = 0$	10 (+)	Concave Up
$0.59 < x$	$x = 2$	-26 (-)	Concave Down

According to above results which got from Second Derivative Test, I've marked those concavity changing points along with the relevant intervals below number line.



At x = -0.35 the concavity changes suddenly from Down to Up.

At x = +0.59 the concavity changes suddenly from Up to Down.

According to above number line at x = -0.35 and x = +0.59, the concavity changes suddenly.

Hence, the x = -0.35 and x = +0.59 are the Inflection Points (Concavity Changing Points).

Graphing the Function →

To graph this function, I've used below Online Graphing Tool and then gave the original function as the input.

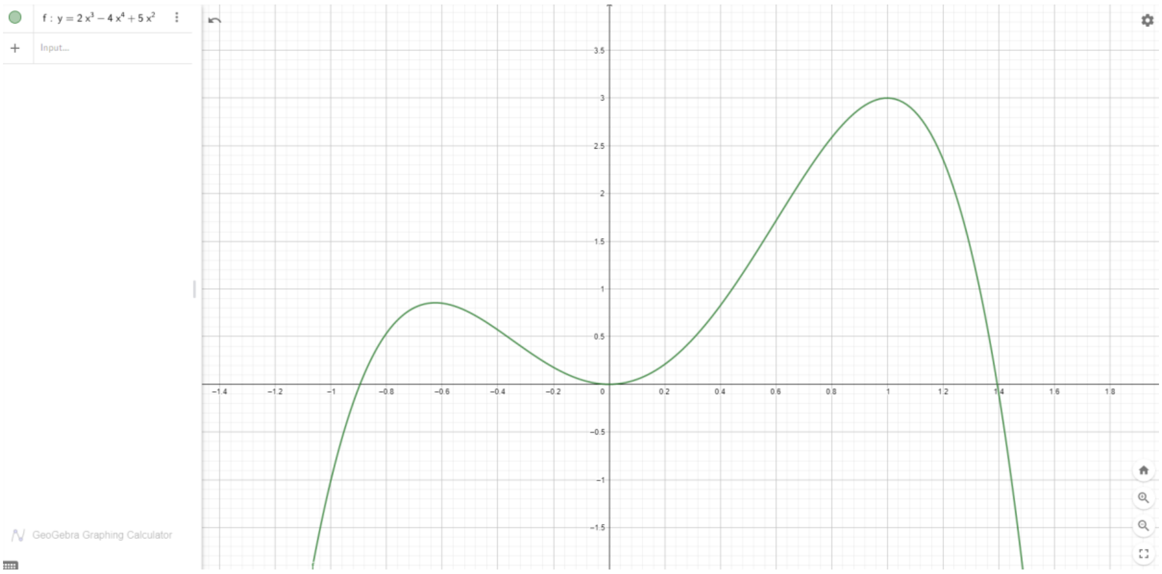


Figure 4. 7 Graphing the Function

https://www.geogebra.org/graphing?lang=en

Finding "x" axis intersect points →

To find "x" axis intersect points, I equalled "y" value to 0

So, I've set original $f(x)$ function's "y" value to 0 and solve it to find all the values which are intersecting "x" axis.

$$f(x) = 2x^3 - 4x^4 + 5x^2$$

To find "y" axis intersect points set $f(x) = 0$

$$0 = 2x^3 - 4x^4 + 5x^2$$

$x = -0.89$ or $x = 0$ or $x = 1.39$

To finding corresponding x values I've used this online algebra calculator : https://www.symbolab.com/geometry

Finding "y" axis intersect points →

To find "y" axis intersect points, I equalled x value to 0

So, I've set original $f(x)$ function's "x" value to 0 and solve it to find all the values which are intersecting "y" axis.

$$f(x) = 2x^3 - 4x^4 + 5x^2$$

To find "x" axis intersect points set $x = 0$

$$f(x) = 2(0)^3 - 4(0)^4 + 5(0)^2$$

$y = 0$

To finding important point values of the graph I've used this Online Graphing Tool:

https://www.transum.org/Maths/Activity/Graph/Desmos.asp

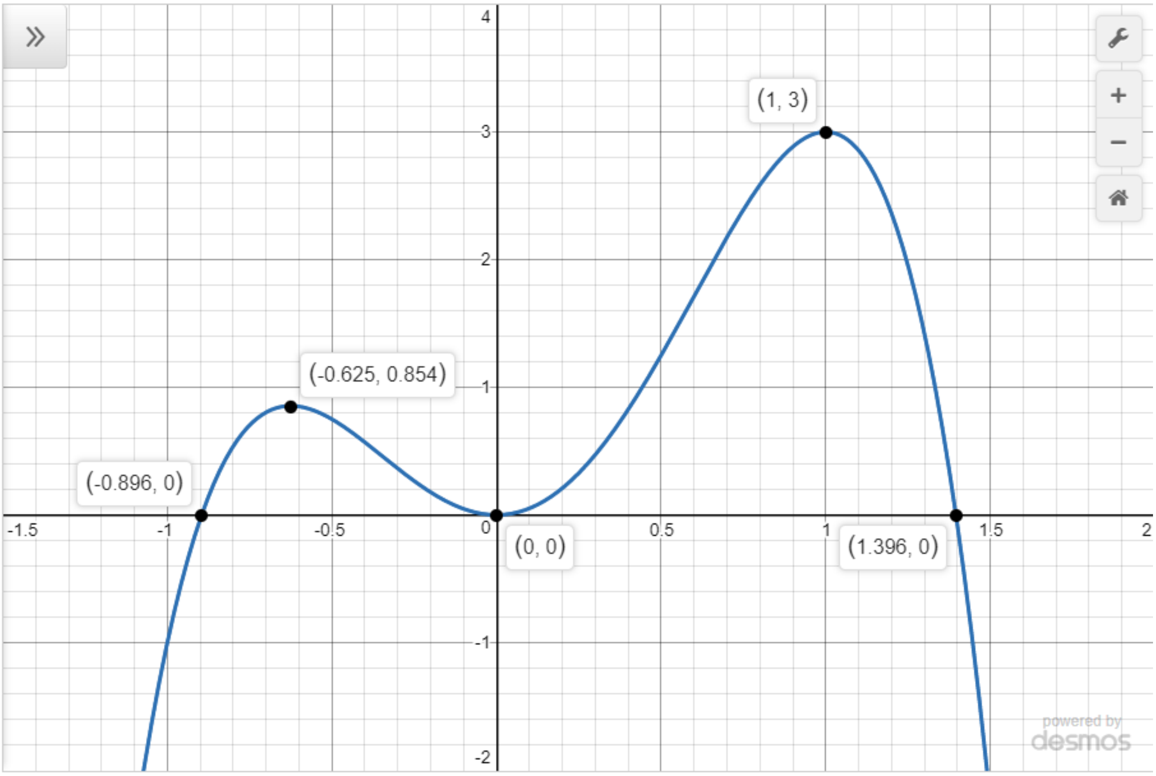


Figure 4. 8 Finding important point values of the graph I've used this Online Graphing Tool

Conclusion

This entire assignment is based on basics of using applied number theory in practical computing scenarios, analyzing events using probability theory and probability distributions, determining solutions of graphical examples using geometry and vector methods and evaluating problems concerning differential and integral calculus. I've used various of graphing and calculating tools to justify the answers' accuracy and have given small theory parts for each question to understand the question and the answer easily.

References

Greatest Common Divisor | Brilliant Math & Science Wiki. [no date].

Available at: https://brilliant.org/wiki/greatest-common-divisor/#:~:text=The%20GCD%20is%20used%20for,applications%2C%20such%20as%20simplifying%20fractions.

[Accessed: 3 August 2022].

Greatest Common Divisor (GCD) | Find GCD with Examples. [no date].

Available at: https://byjus.com/maths/greatest-common-divisor/

[Accessed: 3 August 2022].

Factors of 16 - Find Prime Factorization/Factors of 16. [no date].

Available at: https://www.cuemath.com/numbers/factors-of-16/

[Accessed: 3 August 2022].

What is the Greatest Common Factor and Least Common Multiple? 2013. Available at: https://www.mometrix.com/academy/greatest-common-factor/

[Accessed: 3 August 2022].

LCM (Least Common Multiple) - How to Find LCM? Examples. [no date].

Available at: https://www.cuemath.com/numbers/lcm-least-common-multiple/

[Accessed: 3 August 2022].

Definition and Examples of Sequences. [no date].

Available at: https://www.cliffsnotes.com/study-guides/algebra/algebra-ii/sequences-and-series/definition-and-examples-of-sequences

[Accessed: 3 August 2022].

Equation of a Circle (Formula & Examples of Circle Equation). [no date].

Available at: https://byjus.com/maths/equation-of-a-circle/

[Accessed: 3 August 2022].

Equation of a Circle. 2022.

Available at: https://www.basic-mathematics.com/equation-of-a-circle.html

[Accessed: 3 August 2022].

Step-by-Step Calculator - Symbolab. [no date].

Available at: https://www.symbolab.com/solver

[Accessed: 3 August 2022].

Geometry Calculator - Symbolab. [no date].

