Heat transfer with phase change – 1D CONDUCTION

Wednesday, June 10, 2020 11:25 AM

$$\frac{\partial(\overline{\rho H})}{\partial t} - \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) = 0$$

$$\overline{\rho H} = \begin{cases} (1-\phi)\rho_r C_r T + \phi \rho_i C_i T, & T \le 0 \\ (1-\phi)\rho_r C_r T + \phi \rho_w (L + C_w T), & T > 0 \end{cases}$$

ϕ = porosity
r – rock, i – ice, w – water

smoothing out the discontinuity:

$$\overline{\rho H} = (1-\phi)\rho_r C_r T + \phi\left[\rho_i C_i T\left(\frac{1}{2} - \frac{1}{2}\tanh(BT)\right) + \rho_w (L + C_w T)\cdot\left(\frac{1}{2} + \frac{1}{2}\tanh(BT)\right)\right]$$

(B = big number, 10 or 100)

$$k = (1-\phi)k_r + \phi\left[\left(\frac{1}{2} - \frac{1}{2}\tan(BT)\right)k_i + \left(\frac{1}{2} + \frac{1}{2}\tanh(BT)\right)k_w\right]$$

semi-implicit formulation

lag nonlinearity in k and $\overline{\rho H}$ by one time-step

$$\frac{d(\overline{\rho H})}{dT}\frac{\partial T}{\partial t} - \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) = 0$$

$$\underbrace{(\overline{\rho H}')^n_j}_{\text{call this } (ARC)^n_j}\frac{T_j^{n+1} - T_j^n}{\Delta t} - \frac{1}{\Delta z}\left[k^n_{j+1/2}\frac{T^{n+1}_{j+1} - T^{n+1}_j}{\Delta z} - k^n_{j-1/2}\frac{T^{n+1}_j - T^{n+1}_{j-1}}{\Delta z}\right] = 0$$

Should be a negative sign

- $$-\frac{k^n_{j-1/2}}{\Delta z^2}T^{n+1}_{j-1} + \left[\frac{(ARC)^n_j}{\Delta t} + \frac{k^n_{j-1/2} + k^n_{j+1/2}}{\Delta z^2}\right]T^{n+1}_j + \frac{k^n_{j+1/2}}{\Delta z^2}T^{n+1}_{j+1} = \frac{(ARC)^n_j}{\Delta t}T^n_j$$

  ($j = 2$ to $J-1$)

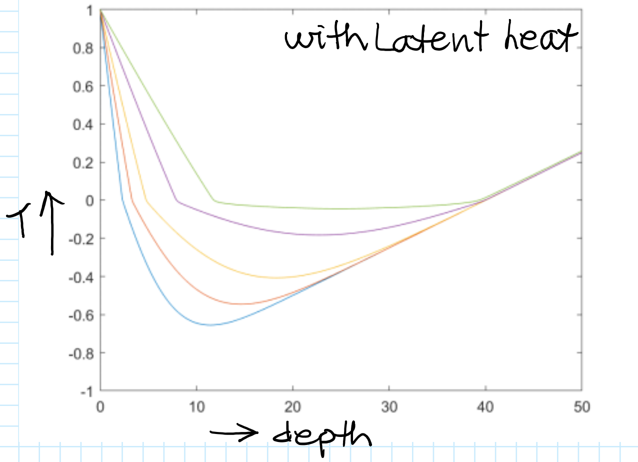
- with Boundary conditions

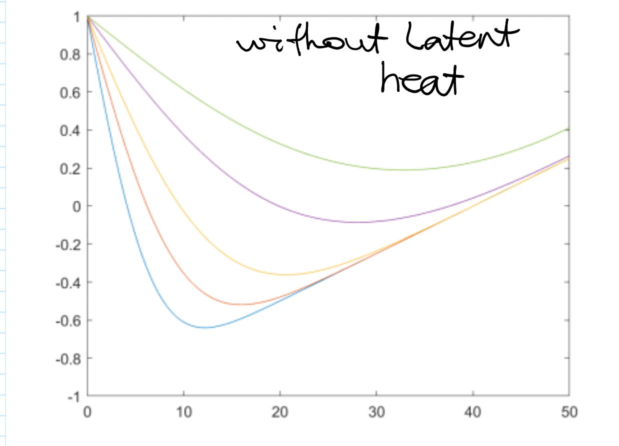
  specified temperature on surface ($j=1$)

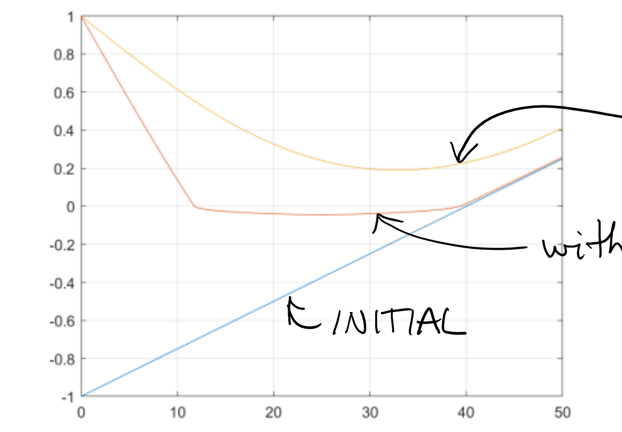
  geothermal gradient at depth ($j=J$)

geothermal gradient at depth (d = J)

Example Simulation: Initial Temperature = -1 Degrees at the surface, and geothermal gradient below (i.e. linear). Surface Temperature abruptly increased to 1 Degrees C at the surface. Bottom Boundary maintained at geothermal gradient. 50m thick domain. Properties (see code) - 2% porosity Granite, standard properties for ice and water







COMPARISON AFTER (30×100) DAYS

<u>NOTE</u>: ONLY 2% POROSITY