EDS 212: Day 1, Lecture 1

Course intro, algebra refresher

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Welcome to EDS 212 - Essential Math in Environmental Data Science

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- **Course hours:** 10am - 4:30pm PST
- **Location:** NCEAS 1st Floor Classroom

Let's take a look at the course syllabus together.

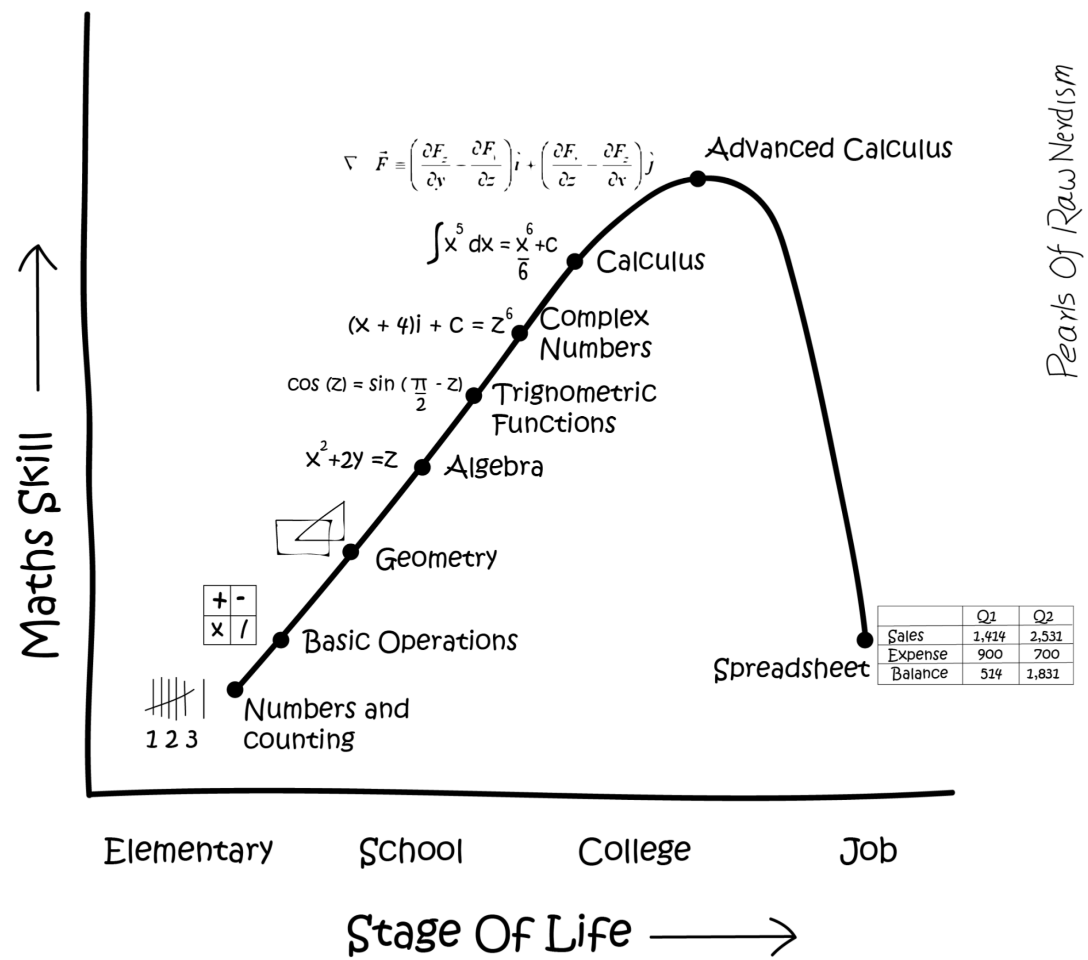
Course description

- **Units:** 2
- **Grading:** Satisfactory/Unsatisfactory
- **Description:** Quantitative skills are critical when working with, understanding, analyzing and gleaning insights from environmental data. In the intensive EDS 212 course, students will refresh fundamental skills in math (algebra, uni- and multivariate functions, units and unit conversions), summary statistics and basic probability theory, derivative and differential equations, linear algebra, and reading, writing and evaluating logical operations.

