EDS 212: Day 1, Lecture 2

Exponential functions, logarithms, graphs, average slope

August 5th, 2024

Math brain warm up

- Algebra blitz
- **Exponentials and logarithms**
- Common units and unit conversions
- Functions
- Understanding graphs
- Interpreting equations

The Natural Exponential

In previous examples, we evaluated exponentials with different bases that were variables (e.g.) and rational numbers (e.g.).

$$x^5$$

Here, we'll learn about the *natural exponential*, , which appears frequently in environmental science and modeling.

Where does come from?

- The value is from continuous compounding over infinite intervals:

- The **e** is from Leonard **Euler**, Swiss mathematician who proved the value was irrational

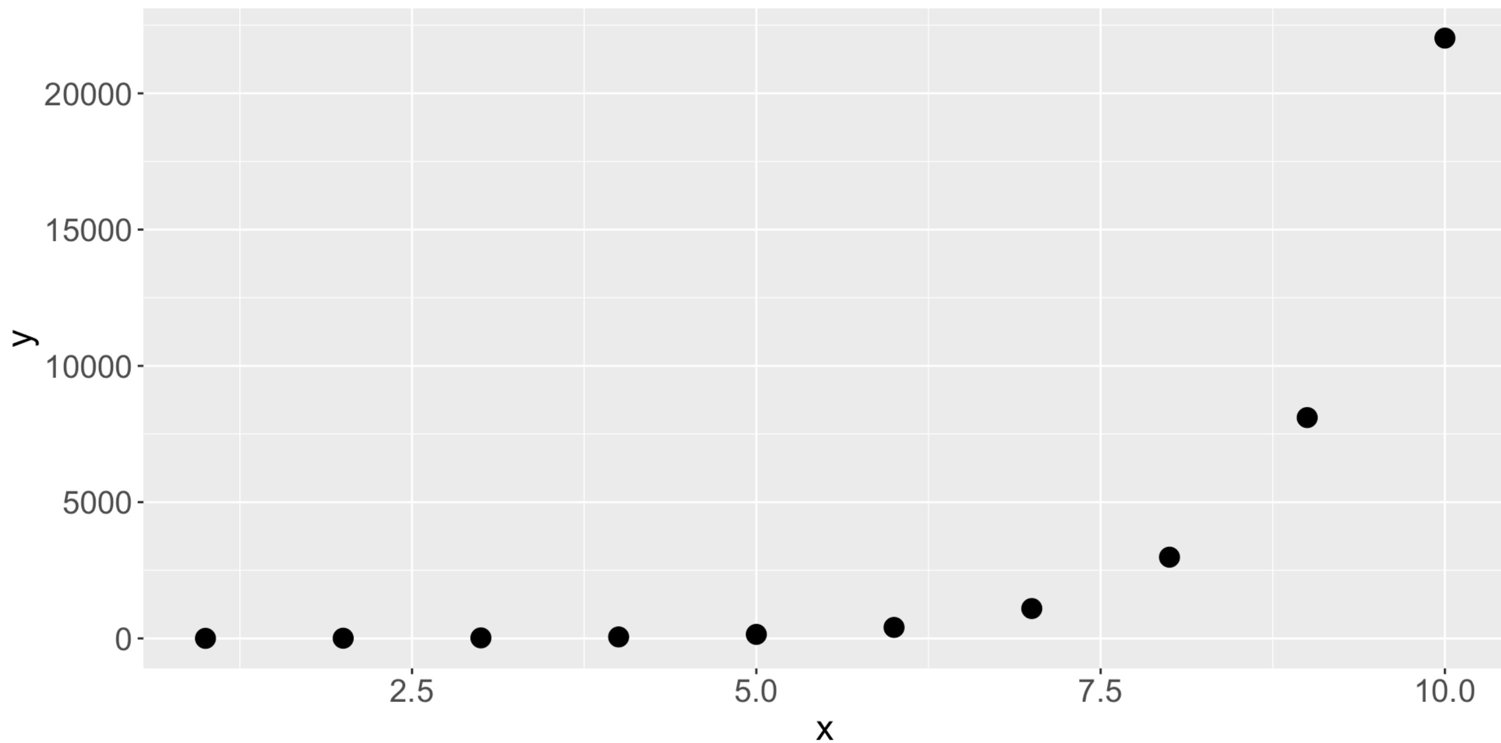
is a *number*, not a variable

It is an irrational number, yes - meaning it can’t be expressed by a simple ratio of integers - but a number nonetheless. With infinite decimal places.

