EDS 212: Day 4, Lecture 1

Linear algebra continued

August 8th, 2024

Part 1: Linear algebra continued

- Refresher: vectors and working with them
- Matrices: notation, language, basic algebra
- Representing systems of equations with matrices
- Linear algebra in environmental science

Matrices

A matrix is a table of values (multiple vectors in combination). A vector, therefore, can be thought of as a matrix with a single column.

- **Dimensions:** the size of the matrix, in rows x columns (*m* x *n*)
- **Elements:** values in a matrix, often denoted symbolically with a subscript where the first number is the *row* and the second number is the *column* (e.g. a_{34} indicates the element in row 3, column 4)

Matrix algebra (add / subtract)

Add or subtract the corresponding elements (by matrix position) to create a new matrix of the same dimensions.

$$\begin{bmatrix} 1 & -3 \\ 0 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 4 \\ 5 & 6 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 5 & 8 \end{bmatrix}$$

Scalar multiplication

To multiply a matrix by a *scalar*, multiply each element in the matrix by the scalar to get a scaled matrix of the same dimensions.

For example:

$$6 \times \begin{bmatrix} 1 & -3 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 6 & -18 \\ 0 & 12 \end{bmatrix}$$

Recall: dot product

The dot product of two vectors is the sum of their elements multiplied:

For $\vec{a}$ and $\vec{b}$:

$$\vec{a} = [1, 5] \quad \vec{b} = [2, -3]$$

$$\vec{a} \cdot \vec{b} = (1)(2) + (5)(-3) = -13$$

Matrix multiplication

We find the *dot product* of row column vectors:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \times \begin{bmatrix} E & F \\ G & H \end{bmatrix} = \begin{bmatrix} AE + BG & AF + BH \\ CE + DG & CF + DH \end{bmatrix}$$

Practice problems

$$\begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix} \times \begin{bmatrix} 1 & 5 \\ 6 & -3 \end{bmatrix} = ?$$

Critical thinking: Matrices with unequal dimensions

What do you think the output matrix would contain if you were multiplying the following?

$$\begin{bmatrix} A & B & C \\ D & E & F \end{bmatrix} \times \begin{bmatrix} G & H \\ I & J \\ K & L \end{bmatrix} =$$

Let's try one!

$$\begin{bmatrix} 0 & 2 & -1 \\ 3 & 4 & -2 \end{bmatrix} \times \begin{bmatrix} -1 & 0 \\ 5 & 1 \\ -3 & 6 \end{bmatrix} = ?$$

Diagonal matrix

A **diagonal matrix** is (almost always) a square matrix (=) where only elements on the diagonal are non-zero values.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

What happens when we multiply a matrix by a diagonal matrix?

A diagonal matrix is also called a *scaling matrix* because it scales rows proportionally, but not by the same value:

$$\begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix} \times \begin{bmatrix} 3 & 8 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 6 & 16 \\ 8 & 4 \end{bmatrix}$$

Matrices as systems of equations

Often in environmental data science, we have multiple equations representing processes. Matrices give us a way to express these *systems of equations* in data structures that are easy to store and work with in data science. For example, let's say we have a system:

How can we write this using matrices?

Rewriting in matrix form:

The matrix form of this system of equations looks as follows:

$$\begin{bmatrix} 3 & -8 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \end{bmatrix}$$

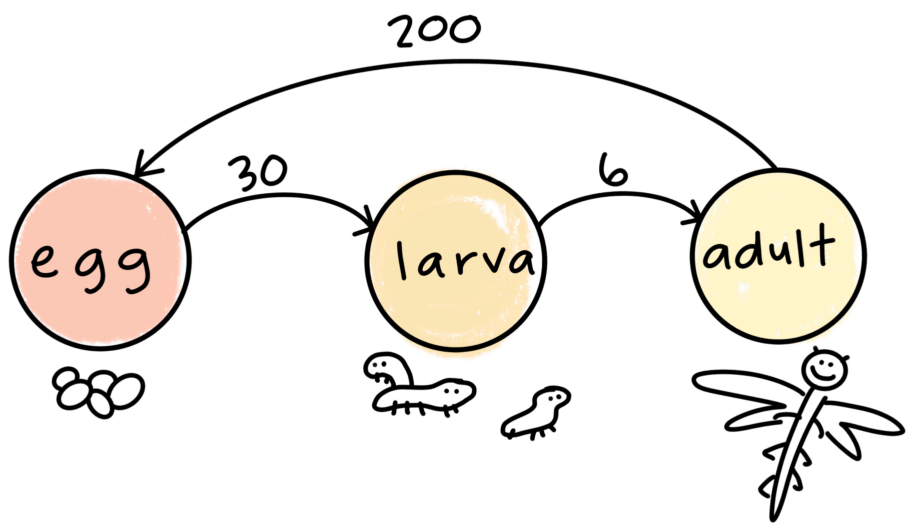
Example: matrices and linear algebra in environmental science

Leslie Matrix: Population ecology

A matrix model that accounts for survival / fecundity rates at different life stages for a species.

Overview:

- Define life stages
- Estimate probability of survival / reproduction at different life stages to create a matrix over time
- Combine into a matrix that allows calculation at the next time step



Writing estimates as equations:

For our species, each adult female will lay ~600 eggs during each cycle (let's say that's a year). Which means that the eggs at time can be estimated by the number of adult females * 600:

Writing estimates as equations:

We also estimate that 20% of eggs survive to reach larval stage:

Writing estimates as equations:

We also estimate that 8% of those that reach larval stage will survive to become reproducing female adults:

How can we write this in matrix form?

How can we write this in matrix form?

$$\begin{bmatrix} E_{t+1} \\ L_{t+1} \\ F_{t+1} \end{bmatrix} = \begin{array}{c} \begin{array}{ccc} \text{Egg stage} & \text{Larval stage} & \text{Adult stage} \end{array} \\ \begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix} \end{array} \begin{bmatrix} E_t \\ L_t \\ F_t \end{bmatrix}$$

Leslie matrix

$$\begin{bmatrix} E_{t+1} \\ L_{t+1} \\ F_{t+1} \end{bmatrix} = \begin{array}{c} \begin{array}{ccc} \text{Egg stage} & \text{Larval stage} & \text{Adult stage} \end{array} \\ \begin{bmatrix} 0 & 0 & 600 \\ 0.2 & 0 & 0 \\ 0 & 0.08 & 0 \end{bmatrix} \end{array} \begin{bmatrix} E_t \\ L_t \\ F_t \end{bmatrix}$$

Leslie matrix

$$
\begin{bmatrix} E_{t+1} \\ L_{t+1} \\ F_{t+1} \end{bmatrix}
=
\begin{array}{c}
\begin{array}{ccc} \text{Egg stage} & \text{Larval stage} & \text{Adult stage} \\ \downarrow & \downarrow & \downarrow \end{array} \\
\begin{bmatrix} \boxed{0} & \boxed{0} & \boxed{600} \\ \boxed{0.2} & \boxed{0} & \boxed{0} \\ \boxed{0} & \boxed{0.08} & \boxed{0} \end{bmatrix}
\end{array}
\begin{bmatrix} E_t \\ L_t \\ F_t \end{bmatrix}
$$