Topics overview

- **Day 1:** Course introduction & math basics refresher
- **Day 2:** Derivatives
- **Day 3:** Differential equations, intro to linear algebra
- **Day 4:** Linear algebra, summary statistics
- **Day 5:** Basic probability theory, Boolean algebra

Maybe you're thinking…



Why am I taking a math class?

or,

Why do I have to know math when a computer will do it for me?

A quantitative brain warm-up

MEDS students have diverse work & academic histories

This course (re)introduces math concepts and tools that:

- Focus on applications to environmental data science
- Are specifically relevant for MEDS projects, coursework
- Provide an entryway into building computational skills
- Refresh quantitative thinking skills generally
- Refresh essentials like units, conversions, notation, language

Math brain warm up

- Algebra blitz
- Exponentials and logarithms
- Common units and unit conversions
- Functions
- Understanding graphs
- Interpreting equations

Math brain warm up

- **Algebra blitz**
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Algebra blitz

You can get far with a few rules:

1. Order of operations
2. Equations are already solved (but sometimes we need them in a different format)
3. Do whatever you want but do the same thing to both sides

1. Order of operations (P-E-MD-AS)

P - Parentheses

E - Exponents

M/D - Multipication / division

A/S - Addition / subtraction

1. Order of operations practice problems

Simplify the following: $(12-2)/5+5(3+2)/6$

Simplify the following: $\frac{4-6}{2}(3+1)-\frac{1+2*4}{3}$

Simplify the following: $3x+4(8x-6x)-(2y-5)+\frac{2x(1-3)}{2}$

1. Notation matters

Simplify the following: $6 \div 3(4+2)$

What would be a harder-to-misinterpret way to write this?

1. An important takeaway:

Being readable & hard to incorrectly interpret is often as important as being technically “correct”

When designing things, it’s important to **consider the different ways that users might misuse or misunderstand it** - then **build in safeguards** to help them use it correctly. **Clear communication** and **user-centered design** is critical in environmental data science.

2. Equations are solved

$$2x - 5y + 3.9 = 8x^2 - 100.7x$$

Provides solutions for the questions:

1. “What is the value of $2x - 5y + 3.9$?” and
2. “What is the value of $8x^2 - 100.7x$?”

2./3. It is often helpful to reorganize things

In the equation on the previous slide (shown below), we might want to solve for *y*:

$$2x - 5y + 3.9 = 8x^2 - 100.7x$$

The one rule to rule them all:

You can do whatever you want to an equation, as long as you do **the exact same thing to both sides.** That includes ensuring that you are applying something entirely to each side.

3. We're really just doing the same thing to both sides until we're happy with the format

Example:

Apply the same operation to each side of the following equation step-by-step to isolate x on one side. **Write out all steps.**

$$4x + 8 = 5 - 2x$$

3. We're really just doing the same thing to both sides until we're happy with the format

Example:

Apply the same operation to each side of the following equation step-by-step to isolate a on one side. **Write out all steps.**

$$\frac{2(a+1)}{3a}+4=6$$

Math brain warm up

- Algebra blitz
- **Exponentials** and logarithms (this afternoon)
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Exponents & how to do math with them

$$x^n = x \times x \times \ x \times x ... (n \ times)$$

Evaluate the following to find a value for y:

1. $y = 12 - 2^4$
2. $2y + 30 = y + 3^3$

Exponent rules

- $x^a x^b = x^{a+b}$; **Example:** $x^5 x^3 = x^{5+3} = x^8$; **Example:** $2^2 \times 2^2 = (2 \times 2) \times (2 \times 2) = 2^4$
- $\frac{x^a}{x^b} = x^{a-b}$; **Example:** $\frac{z^5}{z^3} = z^{5-3} = z^2$; **Example:** $\frac{2^3}{2^2} = \frac{2 \times 2 \times 2}{2 \times 2} = 2^1$
- $\frac{1}{x^a} = x^{-a}$; **Example:** $b^{-4x} = \frac{1}{b^{4x}}$
- $(x^a)^b = x^{ab}$; **Example:** $(2^3)^2 = 2^{3*2} = 2^6 = 64$
- $(\frac{x}{y})^a = \frac{x^a}{y^a}$; **Example:** $(\frac{y}{2^2})^2 = \frac{y^2}{(2^2)^2} = \frac{y^2}{2^4} = \frac{y^2}{16}$
- $(xy)^a = x^a y^a$; **Example:** $(3x)^2 = 3^2 x^2$