It is always the same value.

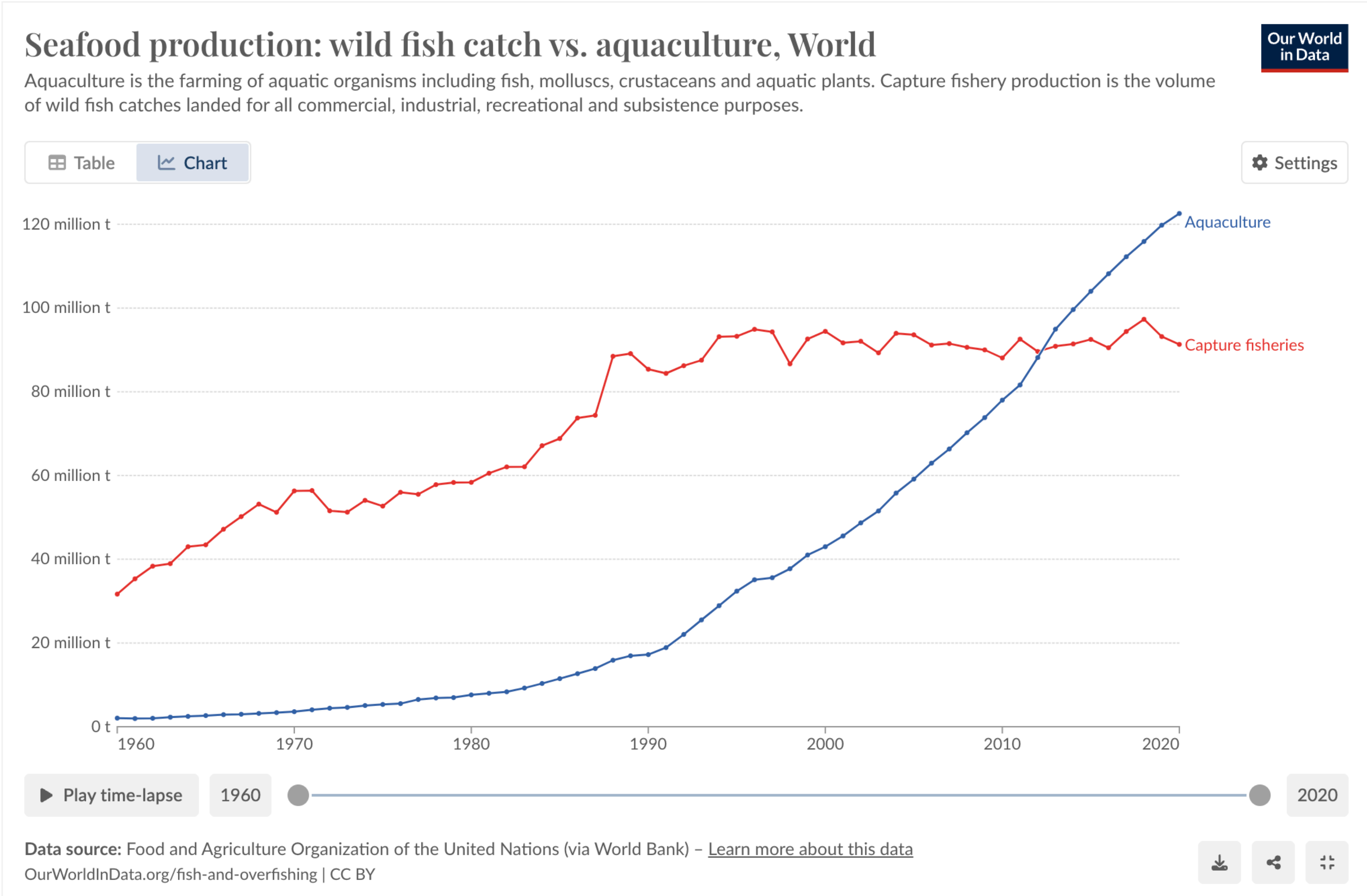
Why is so common?

