mathematics

Logarithmic Differentiation

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Abstract: This informative paper discusses how and when to use logarithmic differentiation.

1. Introduction

Logarithmic differentiation enables us to take derivatives of functions raised to the power of other functions. It is imperative to know when and how to use logarithmic differentiation for the study of calculus and mathematics. A method will be developed for standard cases of logarithmic differentiation.

2. When to use logarithmic differentiation

When attempting to differentiate a function that is raised to another function, logarithmic differentiation is necessary.

3. A Simple Property of Logarithms

The Power Property of logs tells us that when we have a coefficient in-front of a log, we can bring that coefficient inside the log and it becomes the exponent. The same works in reverse.

$$\log(x^a) = alog(x)$$
$$alog(x) = \log\,(x^a)$$

4. How to use logarithmic differentiation

$$f(x) = x^x$$

If we take the function above, we can't simply use the power rule to take its derivative. We must use logarithmic differentiation.

The first step in logarithmic differentiation is to set the function equal to 'y'.

$$y = x^x$$

Next, we take the natural log of both sides. This enables us to bring the power of x to the front.

$$\ln(y) = xln(x)$$

Now, we can use implicit differentiation and the product rule to solve for $\frac{dy}{dx}$

$$\frac{1}{y}\frac{dy}{dx} = (1)\ln(x) + (x)(\frac{1}{x})$$

Simplifying:

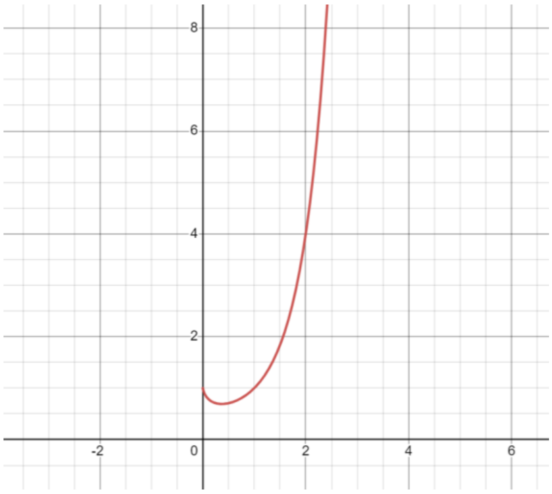
$$\frac{1}{y}\frac{dy}{dx} = \ln(x) + 1$$

Multiplying both sides by y we get:

$$\frac{dy}{dx} = y(\ln(x) + 1)$$

Since we know what y is, we can plug in:

$$\frac{dy}{dx} = x^x(\ln(x) + 1)$$



Graph of x^x

5. A General Formula for Derivatives of x^x

Using the previous example, we can derive a generic formula for the derivatives of $(x+c)^x$

$$y = (x+c)^x$$

$$\ln(y) = xln(x+c)$$

$$\frac{1}{y}\frac{dy}{dx} = (1)(\ln(x+c)) + (x)(\frac{1}{x+c})$$

$$\frac{dy}{dx} = y(\ln(x+c) + \frac{1}{x+c}$$

$$\frac{dy}{dx} = (x+c)^x(\ln(x+c) + \frac{1}{x+c}$$

This general formula can be applied to any function of x plus a constant raised to the power of x.