Exponent practice

Simplify the following expressions using the rules of exponents:

1. $3x^5 x^8 x^{-11}$

2. $\frac{-8x^6}{2x^4} + 7x^2$

3. $\frac{3x}{x^5} - 3.8x^4 \frac{x^3}{x^6} + 8.1x - 11.2$

Multiplying expressions (FOIL)

First, **O**utside, **I**nside, **L**ast

Example:

$$(2x + 5)(x - 3)$$

$$= (2x \times x)(2x \times -3)(5 \times x)(5 \times -3)$$

$$= 2x^2 - 6x + 5x - 15$$

$$= 2x^2 - x - 15$$

Math brain warm up

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- **Common units and unit conversions**
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UNITS. UNITS. UNITS.

Think about these statements, which all contain the same *value* of 4:

- There are four in the refrigerator.
- There are four **burritos** in the refrigerator.
- There are four **roaches** in the refrigerator.
- There are four **million dollars** in the refrigerator.

Units are critical in environmental data science

We cannot responsibly work with data without knowing the units of **each variable we're working with**.

That means we need to always **familiarize ourselves with metadata**, carefully **check units and any unit conversions**, and understand **how units combine into the units of a dependent variable**.

Dimensional analysis for unit conversions

In dimensional analysis, we multiply initial units by a sequence of conversion factors to arrive at the final desired units.

For example, to convert $100\frac{g}{cm^3}$ into units of $\frac{kg}{in^3}$, given that 1 cm^3 = 0.061 in^3.

$$100\frac{g}{cm^3} * \frac{1kg}{1000g} * \frac{1cm^3}{0.061in^3} = 1.639\frac{kg}{in^3}$$

Unit conversion practice

Practice dimensional analysis to perform the following conversions:

1. Convert $\mathbf{8.1}\frac{km}{s}$ to miles per hour, given that 1 km = 0.621 miles.

2. Convert a mass flux of $\mathbf{3.2}\frac{g}{min{\cdot}m^2}$ to $\frac{mg}{s{\cdot}cm^2}$.

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Functions

Functions are mathematical expressions that tell us how input values are related to output values.

For example, $y = 3x - 5$ is a function that tells us the **value of y** at **any value of x**. In this scenario, we would probably say **y is a function of x**.

Could you also rewrite it and say **x is a function of y**? Here, with no knowledge of what's an input and what's an output, sure - but usually in environmental data science we specify the input variable(s), and the output variable(s) carefully. What follows is the expression in the format of: "**[output variable(s)] is/are a function of [input variable(s)]**".

Thinking about inputs and outputs

For the following combinations of related variables, which do you expect would be the **input** and the **output** in a function describing how they are related? Say your answer in a sentence, e.g. "Evapotranspiration is a function of air temperature."

1. fuel (biomass) / slope / wildfire severity / windspeed / air temperature
2. wind speed / power generated by wind turbine
3. soil C:N ratio / bacterial biomass / soil water content / leaf litter decomposition rate

Function notation

Single variable (univariate) function:

$f(x) = [expression\ containing\ x]$

Multivariate function:

$g(a, T, z) = [expression\ containing\ a, T,\ and\ z]$

Evaluating functions

For continuous functions, we evaluate them by plugging in variable values.

Example:

Evaluate $g(x, t) = 2.4x + 0.5t^2$ at $x = 3$ and $t = 10$

$$g(3, 10) = 2.4(3) + 0.5(10^2) = 57.2$$