- **One reason:** Exponential trends show up a LOT in environmental science (the proportional change is the same over each time span)
- **Math reason:** Turns out it's a very useful value for calculus

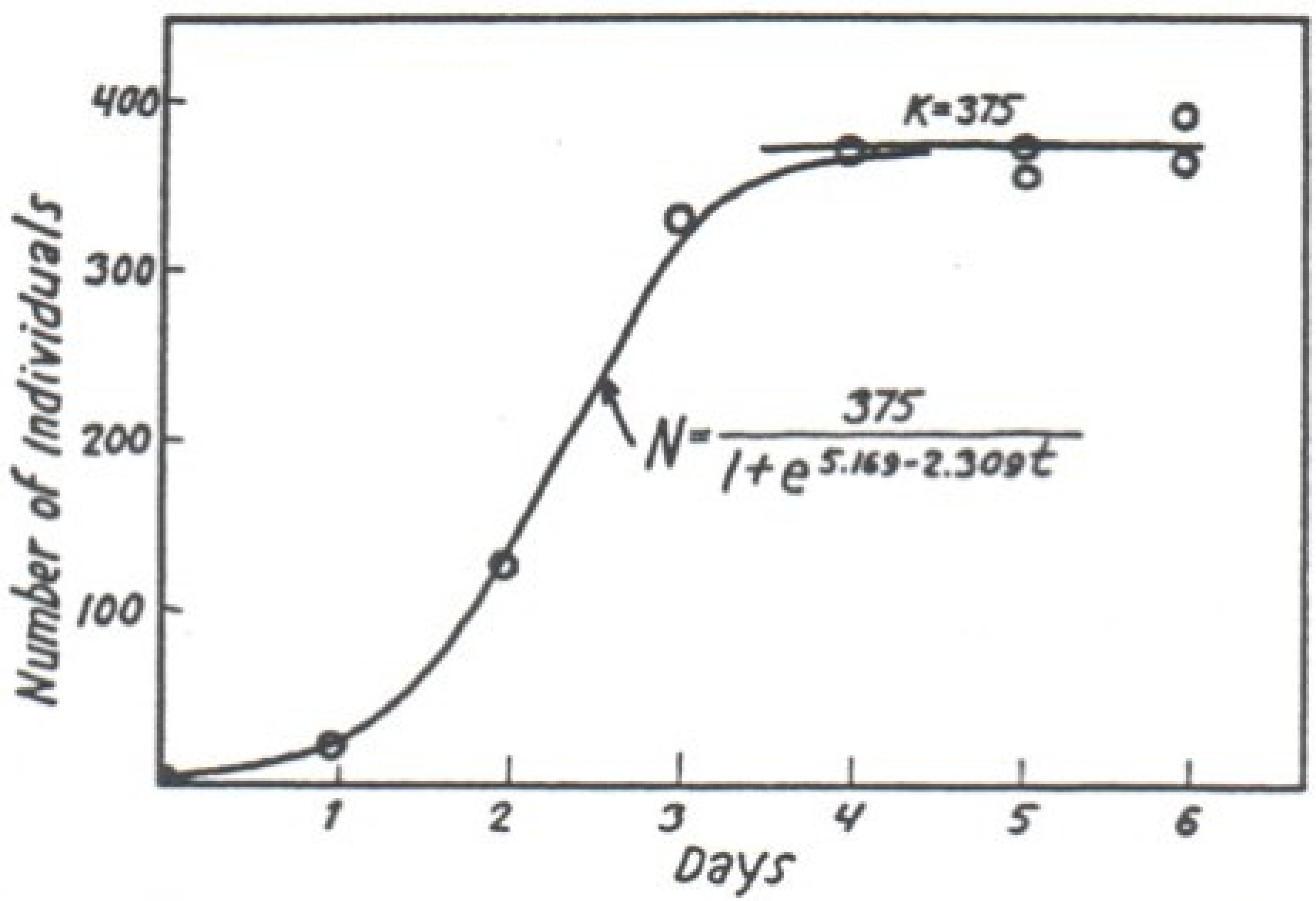


	x	y	previous_y	percent_change_y
1	1	2.718282	NA	NA
2	2	7.389056	2.718282	63.21206
3	3	20.085537	7.389056	63.21206
4	4	54.598150	20.085537	63.21206
5	5	148.413159	54.598150	63.21206
6	6	403.428793	148.413159	63.21206
7	7	1096.633158	403.428793	63.21206
8	8	2980.957987	1096.633158	63.21206
9	9	8103.083928	2980.957987	63.21206
10	10	22026.465795	8103.083928	63.21206

A real-world example:



But populations can't grow exponentially forever . . .



The growth of population of *Paramecium caudatum*

Gause, G. F. 1934. The Struggle for Existence. Baltimore: Williams and Wilkins.

Logistic growth

Where is the population size at time , is the carrying capacity, is the initial population size, and is a growth rate.

We should always think about *why an equations has the shape it has* - both conceptually and mathematically.

