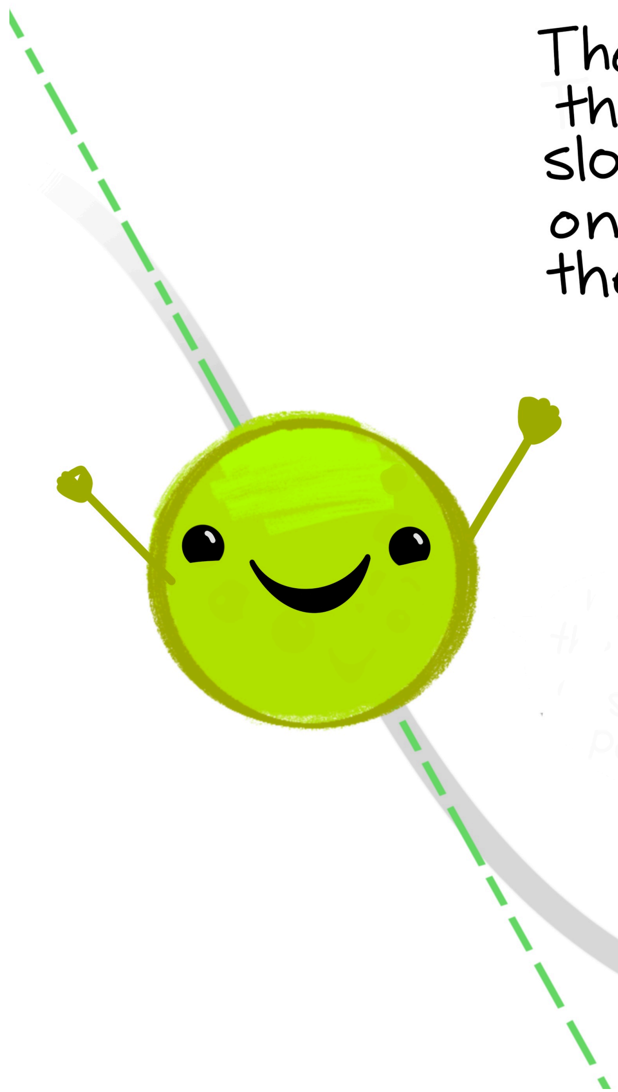
EDS 212: Day 2, Lecture 2

Derivatives continued - higher order, partials, computation, & application

August 6th, 2024



The expression for the instantaneous slope at any point on a function, aka the **derivative**

IS FOUND BY:

① Finding an expression for the **slope** between 2 points separated by Δx...

$$\frac{df}{dx} = \underbrace{\lim_{\Delta x \to 0}}_{②} \overbrace{\frac{f(x+\Delta x)-f(x)}{\Delta x}}^{①}$$

② & evaluating that slope as the points get infinitely close together.

$$\frac{df}{dx} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) -}{\Delta x}$$

(Alternatively, sub for)

Why derivatives in an EDS program?

Describing rates of change is common in environmental science (rate of pollutant concentration change, rate of population growth, rate of energy consumption)

Practice interpretation / math think

Imagine you have a bowl of soup and you want to describe how quickly it cools off.

- What do you think is the *main driver* of cooling?
- What if you have a hot plate?
- What does this look like in an equation?

Newton's Law of Cooling

What do you think is wrapped up in ?

Higher order & partial derivatives

Higher order derivatives

Higher order derivatives are derivatives of derivatives.

Notation:

- First derivative: or
- Second derivative: or
- Third derivative: or

Higher order derivative example:

Find the 3rd derivative of:

Partial derivatives

When we find a partial derivative, we find an expression for the slope with respect to *one variable* in a multivariate function.

Mathematically: Find the derivative with respect to a single variable, *treating all others as constants.*

Notation: the partials of are , , and

Partial derivatives example:

Find all partials of:

OK but what do partials actually mean?

The slope with respect to one variable if other variables are held constant. Let's think about a roller coaster.

How can we describe our orientation? Let us count the ways...

[Commence drawing & handwaving extravaganza!]

Let's try one

The temperature (in Celsius) across a surface (where x and y are in meters) is described by:

At what "rate" is temperature changing (with respect to distance):

- In the direction, at the point (1,3) on the surface?
- In the direction, at the point (0,2) on the surface?

Another super real example

A dragon's breath temperature (, in degrees Celsius) is modeled as a function of its wingspan (, in meters) and length (, also in meters):

- At what rate is breath temperature changing with respect to length for a dragon that is 4.1m long, with a wingspan of 4.5m?

- At what rate is breath temperature changing with respect to wingspan, for the same dragon?

Example: higher order & partial derivatives in environmental data science

The **Advection-Dispersion-Reaction Equation** for solute transport models the change in a solute concentration over time , where groundwater is flowing in direction :

Let's break it down.

- **Left-hand-side:** Rate of concentration change (over time)
- **Right-hand-side first 3 terms:** Concentration change due to dispersion in , , and directions
- **Right-hand-side fourth term:** Concentration change due to groundwater transport (in groundwater flow direction, x)
- **Right-hand-side final term:** Reaction term (e.g. biodegradation / abiotic degradation)