RAFFLES INSTITUTION

RAFFLES PROGRAMME 2021
Year Three Mathematics
Topic 9: Exponential and Logarithmic Equations and Functions

Remedial Worksheet (Logarithmic and Exponential Graphs 01)

Name: ( ) Class: 3( ) Date:

Things to know:

- Graphs of exponential functions of the form $y = a^{bx+c}$.
- Graphs of logarithmic functions of the form $y = \log_a(bx+c)$.

Level 1

1. Sketch the graph of $y = 2 - 2e^{-0.5x}$.

2. Sketch the graph of $y = \ln(3x+1)$ for $x > -\frac{1}{3}$.

3. (a) Sketch the graph of $y = 3\ln(x-1)$, indicating clearly the asymptote and the intercept(s).
   (b) By drawing an additional straight line on the same axes, state the number of solutions of the equation $x = e^{\frac{x-1}{3}} + 1$.

Level 2

4. (a) Sketch the graph of $y = \ln(3x+2)$, indicating clearly the asymptote(s) and intercept(s).
   (b) By drawing an additional straight line on the same axes, state the number of solutions of the equation $e^{2x-1} - 2 = 3x$.

5. (a) Sketch the graphs of $y = \ln\sqrt{x+4}$ for $x > -4$, indicating clearly the asymptote(s) and intercept(s).
   (b) By drawing an additional straight line on the same axes, state the number of solutions of the equation $xe^2 = e^{2x} - 4e^2$.

6. (a) Sketch the graph of $y = e^{-2x}$, indicating clearly the asymptote(s) and intercept(s).
   (b) Find the equation of the straight line required to be drawn on the graph of $y = e^{-2x}$ to obtain a solution of the equation $x + \ln\sqrt{5x-2} = 0$.

7. (a) Sketch the graph of $y = \ln\sqrt{2x-1}$ for $x > \frac{1}{2}$, indicating clearly the asymptote(s) and intercept(s).
   (b) Find the equation of the straight line required to be drawn on the graph of $y = \ln\sqrt{2x-1}$ to obtain a solution of the equation $e^{2-x} + 1 = 2x$.

8. The curve $y = 8 - 2e^{3x}$ intersects the x-axis at P and y-axis at Q.
   (a) The line PQ passes through the point $(2\ln 2,\ k)$. Find the value of k.
   (b) Find the equation of the straight line required to solve the equation $x = \ln \sqrt[3]{\frac{11-2x}{2}}$.

9. The mass, m grams, of a radioactive substance present at time, t days, after first being observed is given by the formula $m = ae^{-kt}$, where a and k are constants. Initially, 36 g of radioactive substance was observed. After 40 days, its mass was reduced to 16 g.
   (a) Find the value of a and of k.
   (b) Sketch the graph of m against t.

10. In the recent years, the release of greenhouse gases has accelerated the melting of glaciers and thus, resulted in a rise of global temperatures. With minimal actions taken to prevent global warming, the average temperature, $T°\text{C}$, projected to rise after x years is given by $T = 31(1.5)^{0.1x}$. The average temperature in Singapore in 2016 is expected to be $31°\text{C}$.
   (a) Find the projected average temperature in Singapore in 2018.
   (b) Find the length of time, to the nearest year, for the average temperature in Singapore to increase by at least 10% of its initial temperature in 2016.
   (c) Sketch the graph of T against x.

11. In a simplified prey-predator model, fifty wolves were deliberately introduced to an island to curb the population of wild rabbits. The decline in population of rabbits, R, was given by $R = 300 + 6000e^{-0.02t}$ where t represented the number of days since the introduction of wolves.
   (a) State the initial number of rabbits.
   (b) Find how long it would take before the population of rabbits dropped below half the original population, correct to the nearest day.

12. A certain radioactive material, radium-226, decomposes according to the formula $A = A_0e^{kt}$ where A is the remaining mass after decomposition, A_0 is the original mass, t is the time in years and k is a constant. A radioactive substance is often described in terms of its half-life, which is the time required for half the material to decompose.
   (a) Given that after 500 years, a sample of radium-226v has decayed to 80.4% of its original mass, find the value of k.
   (b) Hence, find the half-life of radium-226.
   (c) Sketch the graph of A against t.

[Answer]

(3b) $y = 1 - x$ (4b) $y = 2x - 1$, 2 solutions

(5b) $y = x - 1$, 2 solutions (6b) $y = 5x - 2$, 1 solution

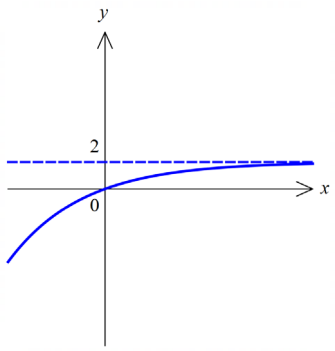
(7b) $y = 1 - \dfrac{x}{2}$ (8a) $k = -12$ (b) $y = 2x - 3$

(9a) $a = 36, \quad k = 0.0203$ (10a) 33.6°C (b) 3 years

(11a) 6300 (b) 38 days

(12a) $k = -0.000436$ (b) 1590 years

(1)



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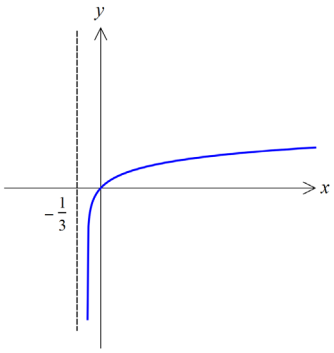