Available at: https://www.symbolab.com/geometry#

[Accessed: 3 August 2022].

olve systems of equations by graphing (Pre-Algebra, Graphing and functions) – Mathplanet. [no date].

Available at: https://www.mathplanet.com/education/pre-algebra/graphing-and-functions/solve-systems-of-equations-by-graphing#:~:text=To%20solve%20a%20system%20of,where%20the%20two%20lines%20intersect.&text=The%20two%20lines%20intersect%20in,to%20this%20system%20of%20eq

[Accessed: 3 August 2022].

What is Modular Arithmetic? - Definition from Techopedia. [no date].

Available at: https://www.techopedia.com/definition/20820/modular-arithmetic

[Accessed: 3 August 2022].

Modular inverses (article) | Cryptography | Khan Academy. [no date].

Available at: https://www.khanacademy.org/computing/computer-science/cryptography/modarithmetic/a/modular-inverses

[Accessed: 3 August 2022].

What is Inverse? - Definition Facts and Examples. 2022.

Available at: https://www.splashlearn.com/math-vocabulary/multiplication/inverse

[Accessed: 3 August 2022].

What is modular arithmetic? (article) | Khan Academy. [no date].

Available at: https://www.khanacademy.org/computing/computer-science/cryptography/modarithmetic/a/what-is-modular-arithmetic

[Accessed: 3 August 2022].

What is a prime number? 2020.

Available at: https://www.techtarget.com/whatis/definition/prime-number#:~:text=A%20prime%20number%20is%20a,factors%20are%20called%20composite%20numbers.

[Accessed: 3 August 2022].

What is a prime number? 2020.

Available at: https://www.techtarget.com/whatis/definition/prime-number#:~:text=A%20prime%20number%20is%20a,factors%20are%20called%20composite%20numbers.

[Accessed: 3 August 2022].

Prime Numbers - Definition, Prime Numbers 1 to 100, Examples | Prime Numbers List. [no date].

Available at: https://www.cuemath.com/numbers/prime-numbers/

[Accessed: 3 August 2022].

@mashable 2020. Why should we care about prime numbers? | Mashable.

Available at: https://mashable.com/article/why-should-we-care-about-prime-numbers#:~:text=Most%20modern%20computer%20cryptography%20works,used%20to%20decrypt%20it%20again.

[Accessed: 3 August 2022].

Prime numbers and their importance to modern life | Blog | CodeCoda. 2021.

Available at: https://codecoda.com/en/blog/entry/prime-numbers-and-the-importance-to-modern-life

[Accessed: 3 August 2022].

Definition and Examples of Sequences. [no date].

Available at: https://www.cliffsnotes.com/study-guides/algebra/algebra-ii/sequences-and-series/definition-and-examples-of-sequences

[Accessed: 3 August 2022].

Conditional Probability Definition. 2022.

Available at: https://www.investopedia.com/terms/c/conditional_probability.asp

[Accessed: 3 August 2022].

Conditional Probability. [no date].

Available at: https://www.mathsisfun.com/data/probability-events-conditional.html

[Accessed: 3 August 2022].

Random Variable - Definition, Meaning, Types, Examples. [no date].

Available at: https://www.cuemath.com/data/random-variable/

[Accessed: 3 August 2022].

Discrete Random Variable - Definition, Formula, Differences, Example, FAQs. [no date].

Available at: https://www.cuemath.com/algebra/discrete-random-variable/

[Accessed: 3 August 2022].

Continuous Random Variable - Definition, Formulas, Mean, Examples. [no date].

Available at: https://www.cuemath.com/data/continuous-random-variable/

[Accessed: 3 August 2022].

Difference between Discrete and Continuous Random Variables. 2022.

Available at: https://www.onlinemath4all.com/difference-between-discrete-and-continuous-random-variables.html

[Accessed: 3 August 2022].

Preshing, J. 2011. Hash Collision Probabilities.

Available at: https://preshing.com/20110504/hash-collision-probabilities/

[Accessed: 3 August 2022].

What is hashing and how does it work? 2021.

Available at: https://www.techtarget.com/searchdatamanagement/definition/hashing

[Accessed: 3 August 2022].

Equation of a Sphere – Examples and Diagram. 2022.

Available at: https://mathmonks.com/sphere/equation-of-a-sphere

[Accessed: 3 August 2022].

Equation of a Sphere - Expii. [no date].

Available at: https://www.expii.com/t/equation-of-a-sphere-1321#:~:text=of%20a%20Sphere-,The%20general%20equation%20of%20a%20sphere%20is%3A%20(x%20%2D%20a,the%20surface%20of%20the%20sphere.

[Accessed: 3 August 2022].