Logistic growth

1. Why might we expect logistic growth for many populations?
2. What variables *besides* time would influence the actual population?

Logarithms

Logarithms ask a question: asks “to what power do I have to raise to get a value of ?

For example:

- asks “to what power do I have to raise 2, to get a value of 8?”
- asks “to what power do I have to raise to get a value of ?”
- asks “to what power do I have to raise to get a value of ?”

The *natural log* = “log base ” = =

So based on what we learned in the previous slide, what is:

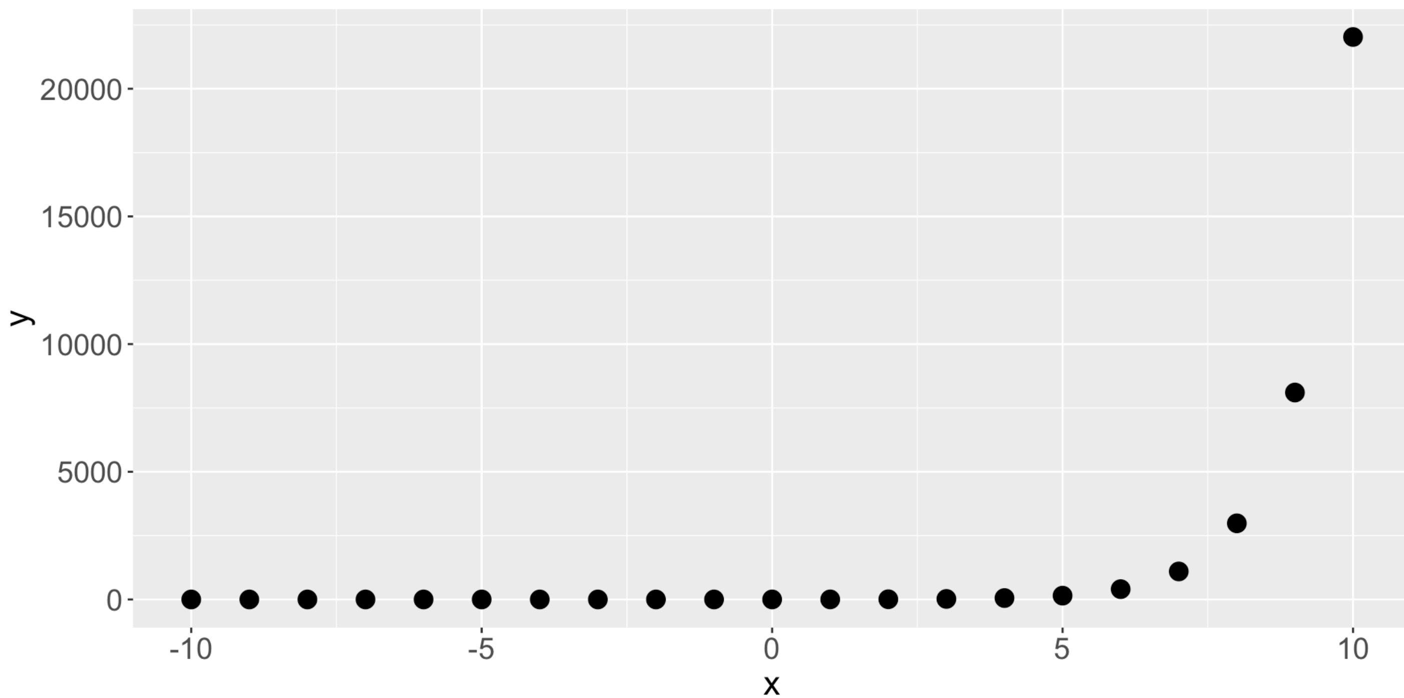
- = ?
- = ?
- = ?

Some log rules

Critical thinking questions

Can the value within a expression ever be 0, or negative? Why?

Can the solution to a natural log expression ever be negative? How?



Working with and in equations

We can think of these as inverses of each other:

-
-

...and use that as a tool for escaping variables from exponents & logs (remembering we can do whatever we want to an equation, as long as we do **the same exact thing to both sides**)

Examples:

- Find given

  Exponentiate both sides: ; simplify left-hand side to get:

- Find given

  Take natural log of both sides: ; simplifies to: , so

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Graphs: visualizing & thinking about data

Graphs are a way for us to more easily process trends or patterns that may be more challenging to understand in a table or list.

