

Currency Arbitrage Optimization

MSE 426 Engineering Design Optimization Project Report

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Contents

1	Introduction	4
1.1	Currency Arbitrage	4
1.2	The Idea	4
2	Problem Description	5
2.1	Problem Variables	5
2.1.1	2-way Arbitrage	5
2.1.2	3-way Arbitrage	5
2.1.3	3-way Arbitrage with Variable Rates	6
2.2	Objective Functions and Constraints	6
2.2.1	2-way Arbitrage	6
2.2.2	3-way Arbitrage	7
2.2.3	3-way Arbitrage with Variable Rates	8
3	Results and Methodology	9
3.1	2-way Arbitrage	9
3.2	3-way Arbitrage	9
3.3	3-way Arbitrage with Variable Rates	10
4	Conclusion	12
4.1	Practicality	13
4.1.1	Brute Force Method	13
4.1.2	Rate Correlation	13

List of Figures

1	USD2CAD Exchange Rate, 3 Months [1]	4
2	Arbitrage Flow	5
3	TransferWise Transfer Fees, USD and CAD [2]	7

List of Tables

1	Exchange Rates	6
2	Problem Variables, 3-way	6
3	Problem Variables, 3-way, Variable Rates	6
4	Variable Exchange Rates	8
5	2-way Arbitrage - fmincon Results	9
6	2-way Arbitrage - ga Results	9
7	2-way Arbitrage Results Comparison	9
8	3-way Arbitrage - fmincon Results	9
9	3-way Arbitrage - ga Results	10
10	3-way Arbitrage Results Comparison	10
11	3-way Arbitrage, Variable Rates - fmincon Results	10
12	3-way Arbitrage, Variable Rates - fmincon Exchange Rates	10
13	3-way Arbitrage, Variable Rates - ga Results	11
14	3-way Arbitrage, Variable Rates - ga Exchange Rates	11
15	3-way Arbitrage, Variable Rates Results Comparison	11
16	3-way Arbitrage, Variable Rates GA Runs	12

1 Introduction

1.1 Currency Arbitrage

Currency arbitrage is a moderately complex optimization problem that enables banks and larger financial institutions to trade capital via a range of different currencies in order to make a gain or profit. As defined by Investopedia;

A currency arbitrage is a forex strategy in which a currency trader takes advantage of different spreads offered by brokers for a particular currency pair by making trades. Different spreads for a currency pair imply disparities between the bid and ask prices. Currency arbitrage involves buying and selling currency pairs from different brokers to take advantage of the miss priced rates. [3]

Essentially, differences in two or more currency exchange rates, for example the exchange of the US dollar for the Canadian dollar, **USD2CAD**, can enable an individual to make a profit by exchange one currency for another, and back. In the case of three or more currencies, there are different combinations of trading 'routes' or 'loops' that can be taken to maximize one's profit.

1.2 The Idea

After completing over a year long internship in California, USA, I began to observe the trend of the **USD2CAD** exchange rate in order to maximize the value of my earnings once transferred to CAD currency. The following figure displays the rate's trend over the last 3 months.



Figure 1: USD2CAD Exchange Rate, 3 Months [1]

After formulating and solving the optimization problem to transfer USD to CAD, and then back to USD at a lower exchange rate, it is clear that the final amount will be the initial amount scaled by a ratio of the 2 exchange rates. However, this is too simple of a problem that only contains 4 control variables, **USD transfer amount**, **CAD transfer amount**, **rate 1**, **rate 2**. After some consideration, why not add another currency and find the optimal path? Thus, we have currency arbitrage.

2 Problem Description

Fundamentally, currency can be exchanged between the 3 currencies according to the following diagram. As shown, there are 6 links, meaning 6 exchange rates, where arrows in opposite directions are the inverses of each other. For example, going from $USD2CAD = 1.41$, and thus $CAD2USD = 1/1.41 = 0.709$. Hence, there are 3 independent exchange rates.

