Tutorial 3 Probability Distribution Exercises.

Question 1:

(a) In laptop assembly factory, there is a 5% chance that a laptop is defective. If a shop ordered 4 laptops at €300 each, and it cost the factory €200 to produce the laptops, calculate the following:

(i) Complete the probability distribution for the factory profits for selling the four laptops, given that the factory has to take the loss if the laptop is defective.

https://www.khanacademy.org/math/cc-seventh-grade-math/cc-7th-probability-statistics/cc-7th-theoretical-and-experimental-probability/e/probability-models
https://www.khanacademy.org/math/ap-statistics/random-variables-ap/binomial-random-variable/e/identifying-binomial-variables
https://www.khanacademy.org/math/ap-statistics/random-variables-ap/binomial-random-variable/e/binomial-probability
https://www.khanacademy.org/math/ap-statistics/random-variables-ap/binomial-random-variable/e/calculating-binomial-probability

(ii) What is the probability that the factory makes some profit on this sale?

https://www.khanacademy.org/math/precalculus/x9e81a4f98389efdf:prob-comb/x9e81a4f98389efdf:probability-distributions-introduction/e/probability-discrete-random-variables

(iii) What is the factory's expected profit?

https://www.khanacademy.org/math/precalculus/x9e81a4f98389efdf:prob-comb/x9e81a4f98389efdf:expected-value/e/mean-expected-value-discrete-random-variable

Question 2:

Tweets with the hashtag #gameon occur at a rate of three per second during a major gaming event. Use the Poisson Distribution to calculate the probability that;

(i) one tweet with this hashtag will be tweeted during a one second interval.

(ii) at least one tweet with this hashtag will be tweeted during a one second interval.

Question 3:

The length of time taken for queries to a mobile network company to be resolved is approximately normally distributed with a mean of 20 hours and a standard deviation of 5 hours.

Determine the proportion of queries which take:

https://www.khanacademy.org/math/ap-statistics/sampling-distribution-ap/xfb5d8e68:the-normal-distribution-revisited/e/probability-normal-density-curves

(i) less than 27.5 hours to resolve;

(ii) longer than 25 hours to resolve;

(iii) less than 18 hours to resolve;

(iv) between 15 and 30 hours.

Determine:

(v) the number of hours within which 80% of queries are resolved.

Formulae:

- **Probability:**

  $P(E_1 \text{ OR } E_2) = P(E_1) + P(E_2) - P(E_1 \text{ AND } E_2)$

  $P(E_1 \text{ AND } E_2) = P(E_1) \times P(E_2|E_1) = P(E_2) \times P(E_1|E_2)$.

  $E(X) = \sum_{i=0}^{n} x_i \, . P(x_i)$

- **Bayes Theorem** : $P(A|B) = \frac{P(B|A)}{P(B)} P(A)$

- **Binomial Distribution:**

  For *n* independent experiments the probability of success is *p*

  $P(x \text{ successes}) = {}^{n}C_x \, p^x (1-p)^{(n-x)}$, $\quad E(X) = np$

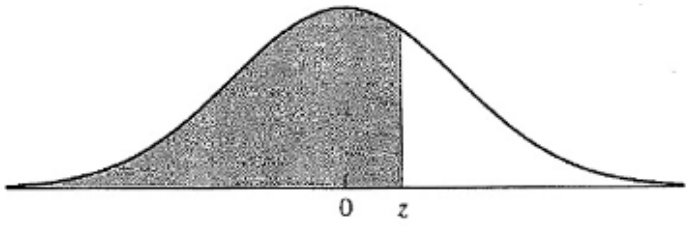
- **Poisson Distribution:**

  $P(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}$, $\quad E(X) = \lambda$

- **Normal Distribution, Z score:**

  Z-score: $\quad Z = \frac{x - mean}{st.dev.}$

TABLE A.2 Cumulative normal distribution (continued)



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998
3.6	.9998	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999