Physics 567
Homework 2
Due Friday, January 31, 2020.

1. In class, I asserted that the truncation error from "Tres Hermanos" quadrature of a single interval is of order $f^{(4)}(x)\,h^5$. Show that this is true, starting from

$$\int_{-\frac{h}{2}}^{\frac{h}{2}} f(x)\,dx \approx h \times \left(\frac{1}{24}f_1 + \frac{11}{12}f_2 + \frac{1}{24}f_3\right).$$

   Use this result to argue that the extended Tres Hermanos rule should converge as $1/N^4$. For simplicity, you may assume all N steps, including the ends, are evaluated by the above formula.

2. Evaluate numerically:

$$\int_0^1 \frac{(1-x)^{\frac{1}{3}}}{\ln x}\,dx.$$

3. Plot the rate of convergence ($\log|\text{error}|$ vs. $\log N$). Use the results to estimate the accuracy you achieved.

Please turn in electronically to the grader:

- Electronic answer to the first question. This is an opportunity to dust off your LaTeX (or MathType, or whatever) skills, which you will put to use for the final project. Note that the latex source for this document (`integration.tex`) is provided as an example. As a last resort, you may turn in a scanned pencil and paper solution.
- Well-commented source code (one or more .m files). Indicate which language you used.
- Report a result that is accurate to at least 10 significant figures.[1] You may need to use the command: `format long`.
- All commands used to complete the assignment.
- Well-labeled plots.

[1] It is possible to reach 15-figure accuracy by the methods we have been using in class.

In this and all future homework assignments, I encourage you to make use of existing code including my examples and the built-in functions in Octave and Matlab.