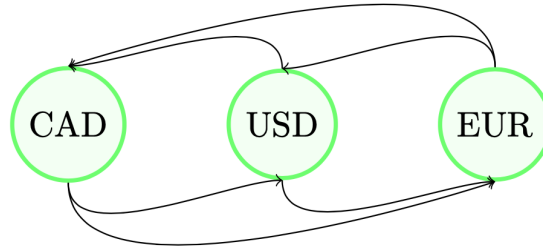


Figure 2: Arbitrage Flow

2.1 Problem Variables

2.1.1 2-way Arbitrage

For simplicity, we assume a starting amount of 10, which can be in thousands, millions, etc. For the most basic problem, 2-way arbitrage, we start with the initial amount in one currency, convert it to the other currency and back again. Hence, we have 2 variables for the amounts to send. We also have 2 more variables for the rates at which to send these amounts.

- x_1 = USD to CAD transfer amount
- x_2 = CAD to USD transfer amount
- x_3 = USD to CAD transfer rate
- x_4 = CAD to USD transfer rate

2.1.2 3-way Arbitrage

To complicate the previous problem, we will add another currency, **EUR**, the Euro. One can see that this problem immediately becomes a problem of 9 variables, n , rather than 4. Thus, the formula for $n = (num.of currencies)^2$. The following table will be used to tabulate the associated transfer rates and amounts, where the **(col, row)** value represents the exchange rate **from col to row**. For example, the bottom center value of **0.9260** represents the USD2EUR exchange rate.

	CAD	USD	EUR
CAD	1	1.4113	1.52448
USD	0.7085	1	1.0799
EUR	0.6559	0.9260	1

Table 1: Exchange Rates

Similarly, a table of the same format will be used for the transfer amounts from each currency to another, which are the problem variables.

	CAD	USD	EUR
CAD	x_1	x_2	x_3
USD	x_4	x_5	x_6
EUR	x_7	x_8	x_9

Table 2: Problem Variables, 3-way

The chosen rates for the solution of the problem will be selected at any given time to see if there is potential for positive gains via arbitrage. In this problem, the rates selected in table 1 were pulled from an online exchange rate monitoring tracker, see [4]. This is the a common model that modern day banks use for **Arbitrage Detection**, essentially constantly checking rates across many currencies to detect potential opportunity for profit.

2.1.3 3-way Arbitrage with Variable Rates

Finally, one can consider what potential gains could be made off optimized exchange rates rather than those given at the current time. The encompassing question is: if the relationship between said currencies is at an optimal point, what is the point (what are the rates), and how much should be routed through each currency? The following table displays our new set of variables.

	CAD	USD	EUR
CAD	x_1	x_2	x_3
USD	x_4	x_5	x_6
EUR	x_7	x_8	x_9
Rates	x_{10}	x_{11}	x_{12}

Table 3: Problem Variables, 3-way, Variable Rates

2.2 Objective Functions and Constraints

2.2.1 2-way Arbitrage

Firstly, there are transaction fees associated with international bank wire transfers. They will be taken into account for this problem, but ignored for the 3-way arbitrage since they are minuscule compared to bank transfer sums. The following figure displays said transfer fees for USD and CAD transfers.

