PHSX 567: HW #2

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Problem 1

We start with the "Tres Hermanos" numerical method:

$$\int_a^b f(x)dx \approx h \cdot \left(\frac{1}{24}f_0 + \frac{11}{12}f_1 + \frac{1}{24}f_2\right) \tag{1}$$

where $\{a, b\}$ are the endpoints of an integral, and $f_i = f(x_i)$. The set of x_is correspond to $\left\{\frac{(b-a)}{6}, \frac{(b-a)}{2}, \frac{5(b-a)}{6}\right\}$. For an extended rule, where $\{a, b\}$ are the endpoints of the integral, and is divided into N segments, our $(b-a) \rightarrow (x_{i+1} - x_i)$.

It is given that the error of the "Tres Hermanos" rule has a truncation error $E_s \sim h^5 f^{(4)}$, so we can assume that, since we are applying this rule to non-overlapping subintervals, we have:

$$E_{ex} \sim \sum_{i=0}^{N-1} f^{(4)}(x_i) \cdot h^5 \tag{2}$$

Following the assumption in equation. 2, we can derive the dependence of the extended error with respect to N:

$$\begin{aligned}
E_{ex} &\sim h^5 \cdot \sum_{i=0}^{N-1} f^{(4)}(x_i) && \text{(3a)}\\
&\sim h^5 \cdot N \cdot \frac{\sum_{i=0}^{N-1} f^{(4)}(x_i)}{N} && \text{(3b)}\\
&\sim h^4 \cdot N \cdot h && \text{(3c)}\\
&\sim h^4 \cdot (b-a) && \text{(3d)}\\
&\sim \frac{1}{N^4} && \text{(3e)}
\end{aligned}$$

where the definition of an average was used in line 3b ($f^{(4)}$ has been omitted from the subsequent lines). The equation $h = \frac{(b-a)}{N}$ was used in line 3c to eliminate the remaining h, thus making the "$\sim h^4$" more obvious.

This thus proves both the first and second questions, because the composite error is proportional to N times the single-interval error. Removing this N gives us back the h^5 dependence.