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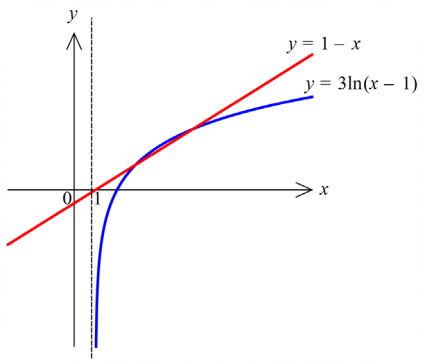

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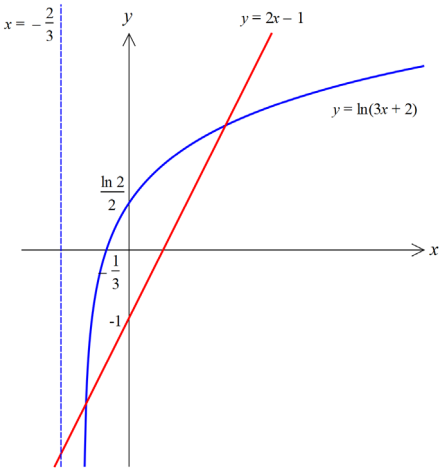

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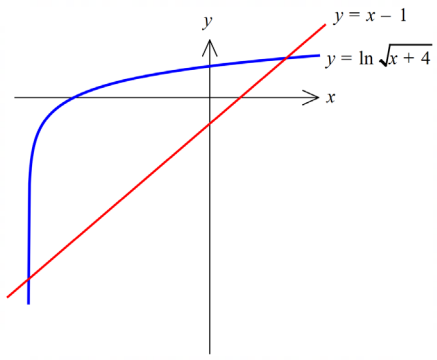

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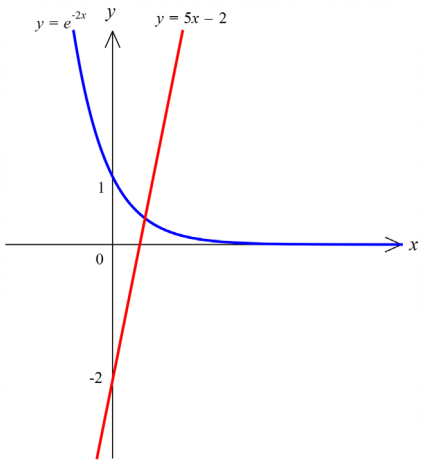

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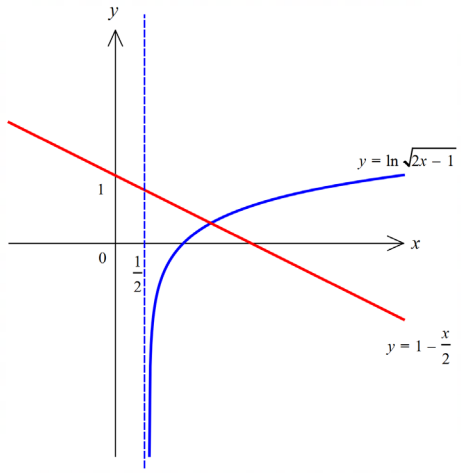
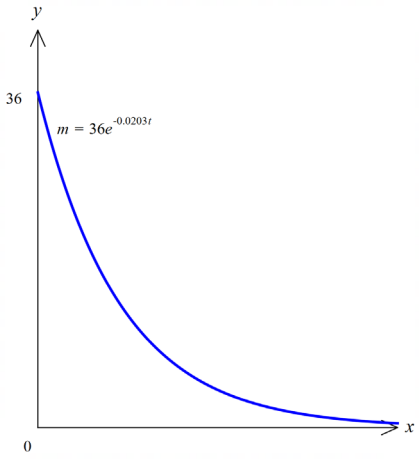
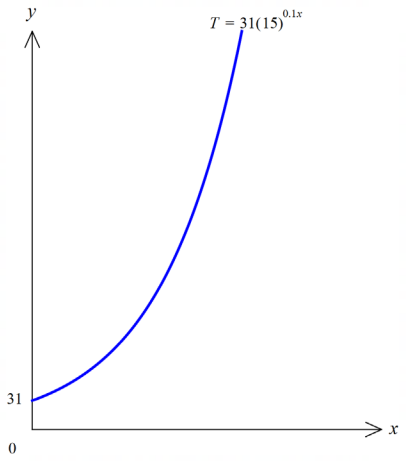

y
x
1
0
1/2
y = ln √(2x − 1)
y = 1 − x/2

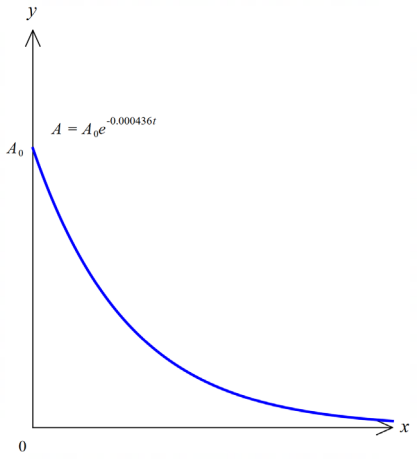
(9)


y
36
m = 36e^-0.0203t
x
0

(10)


y
T = 31(15)^0.1x
31
x
0

(12)


y
A = A0e^-0.000436t
A0
x
0