When you look at graphs, the first things you should ask:

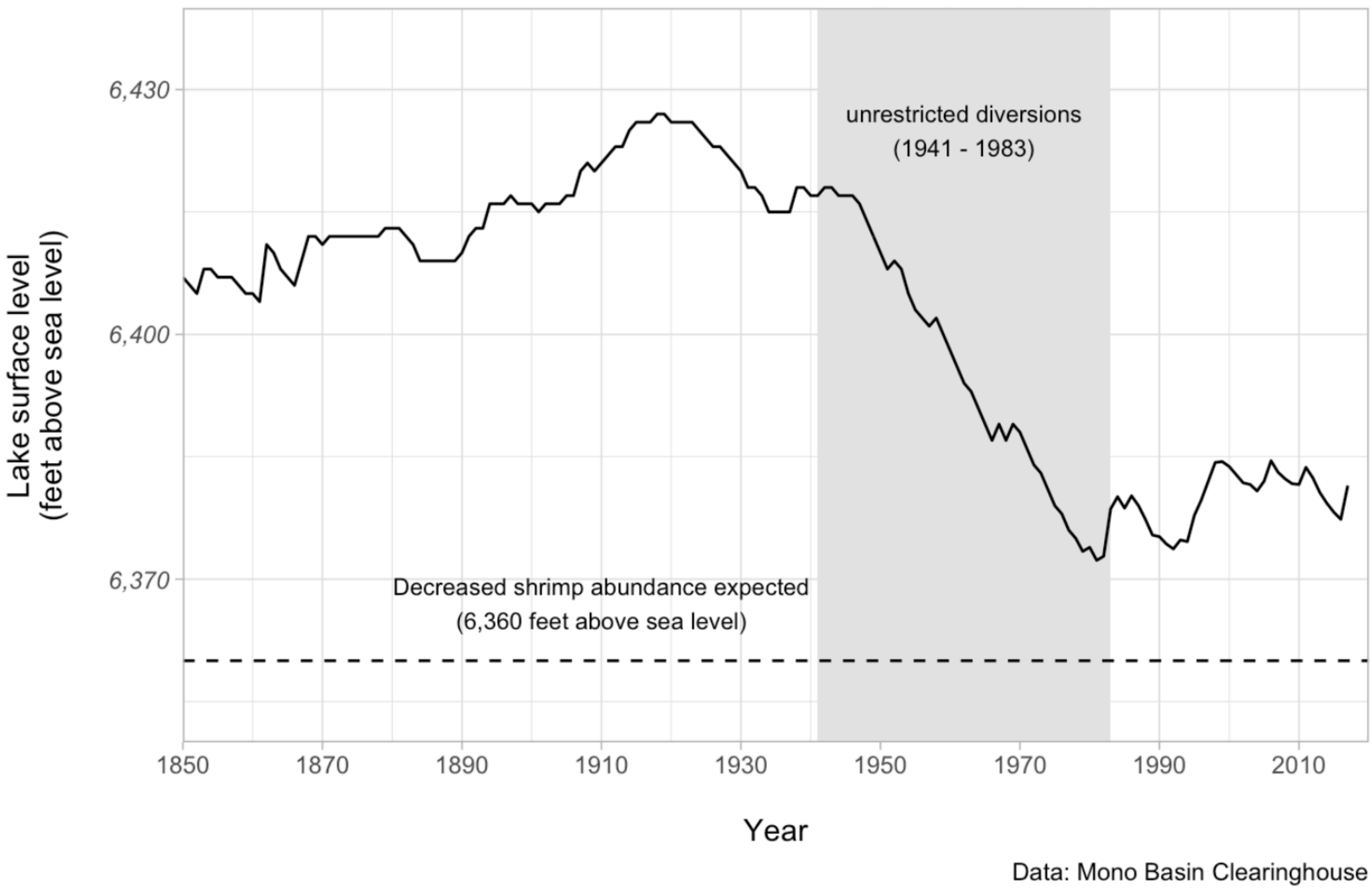
- What variables are plotted (e.g. x- & y-axis, including units)?
- What values are plotted (e.g. raw values, transformed, means, etc.)?
- What are the overall takeaways and am I understanding them responsibly?

Practice saying these things OUT LOUD as if presenting the graph to an audience

> "This figure shows the [change/pattern/relationship] between [x-variable], shown on the x-axis in units of [units] and [y-variable], shown on the y-axis in units of [units]. Overall [overall statement of pattern / trend / findings]."

Possibly with additional context as useful for the audience to put those findings into perspective (e.g. "this reduction represents an 82% decline in rainbowfish stocks along the Narnia Coast since 1991").

