MATH 135 Fall 2023: Written Assignment 10
Ungraded
Covers the contents of the entire course

Q1. One root of the polynomial $f(x) = x^4 + x^3 + 3x^2 + x + 2$ over $\mathbb{C}$ is $-i$.

(a) Write $f(x)$ as a product of irreducible polynomials in $\mathbb{C}[x]$. Show your work.

(b) Write $f(x)$ as a product of irreducible polynomials in $\mathbb{R}[x]$. Show your work.

Q2. Find all solutions to the equation $z^{10} - 2z^5 + 2 = 0$ in complex numbers z. Express each of your solutions in polar form.

Q3. Let $\mathbb{F}$ denote $\mathbb{R}$ or $\mathbb{C}$, and consider $x^3 + ax + b, x^2 + cx - 1 \in \mathbb{F}[x]$. Prove that

$$(x^2 + cx - 1) \mid (x^3 + ax + b)$$

if and only if $c = b$ and $a = -1 - b^2$.

Q4. Find polynomials $p(x)$ and $q(x)$ in $\mathbb{R}[x]$ such that

$$\tan(4\theta) = \frac{p(\tan\theta)}{q(\tan\theta)}$$

for every $\theta \in \mathbb{R}$ where both $\cos\theta$ and $\cos(4\theta)$ are non-zero.

Q5.

(a) Prove that, for every $n \in \mathbb{N}$, $(x - 1) \mid (x^n - 1)$.

Hint: Using your favourite computer algebra system compute $\frac{x^2-1}{x-1}$, $\frac{x^3-1}{x-1}$, $\frac{x^4-1}{x-1}$, and so on. Notice something?

(b) Let $z = a + bi$ be such that $a, b \in \mathbb{Z}$ and $b \neq 0$. Prove that, for every $n \in \mathbb{N}$, the number

$$\frac{z^n - (\overline{z})^n}{z - \overline{z}}$$

is an integer.

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