Time Series Analysis

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***Abstract*—Time series data have become increasingly important in different areas. This document contains background knowledge of time series analysis. Basic properties, methods of forecasting and application areas are presented in overview.**

***Index Terms*—time series, forecasting, analysis, models**

I. Introduction

Time series is an ordered sequence of values of a variable at equally spaced time intervals. Such sequences can be found in many areas and domains and usually forecasting of future values of these sequences is a main goal. To make accurate predictions, we need to learn basic properties of time series and forecasting methods. In this document methods like moving average (MA) and exponential smoothing (ES) are presented. Finally, last lines devoted to practical usage of time series analysis. For example, time series analysis is used for many applications such as Economic Forecasting, Stock Market Analysis, Sales Forecasting, Yield Projections, etc.

II. Basic properties of Time Series

A. Trend

The trend shows the general tendency of the data to increase or decrease during a long period of time. For instance, the amount of air travels increasing every year, so time series of this amount will show trend.

B. Seasonality

Seasonality means periodic fluctuations. For example, retail sales tend to peak for the Christmas season and then decline after the holidays. So from September through December time series of retail will typically show increasing and decreasing sales in January and February.

C. Stationarity

Stationarity means a flat looking series with no trend, constant variance over time and no seasonality. We need this definition, because many time series techniques assume that the data are stationary. If the time series is not stationary, we can often transform it to a stationary one using suitable methods such as taking the logarithm or square root and so on. These methods eliminate trend, seasonality or non-constant variance.

III. Moving Average and Smoothing Techniques

Unfortunately, some forms of random variation are inevitable in the collection of data. So we need methods to reveal more clearly trend and seasonal components. An often-used technique in industry is ”smoothing”. There are two basic groups of smoothing methods:

- Averaging Methods
- Exponential Smoothing Methods

To estimate the accuracy of some method we shall compute the mean squared error (MSE). It can be shown mathematically that the estimator with the smallest MSE is the best.

A. ”Simple” average of all data

It is the easiest way to smooth the data. But is it accurate enough? Unfortunately, this method doesn’t fit for predictions. It’s impossible to use the mean to forecast income if we suspect a trend. In summary, we state that the mean of all past observations is only a useful estimate for forecasting when there are no trends and the weights of all observations are equal. Then we will discuss methods where the more recent events have more weight.

B. Moving Average

Another way to summarize the past data is to compute the average of smaller sets. For example, we count the mean of first three observations and replace the third one with the result, then the next three and so on. This is called Moving Average. Unfortunately, both of the considered methods are not able to cope with significant trend when used for forecasting.

IV. Exponential Smoothing

We will pay more attention for these methods, because they are more accurate and more suitable for making predictions. As already mentioned, in Moving Average methods all past observations are weighted equally, so Exponential Smoothing uses decreasing weights of observations as they get older. In this method there are one or more smoothing parameters, which respond for decreasing weights and the whole method. We will discuss Single, Double and Triple Exponential Smoothing.

A. Single Exponential Smoothing

Consider the sequence y_1, y_2, ..., y_i, where y_i stands for a value of the original observation, and the sequence S_1, S_2, ..., S_i, where S_i stands for a value of the smoothed observation. This smoothing method begins by setting S_2 to y_1 and for any time period t, the smoothed value S_t can be found by computing

$$S_t = \alpha * y_{t-1} + (1-\alpha) * S_{t-1} \tag{1}$$

This is the basic equation of exponential smoothing and the constant or parameter $0 < \alpha < 1$ is called the smoothing constant. We choose the best value for α so the value which results in the smallest MSE. To make a forecast use the following formula

$$S_{t+1} = \alpha * y_t + (1-\alpha) * S_t \tag{2}$$

If you want to forecast next value having no actual y_t observation, compute

$$S_{t+2} = \alpha * y_t + (1-\alpha) * S_{t+1} \tag{3}$$

But you should remember that single exponential smoothing doesn't give accurate result when there is a trend. You can fix this situation by using double exponential smoothing.

B. Double Exponential Smoothing

This method uses one more constant $0 < \gamma < 1$. Using new parameter b_t this method easily monitors a trend. So new equations for S_t and b_t are

$$S_t = \alpha * y_{t-1} + (1-\alpha) * (S_{t-1} + b_{t-1}), \tag{4}$$

$$b_t = \gamma * (S_t - S_{t-1}) + (1-\gamma) * b_{t-1} \tag{5}$$

In this method $S_1 = y_1$ and $b_1 = \frac{y_n - y_1}{n-1}$ for some n. Second equation b_t responds for the trend component and makes double exponential smoothing more accurate than single one. Also we choose the best values for α and γ which result in the smallest MSE. Finally, the forecasting in this method uses formula

$$F_{t+m} = S_t + m * b_t \tag{6}$$

Like single exponential smoothing double one isn't perfect, because it is able to monitor seasonality of time series. So then data show the seasonality we are forced to use triple exponential smoothing.

C. Triple Exponential Smoothing

Like in double exponential smoothing we add new equation to take care of seasonality. So basic equations for this method are:

$$S_t = \alpha * \frac{y_t}{I_{t-L}} + (1-\alpha) * (S_{t-1} + b_{t-1}), \tag{7}$$

$$b_t = \gamma * (S_t - S_{t-1}) + (1-\gamma) * b_{t-1}, \tag{8}$$

$$I_t = \beta * \frac{y_t}{S_t} + (1-\beta) * I_{t-1}, \tag{9}$$

$$F_{t+m} = (S_t + m * b_t) * It - L + m, \tag{10}$$

where y is the observation, S is the smoothed observation, b is the trend factor, I is the seasonal index, F is the forecast at m periods ahead, L is the amount of periods in a complete season's data, $0 < \beta < 1$ is the new coefficient for triple exponential smoothing. To initialize this method you need at least one complete season's data. At the end of discussing these exponential smoothing methods you should know that sometimes it happens that best coefficients for trend (γ) or for seasonality (β) can equals to zero, but it doesn't mean that there is no trend or seasonality. It means that zero coefficients give the smallest MSE.

V. Applications

Now lets consider practical usage of time series analysis in some spheres.

A. Yield Projections

Usually time series analysis is not so useful in this sphere, but it can give people many information. For instance, somebody can predict the number of harvest in the next year, so it helps to forecast his possible profit. But usually our world is unpredictable and there are a lot of factors which effect on the amount of harvest. As practice shows, time series analysis isn't accurate enough in this domain.

B. Economic Forecasting

Economic forecasting has a great importance in our world. Who can make successful predictions in economic stands at the top of the commercial world. Time series analysis is commonly used in pair trading and in creating high-frequency trading algorithms. Specialists in the of time series analysis are in high demand in many organizations. Time series analysis was born thank to economic and it continues to develop.

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