Mono Lake levels (1850 - 2017)



Slope (average)

Sometimes, it can be useful to find the **average rate of change** of a function. Between any two points on a function and the slope is found by:}

Get into the practice of saying the meaning out loud

- As if you're explaining it to someone unfamiliar with the data
- Including units
- Without overstating certainty

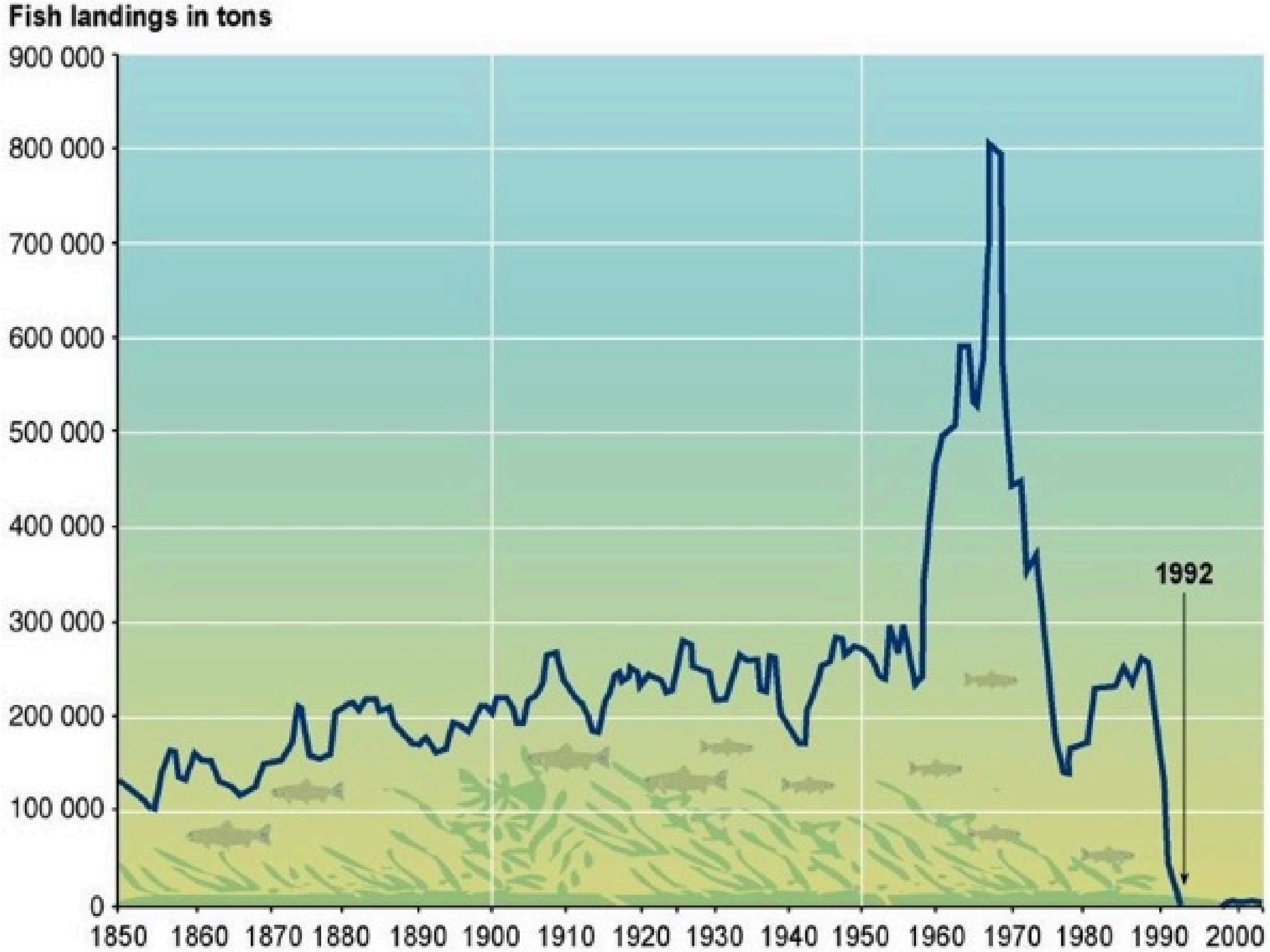
For example:

> "Between 1972 and 2020 the price of hobbit homes increased by an average of $2,450 per year"

differs from

> "Between 1972 and 2020 the price of hobbit homes increased by $2,450 per year."

The *average* slope of a continuous function rarely tells the whole story



Source: Millennium Ecosystem Assessment