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CS 5785 – Applied Machine Learning

**Homework 2 – Written Portion**

1. Here we have a prime example of where to apply the Lagrange multiplier with the constraint entered as :

The minimum can be found when the derivative of this function is 0

Which is the standard eigenvalue problem.

1. With features , a two-class response and class sizes coded as , .
   1. We know and and that we will classify as class 2 when
   2. Because this is a two-class problem we can create where is a vector of 1s where the sample is of class 1 and 0 otherwise and vice versa for .

Can be rewritten as

Where . To minimize, we take the derivative and set it to 0.

And

Note that and the same for class 2.

We know that the covariance matrix is

From the problem we’ll define

Note that and thus . Therefore:

* 1. We know that

Where . And thus, is in the direction . We can plug this into our equation in part b

And thus, the least-squares regression coefficient is identical to the LDA coefficient up to a scalar multiple.

* 1. If we say and then we have

Which holds for any encodings for and .

* 1. Note that . Therefore:

With the decision rule we get:

If we’re classifying to class 2 when then

And from part c we know

And if then

Which is the same as the LDA classification rule for class 2.

1. Given M

[ 1 0 3]

[ 3 7 2]

[ 2 -2 8]

[ 0 -1 1]

[ 5 8 7]

* 1. is given by

[ 39 57 60]

[ 57 118 53]

[ 60 53 127]

And is given by

[ 10 9 26 3 26]

[ 9 62 8 -5 85]

[ 26 8 72 10 50]

[ 3 -5 10 2 -1]

[ 26 85 50 -1 138]

* 1. The eigenvalues for are

214.670489196, 9.32587340685e-15, 69.3295108039

Note that the middle element is essentially 0.

The eigenvalues for are

214.670489196, -8.881784197e-16, 69.3295108039, -3.34838280783e-15, 7.47833226761e-16

Note that all but 214.67048916 and 69.3295108 are essentially 0.

* 1. The eigenvectors for are

214.670489196: [ 0.42615127 0.61500884 0.66344497]

69.3295108039: [-0.01460404 -0.72859799 0.68478587]

The eigenvectors for are

214.670489196: [-0.16492942 -0.47164732 -0.33647055 -0.00330585 -0.79820031]

69.3295108039: [ 0.24497323 -0.45330644 0.82943965 0.16974659 -0.13310656]

* 1. We know that is the diagonal matrix of the square root of ’s eigenvalues

[ 14.65163776 0. ]

[ 0. 8.32643446]

And is composed of the eigenvectors of , thus is

[ 0.42615127 0.90453403 -0.01460404]

[ 0.66344497 -0.30151134 0.68478587]

And we know that which means which is

[ 0.16492942 0.24497323]

[ 0.47164732 -0.45330644]

[ 0.33647055 0.82943965]

[ 0.00330585 0.16974659]

[ 0.79820031 -0.13310656]

* 1. We can find the one-dimensional approximation to by taking which gives:

[ 1.02978864 1.48616035 1.60320558]

[ 2.94487812 4.24996055 4.58467382]

[ 2.10085952 3.031898 3.27068057]

[ 0.02064112 0.02978864 0.0321347 ]

[ 4.9838143 7.19249261 7.75895028]

NOTE: All matrix arithmetic was done using python. See *main.ipynb* “Written 3.A” through “Written 3.E” for calculations.