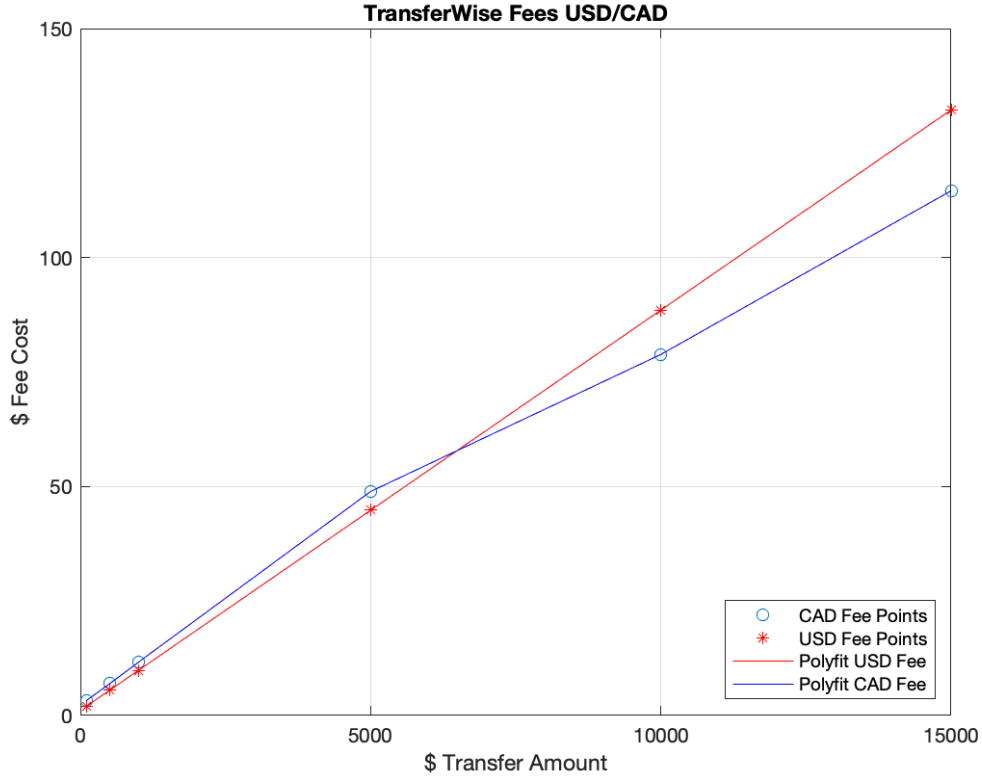


Figure 3: TransferWise Transfer Fees, USD and CAD [2]

To begin, we set the following objective function, subject to practical constraints.

$$\max_x f = -[(x_1 - \text{usdFee}(x_1))x_3 + (x_2 - \text{cadFee}(x_2))/x_4 - x_1] \quad (1)$$

$$\text{subject to:} \quad x_2 - (x_1 - \text{usdFee}(x_1))x_3 \leq 0 \quad (2)$$

Where $\text{usdFee}()$ and $\text{cadFee}()$ calculate the transfer fees, and the non-linear constraint (1) ensures that the amount sent back to USD from CAD, x_2 , cannot be more than the amount received, x_1 . Lastly, the following bounds are set, according to the 5 year high's and low's of the exchange rates for USD to CAD found on [4], and x_0 is the initial amount of \$10,000 USD.

$$\begin{aligned} x_{lb} &= [0, 0, 1.19588, 1.19588] \\ x_{ub} &= [x_0, 60000, 1.4652, 1.4652] \end{aligned} \quad (3)$$

2.2.2 3-way Arbitrage

In this problem of 9 variables, we will neglect transaction fees, since one cannot calculate from TransferWise or other online sources the potential fees for transfers of millions of dollars. Also, we will be running the problem with $x_0 = 10$ in CAD, whether this be thousands or millions, the

relative gains will be the same. The problem to be optimized is:

$$\max_x f = -(x_0 + (EXrates(1, 2) * x_2 + EXrates(1, 3) * x_3) - (x_4 + x_7)) \quad (4)$$

$$\text{subject to:} \quad x_2 + x_8 - [EXrates(2, 1) * x_4 + EXrates(2, 3) * x_6] = 0 \quad (5)$$

$$x_3 + x_6 - [EXrates(3, 1) * x_7 + EXrates(3, 2) * x_8] = 0 \quad (6)$$

Our 3 by 3 matrix, *EXrates*, can be seen in table 1, where elements are selected in MATLAB as *EXrates(row,col)*. The two constraints listed ensure that no capital is left in the other currencies EUR and USD, inputs = outputs. The following bounds are also specified. Notice that x_1, x_5, x_9 can only be 0, similar to the diagonals of table 1, since CAD2CAD, and so forth, would be point-less.

$$\begin{aligned} x_{lb} &= [0, 0, 0, 0, 0, 0, 0, 0, 0] \\ x_{ub} &= [0, 10, 10, 10, 0, 10, 10, 10, 0] \end{aligned} \quad (7)$$

2.2.3 3-way Arbitrage with Variable Rates

Finally, rather than using constant rates that reflect the market at only one instance in time, we will add 3 variables for said exchange rates, to find potential optimal arbitrage opportunities. Our objective function remains the same, however the constraints are now non-linear, since elements of *EXrates* all depend on x_{10}, x_{11}, x_{12} .

$$\max_x f = -(x_0 + (EXrates(1, 2) * x_2 + EXrates(1, 3) * x_3) - (x_4 + x_7)) \quad (8)$$

$$\text{subject to:} \quad x_2 + x_8 - [EXrates(2, 1) * x_4 + EXrates(2, 3) * x_6] = 0 \quad (9)$$

$$x_3 + x_6 - [EXrates(3, 1) * x_7 + EXrates(3, 2) * x_8] = 0 \quad (10)$$

$$-(EXrates(1, 2) * x_2 + EXrates(1, 3) * x_3) - (x_4 + x_7) \leq 0 \quad (11)$$

Constraint (11) is implemented to ensure that strictly positive gains are found, as it can be seen later in this report that **ga** finds many optima, even those with negative gains. The values of *EXrates* are:

	CAD	USD	EUR
CAD	1	$1/x_{10}$	$1/x_{11}$
USD	x_{10}	1	$1/x_{12}$
EUR	x_{11}	x_{12}	1

Table 4: Variable Exchange Rates

Subject to the following bounds, with rates based off the 5-year all time high's and low's found via [4]:

$$\begin{aligned} x_{lb} &= [0, 0, 0, 0, 0, 0, 0, 0, 0, 0.68250, 0.61961, 0.79937] \\ x_{ub} &= [0, 10, 10, 10, 0, 10, 10, 10, 0, 0.83642, 0.76188, 0.96242] \end{aligned} \quad (12)$$

3 Results and Methodology

To solve the aforementioned problems, **MATLAB R.2019b** will be used, with the implementation of the *Optimization Toolbox* and *Global Optimization Toolbox*. Primary functions used will be **fmincon** and **ga**. The results for each type of optimization algorithm run on each problem can be seen in the following sections.

3.1 2-way Arbitrage

Code for this solution can be seen in *arb2.m*. As mentioned earlier, x_0 of \$10,000 USD will be used. The following tables outline the results.

Optimum	Profit	Gain	Iterations	FuncEvals
-16577.8855	6577.8855	0.65779	15	80

Table 5: 2-way Arbitrage - **fmincon** Results

Optimum	Profit	Gain	Iterations	FuncEvals
-15256.2012	5256.2012	0.52562	4	42656

Table 6: 2-way Arbitrage - **ga** Results

X	FMOptima	GAOptima
\$USD2CAD	10000	9806.3548
\$CAD2USD	14522.2859	13307.1288
Rate USD2CAD	1.4652	1.4634
Rate CAD2USD	1.1956	1.2187

Table 7: 2-way Arbitrage Results Comparison

It can be seen that **fmincon** returns a gain of 66%, whereas **ga** returns only 53%. To summarize the results of **fmincon**, one would send \$10k USD to CAD, pay fees, at a rate of 1.4652, and then once received, send \$14,222.29 CAD back to USD at a rate of 1.1956 (see table 7). However, this is extremely impractical since those rates were seen only once in 5 years. Furthermore, **ga** does not solve as well as **fmincon** in this case, most likely because of the non linear constraint.

3.2 3-way Arbitrage

Code for this solution can be seen in *arb3.m*. Moving forward, x_0 of 10 will be used, in CAD (see section 2.2.3). The following tables outline the results. Let's assume the units are in millions, so as to acquire meaningful gains.

Optimum	Profit	Gain	Iterations	FuncEvals
-10.0017	0.0016696	0.00016696	45	480

Table 8: 3-way Arbitrage - **fmincon** Results

Optimum	Profit	Gain	Iterations	FuncEvals
-9.9979	-0.0021135	-0.00021135	51	9900

Table 9: 3-way Arbitrage - **ga** Results

Clearly, **ga** does not return a viable solution, however **fmincon** does, and yields a gain of 0.017%, or \$1,700 CAD. Based off the selected exchange rates for this arbitrage detection, as shown in table 1, a profit can be made. How to achieve this result? See table 10 below.

X	FMOptima	GAOptima
\$CAD2CAD	0	2.0391e-17
\$USD2CAD	0.0010832	9.1614
\$EUR2CAD	6.561	8.6987
\$CAD2USD	9.9976	14.2436
\$USD2USD	0	0
\$EUR2USD	0.022085	5.8152e-14
\$CAD2EUR	0.00433	11.9491
\$USD2EUR	7.1061	0.93017
\$EUR2EUR	0	0

Table 10: 3-way Arbitrage Results Comparison

Essentially, one would begin with \$10 million CAD. The row names **\$CAD2CAD**, **\$USD2CAD**... and so on, represent the corresponding transfer amounts. Since we start with CAD, one would send \$9.9976 to USD, and \$0.00433 to EUR, and so forth, until all currency is converted back to CAD, resulting in \$10,001,670 CAD (table 8).

3.3 3-way Arbitrage with Variable Rates

Code for this solution can be seen in *arb3vary.m*. Finally, as outlined in section 2.2.3, the optimal arbitrage transfer amounts and rates have been tabulated in the following tables. It can be noted that there are many optima for this problem. For **fmincon**, a starting point of $x_{guess} = x_{ub}$ will be used, since larger trades produce higher gains.

Optimum	Profit	Gain	Iterations	FuncEvals
-18.4314	8.4314	0.84314	1	49

Table 11: 3-way Arbitrage, Variable Rates - **fmincon** Results

	CAD	USD	EUR
CAD	1	1.3035	1.433
USD	0.76716	1	1.1248
EUR	0.69786	0.88905	1

Table 12: 3-way Arbitrage, Variable Rates - **fmincon** Exchange Rates

Optimum	Profit	Gain	Iterations	FuncEvals
-11.2978	1.2978	0.12978	5	66610

Table 13: 3-way Arbitrage, Variable Rates - **ga** Results

	CAD	USD	EUR
CAD	1	1.3856	1.3129
USD	0.72171	1	1.212
EUR	0.76164	0.82512	1

Table 14: 3-way Arbitrage, Variable Rates - **ga** Exchange Rates

Tables 11 to 14 display the optimization results for this problem. Clearly, there exist much higher potential arbitrage gains, 84% and 24% from **fmincon** and **ga**, respectively. After a quick investigation of tables 12 and 14, we can see that such rates are fairly impractical. A discussion of practicality is outlined in section 4.1. The aforementioned optimum occur at the following design points.

X	FMOptima	GAOptima
\$CAD2CAD	0	0
\$USD2CAD	9.01	9.7433
\$EUR2CAD	9.01	1.3427
\$CAD2USD	9.01	7.5513
\$USD2USD	0	0
\$EUR2USD	9.01	5.8678
\$CAD2EUR	9.01	6.4142
\$USD2EUR	9.01	2.818
\$EUR2EUR	0	0
Rate CAD2USD	0.76716	0.72171
Rate CAD2EUR	0.69786	0.76164
Rate USD2EUR	0.88905	0.82512

Table 15: 3-way Arbitrage, Variable Rates Results Comparison

It is interesting to see that **fmincon** finds a solution in which all transfer amounts = \$9.01, and only the rates vary. This is of course different with other trials where x_{guess} is changed. Finally, it was noted that since there are numerous optima, **ga** was run 20 times to observe the many optima and analyse the highest few for better understanding of practicality.

Data	Run1	Run2	Run3	Run4	Run5	Run6
Optimum	-10.41	-13.9648	-11.4928	-13.132	-11.0421	-11.942
\$CAD2CAD	0	0	0	0	0	0
\$USD2CAD	1.2831	9.6016	9.9705	9.9171	3.8731	7.2703
\$EUR2CAD	2.3346	2.6524e-06	4.2076	0.13599	1.545	3.4577
\$CAD2USD	2.9045	0.10344	8.6387	3.0051	0.84417	3.7953
\$USD2USD	0	0	0	0	0	0
\$EUR2USD	0.45489	8.3484	7.219	8.3351	8.1373	5.6448
\$CAD2EUR	1.9874	10	9.9999	8.5721	5.8258	9.4535
\$USD2EUR	1.6189	0.91275	4.5486	2.4013	6.0411	2.3746
\$EUR2EUR	0	0	0	0	0	0
Rate CAD2USD	0.83642	0.6825	0.6825	0.6825	0.68251	0.6825
Rate CAD2EUR	0.61961	0.76188	0.76188	0.76081	0.75835	0.76188
Rate USD2EUR	0.96242	0.79937	0.83718	0.8118	0.87142	0.80016

Table 16: 3-way Arbitrage, Variable Rates GA Runs

Clearly there are many solutions that all yield very high gains with the highest being 39.6% (Run2). The genetic algorithm is able to remain within the constraints and ensure all money is converted back to CAD, while obtaining different levels of profit for different transactions and exchange rates.

4 Conclusion

A significant amount of understanding of currency arbitrage, as well as different optimization algorithms, has been gained throughout the duration of this project. Not only through the many solutions produced, but the implementation of the objective functions, their constraints, linear and non linear, and finally upper and lower bounds, have induced a great deal of learning concerning engineering design optimization. To list the outcomes:

- An in depth understanding of currency arbitrage and it's complexity.
- The constraints and drawbacks to currency arbitrage.
- The many different optimization algorithms, and their pros and cons.
- A valid assessment of arbitrage detection and it's potential in the market.
- 2-way arbitrage (in this report) can provide gains with tangible income, assuming proper monitoring of exchange rates.
- A sound understanding of practicality or the market.

Not only engineering specific applications, but the concept as a whole, as proven through this report in the field of Investment Banking and economy. Optimization can be applied to many different fields and problems.

4.1 Practicality

To better understand how practical this information is, let's recall the optima shown in table 16. After just 6 trials, and minimal tuning of the genetic algorithm due to project time constraints, a few profitable solutions were found. An in depth analysis of the selected exchange rates deems whether or not a solution is practical. In practice, these optima on their own are not practical, since there is a slim chance that the market rates will ever actually equate to these optima. However, 2 potential ideas are discussed below.

4.1.1 Brute Force Method

A simple solution would be to run the genetic algorithms over thousands of iterations to produce numerous optima, and to consistently monitor the market rates in real time. If these rates were to ever connect, an immediate series of transactions could lead to the estimated profit.

4.1.2 Rate Correlation

There are many theories and analyses that prove positive and negative correlation between exchange rates [5]. A proper implementation of this idea could increase the practicality of this arbitrage optimization problem. Specifically, by varying the third rate subject to past market correlation data, while dependant on CAD2USD and CAD2EUR rates. Thus, any proposed solutions could have a higher possibility of ever equating to real time market rates.

References

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